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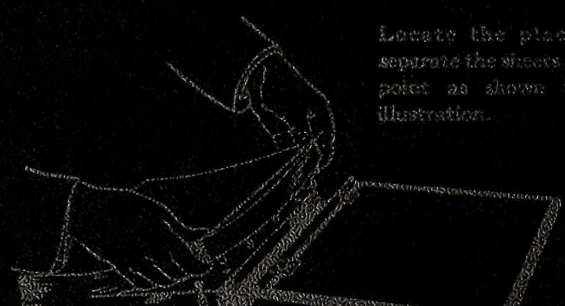

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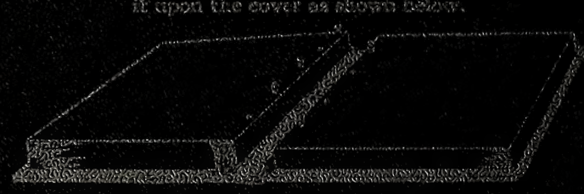

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MANUAL OF AMPHIBIOUS OCEANOGRAPHY

PREFACE

In fulfillment of a contract between the Institute of Engineering Research, University of California and the Office of Naval Research, (Contract N7onr-29535, Project No. NR 252-003) staff members of the Waves Investigations Group collected, compiled and evaluated all available information on waves, surf, beaches, and landing craft. The results of this work are presented in this manual. The intent of the presentation is to make this information available for use by technical personnel who are not necessarily technicians in this field.

This manual has been prepared under the direction of Dean M. P. O'Brien and Professor J. W. Johnson with Mr. R. L. Wiegel as Project Engineer. The writer of each section was, in almost all cases, a specialist either in the field or in a closely related field.

In addition to the authors the following persons made considerable contributions in the preparation of this manual: Mr. R. C. MacCamey and Mr. John Butler who did much work on the theoretical section; Mr. M. Hall and Mr. K. Granthem who helped with the sections on wave recorders; Mrs. Virginia Diller, who performed the non-technical editing; Miss Margarete Lincoln, who prepared all of the illustrations, Mrs. Conklin, Mrs. Kappeler, Mrs. McFate, Mrs. Morison and Mrs. Neunzig, who typed and proofread the manuscript.

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MANUAL OF AMPHIBIOUS OCEANOGRAPHY

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MANUAL OF AMPHIBIOUS OCEANOGRAPHY

i INTRODUCTION:

BY
M. P. O'BRIEN

This Manual presents a summary of the available present knowledge of those aspects of oceanography and coastal engineering which affect amphibious operations. The objective was to prepare a coordinated and consistent treatment of the subject for the benefit of officers engaged in planning or conducting amphibious operations. In so far as seemed feasible, a descriptive, non-technical summary precedes the technical summary, but on some of the subjects, present knowledge is qualitative and only a single treatment was justified.

The term amphibious oceanography is not entirely satisfactory as a designation of the subject matter but no better term was hit upon. Geology, oceanography, geomorphology, fluid mechanics, soil mechanics, structural design, meteorology and many other specialized fields are touched upon and the central theme is one of application rather than scientific investigation. The effort was directed towards amphibious operations and the predominant scientific discipline was that of oceanography; hence the term amphibious oceanography as a compact description of the work.

The manual was written primarily by non-military personnel. A few officers participated in the preparation of certain sections and others contributed by reviewing the draft but, in essence, it is the work of civilians. It should be reviewed and revised by military personnel familiar with both the problems of amphibious operations and with the probable background of the individuals likely to use it.

The authors have stated in each section what they regarded as the most probable facts and in doing so may give an impression of certainty not wholly justified by the data available. This attitude was adopted deliberately in the interest of simplicity of treatment so that the reader might grasp the central facts without being confused by unexplained discrepancies, conflicting theories, and so forth. Application of the equations and empirical relationships presented should be made cautiously and with due regard for the weight of the substantiating evidence.

An important by-product of a compilation of this type is the focusing of attention on gaps in existing knowledge and the formulation of programs of analysis and experiment to fill them. Much remains to be done on almost every subject treated and for this reason the manual should not be regarded as a final statement. It is essentially a progress report which will require revision and extension in the future.

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

Section ii

GLOSSARY OF TERMS

FOR

AMPHIBIOUS OCEANOGRAPHY

BY

R. L. WIEGEL

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GLOSSARY OF TERMS

FOR

AMPHIBIOUS OCEANOGRAPHY

Introduction

For many years, workers in the field of Amphibious Oceanography have needed a standardized definitions of the terms they used. The most complete glossary to date is the "Coast and Beach Glossary for Amphibious Intelligence Use", prepared in 1949 by the U. S. Naval Interpretation Center. This glossary redefined many terms, coined others, and used photographs and sketches to illustrate some difficult concepts.

The University of California's Waves Investigation Group prepared the present glossary as a part of their Manual of Amphibious Oceanography. The staff used the 1949 Glossary, but enlarged its scope, and tried to further clarify many of the definitions. They studied all relevant literature - including government reports, basic texts, and scientific journals - and then worked out the best all round definition for each term included. As these definitions are combinations of so many sources, no specific acknowledgments are made. The most useful sources, however, are included in the Selected List of References in the Appendix.

Suggestions and/or corrections will be appreciated.

GLOSSARY OF TERMS

FOR

AMPHIBIOUS OCEANOGRAPHY

AGROUND: Resting on the bottom of a body of water.

ALLUVIUM: (1) A deposit of sand, mud, etc. formed by flowing water.
(2) Erosional material transported by running water.

APD: High-Speed Transport.

APPROACH: The zone of the sea which extends indefinitely seaward from the shoreline at mean low water springs.

APPROACH-ASSAULT: Area where naval vessels maneuver (when directly engaged in landing or fire support operations).

APPROACH-CONVOY: The area through which the assault convoy passes during the twelve hours immediately preceding an assault, (on its way to the lowering position - which is 7 nautical miles seaward from the beach unless otherwise specified).

ARCHIPELAGO: A group of islands; a sea (or sheet of water) studded with many islands.

ATOLL: A ring-like coral island or islands enclosing a lagoon.

AWASH: (1) (Nautical) The condition of an object which is nearly flush with the water level.
(2) (Common usage) anything tossed about or washed by waves or tide.

BACKBEACH: SEE BACKSHORE

BACKRUSH: The seaward return of the water following the uprush of the waves.

BACKSHORE: (sometimes called BACKBEACH)
...That portion of the beach lying between the foreshore and the coastline.
...It isusually flatter than the foreshore, and is often divided by low scarps formed by cutting during severe storms.
...However, it isusually dry - being acted upon by waves only during exceptionally large storms, especially when these are combined with very high spring tides. (Figure ii-1)

BACKWASH: (1) Backrush;
(2) Water or waves thrown back by an obstruction such as a ship, breakwater, cliff, etc.

BAR: An offshore bank of sand, gravel, or other material, especially at the mouth of a river or harbor; or lying parallel to, and a short distance from, the beach. It may obstruct navigation. (See Figures ii-12, 16, 19)

BARRIER BEACH: An offshore bar which has been built up so that its crest rises a few feet above high level tide. Also called sand reef and offshore barrier.

BARRIER REEF: A coral reef which lies offshore and is separated from the land by a body of water called a lagoon.

BASIN-BOAT: An artificially enclosed body of water where small craft may lie. (See Figure ii-13)

BAY: A recess or inlet in the shore of a sea or lake between two capes or headlands, not as large as a gulf but larger than a cove.
See also: BIGHT
(See Figure ii-15)

BAYOU: A minor and sluggish waterway or estuarial creek, tributary to - or connecting - other streams or bodies of water. It is generally tidal in character, or has a slow or imperceptible current.
Its course is usually through lowlands or swamps.

BEACH: (noun) The zone of unconsolidated material which extends inland from the waterline to the place where there is marked change in material or physiographic form...or to the line of permanent vegetation (usually the effective limit of normal storm waves).
The seaward limit of the beach - unless otherwise specified - is the mean low water springs.
A beach includes FORESHORE and BACKSHORE.
See also LANDING BEACH and LANDING PLACE.
(Figures ii-1, 12, 15)

BEACH (TO): To run or drive (as a boat) upon a beach; to strand.

BEACH FACE: The section of the beach normally exposed to the action of the wave uprush.

BEACH RIDGE: A continuous mound of sand or gravel, a foot or more high, running parallel to the shoreline.

BEACH WIDTH: The horizontal dimension of the beach as measured from the water's edge inland.

BEACHING AREA: The zone bounded by the line where the largest ships touch at lowest tide, and by the line where the smallest landing crafts touch at highest tide.

BENCH: A sloping erosion plane inclined seaward.

BENCH MARK: A mark affixed to a permanent object in tidal observations, or in survey, to furnish a datum level.

BERM: A low, nearly horizontal portion of the beach or backshore formed by the deposit of material by wave action.
It is often soft.
Some beaches have no berms, others have one or several.
(Figures ii-1, 12, 14)

BERM CREST: The seaward limit, generally the highest point of a berm.

BIGHT: A slightly receding bay between two headlands, formed by a long curve of a coastline.
A bend in the coastline, forming an open harbor, or the harbor itself.
(See Figure ii-16)

BILLOW: A wave, especially a great wave or surge of water.

BLIND ROLLERS: Relatively heavy and often dangerous ocean swell as it passes over shoals or approaches land.
See also - GROUND SWELL.

BLUFF: A high slope or bank - irregular and abrupt - which is a major obstacle to vehicle travel.

BOLD (COAST): A prominent land mass that rises steeply from the sea.

BORE: (1) A tidal flood with a high, abrupt front, due to a rapidly narrowing inlet or channel.
(2) Loosely, a very high and rapid tidal flow, as in the Bay of Fundy.

BOTTOM: The ground under any body of water; the bottom of the sea.

BOTTOM, (Nature of): Composition of the bed of an ocean (or other body of water); such as clay, coral, gravel, mud, ooze, pebbles, rock, shells, and shingle.

BOULDER: Rounded rocks more than 256 millimeters in diameter.
Any detached and rounded or worn mass of rock, larger than a cobblestone.
(See Figure ii-17)

BOW: The forward part or head of a vessel.

BREAKER: A wave breaking on the shore, over a reef, etc. Breakers may be (roughly) classified into three kinds, although there is much overlapping:

Spilling breakers
Plunging breakers
Surging breakers

Spilling breakers break gradually over quite a distance; plunging breakers tend to curl over and break with a crash; and surging breakers peak up, but then instead of spilling or plunging they surge up the beach face.
(See Figures ii-2, 3, 6, 8)

BREAKING DEPTH: The still water depth at the point where the wave breaks.
(See Figures ii-2, 3)

BREAKWATER: Anything which breaks the force of waves at a particular place, thus forming protection for vessels, etc.
A solid structure - usually of masonry - protecting a harbor, anchorage, or basin from wind and waves.
Breakwaters are free-standing structures in contrast to seawalls, which have lateral support on the shore side.
(See Figures ii-13, 14)

BROACH (TO): To be thrown broadside on or in surf; or in a seaway.
(See Figure ii-10)

BUOY: A float; especially a floating object moored to the bottom, to mark a channel, anchor, shoal rock, etc.
Some Common types:

A nun or nut buoy is conical in shape
A can buoy is truncated or flat
A spar buoy is a vertical, slender spar anchored at one end
A bell buoy - bearing a bell, run mechanically or by the action of waves, usually marks shoals or rocks
A whistling buoy, similarly operated, marks shoals or channel entrances
A dan buoy carries a pole with a flag or light on it

BUOYANCY: The resultant of upward forces, exerted by the water on an immersed or a floating body equal to the weight of the water displaced by this body.

CANAL: An artificial watercourse for use in navigation.
(See Figure ii-19)

CANYON: (1. oceanographical) A deep submarine depression of valley form with relatively steep slopes.
(2. geographical) A deep gorge or ravine with steep sides, with a river flowing at the bottom of it.

CAPE: A point or extension of land jutting out into the sea, either in the form of a peninsula, or merely as an angle or projecting point on the coast. (See Figure ii-16)

CAUSTIC: In refraction of waves, the name given to the curve or surface to which orthogonals, reflected or refracted by the bottom surface, are tangents.

CAY (or KEY): A low insular bank of sand, coral, etc., awash or drying at low water.

CHANNEL: That part of a body of water deep enough to be used for navigation through an area otherwise too shallow for navigation.

CHARACTERISTIC WAVE HEIGHT: See SIGNIFICANT WAVE HEIGHT

CHART DATUM: The datum to which soundings on a chart are referred. Usually taken to correspond to a low water stage of the tide.
See also: DATUM PLANE and REFERENCE PLANE

CLIFF: Land rising approximately vertically for a considerable distance above water or surrounding land, situated at the seaward edge of the coast. (See Figures ii-1, 14, 15, 17)

COAST: Generally, a strip of land of indefinite width which extends from the shore inland 5 to 25 miles - to the place where there is the first major change in terrain features.

COASTLINE: The line that forms the boundary between the coast and the shore.

COBBLESTONE: Rounded rocks ranging in diameter from 64 to 256 millimeters. (See Figures ii-14, 17)

CLAY: Soil consisting of inorganic material, the grains of which have diameters smaller than .005 millimeters. (U. S. Bureau of Soils Classification)

COMBER: A deep water wave whose crest is pushed over forward by a strong wind.
Also a long-period plunging breaker.

CONTOUR LINE: (1) A line connecting the points, on a land or sea bottom surface, that have the same elevation.
(2) In topographic work, a line connecting all points of equal elevation above or below a datum plane.
(See Figure ii-7)

CONVERGENCE: In refraction phenomena - the decreasing of the distance between orthogonals in the direction of wave travel. (See Figure ii-7)

CORAL: The calcareous or horn-like skeletons of various anthozoans and a few hydrozoans solidifed into a stony mass. Many tropical islands, reefs and atolls are formed from coral.

COVE: A small bay or bay-like recess in the coast, usually affording anchorage and shelter to smaller craft. (See Figures ii-7, 16)

COXSWAIN: The enlisted man in charge of a small boat and serving as steersman.

CREEK: (1) A stream, less prominent than a river in any region, and generally tributary to a river.
(2) Also, a small narrow bay which extends farther inland than a cove and is relatively long compared with its width. It is smaller than a firth.

CREST LENGTH: The length of a wave along its crest.

CREST OF BERM: The seaward limit, generally the highest point of a berm. Also: BERM EDGE

CREST OF WAVE: See WAVE CREST

CURRENT: (1) The movement of water in a horizontal direction.
(2) Ocean currents can be classified as:
periodic currents, due to the effect of the tides
seasonal currents, due to the seasonal winds
permanent flowing currents
littoral or longshore currents, caused by waves breaking at an angle to the beach
(3) Ocean currents are also classified as either drift or stream currents.
Drift currents are broad, shallow and slow moving
Stream currents are narrow, deep and fast moving, and gain their unusual velocity and depth from constriction in a strait.

(See Figures ii-6, 10, 18, 19)

CUSP: One of a succession of nearly semi-circular cutouts occurring along the beach face. (See Figures ii-4, 15, 18)

CUT (noun): Similar to a canal, but shorter, usually not lined.

CYCLOIDAL WAVE: The choppy waves that may spring up quickly in a fairly moderate breeze, and break easily at the crest.

DAILY RETARDATION: The amount by which corresponding tides grow later day by day.

DATUM-CHART: See CHART DATUM

DATUM PLANE: A water surface used as a reference from which to reckon heights or depths.
The plane is called a tidal datum when defined by a certain phase of the tide.
The datum most generally used is based upon mean sea level.
A low water datum is preferred for Hydrographic work - including soundings on charts and tidal predictions. For this purpose, the datum adopted may be: mean low water, lower low water, or low water springs.
Datum planes are referenced to fixed points known as bench marks so that they can be recovered when needed.
(See also REFERENCE PLANES)

DEBARK: To remove to shore from on board a vessel; to land; disembark.

DECAY DISTANCE: The distance through which swell travels after leaving the generating area.
(See Figure ii-9)

DECAY OF WAVES: When waves leave a generating area (fetch) and pass through a calm (or region of lighter winds), they undergo a change. The significant wave length increases and the significant wave height decreases.
(See Figure ii-9)

DEEP(s): (1) An area of the ocean considerably deeper than the surrounding waters.
(2) Secondary and smaller bounded areas within the great ocean basins with depths exceeding 5,000 - 6,000 meters.

DEEP WATER: Where depth is greater than one-half the wave length. Deep water conditions are said to exist when the surface waves are not affected by conditions on the ocean bottom.
(See Figure ii-1)

DELTA: (1) An alluvial deposit, usually triangular, at the mouth of a river.
(2) A tidal delta is a similar deposit at the mouth of a tidal inlet, put there by tidal currents.
(3) A wave delta is a deposit made by large waves which run over the top of a spit or bar beach and down the landward side.
(See Figure ii-19)

DEPTH: Vertical distance from the still water level (or datum as specified) to the bottom.
(See Figure ii-1)

DEPTH OF BREAKING: See BREAKING DEPTH

DEPTH (controlling): The shallowest depth possible for navigation; it controls the draft of vessels using the area.

DEPTH FACTOR (n): The factor by which the apparent depth of the water measured stereoscopically is multipled to give the true depth. This factor is the ratio of the tangent of the incidence angle to the tangent of the refraction angle.

DERELICT: A vessel abandoned on the high seas, and forming a menace to navigation.

DIFFRACTION: Wave diffraction is generally considered to be the phenomenon in which water waves are propagated into a sheltered region formed by a breakwater or similar barrier that interrupts a portion of the otherwise regular train of waves.
(See Figure ii-14)

DIKE (DYKE): A causeway: loose-rubble mound built across shallow water. Also, an earth mound built around a low-lying area to prevent flooding.

DIURNAL TIDE: One high and one low water in a tidal day.

DIVERGENCE: In refraction phenomena - the spreading of orthogonals in the direction of wave travel.
(See Figure ii-7)

DOCK (dry): An artificial basin fitted with gate or caisson, into which vessels may be floated, water pumped out and vessel bottoms made available for painting or repairs.

DOCK (wet): A comparatively large basin into which vessel(s) may be floated at a desired tide. By closing its gate, water can be retained at any desired level.

DOCK (floating): A floating structure which may be sunk to receive a vessel, raised by removing water, and used as a DRY DOCK.

DOLPHIN: A built-up mooring post or buoy usually wooden, erected on shore or in the water.

DOUGLAS SEA SCALE: A series of numbers from 0 to 9 to indicate the state or condition of the sea.

DRIFT (noun): (1) The speed at which a current runs. (Do not confuse with "littoral drift".)
(2) Also, floating material deposited on a beach (driftwood).

DRUMLIN: An unstratified hill of glacial drift. It is compact, and either elongate or oval.

DUKW: (pronounced duck) Amphibian Truck, 2½ ton, 6 x 6.

DUNES: Ridges or mounds of loose, wind-blown material, usually sand.
(See Figures ii-18, 19)

DURATION: In forecasting waves - the length of time the wind blows.

DURATION - MINIMUM: The time necessary for steady state conditions to develop for a given wind velocity over a given fetch length.

EBB CURRENT: The movement of the tidal current away from shore or down a tidal stream.

EBB TIDE: A falling tide.
(See Figure ii-6)

EDDY: A current of air or water running contrary to the main current, especially one moving in a circle; a whirlpool.
(See Figure ii-19)

EMBANKMENT: An artificial bank, mound, dike or the like, built to hold back water, carry a roadway, etc.

EMBAYED: Formed into a bay or bays; as an embayed shore.

EMBAYMENT: A depression in a shoreline forming a large open bay.

ESCARPMENT (SCARP): A more or less continuous line of cliffs or steep slopes facing in one general direction which are caused by erosion or faulting.
(See Figure ii-1)

EXIT: Any natural or artificial feature of the ground by means of which troops and/or vehicles can pass from the beach into the hinterland.

FAIRWAY: The parts of a bay, river, etc. usually traversed by shipping.

FATHOM: A unit of measurement used for soundings on some charts; is equal to 1.83 meters or 6 feet.

FATHOMETER: A special type of sonic depth finder.

FEEDER CURRENT: (1) The currents which flow parallel to shore before converging and forming the "neck" of a surf rip current. (2) That portion of a rip current which flows parallel to the shore and inside the breakers.

FETCH: In forecasting waves, the area of water over which the wind blows. (See Figure ii-9)

FETCH LENGTH: In forecasting waves, the horizontal distance (in a fetch) over which the wind blows.

FIRTH: A narrow arm of the sea; also the opening of a river into the sea.

FJORD (FIORD): A long narrow arm of the sea between high lands, differing from an estuary in having deep depressions along its length.

FLANK: The right or left of any command.

FLOOD CURRENT: The movement of the tidal current toward the shore or up a tidal stream.

FLOOD TIDE: A rising tide. (See Figure ii-6)

FOAM LINE: The front of a wave as it advances shoreward, after it has broken. (See Figures ii-2, 3)

FOLLOWING WIND: A wind blowing the same direction as the swells are travelling.

FORESHORE: The part of the shore, lying between the crest of the berm and the ordinary low water mark, which is ordinarily traversed by the uprush and backrush of the waves as the tides rise and fall. (See Figure ii-1)

FRESHETS: An overflowing of a stream.

FRINGING REEF(s): Coral reefs that are attached to the land.

GENERATION OF WAVES: The growth of waves caused by a wind blowing over a water surface for a certain period of time. The area involved is called the generating area. (See Figures ii-8, 9)

GEODESY (or geodetics): The investigation of any scientific questions connected with the shape and dimensions of the Earth. This is the function of a Geodetic Survey.

GRADIENT: The rate of inclination to the horizontal. Usually expressed as a ratio: such as 1:25, indicating one unit rise in 25 units of horizontal distance - or in a decimal fraction (.04); degrees (2°18'); or percent (4%). Is sometimes expressed by such adjectives as: steep... moderate...gentle...mild...or flat.

GRADUATION (of instrument): A scale divided to indicate small units of measurements.

GRAVEL: Loose, rounded fragments of rock, between 1 and 2 mm. in diameter - (U.S. Bureau of Soils Classification). (See Figure ii-14)

GROIN (GROYNE): A shore-protection and improvement structure (built usually to trap littoral drift or retard erosion of the shore). It is narrow in width (measured parallel to the shoreline); and its length may vary from less than one hundred to several hundred feet (extending from a point landward of the shoreline out into the water).
Groins may be classified as permeable or impermeable: Impermeable have a solid or nearly solid structure; permeable have openings through them.
(See Figures ii-15, 17)

GROUND (TO): To run a craft ashore, to become fast to the bottom.

GROUND SWELL: A relatively heavy ocean swell caused by water in motion meeting lesser depths as it passes over shoals or approaches land. Not usually so high or dangerous as BLIND ROLLERS.

GROUND WATER: Water within the earth, such as supplies wells and springs.

GROUP VELOCITY: The velocity with which a wave group travels.
In deep water, it is equal to one-half (the individual) wave velocity.

GULF: A relatively large (as compared to bay) portion of sea partly enclosed by land.

GULLY: A channel cut by running water; a narrow ravine.

GUT: A channel in otherwise less deep water; generally formed by water in motion.

HEAD-(HEADLAND): A comparatively high promontory with either a cliff or steep face.
It extends into a large body of water, such as a sea or lake. An unnamed head is usually called a headland.
(See Figure ii-14)

HEAD (RIP): The section of surf rip current which has widened out seaward of the breakers.

HIGH TIDE: HIGH WATER(HW): Maximum height reached by each rising tide.

HIGHER HIGH WATER (HHW): The higher of the two high waters of any tidal day.

HIGHER LOW WATER (HLW): The higher of two low waters of any tidal day.

HIGH WATER LINE: In strictness, the intersection of the plane of mean high water with the shore. The shoreline delineated on the nautical charts of the Coast and Geodetic Survey is an approximation of the high water line.
(See Figure ii-1)

HINTERLAND: (1) That zone containing the beach flanks and the area inland from the coastline to a distance of five miles.
(2) The region lying behind the coast district.

HOGBACK: A ridge formed by outcropping edges of tilted strata; any ridge with a sharp summit and steeply sloping sides.

HOLDING GROUND: The condition of the bottom of an anchorage area; called good or bad according to whether or not the material of which the bottom is composed will prevent a ship's anchor from dragging.

HOOK: A spit or narrow cape, turned landward at the outer end, resembling a hook in form.

HYDROGRAPHY: The description and study of seas, lakes, rivers, and other waters.

INDIAN SPRING LOW WATER: An arbitrary tidal datum approximately the level of the mean of lower low waters at spring tides.

INDIAN TIDE PLANE: A convenient way of expressing an approximation to the level low water of ordinary spring tides, but where there is a large diurnal inequality in low waters.

INLET: An arm of the sea (or other body of water), that is long compared to its width, and that may extend a considerable distance inland.
(See Figure ii-19)

INSHORE CURRENT: A current located inside the surf zone.

INSULAR SHELF: The zone of the insular margin extending from the line of permanent immersion to the depth, usually of about 100 fathoms (or 200 meters) where there is a marked or rather steep descent towards the ocean depth.

INTERIOR: The country extending indefinitely inland of the hinder-land.

ISTHMUS: A narrow strip of land, bordered on both sides by water, that connects two larger bodies of land.

JEHEMY: A large frame which can straddle a small boat and retrieve the craft from - or launch it into - shallow water (as along a beach). Is usually towed by a tractor.

JETTY: (1) (U.S. usage) On open seacoasts, a structure extending into a body of water to direct and confine the stream or tidal flow to a selected channel.
Jetties are built at the mouth of a river or entrance to a bay to help deepen and stabilize a channel and thus facilitate navigation.
(2) (British usage) Jetty is synonymous with "wharf" or "pier".
(See Figures ii-6, 13)

KELP: The general name for large species of seaweeds.
A mass or growth of large seaweed or any of various large brown seaweeds.
See also SEAWEED
(See Figures ii-13, 15)

KEY: See CAY

KNOLL: (1) Loosely speaking, a submerged elevation of rounded shape rising from the ocean floor, but less prominent than a seamound.
(2) Underwater mounds, in a line parallel to shore; similar to an intermittent bar.

KNOT: (Symbol kt or kts): The unit of speed used in navigation. Is equal to 1 nautical mile (6,080.20 feet) per hour.

LAGOON: A shallow body of water, as a pond or lake, which usually has a shallow, restricted outlet to the sea.
(See Figures ii-12, 16)

LANDFALL (to make a): First sighting land from seaward.

LANDING BEACH: A beach suitable for the landing and deployment of assault troops and vehicles on a broad front.
See also BEACH

LANDING CRAFT (LC): (1) Any of numerous naval war vessels especially designed for putting ashore troops and/or equipment, especially in amphibious beach assault.
(2) U.S. Navy usage - this term is generally applied to non-oceangoing vessels of less than 150 feet over-all, that are designed for landing operations.
LC designation is therefore used, with appropriate modifications to indicate particular types of these craft.
(See LCI, LCM, etc.)

LANDING PLACE: Any natural or artificial feature adjacent to the shoreline that:
(1) permits the landing of small parties for reconnaissance or other purposes; and
(2) provides secondary facilities for units of main forces.
It is unsuitable for the landing and deployment of assault troops and vehicles on a broad front.

LANDING SHIPS (LS): U.S. Navy usage - this includes oceangoing vessels of over 200 feet over-all, that are specifically designed for landing operations.
LS designation is therefore used, with appropriate modifications to indicate particular types of these ships.
(See LSU, LSD, etc.)

LANDING VEHICLES (LV): U. S. Navy usage - small units used in landing operations that can be used on land or in water.
LV designation is therefore used for these vehicles.
(See LVT)

LANDLOCKED: An area of water enclosed, or nearly enclosed, by land - as a harbor. (Thus, protected from the sea.)

LANDMARK: A conspicuous object - natural or man-made - located near or on land, which aids in locating a landing beach or landing place, and/or fixes a ship's position at sea.

LCI: Landing Craft, Infantry

LCM: Landing Craft, Mechanized

LCVP: Landing Craft, Vehicle, Personnel
(See Figure ii-10)

LCT: See LSU

LEAD LINE: See SOUNDING LINE

LEE (noun): (1) Shelter, or the part or side sheltered or turned away from the wind.
(2) (Chiefly nautical): the quarter or region toward which the wind blows.

LEEWARD: The direction toward which the prevailing wind is blowing; the direction toward which waves are travelling.

LIMIT OF BACKWASH: The seaward limit of the backwash for any given tide stage.

LITTORAL CURRENTS (Longshore Currents): generated by waves breaking at an angle to the shoreline - which move usually parallel to, and adjacent to the shoreline within the surf zone.
(See Figures ii-10, 15)

LITTORAL DEPOSITS: Deposits of littoral drift located between high and low water lines.

LITTORAL DRIFT: The material that moves generally parallel to the shore under the influence of oceanic forces.

LONGSHORE CURRENT: See LITTORAL CURRENT

LOW TIDE (LOW WATER, LW): Minimum height reached by each falling tide; the mean value of all low waters over a considerable period.

LOW TIDE TERRACE: A flat zone of the beach near the low tide level.

LOW WATER DATUM: An approximation to the plane of mean low water that has been adopted as a standard reference plane.

LOW WATER LINE: The position of the still water line at any standard low tide datum. (See Figure ii-1)

LOWER HIGH WATER (LHW): The lower of the two high waters of any tidal day.

LOWER LOW WATER (LLW): The lower of the two low waters of any tidal day.

LSD: Landing Ship, Dock

LSM: Landing Ship, Medium

LST: Landing Ship, Tank

LSU: Landing Ship, Utility (Previously designated LCT)

LVT: Landing Vehicle, Tracked

MANGROVE: A particular kind of tropical tree or shrub, confined to low-lying brackish areas. (Growth is usually dense, causing poor trafficability)

MARGINAL ZONE: The outer ridge of a coral reef, generally the highest portion of such a reef.

MARINE RAILWAY: A cradle, supported on carriages, which runs on rails or racks laid on an incline. The cradle is run out to receive a ship, which it hauls up until clear of the water. Also called PATENT SLIP

MARSH: (1) A tract of soft, wet land.
(2) Salt marsh: flat land periodically flooded by salt water.

MASS TRANSPORT: Non-periodic movement of water in the direction of wave travel.

MEAN HIGH-WATER SPRINGS: The average of high spring tides.

MEAN LOW WATER (MLW): The average height of all low tides in one locality over a 19 year period.

MEAN LOW WATER SPRINGS (MLWS): The average height of the low waters of spring tides. Frequently abbreviated to "LOW WATER SPRINGS".

MEAN LOWER LOW WATER (MLLW): The mean of the lower low waters due to mixed tides. Frequently abbreviated to "LOWER LOW WATER". (See Figure ii-4)

MEDIAN DIAMETER: The size of the sieve opening through which 50% of a sand sample by weight passes.

MIXED TIDES: A combination of co-existing diurnal and semi-diurnal tides; the proportion between them varies for different seas.

MOLE: A massive work of masonry or of large stones laid in the sea. Serves as a breakwater or pier.

MOORING:
(1) The place in which a vessel may be secured;
(2) That which is used to secure a vessel;
(3) That to which a vessel may be secured.

MORAINE: An accumulation of earth, stones etc. deposited by a glacier.

MUD: A fluid-to-plastic mixture of finely divided particles of solid material and water.

MUSKEG: A bog formed in hollows or depressions of the land surface by the accumulation of water and growth of sphagnum mosses, and tussocks.

NAUTICAL MILE: Also know as Geographical mile is 6,080.20 feet...approximately 1.15 times as long as the statute mile of 5,280 feet.

NEAP TIDES: Tides occurring near the time of the quadrature of the moon. The neap range is usually 10 to 30 percent less than mean tidal range.

NEARSHORE: That part or zone of the sea approach lying between the 3 fathom line and the shoreline at mean low water springs.

NECK (Rip): The narrow band (rip) of water flowing seaward through the surf.

NIGGER HEAD: A hard, dark-colored boulder. Large blocks of reef-rock, usually located near the inner limit of the marginal zone.

NIP: The cut made by waves in a shoreline of emergence.

OCEANOGRAPHY: That science treating of the oceans, their forms, physical features and phenomena.

OFFSHORE: In beach terminology, the comparatively flat zone of variable width, extending from the shore face to the continental shelf.
It is continually submerged.
(See Figure ii-1)

OFFSHORE CURRENTS: Currents outside the surf zone.

OFFSHORE WIND: A wind blowing off the land.

OPPOSING WIND: A wind blowing in the opposite direction to that in which the waves are travelling.

ORTHOGONAL: Line drawn at right angles to wave crests on a refraction diagram.
(See Figure ii-7)

OSCILLATION: A periodic movement to and fro, or up and down.

PASS: (1) A navigable channel, especially at a river's mouth.
(2) A passage through or over a mountainous region.

PATENT SLIP: See MARINE RAILWAY.

PEBBLES: Smooth rounded stones ranging in diameter from 2 to 64 mm. (U. S. Bureau of Soils Classification)

PEN: One of a series of parallel jetties for berthing several destroyers, submarines or small craft.

PENINSULA: An elongated portion of land nearly surrounded by water, and connected to a larger body of land, usually by a neck or an isthmus.

PERMAFROST: Permanently frozen subsoil.

PIER: A structure built out into the water on piles for use as a landing place, pleasure resort, etc.
(See Figures ii-5, 7, 13, 15)

PILE: A long substantial pole of wood, concrete or metal, driven into the earth or sea bed to serve as a support or protection.

PINNACLE: A sharp pyramid or cone-shaped rock under water or showing above it.
See also REEF PINNACLE

PLAIN: A low relief region of horizontal strata.

PLAIN, COASTAL: A plain fronting the coast and generally representing a strip of recently emerged sea bottom.
(See Figure ii-19)

PLATEAU: (1) (Geographical): A relatively large elevation of comparatively flat or level land; a table land.
(2) (Oceanographical): An elevation from the bottom of the ocean with a more or less flat top and steep sides.

POINT: The extreme end of a cape, or the outer end of any land area protruding into the water which is less prominent than a cape.
(See Figures ii-5, 12, 15)

PONTOON CAUSEWAY: A causeway made of, or supported by pontoons. For example, a N. L. pontoon causeway used for ship-shore traffic, espcially between LSTs and the beach.

PORT: The commercial part of a harbor, where the quays, wharves, facilities for landing cargo, docks, repair shops, etc. are situated.

PORTSIDE: Left side of a ship looking forward; indicated by a red running light at night when underway.

PROFILE (of beach): Intersection of the surface of the beach with a vertical plane. A sectional elevation through the beach and surf perpendicular to the shoreline.
(For profile changes See Figure ii-4)

PROMONTORY: A high point of land extending into a body of water; a headland.

PROPAGATION OF WAVES: To cause to spread or extend. To carry forward through space.

QUAY (pronounced Key): A stretch of paved bank, or a solid artificial landing place made beside navigable water, for convenience in loading and unloading vessels.

QUICKSAND: Loose, yielding wet sand which offers no support to heavy objects. The upward flow of the water has a velocity that eliminates contact pressures between the sand grains, and causes the sand water mass to behave like a fluid.

RAMP: (1) A strip of stone or concrete built on a beach to facilitate landing, unloading, and/or hauling up of small craft.
(2) A bulkhead hinged at the bottom, which is dropped from the bow or stern of a vessel to discharge passengers and cargo on a beach.
(3) A strip of sand bulldozed to the bow of an LST or LSU.

REEF (noun): A rocky or coral elevation in the sea dangerous to surface navigation; it may or may not be above the sounding datum in elevation.
A rocky reef is always detached from shore.
A coral reef may or may not be connected with shore.

REEF-ATOLL: A ring-shaped, coral reef, often carrying low sand island, enclosing a body of water.

REEF-BARRIER: A reef which roughly parallels land but is some distance offshore, with deeper water intervening.

REEF-PINNACLE (noun): A small coral pinnacle rising within a lagoon, often coming close to the water surface.
See also PINNACLE

REFERENCE PLANE (DATUM): The planes to which sounding and tidal data are referred. The following reference planes are used:
MEAN LOW WATER - United States (Atlantic Coast), Argentina, Sweden and Norway.
MEAN LOWER LOW WATER - Pacific Coast, U. S.
MEAN LOW WATER SPRINGS - Great Britian, Germany, Denmark, Italy, Brazil, and Chile.
LOWEST LOW WATER SPRINGS - Portugal
LOW WATER INDIAN SPRINGS - India and Japan (See INDIAN TIDE PLANE.)
LOWEST LOW WATER - France, Spain, and Greece.

REFERENCE POINT: A specified location (in plan and elevation) to which measurements are referred.

REFERENCE STATION: A station for which tidal constants have previously been determined and which is used as a standard for the comparison of simultaneous observations at a second station.

REFLECTED WAVE: The wave which is returned seaward when a wave impinges upon a very steep beach or cliff.

REFRACTION: (1) When a train of waves approaches a shoreline at an angle, the wave crests are bent because the first portion to reach shallow water travels more slowly than the portion still advancing in the deeper water. This process of bending of waves crests is known as refraction.
(2) Also: the bending of wave crests by currents.
(See Figures ii-5, 6, 7, 11)

REFRACTION COEFFICIENT: The ratio of the refracted wave height to the deep water wave height.

RETRACTING (RETRACTION): The act of running or driving (of a landing craft, etc.) from a beach to sea.

RHINO FERRY: A 6 x 30 N. L. pontoon barge, self-propelled.

RIA: A long narrow inlet, with depth gradually diminishing inward.

RIDGE, BEACH: Essentially continuous mound of beach material that has been heaped up by wave or other action above the upper limit of wave uprush. Ridges may occur singly or as a series of approximately parallel ridges behind the beach.
(See Figures ii-18, 19)

RILL MARKS: Tiny drainage channels in a beach caused by the flow seaward of water left in the sands of the upper part of the beach after the retreat of the tide or after the dying down of storm waves.

RIP: A body of water made rough by the meeting of opposing tides or currents.

RIP, SURF (RIP CURRENT): Narrow current of water flowing seaward through the breaker zone.
A rip consists of three parts:
(1) the "feeder currents" flowing parallel to the shore inside the breakers;
(2) the "neck" - where the feeder currents converge and flow through the breakers in a narrow band or "rip"; and
(3) the "head" - where the current widens and slackens outside the breaker line.
(See Figure ii-18)

RIPPLE MARKS: Small, fairly regular ridges in the sand or silt. As their form is normal to the direction of wind and/or current direction, they indicate both the presence and the direction of winds and/or currents.

RIPRAP: (1) Broken stones used for foundations, etc.
(2) Foundation or wall of stones thrown together irregularly.
(See Figure ii-14)

RIVER: A natural stream of water larger than a brook or creek.

ROADSTEAD (ROAD): (nautical) a sheltered area of water near shore where vessels may anchor in relative safety.

ROCK: A concreted mass of stony material; also, broken pieces of such masses.
(See Figure ii-19)

ROCKS, AWASH Those exposed at any stage of the tide between mean high water and the sounding datum, or exactly awash at these planes.

ROCKS, BARE: Those extending above the plane of mean high water.

ROCKS, SUNKEN: Those covered at the sounding datum, but potentially dangerous to navigation.

ROLLER: Long, high swell.

RUBBLE: Loose, angular, and water-worn stones along a beach.

RUNNEL: A corrugation of the foreshore (or the bottom just offshore), formed by wave and/or tidal action.

SAND: An unconsolidated mixture of inorganic material consisting of small but easily distinguishable grains ranging in diameter from 0.05 - 1.0 mm.: very fine sand
0.1 - 0.25 mm.: fine sand
0.25 - 0.5 mm.: medium sand
0.5 - 1.0 mm.: coarse sand
(U. S. Bureau of Soils size classification)
(See Figure ii-15)

SAND BAR: A bar or ridge of sand built up to, or near to the surface of the water by currents in a river or by wave action in coastal waters.

SCARP: See ESCARPMENT

SCARP-BEACH: An almost perpendicular slope along the beach foreshore which is eroded by wave action.

SEA: Waves caused by wind at the place and time of observation.

SEA (STATE OF): Description of the state of the ocean surface existing in a storm area.

SEAWALL: An artificial structure built along a portion of a coast to prevent erosion by sea water.
Since, on the shoreside, most seawalls are filled in to about the top of the wall, they provide resistance to wave force by the mass of the wall and also by the backfill's pressure resistance.
(See Figure ii-14)

SEAWEED: A characteristics plant growth in sea water. Usually indicates the presence of rocks.
See also KELP and EELGRASS

SEICHE (pronounced sāsh-"a" as in able): The periodic surging (or rhythmical movement) of water in any body of water. (Originally applied only to lakes, but now expanded to oceans etc.)

SEISMIC WAVE: See TSUNAMI

SEMIDIURNAL TIDES: Two highs and two lows per lunar day, with comparatively little diurnal inequality.

SET (of current): The direction towards which it flows.

SHALLOW WATER: Water in which the depth is less than one-half the wave length at the particular time. (See Figure ii-1)

SHELF: The zone extending from the line of permanent immersion to the depth (usually about 65 fathoms) where there is a marked or rather steep descent towards the great depths.

SHELF, CONTINENTAL: A shelf bordering a continent.

SHELF, INSULAR: A shelf surrounding an island.

SHINGLE: (1) Loosely and commonly - any beach gravel coarser than ordinary gravel, especially any having flat or flattish pebbles;
(2) Strictly and accurately - beach gravel of smooth, well-rounded pebbles that are roughly the same size. The spaces between pebbles are not filled with finer materials as they are in ordinary gravel. Shingle gives out a musical note when stepped on.

SHOAL (noun): (1) An elevation of the sea bottom which comes within six fathoms of the surface.
(2) A detached area of any material except rock or coral. The depths over it are a danger to surface navigation.

SHOAL (verb): (1) to become shallow gradually,
(2) to cause to become shallow,
(3) to proceed from a greater to a lesser depth of water.

SHORE: See BEACH

SHORE FACE: The rather steeply sloping zone between the low tide shoreline and the beginning of the comparatively flat zone of variable width known as the offshore.

SHORELINE: The juncture of the sea and land. Unless specifically stated as the "high tide" shoreline, it refers to the juncture at low tide.

SIGNIFICANT WAVE HEIGHT: Average height of the highest one-third of the waves for a given period.

SIGNIFICANT WAVE PERIOD: Average period of the highest one-third of the waves for a given period.

SILT: A clastic deposit (one that has been transported mechanically and deposited) of an inorganic granular material with median diameter between 0.005 mm. and 0.05 mm. It is thus between sands and clays in size. (U. S. Bureau of Soils size classification)

SLACK TIDE: The intermediate period between tidal currents, during which there is no horizontal motion.

SLIP: A space between two piers for berthing a vessel.

SLOPE: See GRADIENT

SLOUGH (pronounced - sloo): (1) A small muddy marshland or tidal waterway which usually connects other tidal areas.
(2) A tide land or bottom-land creek.
(See Figure ii-15)

SOLITARY WAVE: A wave consisting of a single elevation which propagates without change of form (in two dimensions).

SORTING COEFFICIENT: A coefficient used in describing the distribution of grain sizes in a sample of beach material.
It is defined as $S_o = \sqrt{Q_1/Q_3}$
where Q_1 is the grain diameter, having 75% of the sample smaller; and Q_3 is the grain diameter having 25% of the sample smaller.

SOUND: A wide waterway between two sea areas.
See also STRAIT

SOUNDING: The depth measured, or the number indicating the depth on the chart, usually in fathoms.

SOUNDING DATUM: The datum (Reference Plane) to which soundings are referred.

SOUNDING LINE: A line, wire, or cord used in soundings. It is weighted at one end with a plummet (sounding lead).

SPIT: A small point of land or narrow shoal running into a body of water from the shore.
(See Figure ii-14)

STAND: That period at high or low water which marks the transition between tides, during which no vertical change can be detected.

STANDING WAVE (STATIONARY WAVE): A type of wave in which there are nodes, or points of no motion, between which the water oscillates. They are the result of two similar waves travelling simultaneously, but in opposite directions.

STERN: The rear end of a ship or boat.

STILL WATER LEVEL: The surface of the water if all wave action were to cease.
In deep water this level approximates the midpoint of the wave height. In shallow water it is nearer to the trough than to the crest.
(See Figure ii-1)

STRAIT: A relatively narrow waterway between two larger bodies of water.
(See also SOUND)

STREAM: (1) A course of water flowing along a bed in the earth,
(2) A current in the sea formed by wind action etc.
(Gulf Stream).

SURF: The water in the area between the shoreline and the outermost limit of breakers.
(For surf variability See Figures ii-10, 11)

SURF ZONE: The area between the outer most breaker and the limit of wave uprush.
(See Figures ii-5, 10, 11)

SURGE: (1) The name applied to wave motion with a period intermediate between that of an ordinary wind wave and that of the tide; say from one to sixty minutes.
It isusually of low height; perhaps 0.3 feet.
(2) Also: long interval variations, in velocity and pressure in fluid flow, not necessarily periodic, perhaps even transient in nature.
See also SEICHE

SWAMP (noun): A tract of wet spongy land, frequently inundated by fresh or salt water, and characteristically dominated by trees and shrubs.

SWAMP (verb): To swamp: to overset, sink, or fill up a craft with water.

SWASH MARK: The thin wave line of fine sand, mica scales, bits of seaweed, etc. left by the swash when it recedes from its upward limit of movement up the beach face. (For SWASH See UPRUSH)

SWELL: Wind-generated waves which have advanced into regions of weaker winds or calm and are decreasing in height.
(See Figures ii-2, 8)

TABLELAND: A lofty plain; a plateau; an elevated area of land of generally level surface and considerable extent.

TERRACE: A horizontal or nearly horizontal natural or artificial topographic feature interrupting a steeper slope, usually occurring in a series.

TIDAL CURRENT: Currents caused by the horizontal movement of the rise and fall of tide (due in turn chiefly to the attraction of the moon and sun).

TIDAL DATUM: See REFERENCE PLANES, CHART-DATUM, etc.

TIDAL DAY: The variable interval between two alternate high or low waters.

TIDAL DELTA: See DELTA

TIDAL FLATS: Areas bordering the oceans which are covered and uncovered according to the state of the tide.

TIDAL PERIOD: The interval of time from low tide to the next low tide, or from high tide to the next high tide.

TIDAL POOL: A pool of water remaining on a beach or reef after recession of the tide.

TIDAL PRISM: The total amount of water that flows into the harbor and out again with movement of the tide.

TIDAL RANGE: The difference between the level of water at high tide and low tide.

TIDAL STAND: An interval at high or low water when there is no sensible change in the height of the tide. The water level is stationary at high and low water for only an instant; but the change in level at these times is so slow that it is not usually perceptible.

TIDAL WAVE: See TSUNAMI

TIDE: The periodic rise and fall of oceans and bodies of water connecting them, caused chiefly by the attraction of the sun and moon.

TIDE, DIURNAL: See DIURNAL TIDE

TIDE, EBB: See EBB TIDE

TIDE, FLOOD: See FLOOD TIDE

TIDE, MIXED: See MIXED TIDES

TIDE, NEAP: See NEAP TIDES

TIDE, SEMIDIURNAL: See SEMIDIURNAL TIDES

TIDE, SLACK: See SLACK TIDE

TIDE, SPRING: The tides, occurring at new and full moon that rise highest and fall lowest from the mean level.

TOMBOLO: An area of unconsolidated material, deposited by wave action, which connects a rock or island to the main shore.

TOPOGRAPHY: Accurate and detailed description of a locality as regards its elevations, etc.

TRAFFICABILITY: The ability of terrain to sustain the flow of military traffic.

TROUGH, WAVE: See WAVE TROUGH

TRUNCATED LANDFORM: (As a Truncated Drumlin)-Landform cut off, especially by erosion, and forming a steep side or cliff.

TSUNAMI: Wave caused by an underwater seismic disturbance. Commonly misnamed "tidal wave".

TUNDRA: Arctic plain consisting of black, mucky soil and permanently frozen subsoil, and covered by dense mosses, grasses, etc.

TUSSOCK: A dense bunch of grass or sedge.

UDT: Underwater Demolition Team.

UNDERTOW: (1) A current, below water surface, flowing in the opposite direction to water on the surface...(also) the receding water below the surface from waves breaking on a shelving beach.
(2) Actually "undertow" is largely mythical. As the backwash of each wave flows down the beach, a current is formed which flows seaward. However, it is a periodic phenomena. The most common phenomena expressed as "undertow" are actually the rip currents in the surf.

UNDERWATER GRADIENT: The slope of the sea bottom off the beach.

UNDULATION: A continuously propagated motion to and fro, in any fluid or elastic medium, with no permanent translation of the particles themselves.

UPRUSH (SWASH): The rush of water up onto the beach following the breaking of a wave.

VALLEY: An elongated depression, usually with an outlet, between bluffs or between ranges of hills or mountains.

VARIABILITY: See WAVE VARIABILITY

WADI: A permanent intermittent stream bed in an arid region.

WARPING TUG: A tug outfitted primarily for moving (a ship, etc) into some desired place or position by hauling on a rope or warp which has been fastened to something fixed, as a buoy, anchor, or the like.
For example: a N. L. pontoon warping tug used for pulling small craft off the beach.

WATER LINE: The juncture of land and sea.
This line migrates, changing with the tide. Where waves are present on the beach, this line is also known as the limit of backrush.

WAVE: An oscillatory movement in the sea which results in an alternate rise and fall of the surface.

WAVE AGE: The ratio of wave velocity to wind velocity (in forecasting waves).

WAVE CREST: The highest part of a wave.
(See Figures ii-1, 2)

WAVE-CREST LENGTH: See CREST LENGTH

WAVE DELTA: See DELTA

WAVE GROUP: A group of waves in which the distance between successive crests, and the amplitude, vary only slightly.

WAVE GENERATION: See GENERATION OF WAVE

WAVE HEIGHT: Vertical distance between crest and preceding trough. Usually expressed in feet.
(See Figure ii-1)

WAVE LENGTH: Horizontal distance between successive wave crests measured perpendicularly to the crest. Usually expressed in feet.
(See Figure ii-1)

WAVE PERIOD: The time, in seconds, for a wave crest to traverse a distance equal to one wave length.
Also defined as the time for two successive wave crests to pass a fixed point such as a rock or an anchored buoy.

WAVE REFRACTION: See REFRACTION

WAVE STEEPNESS: The ratio of the wave's height to its length.

WAVE TRAIN: A series of waves.
(See Figure ii-8)

WAVE TROUGH: The lowest part between crests.
(See Figure ii-1)

WAVE VARIABILITY: (1) Wave trains are not composed of waves of equal height and periods, but rather of heights and periods which vary in a statistical manner.
This variation is usually termed "wave variability".
(2) Also, the variability in direction of wave travel when leaving the generating area.
(3) Note: The variation in height along the crest is usually called "variation along the wave".

WAVE VELOCITY: Speed at which the individual wave form advances across the ocean.

WHARF: A structure built on the shore of a harbor, river, canal, etc., so that vessels may lie alongside to receive and discharge cargo, passengers, etc.

WHITECAP:	The white froth on crests of waves in a breeze.
WINDWARD:	The direction from which the wind is blowing.
WIND WAVES:	Waves being formed and built up by the wind. (See Figure ii-8)

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GLOSSARY APPENDIX II

Figures
(photographs, diagrams and sketches)

The following index of Figures was prepared to assist in finding illustrations for definitions of terms commonly employed in discussions of waves, surf, and beaches.

Figure ii-1: Definitions of Terms

Wave Terminology, including:
- Deep water
- Depth
- Direction
- Plunge Point
- Shallow water
- Still water level
- Tide Stage
- Wave crest
- Wave height
- Wave length
- Wave trough

Beach Terminology, including:
- Backshore
- Beach
- Berm
- Cliff
- Escarpment (Scarp)
- Foreshore
- High Water Line
- Low Water Line
- Offshore
- Shore

Figure ii-2: Estero Bay

Wave Terminology, including:
- General discussion of waves, swell, and breakers
- Breaking point and depth
- Foam line
- Swell
- Wave crest

Figure ii-3: Breaker Types

Breaker Terminology, including:
- Breakers: plunging, spilling and surging
- Breaking point and depth
- Foam line

Figure ii-4: Profile Changes by Wave Action

Includes:
- Discussion of beach profile change due to wave action
- Cusps
- M.L.L.W.

Figure ii-5: Refraction of Waves

Includes:
- Illustration of variance in width of surf zone
- Pier
- Point
- Refraction

Figure ii-6: Refraction of Waves

Shows:

Breaker angle
Currents
Currents and waves in shoaling water
Ebb tide
Flood tide
Jetty

Figure ii-7: Refraction of Waves, General

Shows:

Contour lines
Convergence
Cove
Divergence
Orthogonals
Pier
Wave fronts

Figure ii-8: Generation of Waves

Shows:

Breakers
Swell
Wave train
Wind waves

Figure ii-9: Synoptic Weather Map of Northeast Pacific

Shows:

Decay distance
Decay of waves
Generation of waves and generating area
Fetch

Figure ii-10: Surf Zone: Longshore Currents

Shows:

Broached landing craft - LCVP and Jeep
Currents and their effects
Longshore currents (or littoral currents)

Figure ii-11: Surf Zone: Variability of Surf

Illustrations and discussions of surf (or wave) variability
Example of wave record
Example of wave refraction

Glossary, Appendix II - Index cont.

Figure ii-12: Lagoon Beaches

Includes:
(Discussion of beach materials)
Bar
Berm
Point

Figure ii-13:

Shows:

Boat basin
Breakwater
Jetty
Kelp beds
Pier
Sand deposit

Figure ii-14:

Shows:
Berm, cobble
Breakwater
Cliff
Cobblestones
Diffraction
Gravel
Headland
Riprap
Seawall
Spit

Figure ii-15:

Shows:
Bay
Causeway
Cliff
Cusps
Groin
Island
Kelp beds
Longshore currents - (or littoral currents)
Pier
Point
Sand beach
Sand deposit
Slough

Figure ii-16:

Shows:

Bar
Bight
Cape
Cove
Lagoon

Figure ii-17:

Shows:

Boulders
Cliff
Cobblestones
Erosion
Groin

Figure ii-18:

Shows:

Cusps
Dunes
Hollow
Ridge beach
Rip current (rip surf)

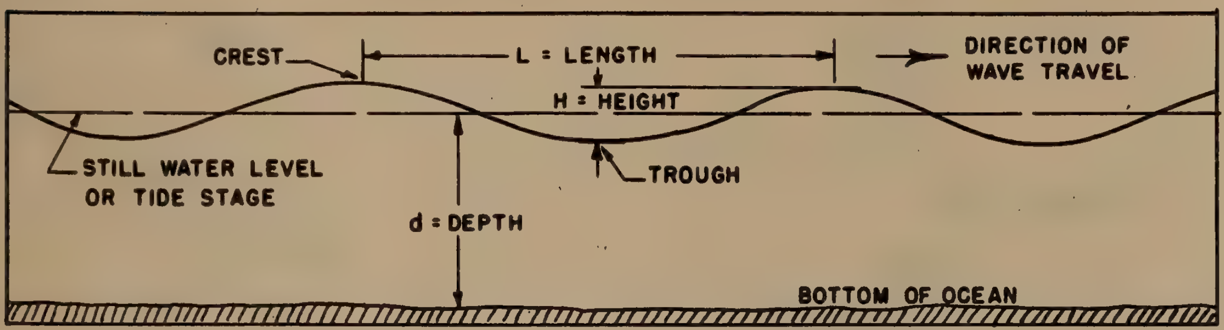
Figure ii-19:

Shows:

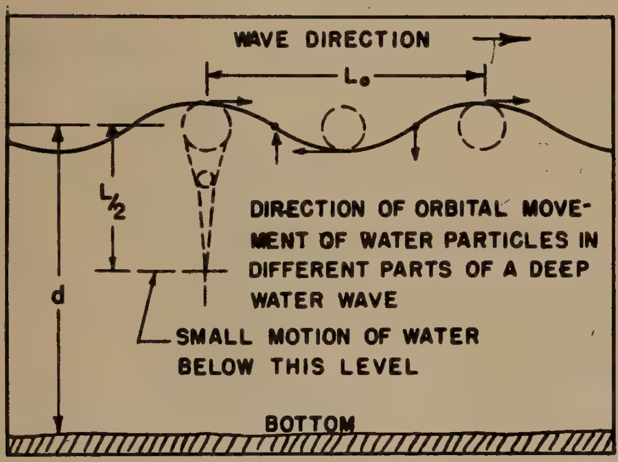
Bar (underwater)
Canal
Delta
Dunes
Eddy currents
Inlet
Plain, coastal
Ridge - beach
Rock

DEFINITIONS

THE FOLLOWING VERTICAL-SECTION SKETCHES ILLUSTRATE TERMS COMMONLY EMPLOYED IN DISCUSSIONS OF WAVES, SURF AND BEACHES

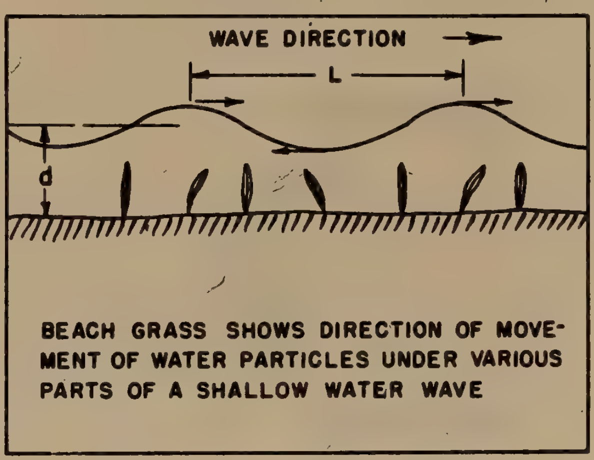


DEFINITIONS OF WAVE CHARACTERISTICS



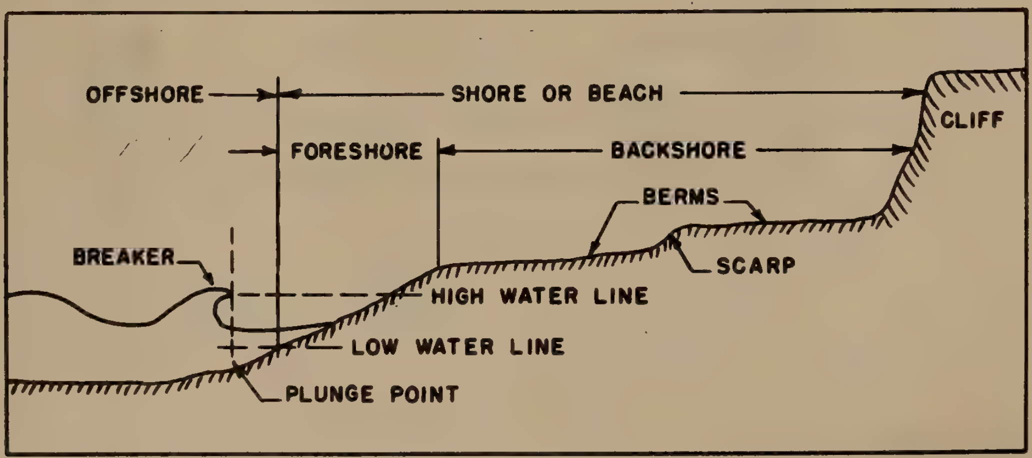
DEEP WATER WAVES

When the water depth is great, the bottom has no effect on the motion of waves; hence such waves are called <u>deep water waves</u>.

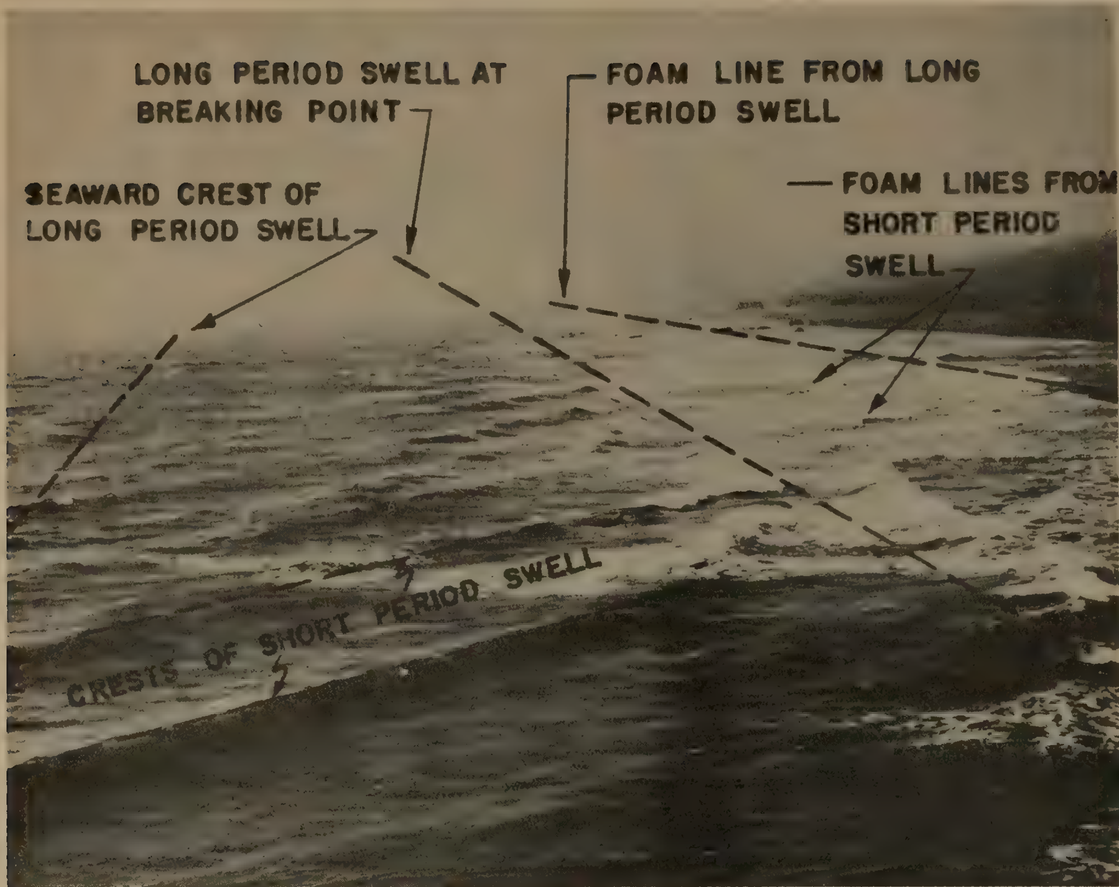


SHALLOW WATER WAVES

When the water depth is small, waves cause the water to move at the bottom.



DEFINITIONS OF BEACH CHARACTERISTICS



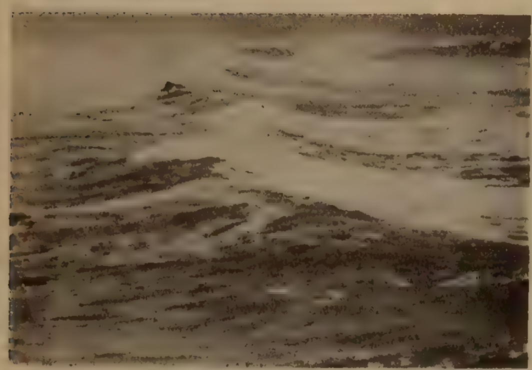
10032 May 17, 1945

Estero Bay. In this photograph long period swell and high short period waves are superimposed. Note that the short period waves break only when the swell reaches the breaking point. Breaking takes place in oblique sections at a line parallel to the swell. Between the long period swell no breaking occurs at the surf zone, and no foam lines carry through. This situation gives rise to high currents, and the oblique breaking sections contribute heavily to broaching of landing craft.

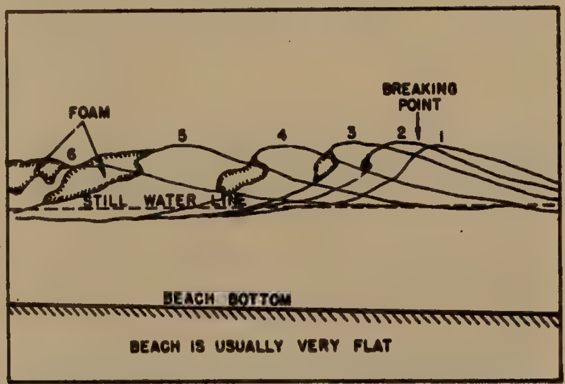
FIGURE ii-2

BREAKER TYPES

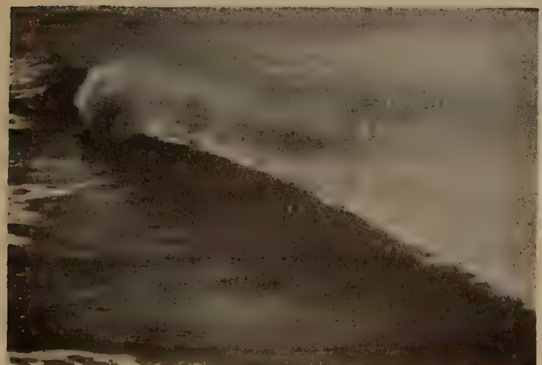
Both photographs and diagrams of the three types of breakers are presented below. The sketches consist of a series of profiles of the wave form as it appears before breaking, during the breaking, and after breaking. The numbers opposite the profile lines indicate the relative times of the occurrences.



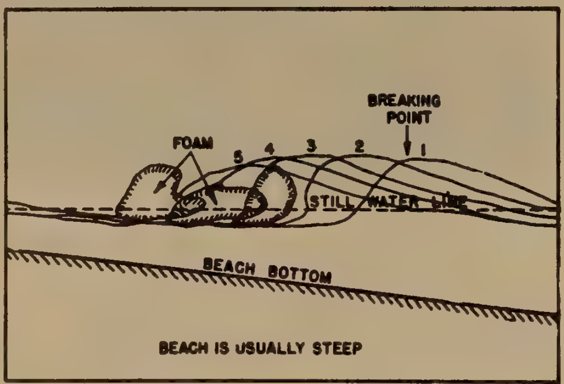
SPILLING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF SPILLING BREAKERS

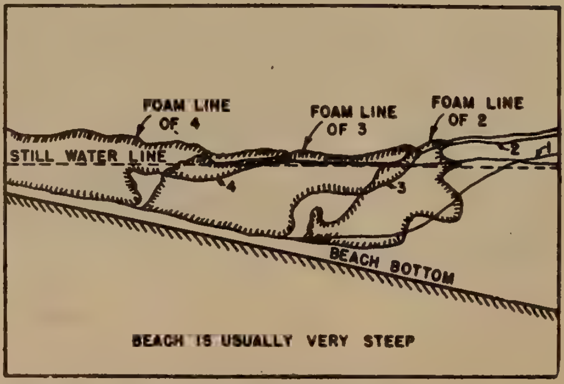


PLUNGING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF PLUNGING BREAKERS

SURGING BREAKER

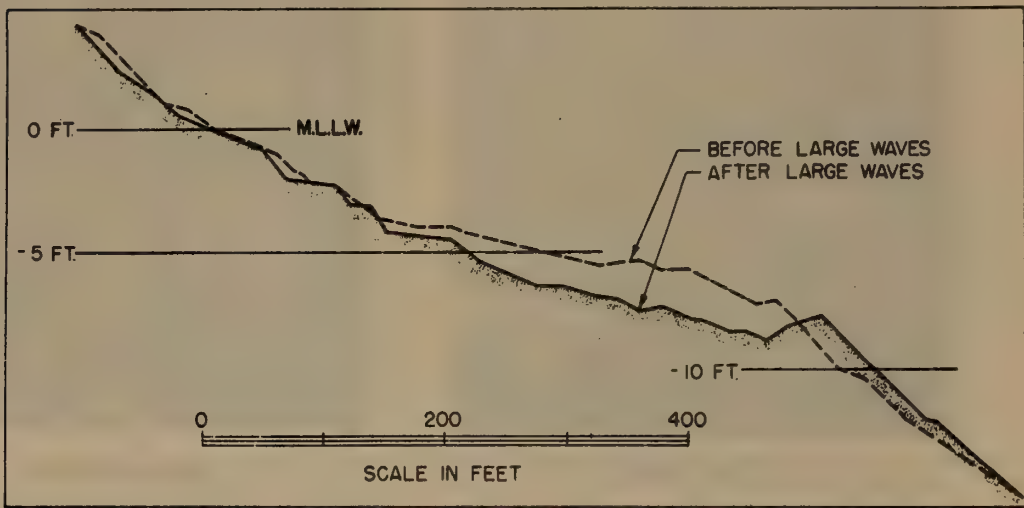


SKETCH SHOWING THE GENERAL CHARACTER OF SURGING BREAKERS

FIGURE ii-3

PROFILE CHANGES BY WAVE ACTION

A sandy shoreline changes in profile and plan as the waves change, and waves are as variable daily, seasonally, and annually as other aspects of the weather. Conversely, changes in its beach cause a change in the character of the surf. During periods of low long waves, the beaches may build up their berms on the foreshore. The beach slope in the shore area tends to steepen and the bars tend to become less pronounced, often nearly disappearing. On the other hand, during periods of high steep waves, the slope in the shore area tends to flatten while the offshore bars tend to increase in size. With a given wave action, the steeper beaches change faster than the flatter beaches. Bars, cusps, gravel deposits and minor forms appear and disappear rapidly and frequently. Recent sand deposits are usually soft but become harder under compaction by wave action and tidal variations. Any beach which is undergoing, or has recently undergone, a distinct change may be a difficult beach upon which to operate.



Profile Changes due to Wave Action

Changes such as this take place more rapidly on a steep beach than on a flat beach

Typical Beach Cusps

Beach cusps may appear and disappear overnight with an abrupt change of wave action. Erosion of cusps may leave a series of steps where the crest of each cusp has been cut away, and soft sand is newly deposited in the troughs of the old cusps. Water rushing off the ridges into the hollows of the cusps and out to the sea make it advisable to bring LVT's and Dukws in on the ridges but LCVP's and LCM's in on the hollows.

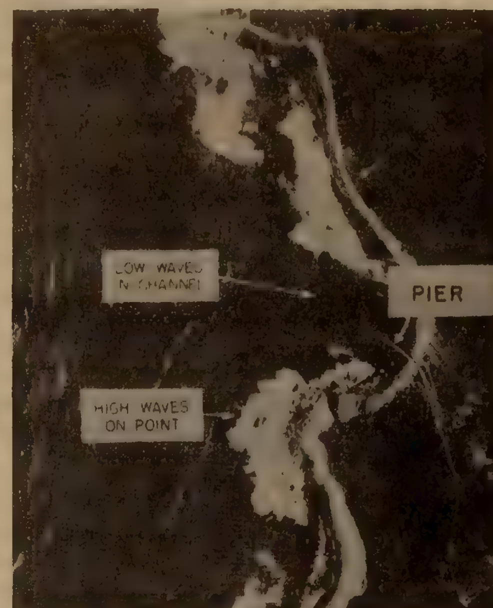
FIGURE ii-4

REFRACTION OF WAVES

As waves move into shallow water and approach the shore at an angle to the bottom contours, the waves are bent. This is known as refraction. The bending of waves may cause a converging or diverging of the crests (depending upon the hydrography) with a subsequent increase or decrease in wave or breaker height. Another effect of waves approaching a beach at an angle is to induce a longshore current inside the breaker zone. This current is effective in transporting sand along the shore and is also a hazard to landing craft operating through the surf zone

Pt. Pinos, California

Waves moving over a submarine ridge concentrate to give large wave heights on a point.

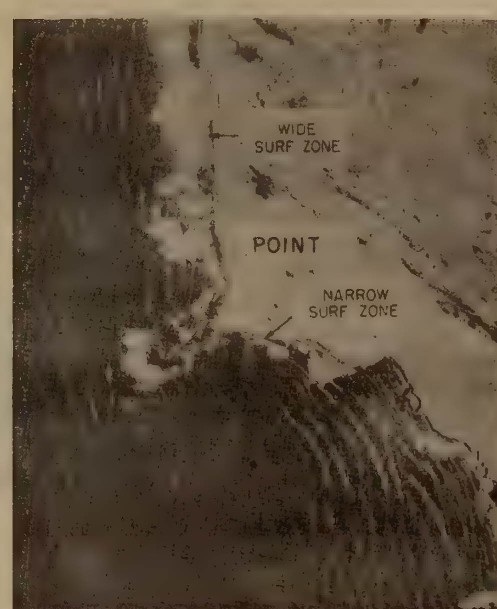


Arena Cove, California

Spreading of waves by refraction produces low wave heights at the pier.

Halfmoon Bay, California

Note the increasing width of the surf zone with increasing degree of exposure to the south.



Purisima Pt., California

Refraction of waves around a headland produces low waves and a narrow surf zone where bending is greatest.

FIGURE ii-5

REFRACTION OF WAVES

Waves in shoaling water refract in such a manner that they tend to become parallel to the underwater contours and, eventually, the shore. However, they usually break before becoming quite parallel to the beach.

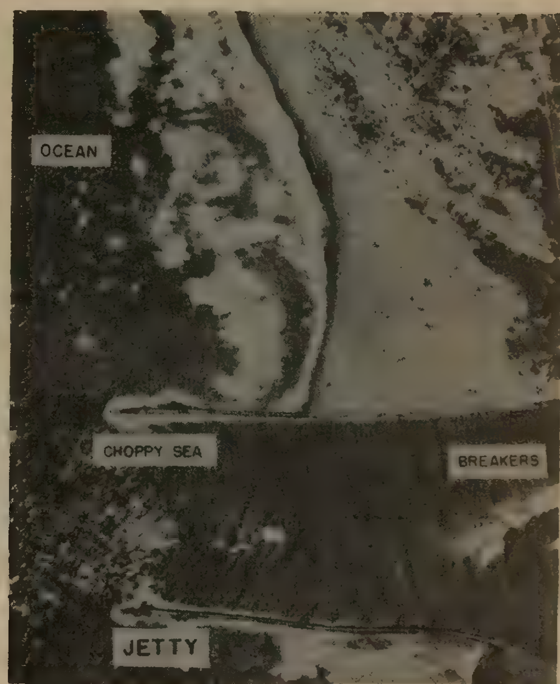
Oceanside, California

Waves tend to become parallel to the beach.

When the waves buck a current, as when they run against an ebb current at a **harbor entrance, they shorten up and grow higher, producing** a choppy sea and, frequently, breakers. **When waves run with a current, as in flood current at a harbor entrance, they** become longer and lower.

Humboldt Bay, California

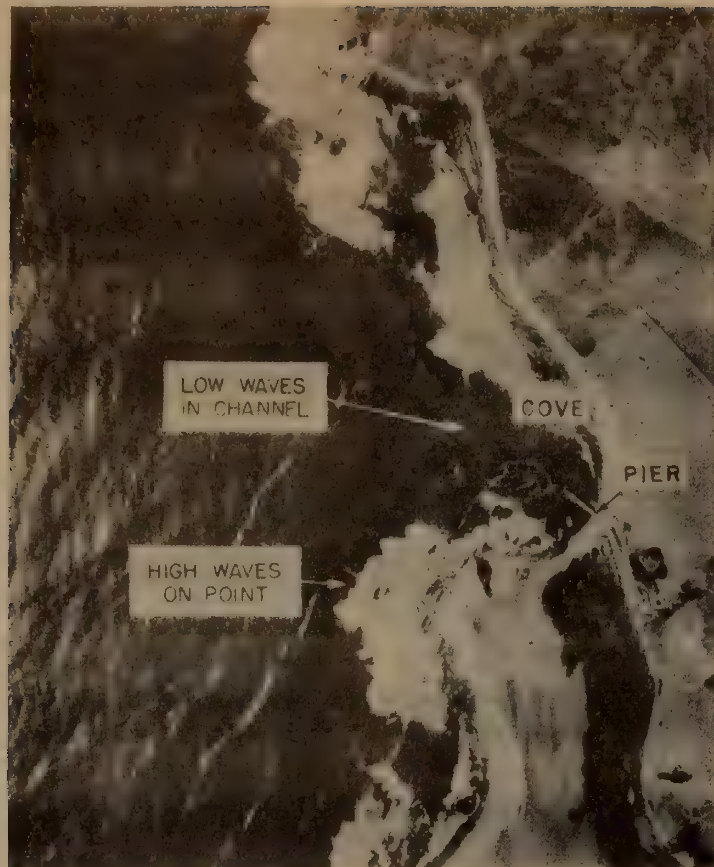
FLOOD TIDE

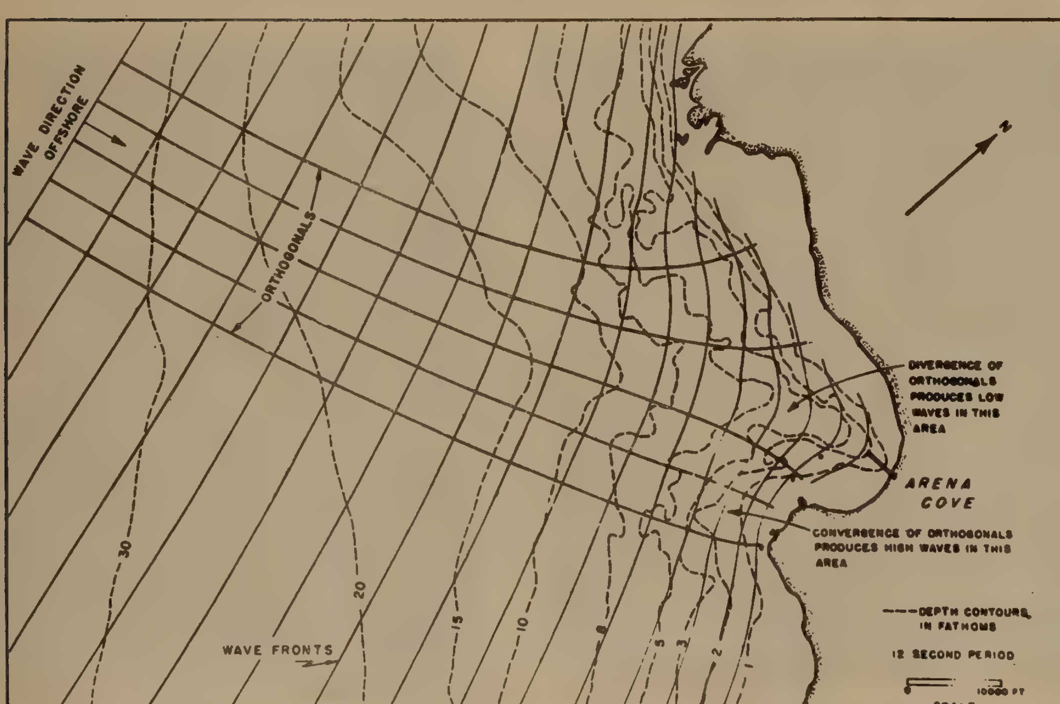


Humboldt Bay, California

EBB TIDE

FIGURE ii-6



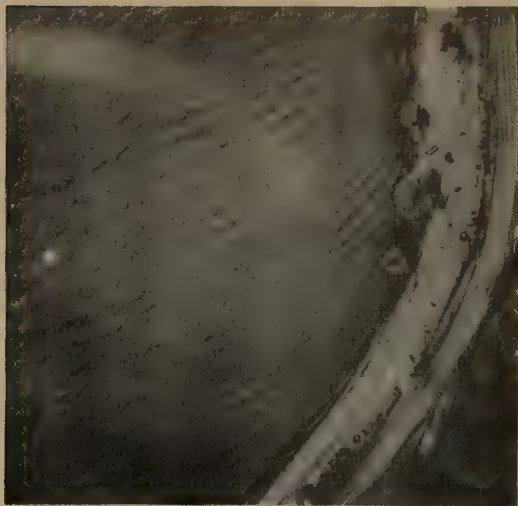


REFRACTION DIAGRAM. Compare with the aerial photograph shown above for illustration of how such diagrams can be utilized to determine points where low and high wave action can be expected.

FIGURE ii-7

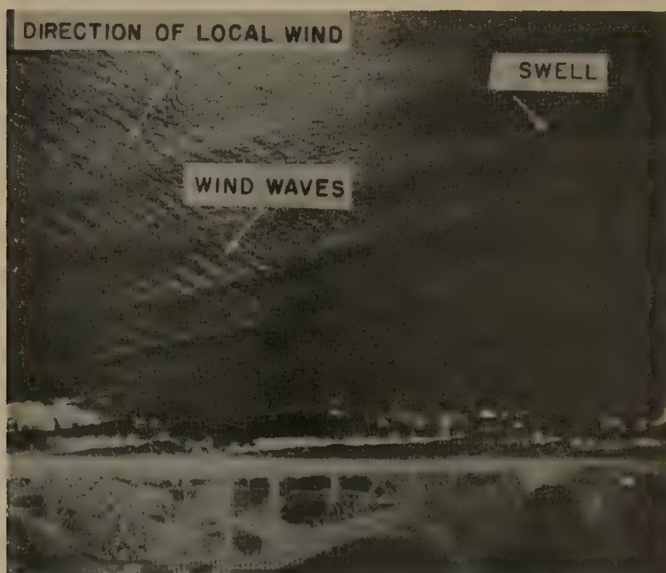
GENERATION OF WAVES

Waves which affect amphibious operations are generated by wind blowing over the ocean surface. At first the waves are very small, as shown on the small lake in the photograph below (left). The greater the length of the area over which the wind blows, the longer it blows, and the higher its speed, the larger are the waves. After the waves leave a storm area, as in the open ocean, they tend to become long and smooth "swells" like those shown in the photograph below (right). Eventually the swell reaches shore peaks up and then breaks. It is the larger waves, or swell, breaking on a beach which make the surf dangerous. An aerologist, with the aid of weather charts and hydrographic charts, can forecast the wind blowing over the ocean and then forecast the waves generated by these winds.



Typical wind waves in a generating area. Note the variability of the wave height, wave length, and the length along the crest.

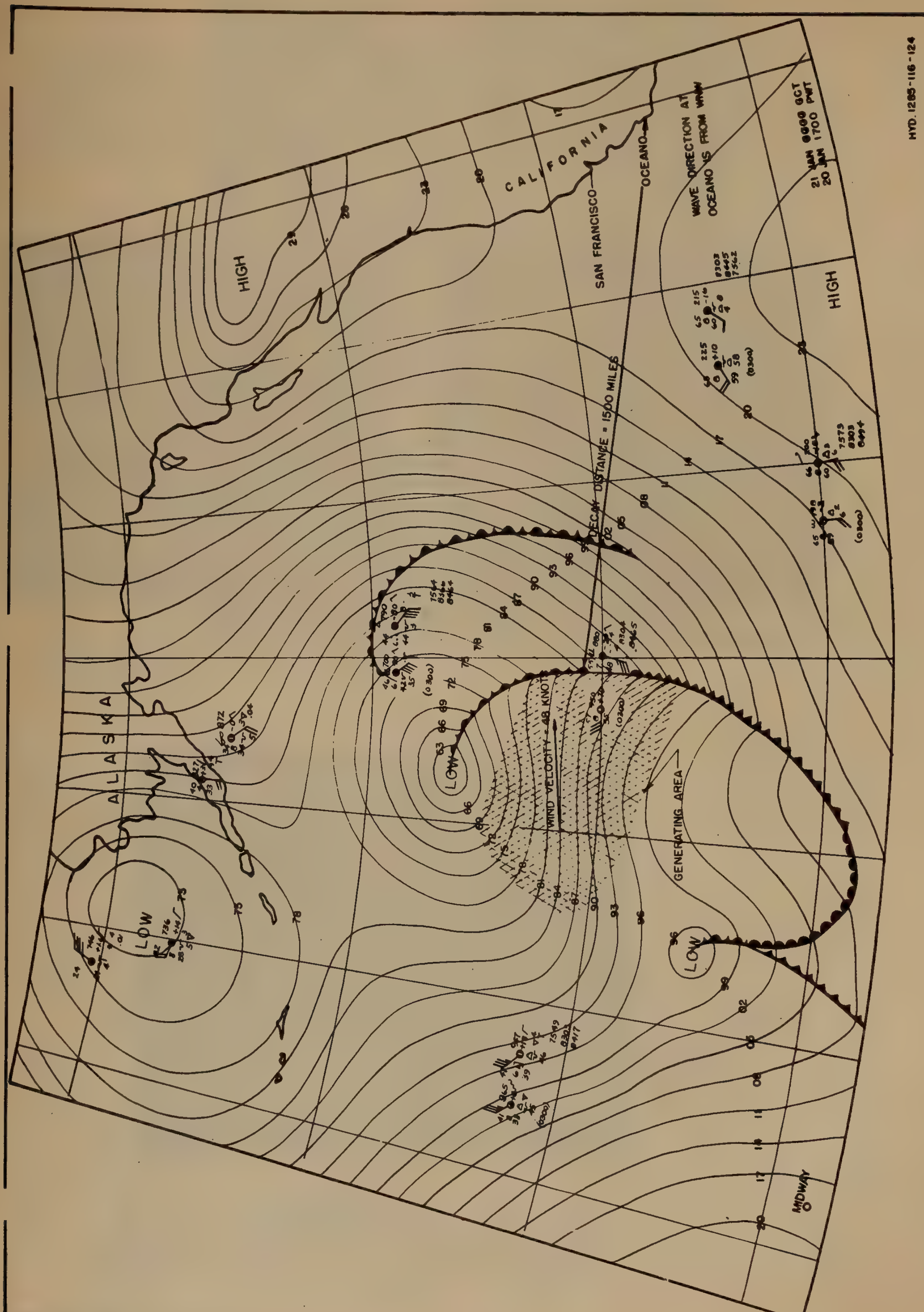
Swell in the vicinity of Oceanside, California resulting from storms thousands of miles to the south. Note the regularity of the swell compared to the waves in a generating area, as shown in the photograph to the left.



Aerial Photograph Showing Two Wave Trains.

The small waves coming from the upper right are being generated by the local wind. The long low swell from the upper left was generated several thousands of miles away. Note that the swell is almost invisible in deep water but peaks up near shore to form the predominant breakers.

FIGURE ii-8

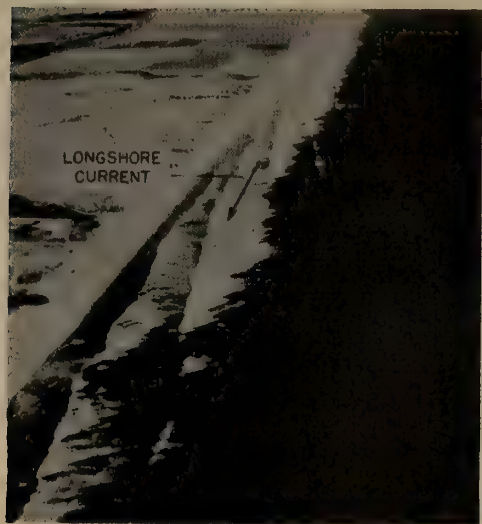


SYNOPTIC WEATHER MAP OF PACIFIC NORTHEAST, 20 JANUARY 1945, 1700 PWT. Note generating area with 500 mile fetch.

FIGURE ii-9

LONGSHORE CURRENT

A longshore current is set up within the surf zone when waves break at an angle with the shoreline. The currents flow parallel to the shoreline inside the breaker zone and are found most commonly along straight beaches. Their velocity increases with increasing breaker height, decreasing wave period, increasing angle of breaker-line with the beach, and increasing beach slope. The longshore current is much less than the velocity of the oscillating currents but it is relatively constant in direction with sustained velocities as high as three knots having been measured. Normally, velocities of from 1/4 to one knot can be expected. Strong longshore currents are effective in causing landing craft to broach.



Aerial photograph of swell breaking at an angle to the shoreline, thus causing a longshore current in the direction shown.

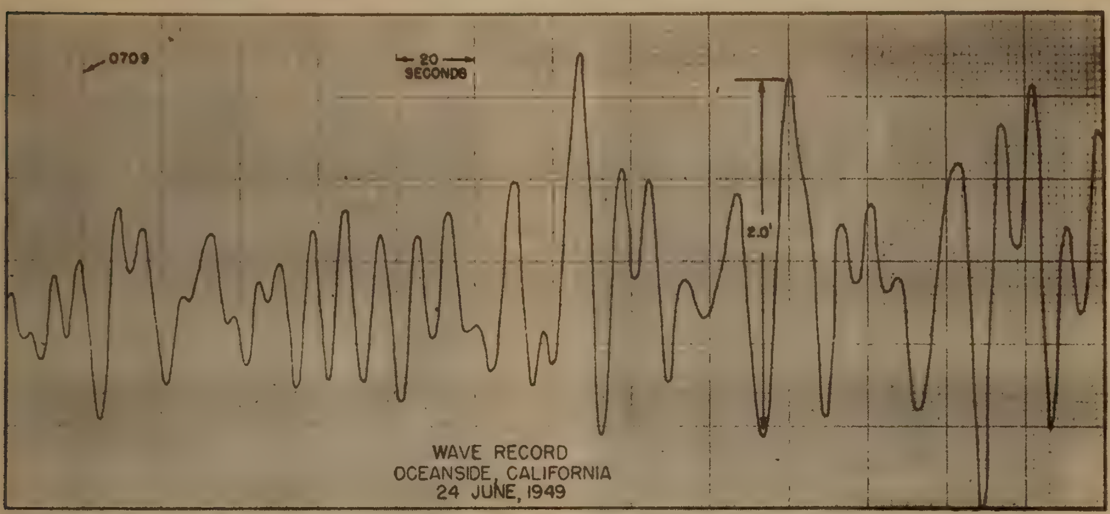
Photograph of landing craft broached as a result of the longshore current.

Some difficulty was encountered while attempting to drive jeeps from LCVPs which were careened by the surf, sometimes violently, endangering personnel. An attempt was made to steady and straighten out the craft by hawsers held by men on the beach. (Notice men standing between the jeep and side of LCVP.) November 25, 1946

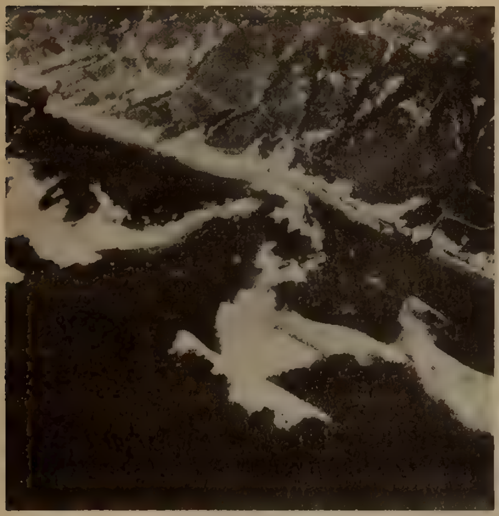
FIGURE ii-10

VARIABILITY OF SURF

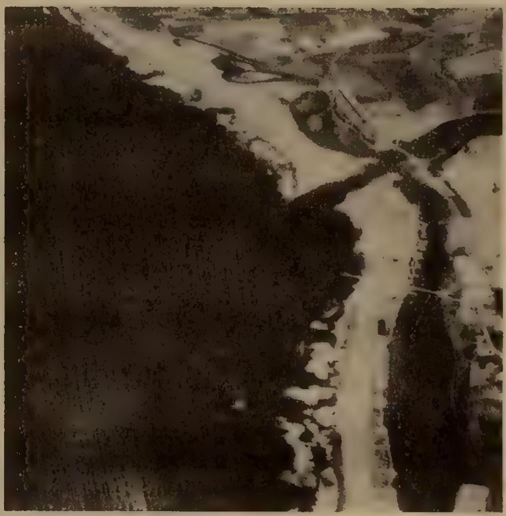
Surf is extremely variable both with position along the shore and with time. Refraction effects due to changes in the alignment of a coast, as well as variable bottom conditions along a beach, may cause marked variation in the character of the surf from point to point along a shoreline. Of equal importance is the fact that the height of waves varies considerably from wave to wave. As illustrated by the following chart from a wave recorder, wave trains are usually characterized by groups of high waves separated by periods of relative calm. Waves vary appreciably both as to height and period. If it is desired to traverse the surf in either direction, observe the sequences of the highest waves, then start immediately after the occurrence of a set of high waves because a series of low waves usually occurs in the succeeding minutes.



This record demonstrates the fallacy of the old statement that every seventh wave is a high wave. Passage through the surf should be timed to follow immediately after the occurrence of the higher waves.



Point Loma, California



Moss Landing, California

VARIATION OF SURF ALONG A BEACH

An appreciation of the principles of wave refraction permits the selection of a locality where the surf is relatively low.

FIGURE ii-11

Freshwater Lagoon from the highway. Very heavy surf causes the haze over the breaker zone.Note the drift on the back of the bar.

Same scene as at the left but with a milder surf. (too rough for test operations) Sharp point in the distance.

Big lagoon beach with storm seas breaking on the beach face. Author standingat uprush to show scale.Foot prints are ankle deep.

LAGOON BEACHES

The California lagoon beaches lie some 25 miles north of the Humboldt Bay entrance. They face WNW and are fully exposed to the sea. The total length of Big Stone and Freshwater lagoon beaches is about 8 miles, they appear to be similar to some other lagoon beaches such as Garrison Lake at Port Orford, Oregon. Their value as experimental beaches lies in the fact that they are the steepest beaches of any length on the coast. Beach face slopes 1/9 and, though never surveyed, the bar face seems to be quite steep too. Seas are of the same intensity as those encountered at Humboldt and Pelican Bays but the steepness here makes the waves peak up and break more rapidly. This beach will be very dangerous for any craft in surf over 6 feet high. In heavy weather, breakers on the beach face may be 8 or more feet high.

The beach material, too, is unique, being made up of polished small pebbles and large sand grains. When long uprushes lubricate the beach face, a man will sink to his ankles in this material. Dukws with tire pressures as low as 6 psi were unable to move about on the seaward face of the beach. It will be interesting to know if an LVT can operate on such a beach, but extreme care must be exercised entering and leaving the water.

The barrier bar which dams the lagoon is some 500 feet wide and has some large drift on it (not enough to seriously hinder movements). This bar ends eastward in the lagoon which is freshwater and of remarkably uniform depth (20-30 ft. - depending on the season).

The LST could not unload the craft close by nor should the LVT's be landed through the surf. The coast highway, however, touches the edge of the lagoon to the east and a test vehicle or two could be brought in on a lowboy tractor-carrier at small expense. Dukws could, of course, travel on the road. Although this would be an unusual procedure, the information gained from this maximum slope, maximum sand, minimum traction beach would probably make the effort worthwhile. Various freshwater tests could be conveniently run in the lagoon

Bar is government controlled and was used for aircraft rocket target practice during the war. Near the old target area, duds may be a hazard. LST could use harbor at Eureka or Crescent City.

Erosion of the berm on Big Lagoon be shows the size and distribution of the unusually large sand grains. (1/8" to 3/8")

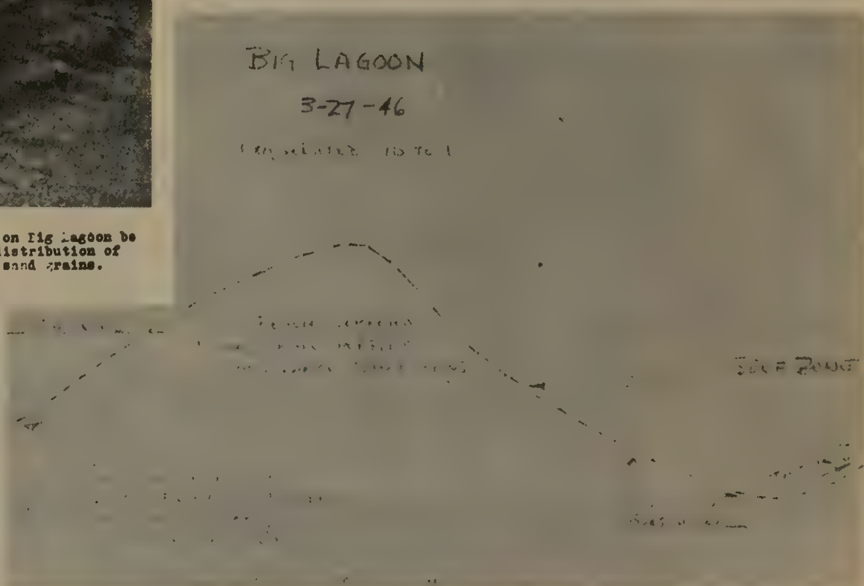


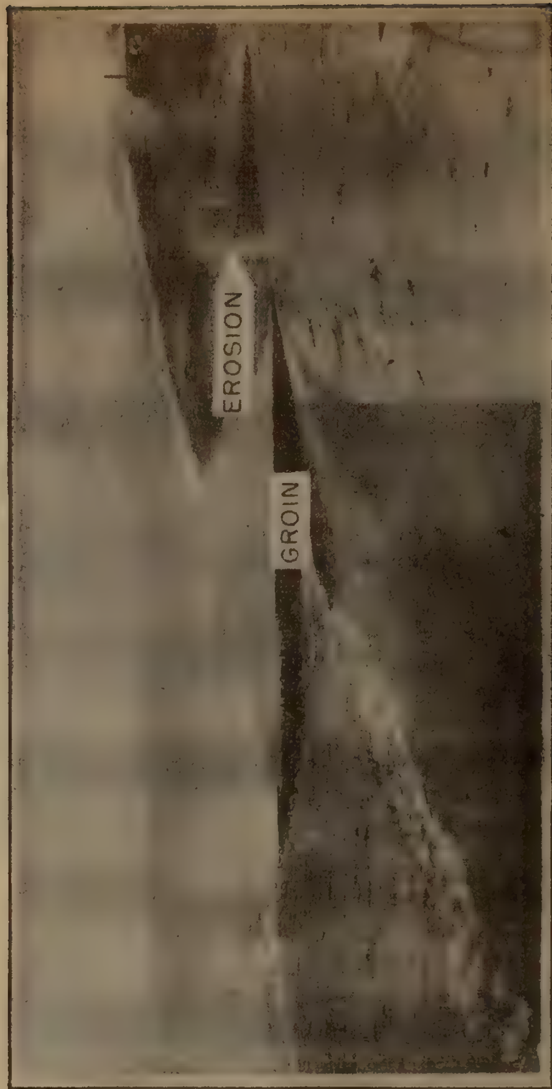
FIGURE ii-12

FIGURE II-13

FIGURE ii-14

[redacted]

FIGURE ii-16



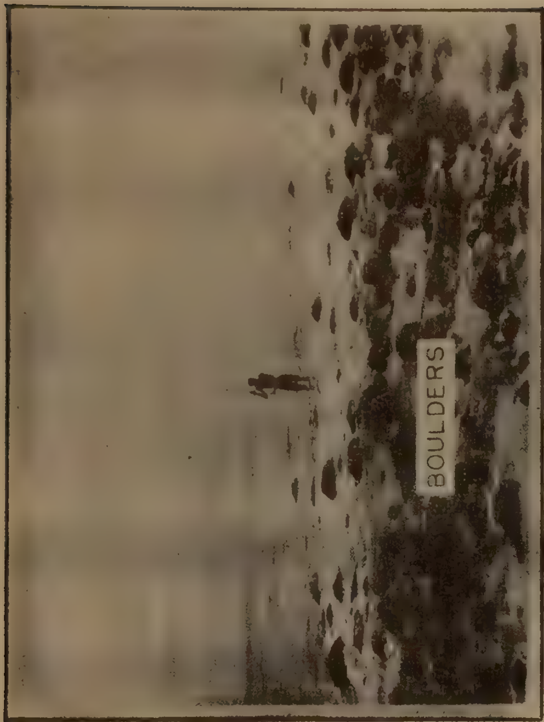


FIGURE ii-17

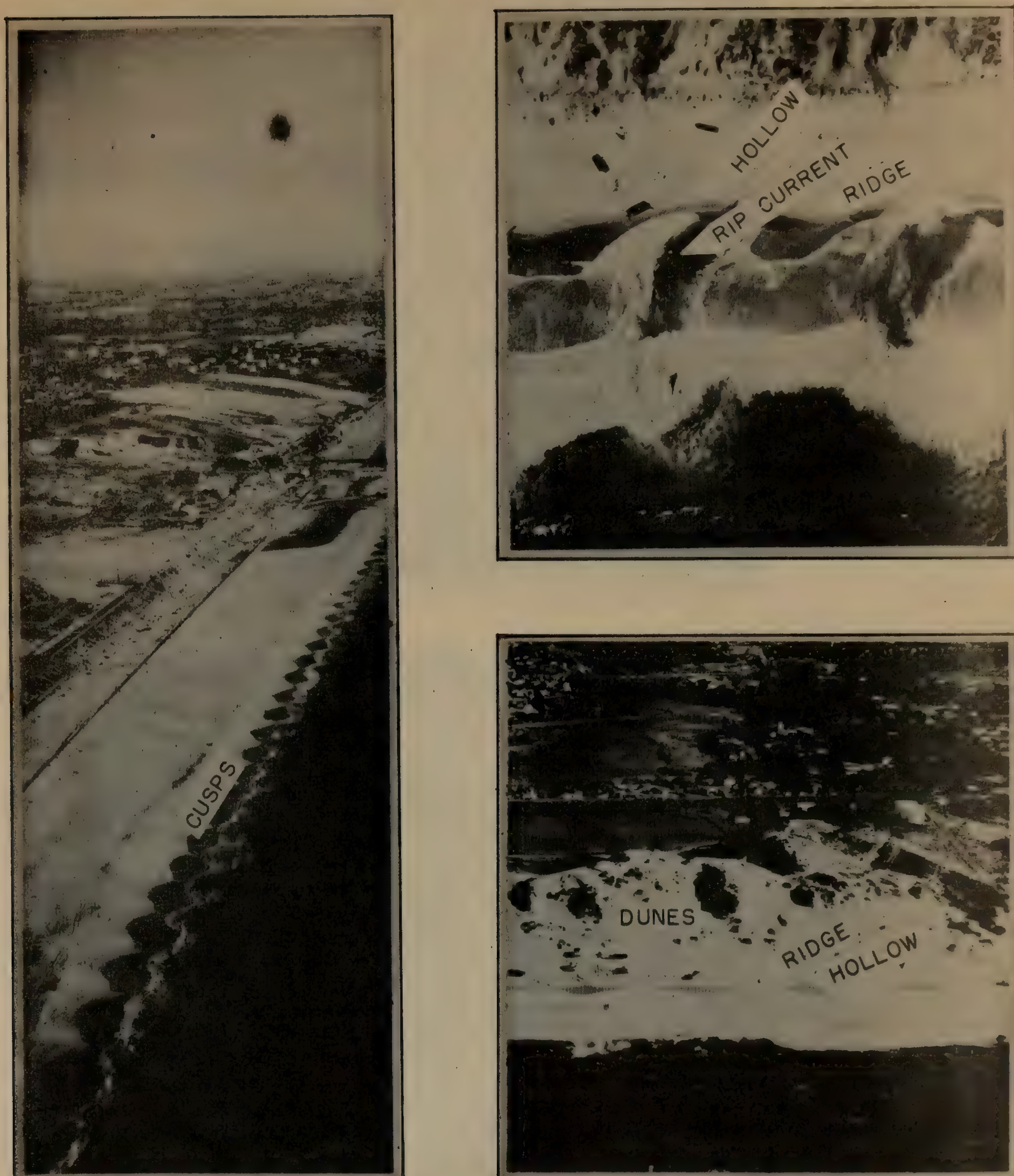


FIGURE ii-18


ROCK
DUNES
INLET
EDDY CURRENTS


COASTAL PLAIN
DELTA
CANAL
UNDERWATER BAR
RIDGE - BEACH

FIGURE ii-19

SECTION iii

LIST OF STANDARD SYMBOLS

BY R. L. WIEGEL

Symbol	Definition	Units
A	Area	feet^2
A_p	Projected Area of object	feet^2
a	Acceleration Also: amplitude	ft/sec^2 feet
a'	Length of semi-major axis of orbit of water particle	feet
$\_\_{av}$	Subscript "av" refers to average	
B	Breakwater gap width	feet
B.F.	Beaufort wind force	none
b	Length of wave crest between orthogonals (measured perpendicular to the local direction of travel)	feet
b'	Length of semi-minor axis of orbit of water particle	feet
$\_\_b$	Subscript "b" refers to breaking conditions	
C	Wave velocity	ft/sec unless designated as knots
C_d	Coefficient of drag	none
C_g	Group velocity	ft/sec
C_H	Velocity of waves of finite height	ft/sec
C_i	Instrument calibration constant	$\frac{\text{ft. of water}}{\text{chart division}}$
C_M	Coefficient of mass	none
C_o	Deep water wave velocity	ft/sec (unless specified as knots
c	Number of columns in rectangular array of craft	none
Cos	Cosine	none
Cosh	Hyperbolic cosine	none
D	Decay distance Also: Diameter	Nautical miles feet

Symbol	Definition	Units
D_d	Coefficient of shoaling	none
D_e	Effective decay distance	nautical miles
d	Depth of water, measured from the still water level to the bottom	feet unless designated as fathoms
E	Mean total energy of wave per unit length of crest per wave	ft-lbs/ft of crest
E_k	Mean kinetic energy of wave per unit length of crest per wave	ft-lbs/ft of crest
E_p	Mean potential energy of wave per unit length of crest per wave	ft-lbs/ft of crest
F	Force Also: a function of one or more variables, as $F(x, y)$. Also: Fetch length	pounds none nautical miles
F(H)	Percent of wave heights below the height H.	none
F_H	Horizontal component of force	pounds
F_m	Minimum fetch length	nautical miles
F_v	Vertical component of force	pounds
$F_x(x)$	Prob. $(X \leq x)$ = cumulative distribution function of the random variable X	none
f	Function of one or more variables, as $f(x,y)$ Also: Coriolios parameter ($f = 2\Omega \sin\phi$)	none radian/sec
g	Acceleration of gravity Also: Function of one or more variables, as $g(x,y)$	ft/sec^2 none
H	Wave height	feet
$H_{1/10}$	Average height of the highest one-tenth of the waves for a specified period of time	feet
$H_{1/3}$	Average height of the highest one-third of the waves for a specified period of time	feet
H_D	Significant wave height at end of decay distance	feet
H_F	Significant wave height at end of fetch	feet
$(H_b)_C$	Critical breaker height (for craft casualties)	feet
H_{av}	Average of the wave heights for a specified period of time	feet

Symbol	Definition	Units
H_m	Highest wave for a specified period of time	feet
h_w	Effective width of craft	feet
h_l	Effective length of craft	feet
J	Distance between two underwater contours as used in the orthogonal methods of wave refraction coefficient determination	feet
K	Sub-surface pressure response coefficient	none
K_d	Refraction coefficient	none
K_b	Refraction coefficient for breakers	none
K'	Diffraction coefficient	none
i	Beach slope, as specified Also: $\sqrt{-1}$	none none
k	$\sqrt{m^2 + n^2}$, where $m = 2\pi/L$ and $n = 2\pi/L'$	1/feet
L	Wave length (distance between two successive crests in the direction of propagation	feet
L_D	Wave length at end of decay	feet
L_F	Wave length at end of fetch	feet
L'	Wave length in the crest direction (short-crested theory)	feet
ℓ	a length	feet
ℓ_a	Distance from the bottom of an object, or pile, to the elevation at which the total force may be applied	feet
ℓ'	Crest length (the linear distance measured along the wave crest between consecutive intersections of the crest and the still water level).	feet
M	Energy coefficient	none
M_S	Total moment about the level, S	ft-lbs
MLLW	Mean lower low water - a datum	none
MSL	Mean Sea Level - a datum	none
m	$2\pi/L$	1/feet
N	Number of waves in the analysis	none

Symbol	Definition	Units
N_C	Number of craft in the operation	none
N_r	Number of rows in rectangular array of craft	none
n	Ratio of group velocity to wave velocity Also: crest interval in refraction drawings Also: $2\pi/L'$	none none 1/feet
n_b	Number of breakers encountered by craft	none
$\_\_\__o$	Subscript "o" refers to deep water	
P	Power transmitted by a wave per unit length of crest per wave	ft-lbs/sec per foot
p	Probability Also: Sub-surface pressure associated with wave motion Also: Atmospheric pressure	none feet as specified
Prob (A) or p(A)	The probability of a statement A	none
q	Water particle velocity in direction of interest	ft/sec
R	A dimension	feet
R_i	Reading of a wave recording instrument	chart division
R_c	Distance between contours measured along an orthogonal	feet
Re	Reynolds number	none
r	Radius Also: Coefficient of energy partition Also: Distance from end of breakwater to point (x,y) in diffraction theory	feet none feet
S	Distance from the ocean bottom to a water particle Also: Real part of diffraction function $f(-u)=S + iw$	feet none
S_i	Chart scale	none
s	Sheltering coefficient	$1/ft^3$
$\_\_\__s$	Subscript "s" refers to surface terms	
Sin	Sine	none
Sinh	Hyperbolic sine	none
S.W.L.	Still (undisturbed) water level. A datum	none

Symbol	Definition	Units
T	Wave period Also: Temperature	seconds degrees (scale as specified)
T_C	Significant wave period--the average period for the well defined series of highest waves on a record	seconds
T_D	Significant wave period at end of decay	seconds
T_F	Significant wave period at end of fetch	seconds
Tan	Tangent	none
Tanh	Hyperbolic tangent	none
t	time	seconds unless otherwise specified
t_D	Travel time of waves (from end of fetch to end of decay distance)	hours
t_d	Wind duration, interval of time wind blows at constant velocity in generating waves	hours
t_υ	Modulus of wave decay due to viscosity	hours unless otherwise specified
t_{min}	Minimum duration of wind in fetch area	hours
U	Velocity of surface wind	knots, or as specified
$\overline{U}$	Velocity of mass transport	ft/sec
U'	Horizontal velocity of motion left after wave motion has been destroyed (Gerstner Theory). Also: Approximate velocity of surface wind	ft/sec knots
u	Water particle (horizontal component, positive in the direction of wave advance) orbital velocity Also: Upper limit of integral term in solution of diffraction problem	ft/sec none
V	Velocity, other than wave	ft/sec
V_c	Craft velocity	ft/sec unless specified as knots
V_F	Velocity of storm or fetch front	knots
V_g	Geostrophic wind velocity	knots

Symbol	Definition	Units
V_m	Volume	ft^3
v	Water particle (vertical component) orbital velocity	ft/sec
W	Weight Also: Work performed by a wave per unit length of crest per wave	pounds ft-lbs/ft of crest
w	Unit weight of water Also: Imaginary part of diffraction function $f(-u) = S + iw$	lbs/ft^3 none
X	Extraneous force in x direction	Use appropriate units
x	Horizontal coordinate (arbitrary origin), positive in direction of wave advance	feet
x_o	Equilibrium position of particle, horizontal coordinate	feet
$\overline{x}$	Horizontal displacement of particles from equilibrium position	feet
Y	Extraneous force in y direction	Use appropriate units
y	Vertical coordinate (arbitrary origin), positive when measured upwards	feet
y_o	Equilibrium position of particle, vertical coordinate	feet
$\overline{y}$	Vertical displacement of particle from equilibrium position	
Z	Extraneous force in z direction	Use appropriate units
Z_t	Time between successive weather maps Also: Greenwich mean time	hours hours
z	Horizontal coordinate (arbitrary origin) measured perpendicular to direction of wave advance	feet
α (alpha)	Angle of wave crest to bottom contours Also: Angle of wave approach, measured between the shoreline and the line of wave advance Also: Angle between gradient and surface winds	degrees degrees degrees

Symbol	Definition	Units
α (alpha)	Also: Phase difference between axis of "f" and "g" terms in diffraction theory	radians
α_3	Skewness coefficient	none
β (beta)	Anuglar position of the water particle when the maximum horizontal particle velocity occurs.	degrees
	Also: Wave age, the ratio of wave velocity to the velocity of the generating wind	none
γ^2 (gamma)	Resistance coefficient applicable to wind	none
Δ (Delta)	Change	as stated
	Also: Craft displacement	as stated
δ (delta)	Wave steepness, H/L	none
ε (epsilon)	$(R_N + R_T)/R_u$	none
η (eta)	Surface elevation	feet
θ (Theta)	Angular displacement	Radians unless specified as degrees
	Also: Angle of variability in the direction of wave travel	degrees
λ (lambda)	Horizontal distance from center of gravity of craft to the breaker crest center at the instant the wave breaks	feet
λ_c	Critical; i.e., that distance between the center of the breaker crest at the instant the wave breaks and a point shoreward of the wave which, if the c.g. of the craft were within it at the time of breaking would pitch over, providing the breaker were over a critical height	feet
μ (mu)	Arithmetic mean	appropriate units
	Also: Absolute viscosity	lb-sec/ft^2
ν (nu)	Kinematic viscosity	ft^2/sec
π (pi)	3.1416	none
ρ (rho)	Correlation coefficient	none
ρ_a	Density of air	slugs/ft^3

Symbol	Definition	Units
ρ_w	Density of water	$slugs/ft^3$
σ (sigma)	Standard deviation Also: $2\pi/T$	appropriate units 1/sec
σ_w	Surface tension of water	$ft\text{-}lbs/ft^2$
τ (tau)	Drag force per unit area	lbs/ft^2
ϕ (phi)	Velocity potential Also: Phase of diffracted wave	ft^2/sec radians, unless otherwise stated
ψ (psi)	Aximuth of direction of wave travel, in the direction of travel Also: Stream function Also: Direction from which the wave comes	degrees ft^2/sec Compass direction (true)
Ω (omega)	Angular velocity of the earth	radians/sec
ω (omega)	Angular velocity	radian/sec

MANUAL

OF

AMPHIBIOUS OCEANOGRAPHY

SECTION I

WAVES, TIDES AND BEACHES

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION I: WAVES, TIDES, AND BEACHES

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SECTION I - WAVES, TIDES AND BEACHES

A. WIND WAVES, SWELL AND SURF - INTRODUCTION

BY

R. L. WIEGEL

A. WIND WAVES, SWELL AND SURF - INTRODUCTION

The waves which one encounters at sea and which eventually break along beaches, reefs and cliffs are almost always caused by winds. There are other types of surface waves in the ocean such as those formed by a ship under way and the "tidal" waves (In reality, seismic waves (Tsunami) caused by earthquakes or landslides on the ocean bottom, and in no way related to the tides) which occur occasionally and often cause great damage. The tsunami are very rare and the s hip waves are of little significance, except perhaps in harbors. The type which affect amphibious operations are the waves generated by winds blowing over the ocean surface. These are of extreme importance in the open sea debarkation phase and are the ones that peak up and break on shore, causing the surf through which amphibious craft must operate in the ship to shore phases.

Soon after these wind generated waves leave the storm area their height decreases and the small, steep chop disappears; they are transformed into the long, smooth waves known as swell. The swell continue to travel across the ocean. When they near shore and reach a depth equal to about half their length they begin to "feel bottom". They slow down and become shorter.

If they travel into shoaling water at an angle with the bottom contours, then the wave bends in such a manner as to tend to become parallel to the contours. (This bending is due to the fact that each part of the wave travels at a velocity dependent upon the depth of water under it, in shallow water.) This bending process is known as refraction. Because of it waves will bend around an island or into a bay and hence the shelter in the lee is not as great as would be expected.

When the swell traveling in shoaling water near a beach they begin to peak up rapidly. They become unstable, break and then continue to run onto the beach as lines of "foam". It is this surf, the origin of which is usually many miles away, that is of such great concern to personnel engaged in amphibious landings.

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION I - WAVES, TIDES AND BEACHES

B. WIND WAVES AND SWELL IN DEEP WATER

BY
R. L. WIEGEL

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B. WIND WAVES AND SWELL IN DEEP WATER

1. Introduction to Wave Motion:

The waves created by winds blowing over the ocean surface vary considerably in height, length and period. Being created by winds they are very low at first, increasing in size as the winds increase, as the fetch length increases and as the duration of the storm increases; then diminishing in size as the storms blow themselves out.

a. Simplified Theory for Waves of Small Height:

In order to arrive at an understanding of the nature of a phenomenon as complex as ocean waves certain simplifications must be made. Accordingly, the first step is to determine the behavior of a simple series of waves of stable sinusoidal form; uniform height, length and period; and in which all elements of wave profile advance with the same velocity relative to the undisturbed water.

A wave is described by its length L (the horizontal distance from crest to crest or trough to trough), its height H (the vertical distance from trough to crest) and its period T (the time interval, in seconds, it takes two successive crests to pass a fixed point). See Figure IB-1.

(1) Orbital motion: Anyone who has looked at the ocean or at a lake has noticed waves traveling across the water surface. At first it appears as if the mass of water forming the waves actually moves across the water surface. Closer observation reveals the fact that the water particles, of which the waves consist, do not move along continuously. Instead, the particles move in circular paths in deep water (or elliptical paths in shallow water), moving forward as the crest of the wave passes and backwards as the trough passes. This can be seen by watching a cork or a small piece of driftwood floating on the ocean when waves are present. It moves forward as it rises with the crest and backwards as it descends with the trough ending up nearly at its original position, at which time the cycle commences again. This process has been indicated schematically in Figure IB-2 (see also Figure IB-4). Thus, there are two modes of motion of such a wave. One is the progressive motion of the wave form and the other is the motion of the water particles of which the wave consists.

Submariners know that if they dive deep they are no longer subject to motion, no matter how violent the storm. The surface water particles move in orbits with diameters the same as the height of the wave (Figure IB-3). At any instant all particles underneath the surface particles are moving in the same direction as the surface particles, but to lesser degrees. As the distance below the surface increases the orbital diameters decrease very rapidly (Figure IB-6a). In fact, as the distance below the surface increases in arithematical progression the orbital diameters decrease in geometrical progression until at a depth equal to half the length of the wave there is no water motion at all, for practical purposes. This has been shown in Table IB-1, where the depths are shown as percentages of the wave length and orbital diameters as percentages of wave height.

TABLE IB-1

DECREASE OF ORBITAL DIAMETER
WITH DEPTH BELOW WATER SURFACE (IN DEEP WATER).

Depth (% of Wave Length)	0%	5%	10%	20%	40%	60%	80%	100%
Orbit Diameter (% of Wave Height)	100%	73%	53.3%	28.5%	8.1%	2.3%	0.7%	0.2%

The orbital velocities, V, of the surface water particles depend upon the wave height, H (feet), and period T (seconds), in the following manner (see Figure IB-5):

$$V = \pi H/T \text{ feet per second}$$

Table IB-2 gives the orbital velocity of the surface particles for a wave one foot in height for various wave periods. In order to know the velocity of a two-foot high wave these values would be multiplied by 2; for a three-foot high wave, by 3; etc.

TABLE IB-2

SURFACE PARTICLE ORBITAL VELOCITIES
OF A WAVE ONE FOOT HIGH (IN DEEP WATER).

T (seconds)	4	6	8	10	12	14	16	18	20
V (feet per second)	0.79	0.53	0.39	0.31	0.26	0.23	0.20	0.18	0.16

The velocity at any depth, z, below the water surface is

$$V_z = (\pi H/T)\ e^{2\pi z/L}$$

Table IB-3 shows how the velocity decreases with depth for a ten second wave. It can be seen that the velocities decrease as rapidly as do the orbital diameters.

TABLE IB-3

DECREASE OF ORBITAL VELOCITY
WITH DEPTH FOR A TEN-SECOND WAVE

Depth (% of Wave Length)	0%	5%	10%	20%	40%	60%	80%	100%
Orbital Velocity (% of surface orbital velocity)	100%	73%	53.3%	28.5%	8.1%	2.3%	0.7%	0.2%

(2) Wave propagation: Once waves have been formed by the wind they move across the ocean of their own accord rather than being pushed forward by the wind. A person, after throwing a stone into a calm pond, will notice that the small waves run out, continued forward away from the point of initial disturbance. The movement is due to a combination of surface tension and gravity. For waves over a foot in length the effect of surface tension is negligible: the movement is due to gravity. In fact, ocean waves are often called gravity waves. Waves move forward across the ocean with a velocity which is much greater than that at which the water particles move in their orbits. This velocity of wave propagation, C_o , is given by the expression

$$C_o = gT/2\pi$$
$$= 5.12\ T \text{ feet per second}$$
$$= \sqrt{g/2\pi\, L_o}$$
$$= 2.26\sqrt{L_o} \text{ feet per second}$$
$$= 3.03\ T \text{ knots}$$
$$= 1.34\sqrt{L_o} \text{ knots}$$

where g is the acceleration of gravity, π is 3.14, T is measured in seconds and L_o in feet. Some values have been given in Table IB-4.

TABLE IB-4

VELOCITY OF WAVE PROPAGATION IN DEEP WATER
WAVE LENGTH IN DEEP WATER

T (seconds)	4	6	8	10	12	14	16	18	20
L_o (feet)	82	184	328	512	737	1004	1310	1658	2047
Velocity (ft/sec)	20.5	30.7	40.9	51.2	61.4	71.6	81.9	92.1	102.4
Velocity (knots)	12.1	18.2	24.2	30.3	36.4	42.4	48.5	54.5	60.6

(3) Wave length: The defining relationship between the velocity, C, the period, T, and the wave length, L, is

$$L = CT$$

For deep water the wave length, L_o, is

$$L_o = gT^2/2\pi$$
$$= 5.12\ T^2 \text{ feet.}$$

Some values have been given in Table IB-4.

b. Wave of Appreciable Height:

More complex theories have been developed for wave motion which take into consideration the fact that waves have appreciable height. These theories are presented in detail in another section of this Manual. Experiments have shown that most equations for waves of small height continue to be valid, as far as most practical applications are concerned, for waves of appreciable height. Certain of the findings from the theoretical studies are of great interest.

(1) Velocity of wave propagation: Stokes found that the velocity of wave propagation was dependent upon the height of the waves as well as upon its length. Figure IB-7 shows experimental values compared with theoretical values. It appears that the more simple equation, for waves of small height, can be used for most practical purposes.

(2) Wave profile: In the most simple theory the waves are sine-shaped. In the more advanced theories the crests are not similar to the troughs, but rather the crests are narrower than the troughs and the peak of the crest lies a greater distance above the still water level than the bottom of the trough lies below.

(3) Orbital motion: The orbital motion described in the more advanced theories are similar to those described in the simplest theory, except for one important difference. That is, the particle velocity is greater during its forward movement (with the crest) than in its backward movement (with the trough). The result of this is that the forward motion of the particles are not altogether compensated by the backward motion, so, in addition to an orbital motion, there is a small progressive motion in the direction of wave propagation. The orbits are open rather than closed. This motion, known as mass transport, has been verified experimentally (IB-15). Figure IB-8 shows a diagram of this type of orbit.

c. Short-crested Waves:

Previous theories have considered waves which were extremely long crested; that is, the crest lengths were many, many times as great as the distances between successive crests (the wave length). In nature the length along the crests are usually nearly of the same order of magnitude as the distance between crests and the resulting pattern looks diamond-shaped from the air (Figure IB-9).

Recent theoretical work (IB-10) has shown the velocity of short-crested waves to be larger than that for long-crested waves. For waves of small height it was found that

$$C_o = \sqrt{gL_o/2\pi}\left(\sqrt[4]{1+(L_o/L')^2}\right)$$

$$= 2.26\sqrt{L_o}\left(\sqrt[4]{1+(L_o/L')^2}\right) \text{ feet per second}$$

where L_o is the deep water wave length in feet and L' is the distance along the wave crest, in feet. It can be seen that the shorter the wave along its crest compared with the wave length the faster it moves. Table IB-5 presents some values of this increase in wave velocity, compared with the long-crested theory, with increasing relative crest length, L_o/L'.

TABLE IB-5

VELOCITY OF SHORT CRESTED WAVES (% OF LONG CRESTED VELOCITY) RELATED TO RELATIVE CREST LENGTH (L_0/L') FOR WAVES IN DEEP WATER

Relative Crest Length (L_0/L')	1/1	2/3	1/2	1/3	1/4	1/5	1/6
Wave Velocity (% of long-crested velocity)	118.9%	109.6%	105.7%	102.7%	101.5%	101.0%	100.7%

2. Wave Observations:

The basis of all theoretical and applied work on the cause and effects of waves depends upon observations. Almost all observations, until recent years, have been visual, with the inherent errors. Some of the observations, historical and contemporary, are present in this section.

a. Historical Observations:

Most historical observations were made of the maximum wave encountered during severe storms at sea and most of these were of the heights of great waves. The accuracy of these observations have been discussed by Dr. Vaughan Cornish (IB-6) as follows:

"Measurements of waves at sea by means of the eye are not susceptible of great accuracy; but the irregularity of the waves themselves is so considerable, especially in their most important condition, which is during storms, that it is more useful to measure many waves somewhat roughly than to obtain (even if it be possible) the precise measurement of a few. The advantage of the mere number of observations does not, however, apply when we pass from rough measurements by careful observers to mere guessing at the dimensions of waves as seen on board ship. Measurements of the height of waves, for instance, taken in the usual way by finding the height above the ships water line from which a neighboring wave-crest just intercepts the horizon are believed to be accurate to within 1 foot in 10 when made by a practiced observer. This was the estimate of the late Lieutenant Paris, of the French Navy, for his observations, and the late Lord Kelvin informed me that he relied upon his own measurements to the same extent. When, however, an unpracticed observer, judging merely by the look of things from the deck of a ship, guesses the height, and length of waves, it is possible for him to err much more widely than he would on land, where he stands on a firm platform, with objects of known size in the neighborhood to afford a scale. The rolling of the ship, in particular, alters the apparent direction of the vertical so as to mislead the judgement as to height. It is difficult to say how widely these guesses may depart from fact, but I do not think it unlikely that waves of 20 feet high may, according to the circumstances, be guessed by unpractised or careless observers, at anything from 10 to 30 feet. This a range of error of 100 percent, as against 10 per cent of the practised observer."

In regard to the estimation of wave lengths at sea, Dr. Cornish (IB-7) states:

"Watching a small tramp steamer as it was raised and lowered bodily by the swell, I was struck by the fact that when she was on the top of the billow the water lay almost level along her side. This at once suggested to me an explanation of the discrepancy which I had often noted between the wave-lengths calculated from the period (or interval of time between the passage of the crests) and the estimates from the apparent position of the crests along the side of the ship on which I was travelling. My own estimates, and also those of navigating officers with whom I compared notes, were quite out of accord with the calculated length, being invariably much less than the latter. It now occurred to me that the eye was deceived, that attention was fixed upon the steep sholders of the advancing and receding waves, and that the considerable part of the wave-length comprised in the greatly foreshortened, nearly flat, top had escaped observation."

(1) Wave heights: The wave dimension which has been most often noted by observers is the wave height, especially for great waves during severe storms. There have been many accounts in the logs of ships during the eighteenth century of tremendous waves. However, in the light of more reliable observations it is believed that these accounts are in considerable error. Although observers have recorded that they have seen waves fifty and sixty feet high during severe storms of hurricane force they are extremely rare. Captain D. D. Gaillard, U. S. Army, (IB-11) collected the most reliable observations in his book. These have been presented in Table IB-6.

TABLE IB-6

HEIGHT, LENGTH AND PERIOD OF OBSERVED OCEAN WAVES
ARRANGED ACCORDING TO HEIGHT

(1) Date	(2) Locality	(3) Force of wind (Beaufort Scale)	(4) State of sea	(5)	(6) Wave	(7)	(8) Ratio of Length to Height = L_o/H_o	(9) Authority
				Height (H_o) feet (a)	Length (L_o) feet	Period (T) secs.		
	North Pacific							Photographs from Capt. Z.L. Tanner, U.S.Navy
1885	South Pacific		Heavy	46.0	765	16.5	16.6	Abercromby
1848	Atlantic			43.0	559	11.7	13.0	Wm. Scoresby
1894	"			40.0				Officers Normania
	Wick Bay			40.0				Thos. Stevenson
1900	North Atlantic			40.0				Vaughn Cornish
1891	South Atlantic	10.0	Confused	39.4	701	11.7	17.8	Dr. Schott
1900				37.7				Lt. Paris
1900	Peterhead	50-89(b)		35.0	600	15.0	16.0	Wm. Schield
				36.0				Novara
	South Pacific			33-36				Capt. Chuden
	Indian Ocean			33.6	374	7.5	11.1	Lt. Paris
1891	" "	9.0	Confused	32.8	424	9.1	12.9	Dr. Schott
1839	South Pacific			32.0	380	8.7	11.9	Commodore Wilkes

(a) Greater than 50
(b) Miles per hour

TABLE IB-6 (Continued)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1891	South Atlantic	9.0	Confused	29.5 27.0	370	8.5	12.5	Dr. Schott Scott Russell
1892	Indian Ocean	9.0	Confused	26.2	428	8.8	16.3	Dr. Schott
1888	Peterhead			26.0	500	12.2	19.2	Wm. Schield
1886	Atlantic	10.0	Confused	25.0	375	7.5	15.0	Officers, U.S.Navy
1885	Indian Ocean	4.0	"	25.0	450	11.0	18.0	" "
1885	" "	4.0	"	25.0	500	13.0	20.0	" "
1892	" "	5.0	Regular	24.6	1121	14.5	45.6	Dr. Schott
1895	South Atlantic	7.5		24.6	459	11.0	18.7	Lt. Gassenmayr
1885	Indian Ocean	4.0	Confused	24.0	400	12.0	16.7	Officers, U.S.Navy
1886	Pacific	2.0	Long Swell	23.0	350	10.0	15.2	" "
1885	Indian Ocean	4.0	Confused	22.0	400	10.0	18.2	" "
1895	Atlantic	6.5		21.3	394	10.0	13.8	Lt. Gassenmayr
1886	"	8.0	Regular	21.0	328	8.0	15.6	Officers, U.S.Navy
1885	Indian Ocean	4.0	Confused	21.0	400	9.0	19.0	" "
1892	" "	5.0	Regular	19.7	460	9.5	23.4	Dr. Schott
1895	Atlantic	9.0		19.7	262	9.5	13.3	Lt. Gassenmayr
1886	"	7.0	Regular	18.0	318	7.5	17.7	Officers, U.S.Navy
1885	Indian Ocean	4.0	Confused	18.0	300	8.0	16.7	" "
1885	" "	4.0	"	18.0	250	9.0	13.9	" "
1885	" "	4.0	"	18.0	400	12.0	22.2	" "
1891	" "	5.0	Regular	16.4	396	8.0	24.1	Dr. Schott
1885	" "	4.0	Confused	16.0	350	9.0	21.9	Officers, U.S.Navy
1885	" "	1.0	"	16.0	350	11.0	21.9	" "
1885	" "	4.0	"	15.0	200	8.0	13.3	" "
1885	" "	4.0	"	15.0	150	7.0	10.0	" "
1885	" "	4.0	"	15.0	300	9.0	20.0	" "
1885	" "	4.0	"	15.0	250	9.0	16.7	" "
1895	South Atlantic	7.0		14.8	394	8.0	26.6	Lt. Gassenmayr
1895	Atlantic	8.0		14.8	328	8.0	22.2	"
1892	South Atlantic	6.0	Confused	14.8	202	6.0	13.6	Dr. Schott
1895	South Atlantic	6.0		14.4	459	10.0	31.9	Lt. Gassenmayr
1883	China Sea	6.0	Confused	14.0	160	6.0	11.4	Officers, U.S.Navy
1892	South Atlantic	0.0	Regular	13.1	571	10.0	43.6	Dr. Schott
1890	" "	6.0	Confused	13.1	193	6.6	14.7	"
1886	Pacific	4.0	"	12.0	276	5.0	23.0	Officers, U.S.Navy
1895	South Atlantic	3.0		11.5	426	10.0	37.0	Lt. Gassenmayr
1895	" "	5.0		11.5	164	6.0	14.3	"
1895	Atlantic	8.0		11.5	148	7.0	12.9	"
1895	South Atlantic	4.0		11.1	180	5.5	16.2	"
1883	China Sea	5.0	Confused	11.0	167	6.8	15.2	Officers, U.S.Navy
1883	Pacific	8.0	Irregular	10.5	209	6.7	19.9	" "
	Algoa Bay	58.0 (a)		10.0	200	10.0	20.0	Wm. Shield
1895	South Atlantic	5.0		9.8	426	9.5	43.5	Lt. Gassenmayr
1895	" "	6.0		9.8	148	5.6	15.1	"
1895	Atlantic	6.0		8.9	115	6.0	12.9	"
1891	Indian Ocean	0.0	Regular	8.2	305	7.0	37.2	Dr. Schott
1895	South Atlantic	6.0		8.2	131	6.5	16.0	Lt. Gassenmayr
1892	Indian Ocean	6.0	Confused	8.2	145	5.4	17.7	Dr. Schott
1895	South Atlantic	5.0		8.2	262	6.0	32.0	Lt. Gassenmayr
1895	" "	6.0		8.2	131	5.5	16.0	"
	Bishop Rock			8.0	171	7.5	21.4	Sir James N. Douglass

TABLE IB-6 (Continued)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1883	Pacific	6.0	Confused	8.0	191	6.1	23.9	Officers, U.S.Navy
1886	"	1.0	Swell	8.0	400	10.0	50.0	" "
1883	South Pacific	5.0	Confused	8.0	249	7.0	31.1	" "
1885	Pacific	4.0	"	8.0	100	7.0	12.5	" "
1895	South Pacific	6.0		7.5	131	6.0	17.5	Lt. Gassenmayr
1895	" "	6.0		7.5	115	6.0	15.3	"
1885	South Pacific	2.0	Regular	7.0	100	6.0	14.3	Officers,U.S.Navy
1892	North Atlantic	0.0	Regular	6.6	328	7.4	49.7	Dr. Schott
1895	South Atlantic	6.0		6.6	98	6.0	14.8	Lt. Gassenmayr
1892	" "	5.0	Confused	6.6	123	5.0	18.6	Dr. Schott
1892	" "	2.0		6.6	158	6.3	23.9	Lt. Gassenmayr
1885	South Pacific	3.0	Regular	6.0	100	6.0	16.7	Officers,U.S.Navy
1885	Pacific	1.0	"	6.0	80	5.0	13.3	" "
1886	Atlantic	4.0	"	5.0	261	7.7	52.2	" "
1886	"	0.1	"	5.0	314	8.3	62.8	" "
1885	Pacific	1.0	"	5.0	60	5.0	12.0	" "
1895	South Atlantic	4.0		4.9	131	5.0	26.7	Lt. Gassenmayr
1895	" "	5.0		4.9	98	4.5	20.0	"
1895	" "	5.0		4.6	115	4.8	25.0	"
1895	" "	4.0		4.6	148	5.0	32.2	"
1895	" "	4.0		4.3	82	14.0	19.1	"
				4.0	500	0.0	125.0	Deverell
1887	Caribbean	3.0	Regular	4.0	100	8.0	25.0	Officer,U.S.Navy
1885	Pacific	0.0	"	4.0	50	5.0	12.5	" "
1892	Indian Ocean	2.0	"	3.9	134	5.0	34.4	Dr. Schott
1895	South Atlantic	6.0		3.6	115	5.2	31.9	Lt. Gassenmayr
1892	" "	5.0	Confused	3.3	119	4.9	36.1	Dr. Schott
1887	Caribbean	3.0	Regular	3.0	50	4.0	16.8	Officers,U.S.Navy
1887	Atlantic	4.0	"	3.0	30	3.0	10.0	" "
1892	North Atlantic	1.0	"	2.6	162	5.2	62.3	Dr. Schott
1892	Indian Ocean	5.0	Confused	2.6	108	4.6	41.5	"
1887	Caribbean	3.0	Regular	2.0	40	4.0	20.0	Officers,U.S.Navy

Although high waves do occur during severe storms the waves more often encountered are very much smaller. These have been tabulated by Bigelow and Edmunson (IB-2) (after Arnold Schumacher) and are shown in Table IB-7.

TABLE IB-7

RELATIVE FREQUENCY OF WAVES
OF DIFFERENT HEIGHTS IN DIFFERENT REGIONS

Region	Height of Waves, in Feet					
	0 - 3 %	3 - 4 %	4 - 7 %	7 - 12 %	12 - 20 %	< 20 %
North Atlantic, between Newfoundland and England	20	20	20	15	10	15
Mid-equatorial Atlantic	20	30	25	15	5	5
South Atlantic, latitude of southern Argentina	10	20	20	20	15	10
North Pacific, latitude of Oregon and south of Alaskan Peninsula	25	20	20	15	10	10
East equatorial Pacific	25	35	25	10	5	5
West wind belt of South Pacific, latitude of southern Chile	5	20	20	20	15	15
North Indian Ocean, north-east monsoon season	55	25	10	5	0	0
North Indian Ocean, south-west monsoon season	15	15	25	20	15	10
Southern Indian Ocean between Madagascar and northern Australia	35	25	20	15	5	5
West wind belt of southern Indian Ocean on route between Cape of Good Hope and southern Australia	10	20	20	20	15	15

(2) Wave lengths : Gaillard (IB-11) compiles two tables in which observed wave lengths were presented. These have been reproduced for this Manual in Table IB-6 and IB-8.

TABLE IB -8

COMPARISON OF OBSERVED AND COMPUTED VELOCITIES
OF DEEP-WATER WAVES

Wave Height (1)	Wave Length (2)	Wave Velocity (feet per second)		
		Computed (3)	Observed (4)	Difference, columns (3) and (4)
24.6	1,121	75.8	77.3	-1.5
46.0	765	62.5	46.4	+16.1
36.1	650	57.6	58.0	-0.4
37.5	600	55.4	40.0	+15.4
13.1	571	54.1	57.1	-3.0
43.0	559	53.5	47.7	+5.8
25.0	500	50.6	38.5	+12.1

TABLE IB-8 (Continued)

(1)	(2)	(3)	(4)	(3) and (4)
4.0	500	50.6	50.0	+0.6
19.7	460	48.5	48.5	0.0
24.6	459	48.4	41.7	+6.7
14.4	459	48.4	45.9	+2.5
25.0	450	47.9	40.9	+7.0
26.2	428	46.8	48.2	-1.4
11.5	426	46.6	42.6	+4.0
9.8	426	46.6	43.5	+3.1
24.0	400	45.2	33.3	+11.9
22.0	400	45.2	40.0	+5.2
21.0	400	45.2	44.4	+0.8
18.0	400	45.2	33.3	+11.9
8.0	400	45.2	40.0	+5.2
21.3	394	44.9	21.3	+23.6
14.8	394	44.9	49.2	-4.3
32.0	380	44.1	43.6	+0.5
25.0	375	43.8	50.0	-6.2
33.6	374	43.7	49.9	-6.2
23.0	350	42.3	35.0	+7.3
16.0	350	42.3	38.9	+3.4
16.0	350	42.3	31.8	+10.5
21.0	328	40.9	41.0	-0.1
14.8	328	40.9	41.0	-0.1
6.6	328	41.0	44.6	-3.6
18.0	318	40.3	42.4	-2.1
5.0	314	40.0	38.1	+1.9
8.2	305	39.5	39.4	+0.1
18.0	300	39.1	37.5	+1.6
15.0	300	39.1	33.3	+5.8
12.0	276	37.5	55.2	-17.7
19.7	262	36.6	27.6	+9.0
8.2	262	36.6	43.7	-7.1
5.0	261	36.5	33.9	+2.6
18.0	250	35.7	27.8	+7.9
15.0	250	35.7	27.8	+7.9
8.0	249	35.7	35.6	+0.1
10.5	209	32.7	31.2	+1.5
14.8	202	32.2	33.5	-1.3
15.0	200	32.0	25.0	+7.0
13.1	193	31.5	28.9	+2.6
8.0	191	31.2	31.3	-0.1
11.1	180	30.3	32.7	-2.4
8.0	171	29.6	22.8	+6.8
11.0	167	29.2	24.6	+4.6
11.5	164	28.9	27.3	+1.6
2.6	162	28.8	31.2	-2.4
14.0	160	28.6	26.7	+1.9
6.6	158	28.4	25.1	+3.3
15.0	150	27.7	21.4	+6.3
11.5	148	27.5	21.1	+6.4
9.8	148	27.5	26.4	+1.1

TABLE IB-8 (Continued)

(1)	(2)	(3)	(4)	(3) and (4)
4.6	148	27.5	29.8	-2.3
8.2	145	27.3	26.9	+0.4
3.9	134	26.2	26.9	-0.7
8.2	131	25.9	20.2	+5.7
8.2	131	25.9	23.8	+2.1
7.5	131	25.9	21.7	+4.2
4.9	131	25.9	26.2	-0.3
6.6	123	25.1	25.6	-0.5
3.3	119	24.7	24.3	+0.4
8.9	115	24.2	19.2	+5.0
7.5	115	24.2	19.2	+5.0
4.6	115	24.2	24.0	+0.2
3.6	115	24.2	22.1	+2.1
2.6	108	23.5	23.6	-0.1
8.0	100	22.6	14.3	+8.3
7.0	100	22.6	16.7	+5.9
6.0	100	22.6	16.7	+5.9
4.0	100	22.6	12.5	+10.1
6.6	98	22.4	16.3	+6.1
4.9	98	22.4	21.8	+0.6
4.3	82	20.5	20.5	0.0
6.0	80	20.2	16.0	+4.2
5.0	60	17.5	12.0	+5.5
4.0	50	16.0	10.0	+6.0
3.0	50	16.0	12.5	+3.5
2.0	40	14.3	10.0	+4.3
3.0	30	12.4	10.0	+2.4
85.0	2,995.6	2,737.9		385.3 +321.5 - 63.8

Bigelow and Edmondson (IB-2) (adapted from Krummel) also have tabulated some interesting data on wave lengths (see Table IB-9). They state:

"Anyone who has seen ripples grow to whitecaps under a rising wind and who has watched whitecaps develop into a sea knows that the waves grow longer as they gain in height. And the linear distance from crest to crest increases much more rapidly than does the absolute height of the waves, provided the shape of the latter (i.e., the ratio between its length and its height) continues approximately the same, for waves are invariably many times as long as they are high. If a 5-foot wave, 100 feet long (a common proportion of height to length), doubles in size, for example, its length increases by 20 times as much (by 100 feet) as its height (by only 5 feet). And this increase in length of the wave continues not only as long as its height is increasing rapidly, but even after it has attained the maximum height to which the particular wind in question can raise it."

Table IB-9, "abbreviated from one already published, gives at least a rough picture of the average lengths of the waves to be expected out at sea with winds of different strengths."

TABLE IB-9

AVERAGE LENGTH OF WAVES, OBSERVED AT SEA, ACCORDING TO THE STRENGTH OF THE WIND
(Adapted from Krummel)

Wind			Waves, Average length, in feet
Beaufort Scale	Description	Velocity Knots	
2	Light breeze	11	52
4	Moderate breeze	20	124
6	Stiff breeze	30	261
8	Moderate gale	42	383
10	Strong gale	56	827

"The averages presented in [Table IB-9], which were based on a large number of observations made in different regions, show that ocean waves are usually more than 100 feet long from crest to crest, unless the wind is very light. A similar tabulation [Table IB-10], based on other published measurements of waves from 4 to 46 feet high and more than 60 feet long, also shows that storm waves are not ordinarily longer than 450 to 550 feet in the North Atlantic or North Pacific, and perhaps a little longer, though not averaging so, in high latitudes in the South Atlantic and South Pacific. To find really long storm seas, we must turn to the so-called 'Southern Ocean', on the route from South Africa to Australia, where the seas are commonly as much as 600 to 800 feet long in heavy gales. An average of 775 feet has, in fact, been recorded there for an entire day, with occasional waves 1,200 to 1,300 feet long."

TABLE IB-10

LENGTHS OF STORM WAVES OBSERVED IN DIFFERENT OCEANS
(Adapted from Gaillard)

Ocean Area	Wave Length, in feet			Number of Cases
	Maximum	Minimum	Average	
North Atlantic	559	115	303	15
South Atlantic	701	82	226	32
Pacific	765	80	242	14
Southern Indian	1,121	108	360	23
China Sea	261	160	197	3

It should be emphasized that these observations were of waves in the storm area and not of swell which may be much longer (and usually very much lower in height).

(3) Wave steepness: The steepness of a wave is the ratio of the wave height to its length (or vice versa). Gaillard (IB-11) tabulated many observations of wave steepnesses of the highest waves observed (Table IB-6).

(4) Wave velocity: The wave velocity measurements made by Gaillard (IB-11) have been presented in Table IB-8.

(5) Wave period: A tabulation was made by Gaillard (IB-11) of the observed periods of the greatest waves by various observers. This has been shown in Table IB-6.

b. Contemporary Observations:

Within the last twenty years, or so, considerable advances have been made in the observation and measurements of waves. Some of the observations have been made from lightships, weather ships, etc. Measurements have been made by stereophotogrammetric means, by motion pictures of waves moving past graduated poles and spar buoy, by electrical surface gages and by bottom pressure recorders (discussed in another section of this manual). A brief discussion of some of these measurements is presented in this section.

(1) Stereophotogrammetric studies: During the years 1925-27, Dr. Arnold Schumacher (IB-18) took a large number of stereophotographs from which he made detailed contours of storm waves at sea. These show the complexity of waves in the generating area. Figure IB-10 shows one of the wave contours that he developed.

Additional work along these lines was performed by Weinblum and Block (IB-23) during 1934 aboard the ship "San Francisco". Figure IB-11a shows the stereogrammetric plot of one set of photographs (set 24) and Figure IB-11b shows the wave contours through sections AA, BB and CC of the plot.

(2) Graduated poles: A few measurements of waves in deep water were made by the Waves Investigation Group of the University of California during January, 1945 (IB-9). The trip was undertaken in order to test the practicability of certain instruments for recording waves in deep water. Two sample records of waves passing a graduated pole are shown in Figure IB-12, together with photographs of the sea at that time.

(3) Weather and light ships:

(a) Columbia River Light Vessel: During the period from August 1933 to August 1936 observations were made twice daily by officers of the Columbia River Light Vessel (located several miles off the mouth of the Columbia River in the Pacific Northwest). Wave heights and lengths were estimated, using the known dimensions of the ship as a scale; the wave period was measured by counting the number of wave crests passing a fixed point on the ship during a constant time interval; the wind velocities were estimated in terms of the Beaufort scale; and the wave and wind directions were reported to the nearest point of the compass. These observations have been collected and summarized by M. P. O'Brien (IB-16). Table IB-11 is from this report.

TABLE IB-11

Month	Percentage of Total Observations Exceeding Figure Specified								
	20			50			80		
	H_o	L_o	T	H_o	L_o	T	H_o	L_o	T
January	8.4	310	8.9	5.3	187	7.2	2.9	68	5.6
February	6.6	280	8.4	3.8	130	7.0	1.9	82	4.8
March	8.4	326	9.5	4.4	242	7.5	2.5	159	6.1
April	4.5	227	10.0	2.7	112	7.5	1.3	65	4.8
May	6.2	252	7.9	3.9	172	6.4	2.1	88	5.0
June	5.7	192	7.6	3.3	125	6.0	1.3	71	4.2
July	4.4	275	9.0	2.5	178	6.7	1.2	45	4.0
August	6.1	193	8.1	3.6	168	6.1	1.6	134	4.1
September	6.4	238	8.1	3.8	180	6.5	1.8	78	4.6
October	7.9	293	9.5	4.9	210	6.9	2.4	110	4.6
November	9.9	296	8.5	4.8	223	7.0	2.7	177	4.3
December	10.6	325	9.2	6.3	239	7.2	4.0	153	5.5

H_o = wave height in feet L_o = wave length in feet T = wave period between crests in seconds

O'Brien states that "The data were not obtained by trained observers and the methods used were crude but the results are believed to be at least relatively, if not absolutely, correct since the entire record was obtained by only a few different observers. . . . The depth of the water at the light vessel is sufficient that all waves were of the deep-water type; i.e., depth greater than half the wave length."

O'Brien further states "In considering these data, one must recall that any point on the surface of the ocean is usually disturbed by more than one set of waves and that an observer is unable to distinguish between them unless they are markedly different in length and height. Since the waves are trochoidal rather than sinusoidal, superposition will result in reinforcement of coincident crests but where a crest and trough coincide, the crest will still be evident and an observer will report periods and lengths which are less than the correct value for either set of waves. Studies of the effect of superposition (Defant, 1929)(IB-8) show very clearly why waves are commonly observed to break in "sets" and why the observed period is less than the true period of any of the component waves. It is easily seen that the local wind waves might completely mask a long, low swell from a distant storm, if the observations are made in deep water, but would have no effect on the period of the breakers which would be fixed by the swells because of their greater energy. One would expect, then, that on the average the period observed in deep water would be less than the period of the breakers and this situation is found to hold true at the Columbia River."

O'Brien states that due to the tendency to underestimate wave lengths, and taking into account the deep water wave relationship of $L_o = 5.12T^2$ (where T is seconds, and L_o is feet) that the wave lengths, as reported in Table IB-11, should be increased by one-third. He states that as "the wave heights were an estimated average over an interval and the effect of

superposition would tend to cancel out. However, an observer would probably give greater weight to the large waves resulting from reinforcement of wave crests but no estimate of this error is possible."

The observers also noted the directions from which the waves were coming. These data have been summarized in Table IB-12. One column lists the percentage of observations from various directions while the other column lists the percentages weighted in proportion to the square of the observed wave heights (i.e., according to wave power).

TABLE IB-12

Direction	Percentage of Total Observations	Percentage Weighted in Proportion to H_o^2
N	0.73	0.57
NE	1.80	1.44
E	3.18	1.26
SE	2.38	3.30
S	15.02	25.14
SW	18.74	36.36
W	30.03	23.70
NW	16.57	8.24
Calm	11.54	-

(b) German Lightship observations in the North and Baltic Seas: During the winter of 1936-37 observations of the state of the sea were made by the captains of several German lightships. A summary of their findings (IB-14) are presented in this section. Most of the data presented in the reference was obtained by experienced captains on anchored lightships and hence are undoubtedly more reliable than the great number of observations from moving ships at sea. For a time a DVL wave meter was available. Figure IB-13 shows the comparison of the observations of one of the ships masters with the recorded dimensions. The comparison of other observers were similar.

Figure IB-14 shows the locations of the observation stations while Figure IB-15 presents wave height observations taken from many of these stations. Figure IB-16 presents the frequency diagrams for wave heights and state of the sea for three of the light ships. Figure IB-17 presents the comparison of estimated wave heights and wave lengths for one of the lightships.

(4) Wave recorders: During recent years several types of wave recorders have been developed and installed in various locations. These all record shallow water waves. Deep water conditions have to be computed from these records by means of theoretical relationship. These recorders have been built and operated by the Admiralty Research Laboratory in Great Britain; and by the University of California, Scripps Institution of Oceanography, Beach Erosion Board (Corps of Engineers, U. S. Army) and the Woods Hole Oceanographic Institution, in the United States. All of these instruments measure, in effect, the height and period of the waves. The various instruments used are described in another section of this Manual.

A sample of the data that has been obtained is shown in Figure IB-18.

c. Wave climates:

Just as there is a weather climate for a region so there must be a wave climate. It is even more important, in amphibious operations, to know the wave climate than it is to know the weather climate. It is necessary to accumulate wave records for many years (or to hindcast wave conditions from historical weather maps) to develop the wave climate for a region. A typical example of some of the data necessary is shown in Figures IB-18 and 19, which is for Camp Pendleton, Oceanside, California, (IB-22)

3. State of the Sea

Often one hears the term "state of the sea" used by sea-faring men. The term "sea" is used to describe the state of the ocean surface existing in a storm area (i.e., in the fetch); "sea" being the waves caused by the wind at the place and time of observation. The state of the sea is normally reported according to the Douglas Sea Scale. This scale, together with the wind velocities (in knots and according to the Beaufort Scale) which probably are associated with the given "sea" have been given in Table IB-13*.

Often the only waves present in a storm area are those being formed by the winds in that area; however, there are other times when "swell" (waves which were formed by past winds or by winds at a distance from the area in which the observation is being made), a relatively low, undulating sea surface, is present, perhaps moving in the same direction as the "sea" or perhaps in some other direction. Because there may be a combination of "sea" and "swell" in a particular storm area the question of how to describe the state of the sea has arisen.

According to "Instructions to Marine Meterological Observers"(IB-21) (U.S. Weather Bureau, Circular M, 6th ed., 1938, pp. 53-55) "The . . . scale [Table IB-13, columns 1, 2 and 3] should be used in classifying the character of the sea disturbances. In recording observations in accordance with this scale, 'sea' may be considered to be composed of swells, combined with waves produced by the winds at the place of observation."

However, the following statement is made in the "Admiralty Weather Manual" (1939)(IB-5) "Careful distinction should be made between sea and swell, sea being the waves caused by the wind at the place and time of observation, while swell is the wave motion due to past wind or wind at a distance. The direction from which the swell comes should be noted to nearest compass point."

Another scale which is often associated with state of the sea descriptions is the Beaufort Wind Scale which describes the deep sea signs for each classification of the wind (IB-3). This scale is presented in Table IB-14**.

*Taken from Table III of "Wind, Waves and Swell, Principles of Forecasting", H. O. Publ. Misc. 11, 275,(Reference IB-19).

**Taken from "American Practical Navigator, p. 52 (Reference IB-3).

TABLE IB-13

STATE OF THE SEA AND CORRESPONDING WAVE HEIGHTS AND WIND VELOCITIES

Douglas Sea Scale		Wave Heights (feet)				Wind Velocity	Wind Force	Wind Force
No. (1)	Term (2)	a (3)	b (4)	c (5)	d (6)	(knots) (7)	Beaufort,e (8)	Beaufort,f (9)
0	Calm	0	0		0			
1	Smooth	1	0-0.5		0-1			
2	Slight	1-3	0.5-2	2.5-4	1-3			
3	Moderate	3-5	2-5	4-6.5	3-6	11-16	4	4
4	Rough	5-8	5-9	6.5-12	6-12	15-24	5,6	5,6
5	Very Rough	8-12	9-15	12-20	12-18	22-30	7	6,7
6	High	12-20	15-24		18-25	28-37	8	7,8
7	Very High	20-40	24-36		25-35	35-46	9,10	8,9
8	Precipitous	>40	>36		>35	>46	>10	>9
9	Confused	-	-		-		-	

a. Instructions to Marine Meteorological Observers. (U. S. Weather Bureau. Circular M, 6th ed., 1938).

b. Admiralty Weather Manual, 1938, pages 50-51.

c. Results of Meteor observations.

d. Computed wave height assigned to the terms of the Douglas Sea Scale.

e. According to scale adopted by International Meteorological Committee.

f. According to Bowditch, H. O. no. 9, 1939, page 52.

TABLE IB-14

THE BEAUFORT WIND SCALE

Beaufort No.	Seaman's description of wind	Deep sea signs	Mode of estimating for average sized sailing trawler	Miles per hour (statute)†	Miles per hour (nautical)	Meters per second	Equivalent pressure in millibars* (10^3 dynes per cm^2)	Terms used in U. S. Weather Bureau Forecasts
0	Calm	Sea smooth as a mirror	No headway	less than 1	less than 1	less than 0.3	less than 0.005	Light
1	Light air	Small waveletlike scales no foam crests	Sufficient to give good steerage way to fishing smacks with "wind free"	1–3	1–3	0.3–1.5	0.005–0.03	Light
2	Light breeze	Waves short; crests begin to break	Fishing smacks with topsails and light canvas, "full and by" make up to to 2 knots	4–7	4–6	1.6–3.3	0.03–0.1	Light
3	Gentle breeze	Foam has glassy appearance, not yet white	Smacks begin to heel over slightly under topsails and light canvas, make up to 3 knots, "full and by"	8–12	7–10	3.4–5.4	0.1–0.2	Gentle
4	Moderate breeze	Waves now longer; many white horses	Good working breeze; smacks heel over considerably on a wind under all sail	13–18	11–16	5.5–8.0	0.2–0.5	Moderate
5	Fresh breeze	Waves pronounced and long; white foam crests	Smacks shorten sail	19–24	17–21	8.1–10.7	0.5–1.0	Fresh
6	Strong breeze	Larger waves form; white foam crests all over	Smacks double-reef gaff mainsail	25–31	22–27	10.8–13.8	1–1.5	Strong
7	Moderate gale	Sea heaps up; wind blows foam in streaks	Smacks remain in harbor, and those at sea lie to	32–38	28–33	13.9–17.1	1.5–2	Strong
8	Fresh gale	Height of waves and crests increasing	Smacks take shelter if possible	39–46	34–40	17.2–20.7	2–3	Gale
9	Strong gale	Foam is blown in dense streaks	----	47–54	41–47	20.8–24.4	3–4.5	Gale
10	Whole gale	High waves with long overhanging crests; large foam patches	----	55–63	48–55	24.5–28.3	4.5–6	Whole gale
11	Storm	High waves; ships in sight hidden in troughs	----	64–75	56–65	28.4–33.5	6–8	Whole gale
12	Hurricane	Sea covered with streaky foam; air filled with spray	----	Above 75	Above 65	33.6 or above	Above 8	Hurricane

*1 millibar equals approximately 10 kilograms per square meter or 2 lbs. per square foot.

†Approximate velocity equivalents at a height of 33 ft. above sea level.

4. Generation of Wind Waves

Consider a calm area of the ocean surface just as a breeze begins to blow. The surface remains glassy until the wind increases to a little over three or four knots. Then small ripples form. As the wind speed increases to about six knots these ripples are formed over the entire area upon which the wind is acting. These ripples are capillary in nature and unless energy is continually added to them by the wind they would soon die out due to damping by the internal viscosity of the water.

The two mechanisms by which it is believed that these ripples are increased in size to become gravity waves are (i) the normal pressure forces (push) and (ii) the tangential shear stress (drag) of the wind blowing over the surface.

Consider a wind blowing over the surface of a small wave. Due to the sheltering effect of the surface of the wave there exists an excess of pressure on the windward side over the lee side. Hence, energy is added to the wave by "pushing" on it. The amount of sheltering depends upon the shape of the wave and the relative velocity of the winds. Also the greater the wind velocity compared to the wave velocity the greater the rate at which energy is added. If the waves are built up so that they are moving at the same speed as the wind then no more energy can be added by this mechanism.

The second mechanism by which energy transfer takes place is tangential drag. The wind blowing over the surface of the wavelets tends to drag the surface layer of water particles along with it. This layer drags the next layer below it (by the mechanism of viscosity) which in turn drags the layer below it and so on, the effect decreasing very rapidly with distance below the water surface. The small waves, then, are not sinusoidal in shape, but rather the peaks are steeper and the troughs flatter than a sine wave; the water particles have greater forward velocity at the crest of their orbital motion than with reverse velocity at the trough. Thus energy is continually added to the wave by tangential drag.

The first paper on this subject (IB-20) stated that the wave velocity could exceed the wind velocity because of energy transfer by tangential stress. Other investigators (IB-17) re-examined the theory and found that the original investigation was inconsistent to the degree of approximation used. They concluded that their "analysis limits the growth of waves to the region where the wave velocity is less than the wind velocity. Equation (37) of Sverdrup and Munk (IB-20) does not limit the wave velocity for positive transfer, and their data show waves being generated with wave velocities exceeding the wind velocity. However, the data recently gathered by Johnson (IB-12) shows that the majority of waves generated are in the region [where the wave velocity is less than or equal to the wind velocity], only a comparatively small number [traveling faster than the wind]."

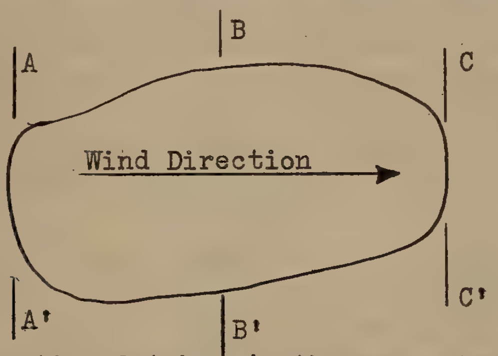
"Two such conditions suggest themselves. Waves may be generated in a region of substantial velocity gradient; hence, measured wind velocities may be lower than the velocity of the undisturbed flow. This would create the impression that the waves are moving faster in relation to the wind than they actually are. Secondly, short period waves in the generating area may disappear, transferring their energy to other waves. Such waves might occasionally exceed the wind velocity."

By means of these two mechanisms energy is continually added to the waves, increasing their size and velocity. Generally, the longer the wind blows, the faster it blows, and the greater the distance over which it blows (the fetch length), the larger the waves grow. This is easy to understand. The faster the wind blows

the greater the rate at which it can impart energy to the wave, the longer it blows the longer it can add energy to the waves and the greater the distance over which it blows, the longer the time it can add energy to the waves. Figure IB-20 shows the increase in wave height with increasing wind velocity for a constant fetch length and steady state conditions (IB-13). Figure IB-21 shows the change in height with increasing fetch length for a constant wind speed and steady state conditions (IB-4).

There are certain limiting factors however; hence, even if the wind velocity, duration time of the wind, or fetch length increases above these limits, the waves will not continue to grow.

In order to understand some of these limitations, consider a steady wind blowing over a constant fetch in the ocean. Small waves will be



formed over the entire fetch. As the waves travel across the fetch they are continually built up by the wind adding energy to them. Hence, at the end of the fetch, CC', the waves at first increase in size with time. However, after a certain time, and as long as the wind has remained fairly constant, the waves at the end of the fetch will reach a steady state condition in that they no longer increase with time. This is because the waves initially formed at AA', the start of the fetch, have had the time to travel to CC', the end of the fetch; i.e., these waves have been acted on for the greatest possible length of time. This time is known as the "minimum duration" and no matter how long the wind blows (unless it changes in velocity) after this time the average height of the waves will no longer increase or decrease with time.

In other words, for a given fetch and wind velocity the height of the waves leaving the fetch area will depend upon the length of time the wind blows until it has blown for the "minimum duration" associated with that fetch. For a wind blowing a long time as is often the case in the ocean, the minimum time is usually exceeded. Hence, the controlling factor is usually the length of the fetch. However, for a given wind velocity there exists a limiting fetch length. This is because energy is added to the wave probably only so long as the wave velocity is less than the geostrophic (not, however, the surface wind) wind velocity. Hence, a wind blowing over an unlimited fetch will continue to build up the waves until this velocity is reached.

The wind doesn't remain constant, but continually varies both in direction and in magnitude so that waves never build up in a simple manner. Instead of a system of regular oscillatory waves leaving the generating area, waves, the dimensions of which vary in a statistical manner, are observed. In addition, laboratory experiments (IB-13) have shown that a constant wind blowing over a contant fetch produces a spectrum of wave heights and periods (Figure IB-22). Because of this the waves must be

5. Travel of Swell in Deep Water:

a. Transformation from "sea" to "swell":

If the demarkation from the generating area to the calm region were sudden, one would observe the same phenomenon as is noticed in the laboratory (IB-4). That is, the waves decrease in height very rapidly and become much smoother than when in the fetch. In the ocean, however, the waves travel through regions of decreasing winds and then finally into a relatively undisturbed area, so the change is more gradual. Even so, the major change probably occurs within a few miles. The waves have transformed into the relatively smooth "swell". As the swell continue to travel through regions of calm they slowly loose energy through air resistance and, to some extent, by the mechanisms of viscosity, and eddy viscosity.

Figure IB-21 shows a series of photographs taken of the formation of waves by winds in a wave channel, together with photographs of these same waves transforming after they leave the generating area. It can be seen that at the end of the fetch small waves are being created upon the larger waves coming from the beginning of the fetch. It can be seen that these very short waves disappear rapidly and that the sea transforms into swell soon after leaving the fetch area.

b. Wave spectrum:

An observer stationed high above the area in which wind waves pass from the fetch into area of calm (the decay area) would notice that the waves in both regions were of a great variety of heights and lengths. If he were to follow a particular wave crest he would notice that it would gradually disappear; he would also notice new crests form.

An observer watching the waves leaving the fetch and measuring the distance between the successive crests as well as the heights would have a true picture of the surface waves at that particular point without having a true picture of the phenomenon. This is because the surface phenomenon is a result of other complex phenomena. Because there is a spectrum of lengths and heights present there must be some sort of group phenomenon; that is, there are no permanent wave forms. Instead, the "humps" (called wave crests) gradually appear and disappear. The longer waves of the "group", travelling with greater velocities than the shorter waves, gradually move ahead, with the shortest waves dropping behind. Hence, a spreading of wave train occurs.

In order to understand what happens in an actual case, where the fetches vary in area, the winds vary in speed and direction and storms that last for different lengths of time one must first consider the simplified case of a fetch of a constant area and location with winds which immediately spring up to a constant velocity and remain at that velocity for a short time (long enough to form a considerable number of waves) and then die down immediately. In addition, the decay distance must be long enough for a complete segregation to take place. After the storm has ceased, a large group of waves will be travelling across the ocean. An observer travelling with the group would notice that the long (high period) waves would gradually move in front of the group and the very short waves would be left at the rear of the group. The longer the group traveled the greater would be this segregation. If the group were to travel many thousands of miles, and were

there no other disturbances, this "stretching" of the group due to segregation would become complete. An observer at a fixed distance from the fetch would notice a steady decrease in period as the waves passed by.

Actually, the duration of the storm and the relatively short decay distances (even several thousand miles) are such that complete segregation never takes place. In local storms relatively no segregation takes place and the lengths (hence, period) between crests of the surface profile are always very short, even if the winds were great, the fetch long and the duration long. As the "decay" distance increases the amount of segregation increases and so the long waves (the so-called "forerunners of storms") reach the coast before the main body of the waves. For a particular storm the longer the decay distance the greater is the time between the arrival of these "forerunners" and the main body of the waves, consisting of the relatively unsegregated section.

The normal case is even more complicated. For example, suppose the storm lasts for three days and that it only takes one day for the medium length waves to reach the section of coast that is being observed. The longer waves are being continually generated as are all the other wave lengths. Thus, even as the first of the shorter waves are arriving at the coast the longer waves, which were being formed after the shorter waves have left the generating area, overtake the shorter waves and arrive at the same time.

It can be seen then that a section of a coast a considerable distance from a storm will be attacked by long, low waves first with the mean period of the waves (with period being the time necessary for two "successive" crests to pass a fixed point) decreasing with time but with the spectrum width about the mean increasing. The wave heights will be increasing because the greatest amount of energy is concentrated in these medium waves. As the last of the waves reach the coast an abrupt decrease in wave period and height will be observed. The average "period" of even these short waves, at the coast, will always be longer than the period observed at the end of the fetch because of the spreading phenomenon and because the smallest wave will have been either "captured" by the large waves or dissapated.

Actually the phenomenon is even more complex than has been described. The winds gradually rise to a maximum, and then decrease again generating longer waves as they increase and shorter waves as they decrease. At the same time usually the fetch varies in length and the storm moves.

As an example consider the wave spectra at Pendeen, England, during the period 14 to 18 March, 1945, which has been presented in Figure IB-23 (IB-1). The wave record at Pendeen was analyzed by a frequency analyzer so that the component period spectra was obtained. It can be seen that the "forerunners" of a new storm first appeared at 1300 on 14 March 1945. The mean of the periods gradually decreased while at the same time the width of the spectra increased.

c. Group velocity:

As has been mentioned previously the velocity of a wave in deep water depends almost entirely upon its length (wave period). This relationship may be seen in Table IB-4. However, this is the velocity of waves of permanent form and not the velocity of the group of waves.

An observer aboard a ship watching the bow waves form will notice that as the group of waves travel away from the ship the leading waves gradually decrease in height while at the same time new waves form at the rear of the group, the total number of waves in the group remaining constant. Hence, the group of waves travels at a different velocity than do the wave forms. In deep water this group velocity is one half that of the individual waves. Because of this it takes twice as long for a group of waves to travel a given distance than it would if the group was traveling at the velocity of individual waves.

At the present time evidence indicates that the waves traveling from a storm area to the coast travel at a velocity equal to the group velocity associated with the period of the waves that arrive at the coast. For example, sixteen second waves (1310 feet long) travel at a velocity of 48.5 knots; however, the group of waves would travel at 24.3 knots. Hence it would take the group 41 hours to travel 1000 nautical miles.

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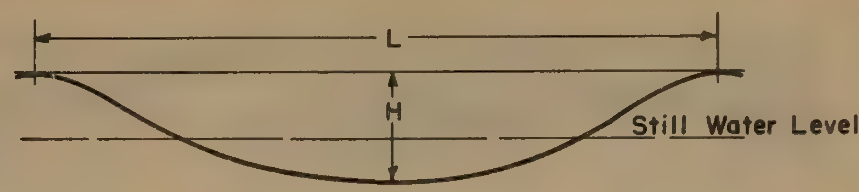
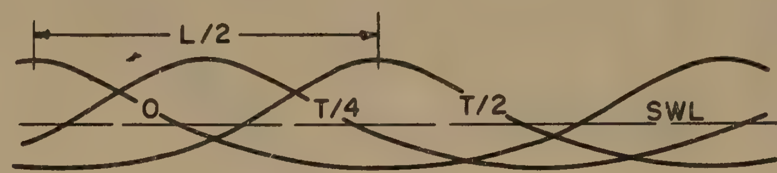
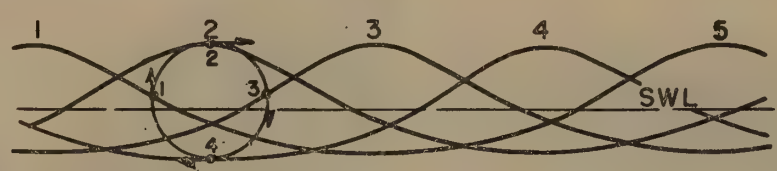


a. Profile of Wave



b. Advance of wave, showing the wave profile at the times t=O, t=T/4 and t=T/2. In the time T/2 the wave has advanced one half wave length, L/2.

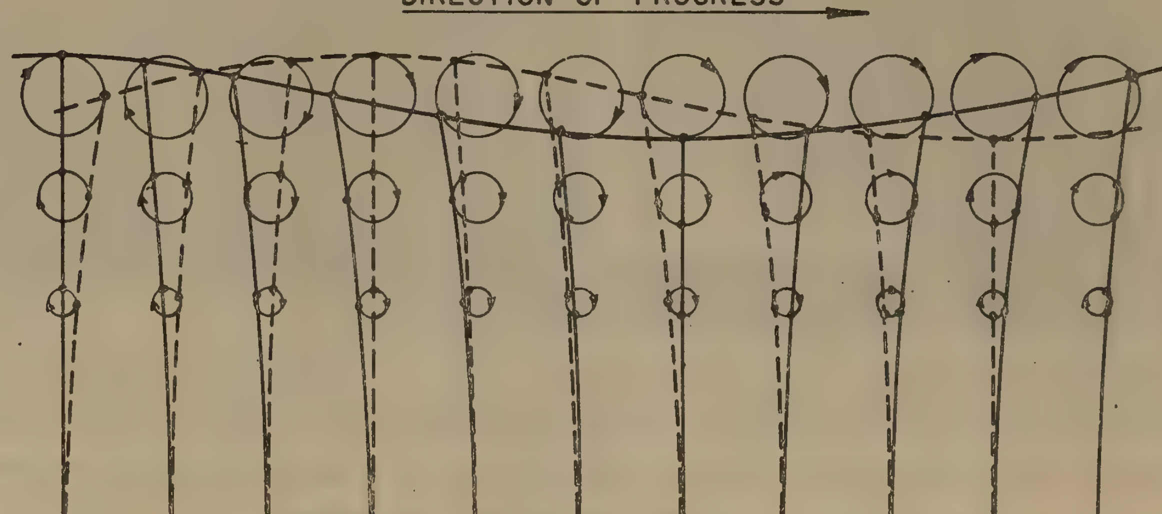
FIGURE IB-1



Successive positions of water particle as wave form advances.

FIGURE IB-2

DIRECTION OF PROGRESS



Movement of water particles in a deep-water wave of very small height. The circles show the paths in which the water particles move. The wave profiles and the positions of a series of water particles are shown at two instants which are one quarter of a period apart.

FIGURE IB-3

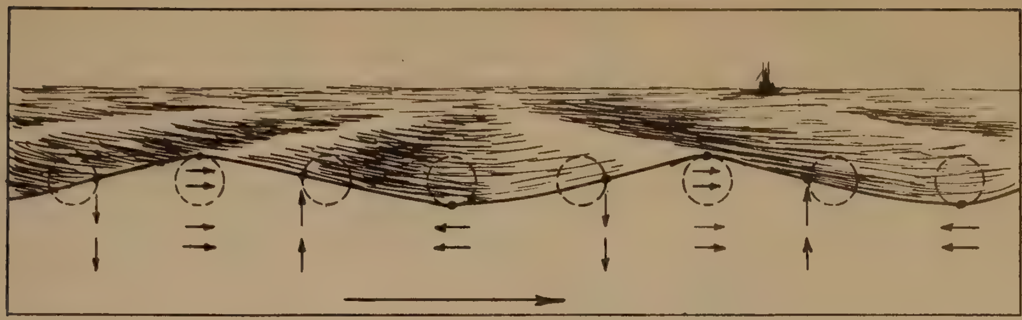


FIGURE IB-4. Directions of orbital movement of water particles in different parts of wind waves advancing in direction of large arrow.

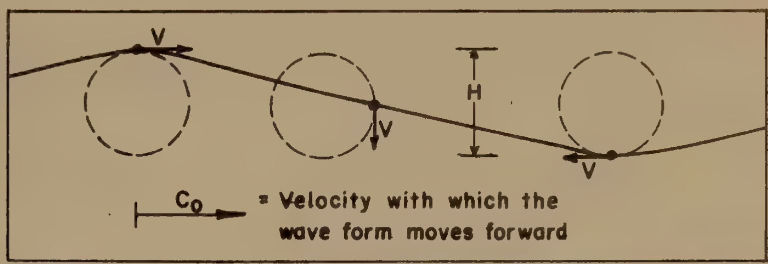


FIGURE IB-5.

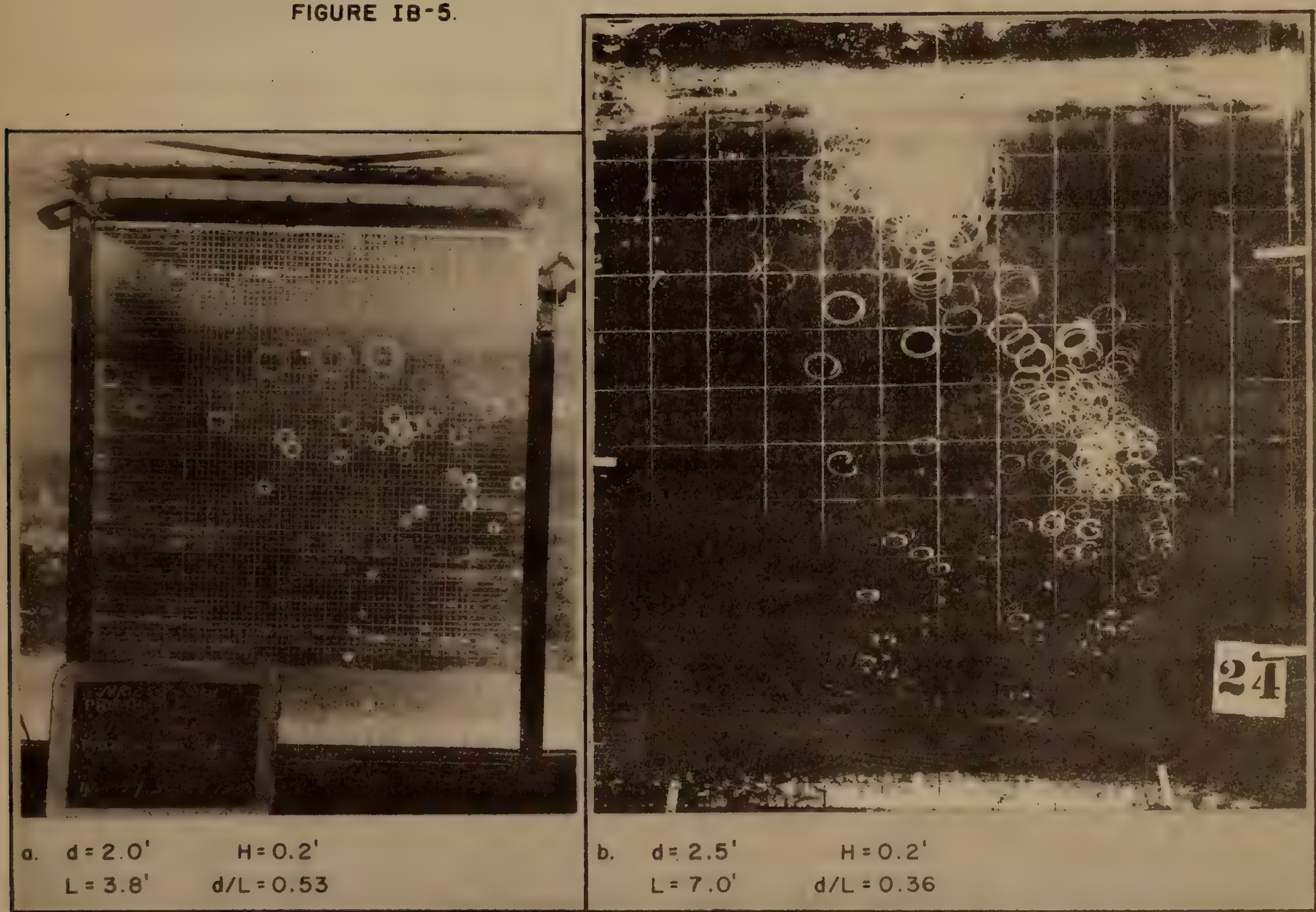


FIGURE IB-6. Time exposures of orbital motion in the Wave Channel, University of California

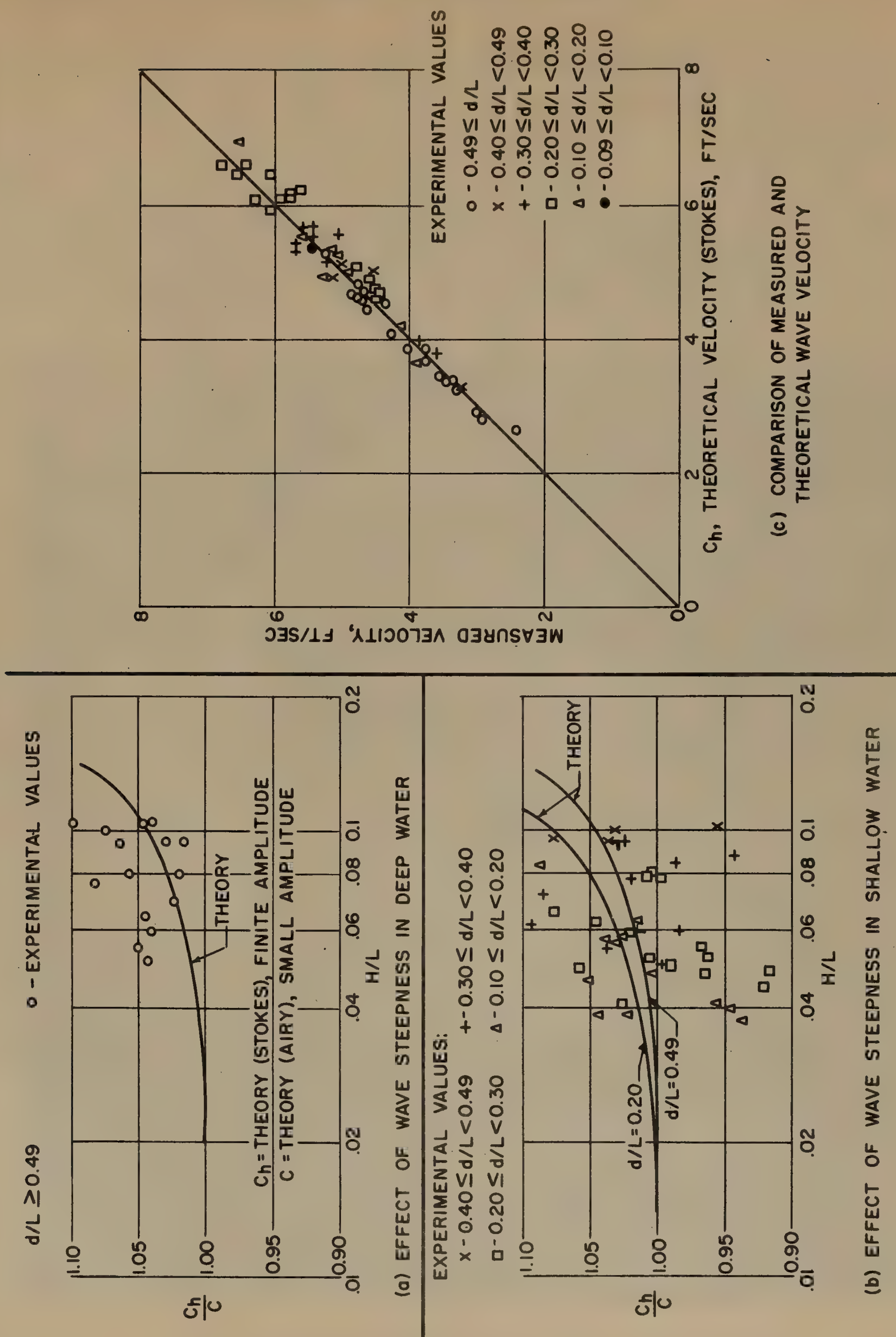


FIGURE IB-7 - VELOCITY OF WAVES OF FINITE HEIGHT

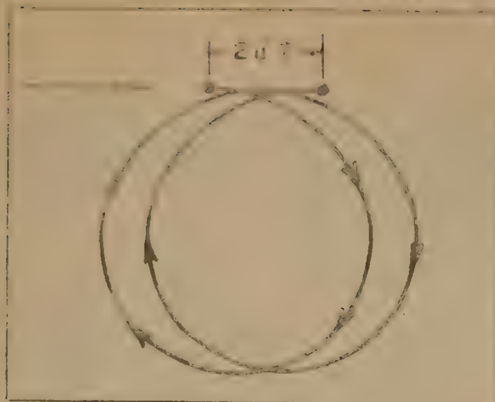
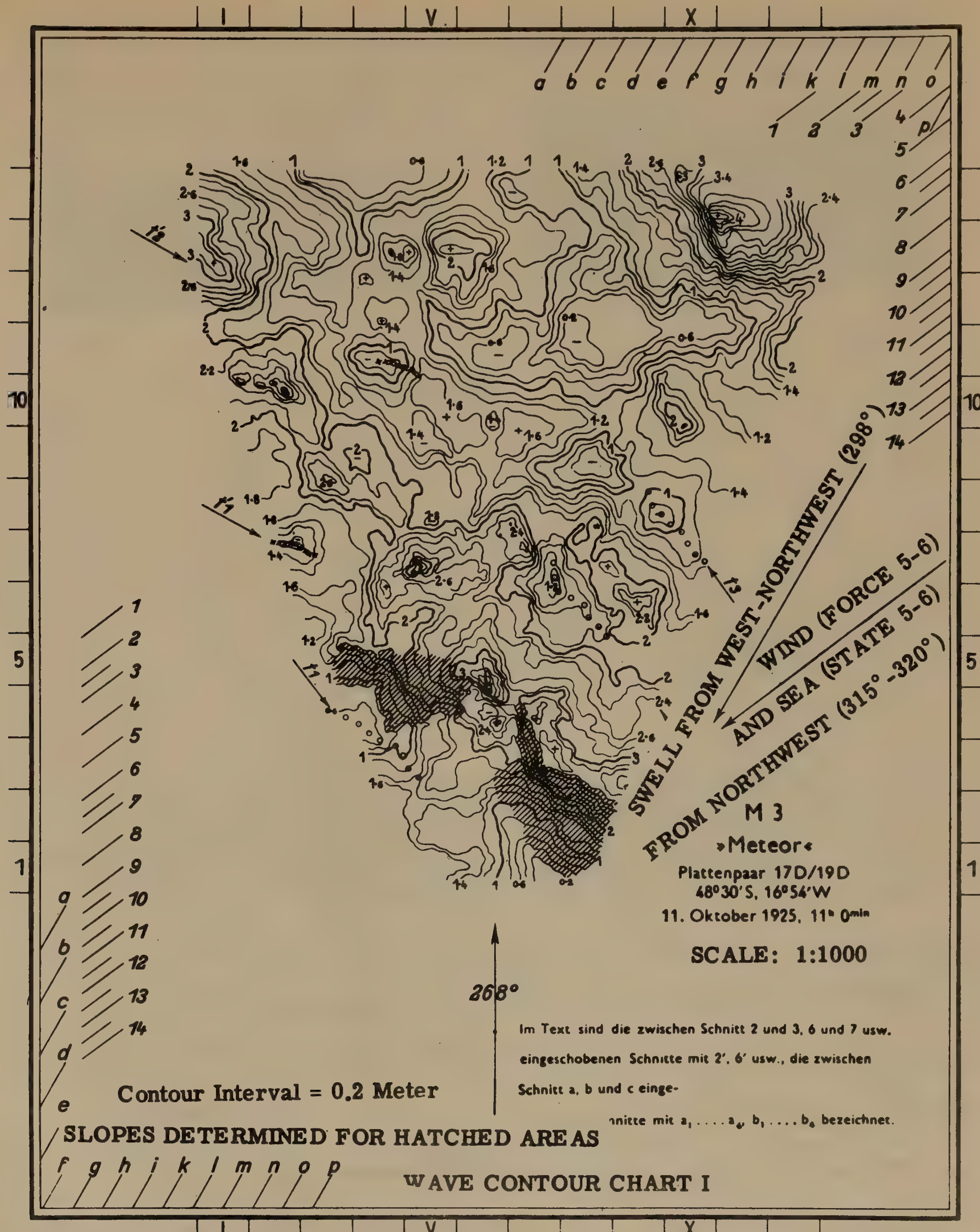


FIGURE IB-8. Orbital motion during two wave periods of a water particle in a deep-water wave of moderate or great height. In two wave periods the forward displacement equals $2\bar{u}T$

FIGURE IB-9. Aerial view of short-crested waves



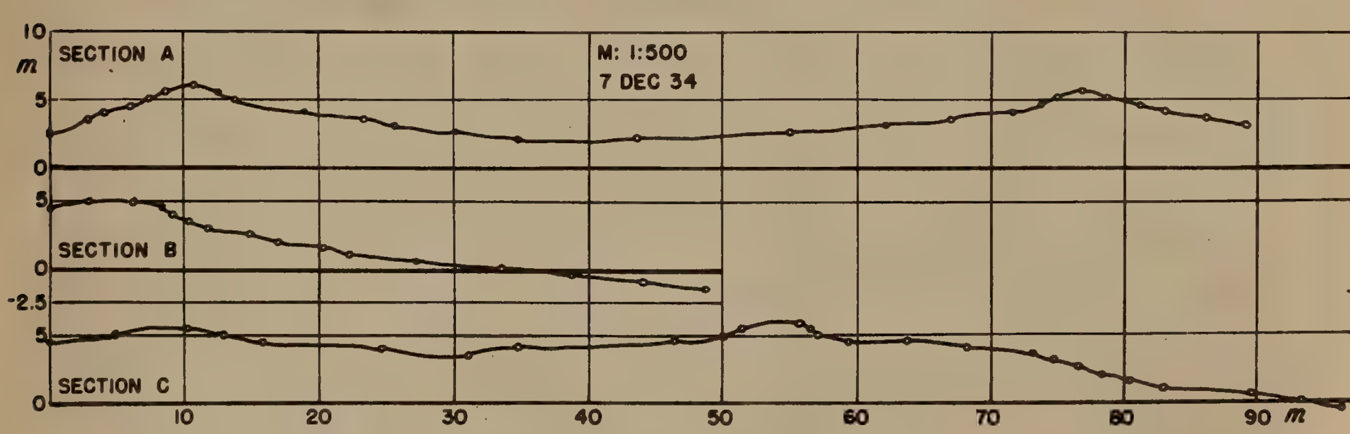
Verlag Walter de Gruyter & Co., Berlin u. Leipzig

Ausmessung. Photogrammetrie G.m.b.H., München

FIGURE IB-10

a. Stereophotogrammetric Plot of Wave Formation from set of photographs.

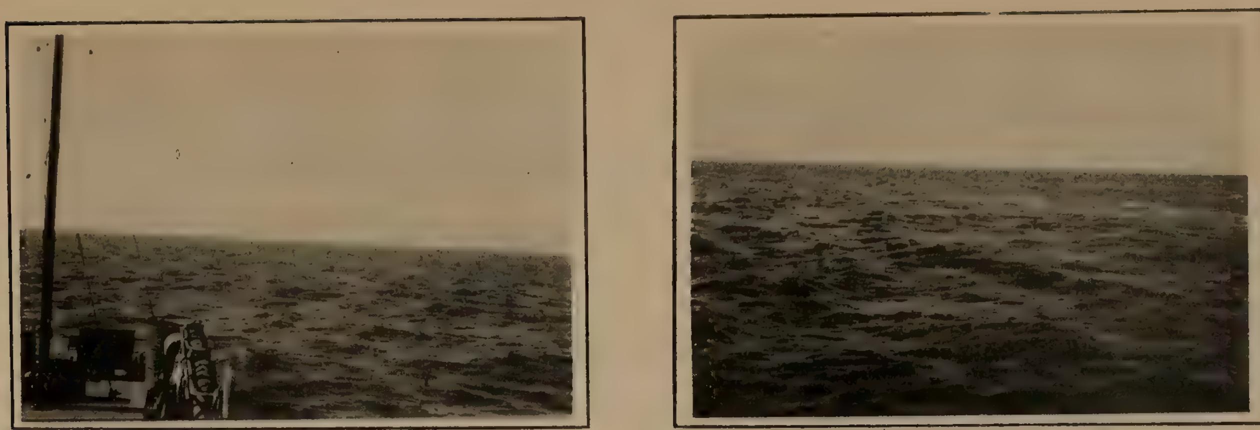
The photograms for the above figure were taken aboard Test Expedition on MS SAN FRANCISCO by Dr.-Ing.habil. Georg Weinblum, Prussian Research Institute for Hydraulics and Shipbuilding. Plotted with aerocartograph by Walter Block, Engineer in charge of measurements. Chair of Photogrammetry at the Technological Institute, Berlin (Prof. Dr.-Ing. Lachman).



b. Wav Profiles from set of photographs.

FIGURE I B-11

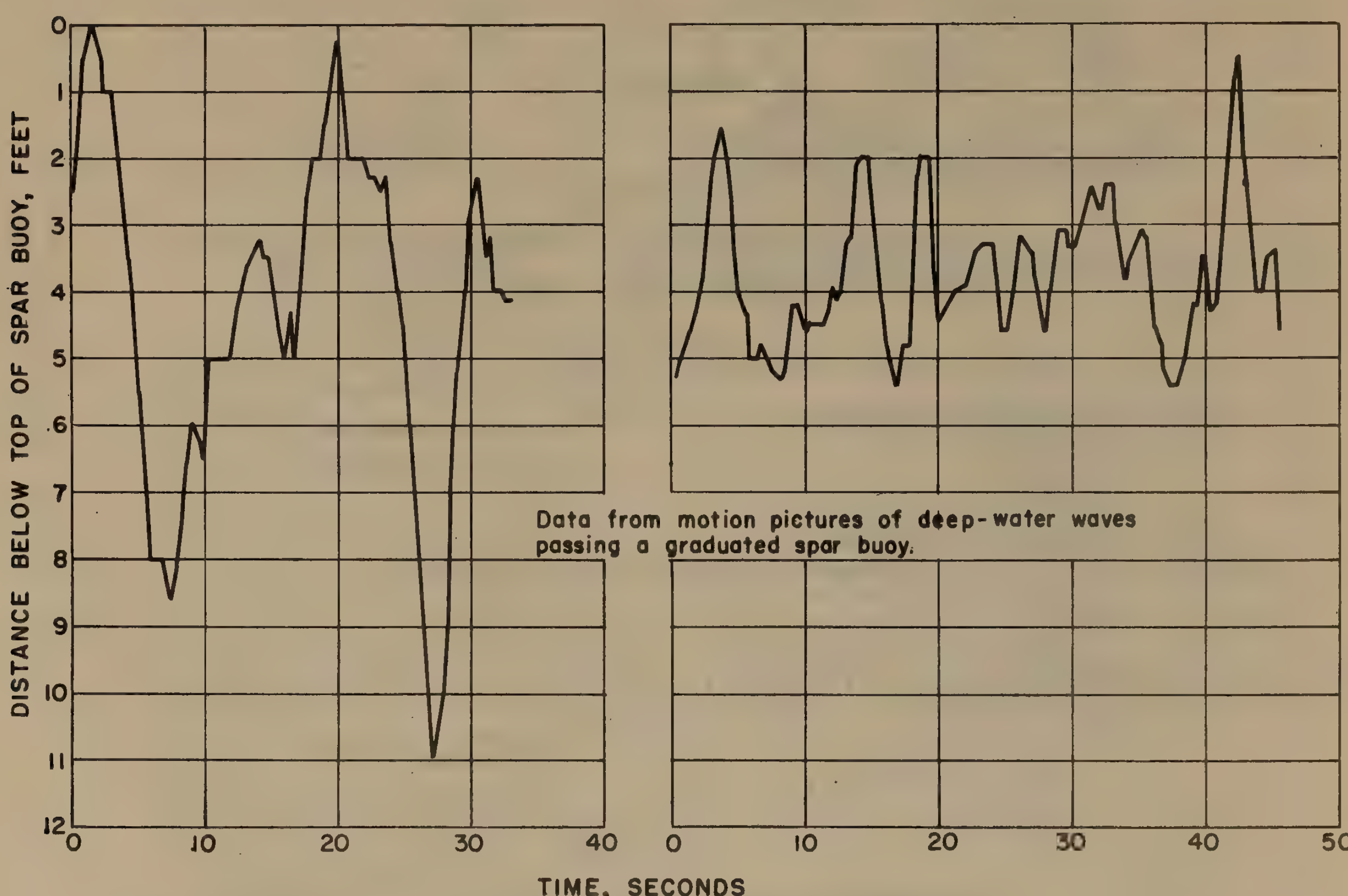
(from "Stereophotogrammetric Wave Photographs" (*Stereophotogrammetrische Wellenaufnahmen*) by Georg Weinblum and Walter Block, translated by F.A. Raven, PhD)



2230 GCT
17 Jan 1945

PHOTOS OF SEA SURFACE

1915 GCT
20 Jan 1945



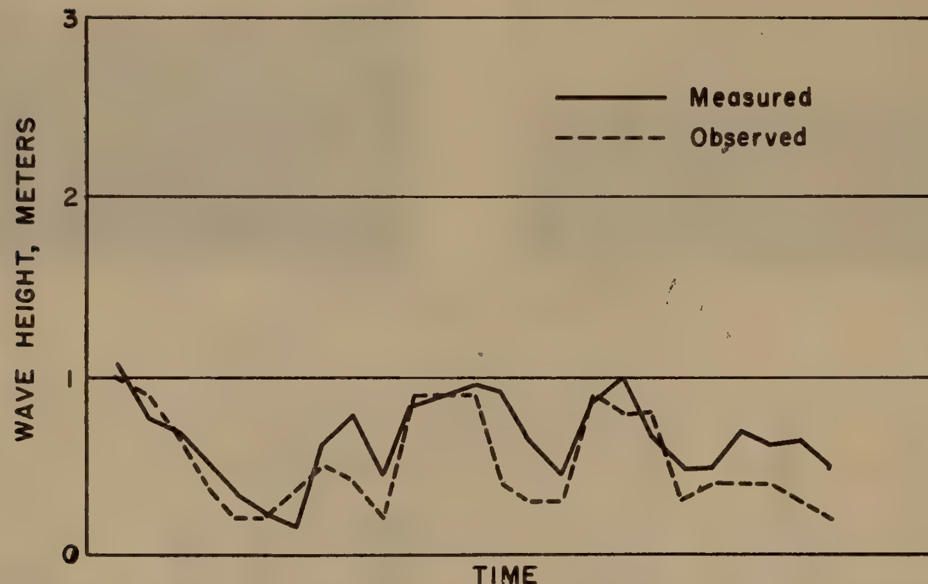
Swell:
H_{ave} = 4.5 ft., T = ——
Wind Waves:
H_{ave} = 3 ft., T = 6 sec.

2230 GCT, 17 Jan 1945

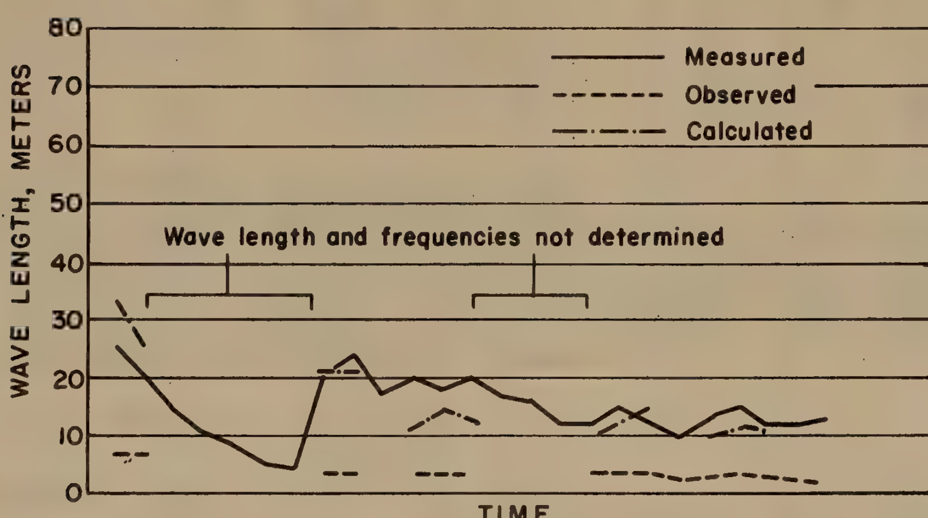
Swell:
H_{ave} = 6.6 ft., T = 12.8 sec.
Wind Waves:
H_{ave} = —— T = 5.5 sec.

1915 GCT, 20 Jan 1945

FIGURE IB-12



a. Comparison of measured and observed wave heights. Observer I of the lightship *Fehmarnbelt*, September 22-30, 1936



b. Comparison of wave lengths. Observer I of the lightship *Fehmarnbelt*, September 22-30, 1936

FIGURE IB-13

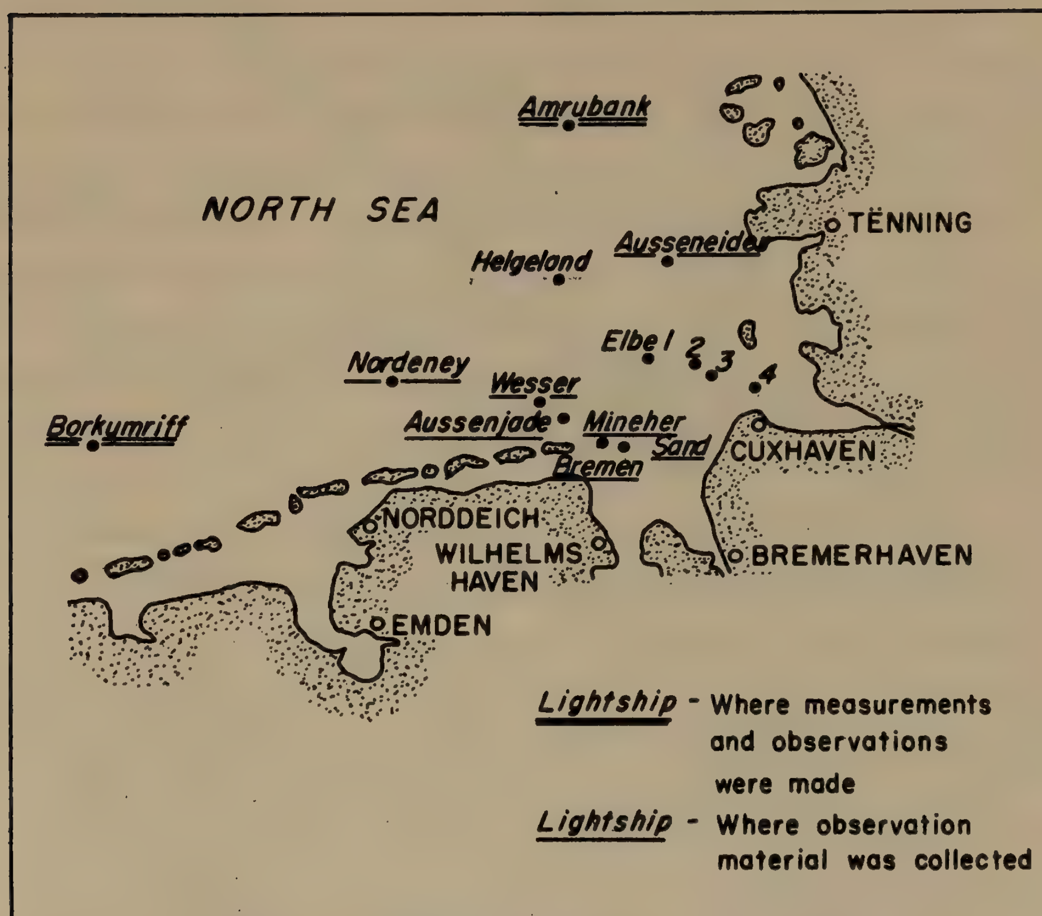


FIGURE IB-14 - LOCATION OF NORTH SEA OBSERVATION STATIONS

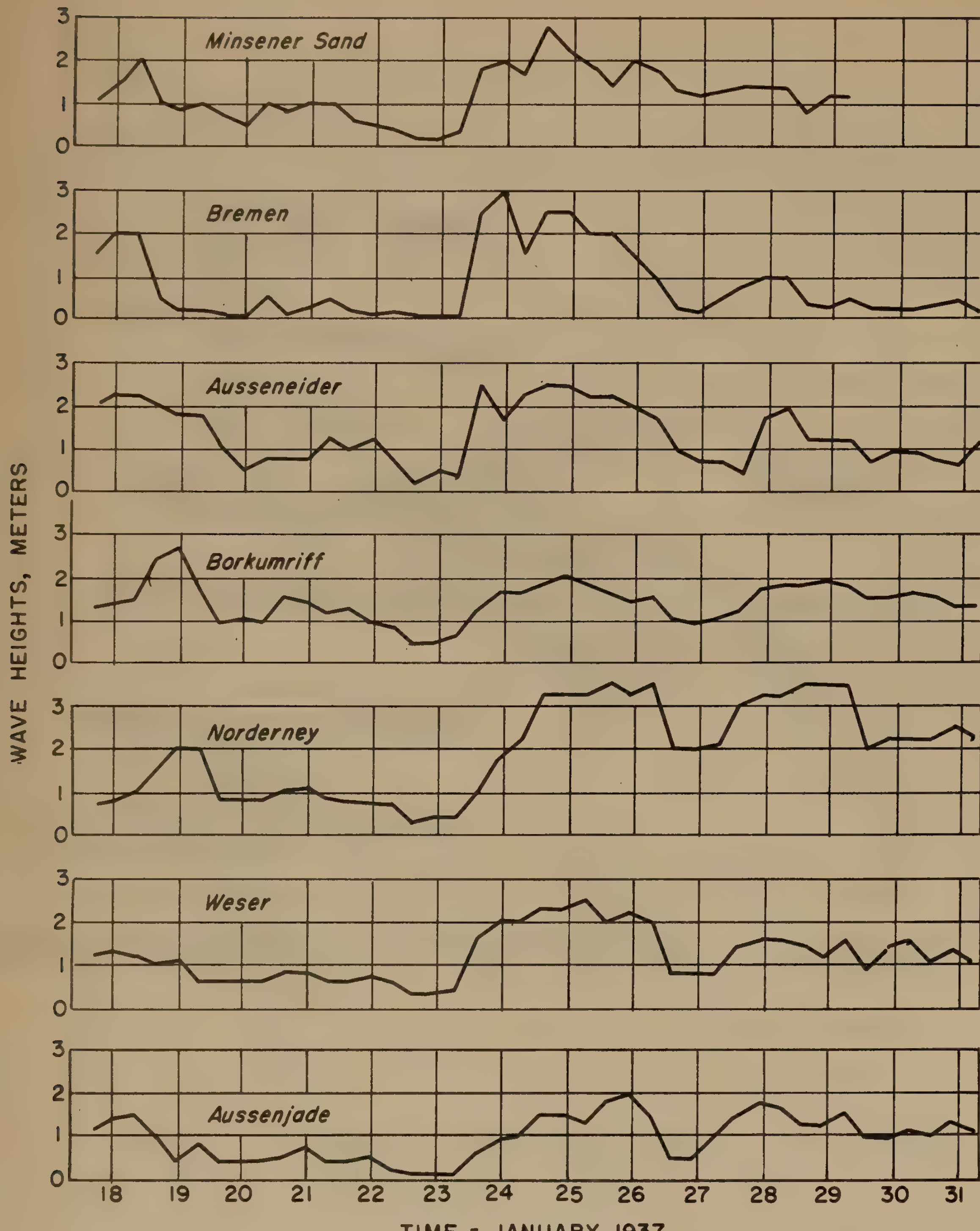
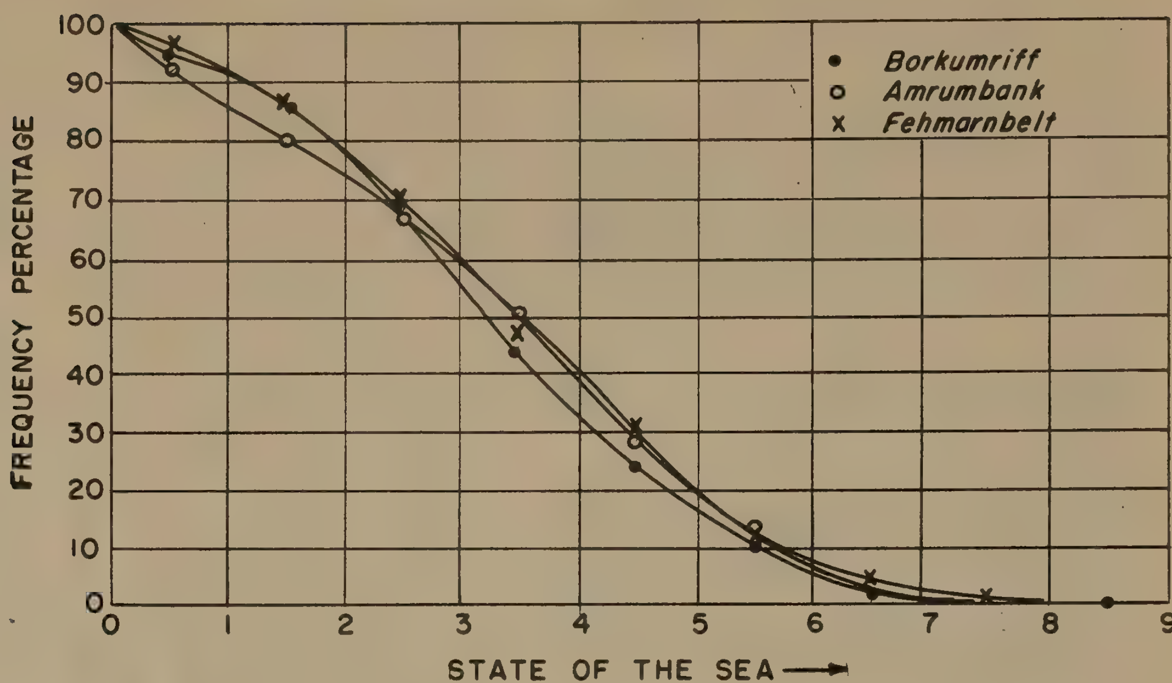
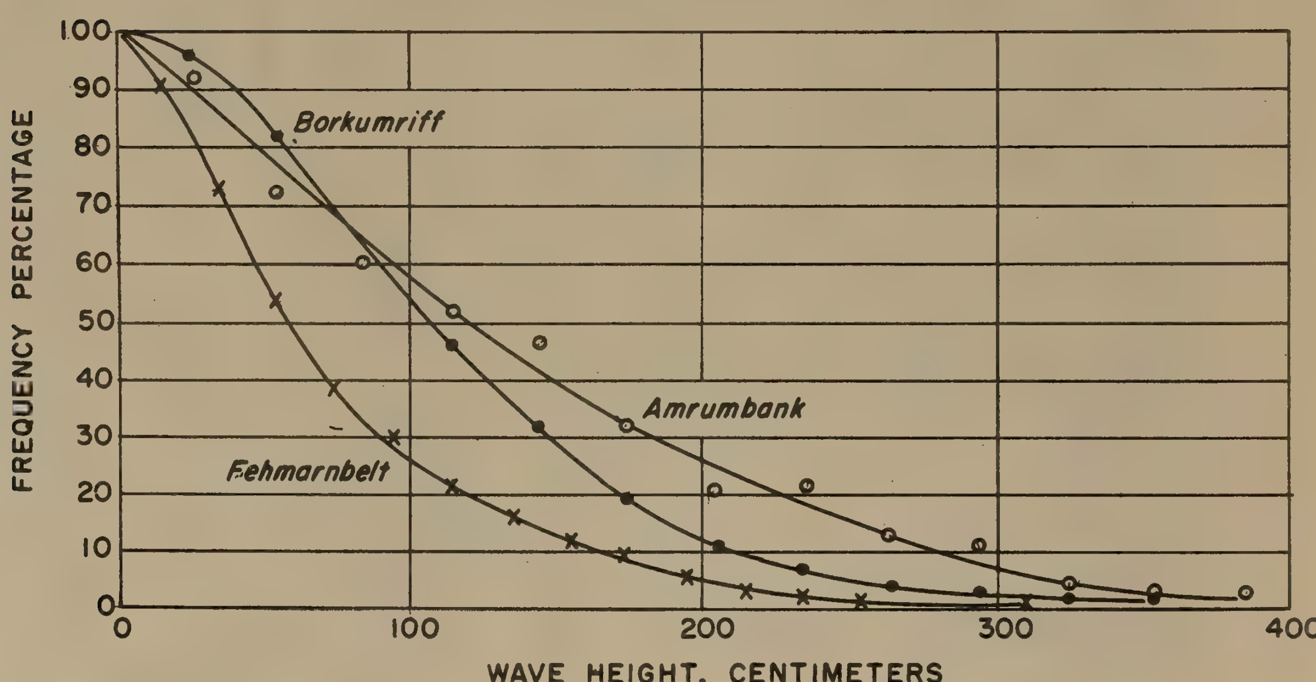


FIGURE IB-15 - COMPARISON OF WAVE HEIGHTS OBSERVED ON DIFFERENT LIGHTSHIPS IN THE NORTH SEA



COMPARISON OF FREQUENCY OF STATES OF THE SEA



COMPARISON OF FREQUENCY OF WAVE HEIGHTS

FIGURE IB-16

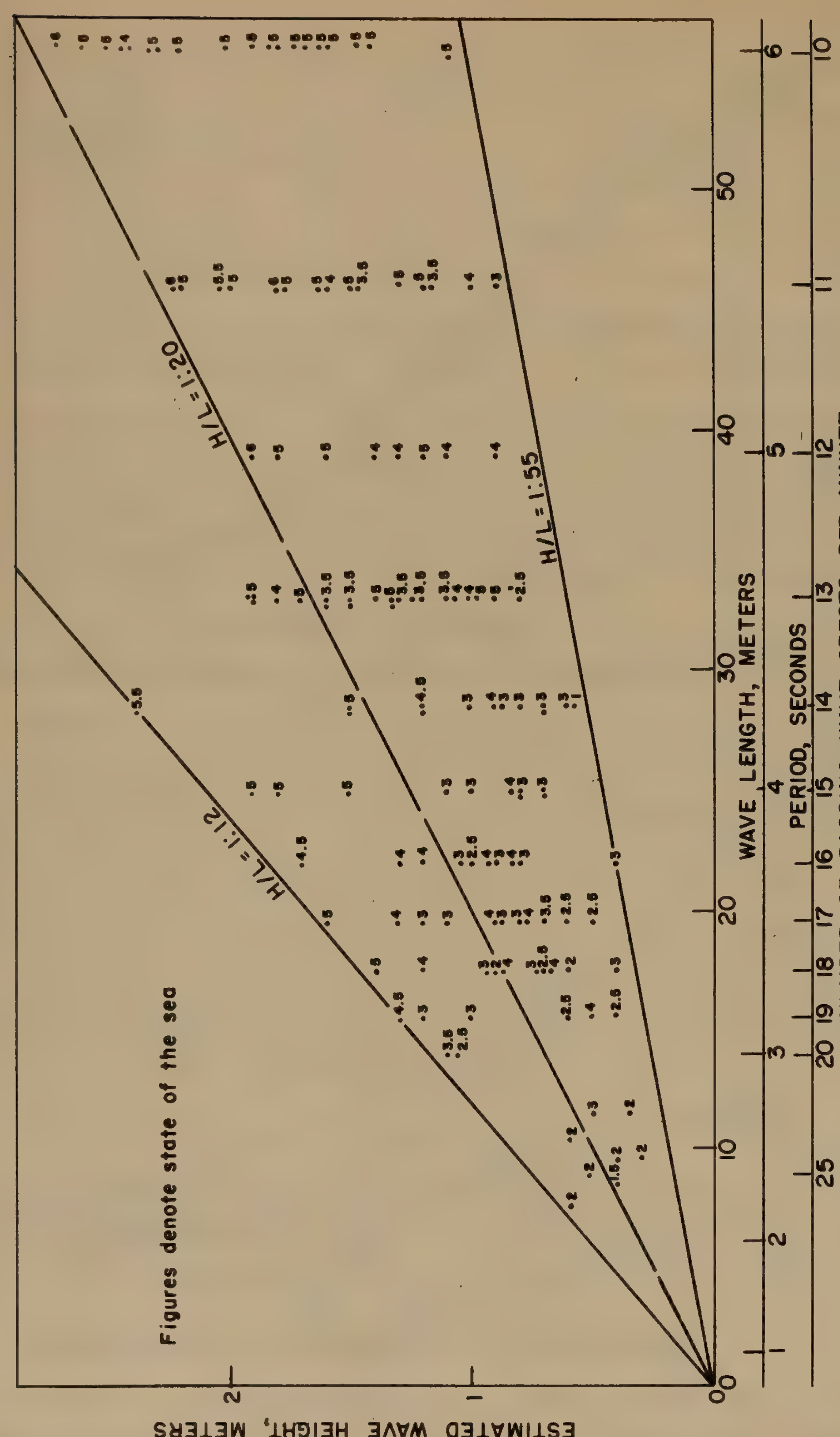


FIGURE IB-17 - WAVE DIMENSIONS AND RELATIVE STATES OF THE SEA
LIGHTSHIP *BORKUMRIFF*, JANUARY 9 - APRIL 6, 1937

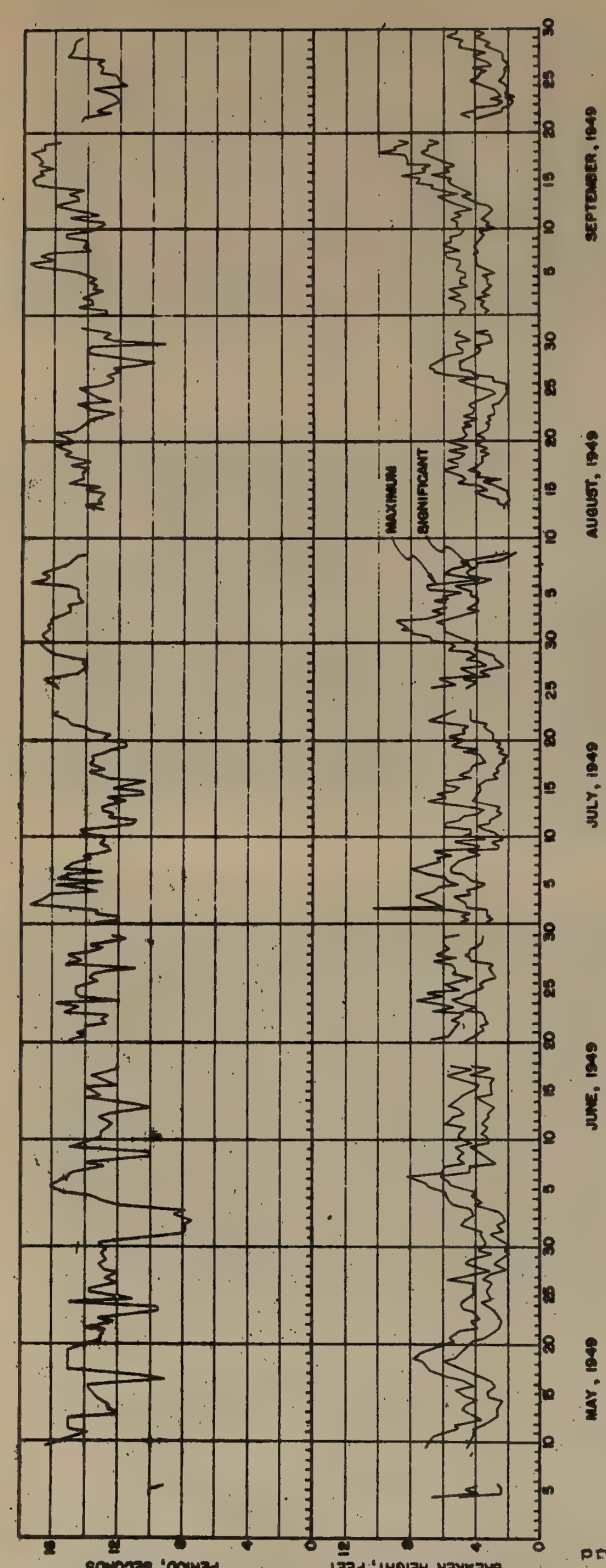


FIGURE IB-18 - BREAKER RECORDS AT CAMP DEL MAR, OCEANSIDE, CALIF., FROM THERMOPILE WAVE METER

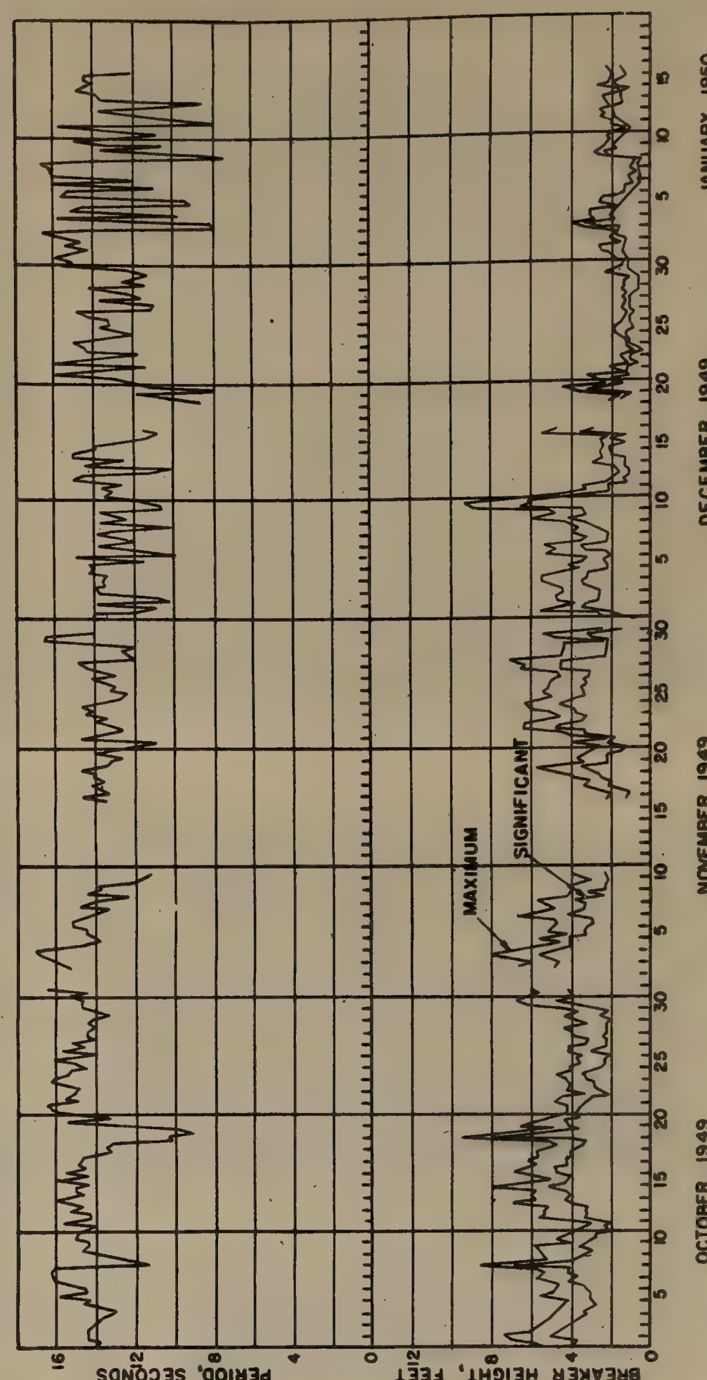
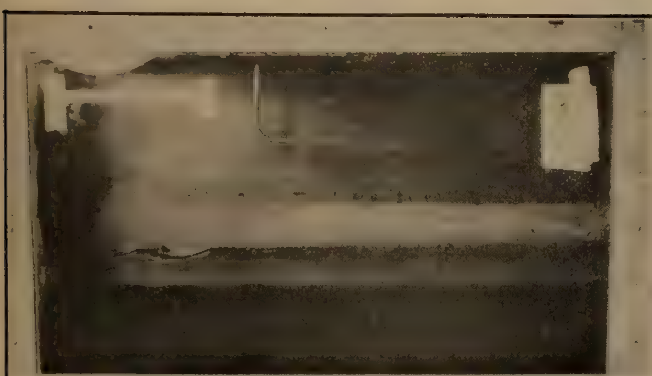


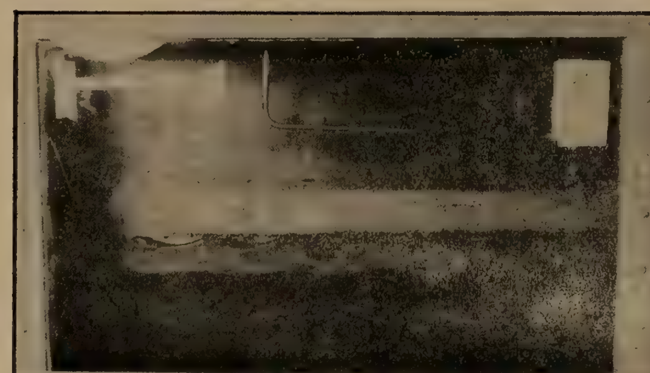
FIGURE IB-19 - BREAKER RECORDS AT CAMP DEL MAR, OCEANSIDE, CALIF., FROM THERMOPILE WAVE METER

a. Wind speed = 6.3 knots

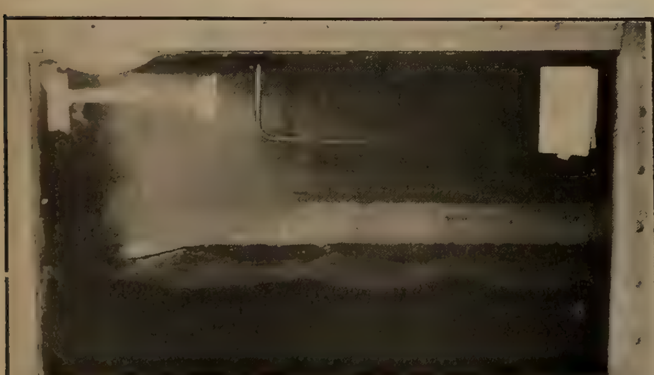
b. 8.4 knots



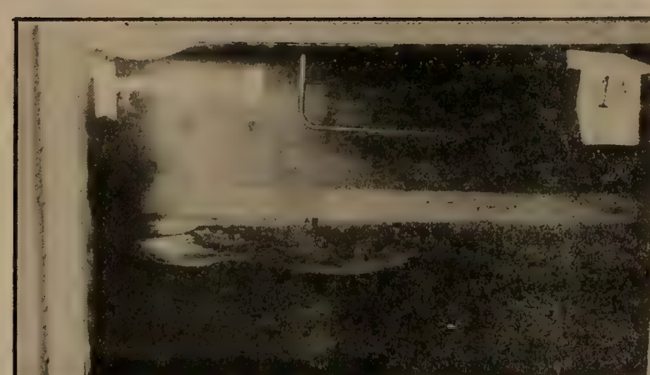
c. 10 knots



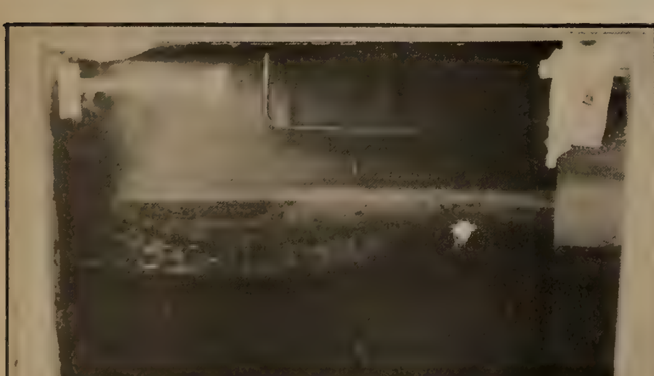
d. 11 knots



e. 13.4 knots



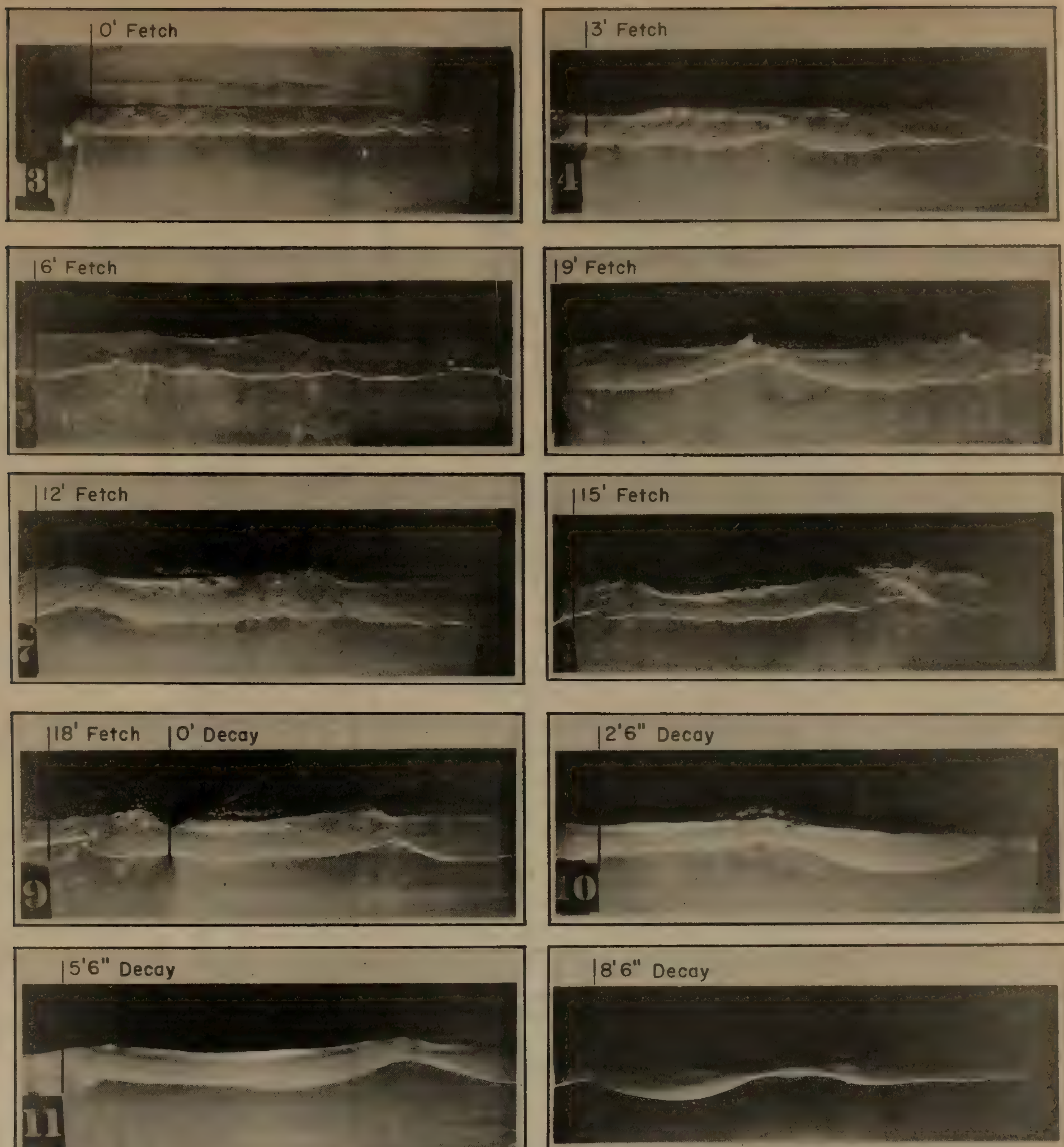
f. 17.0 knots



g. 22.3 knots

FIGURE IB-20 - Relationship between Wind Waves and Wind Velocity for a Constant Fetch and Steady State Conditions

Photographs taken in University of California Wave Channel



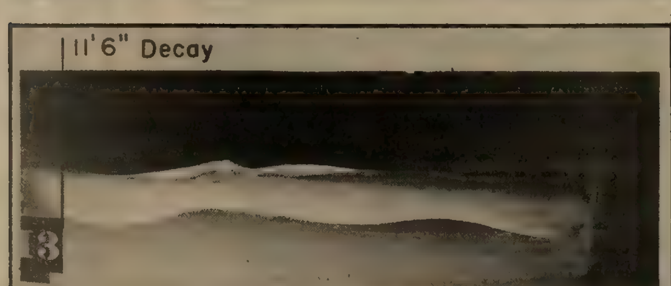


FIGURE IB-21 - Generation and Decay of Wind Waves. Constant wind velocity in the fetch, no wind in the decay area. Steady state conditions. Photographs taken in the University of California Wave Channel.

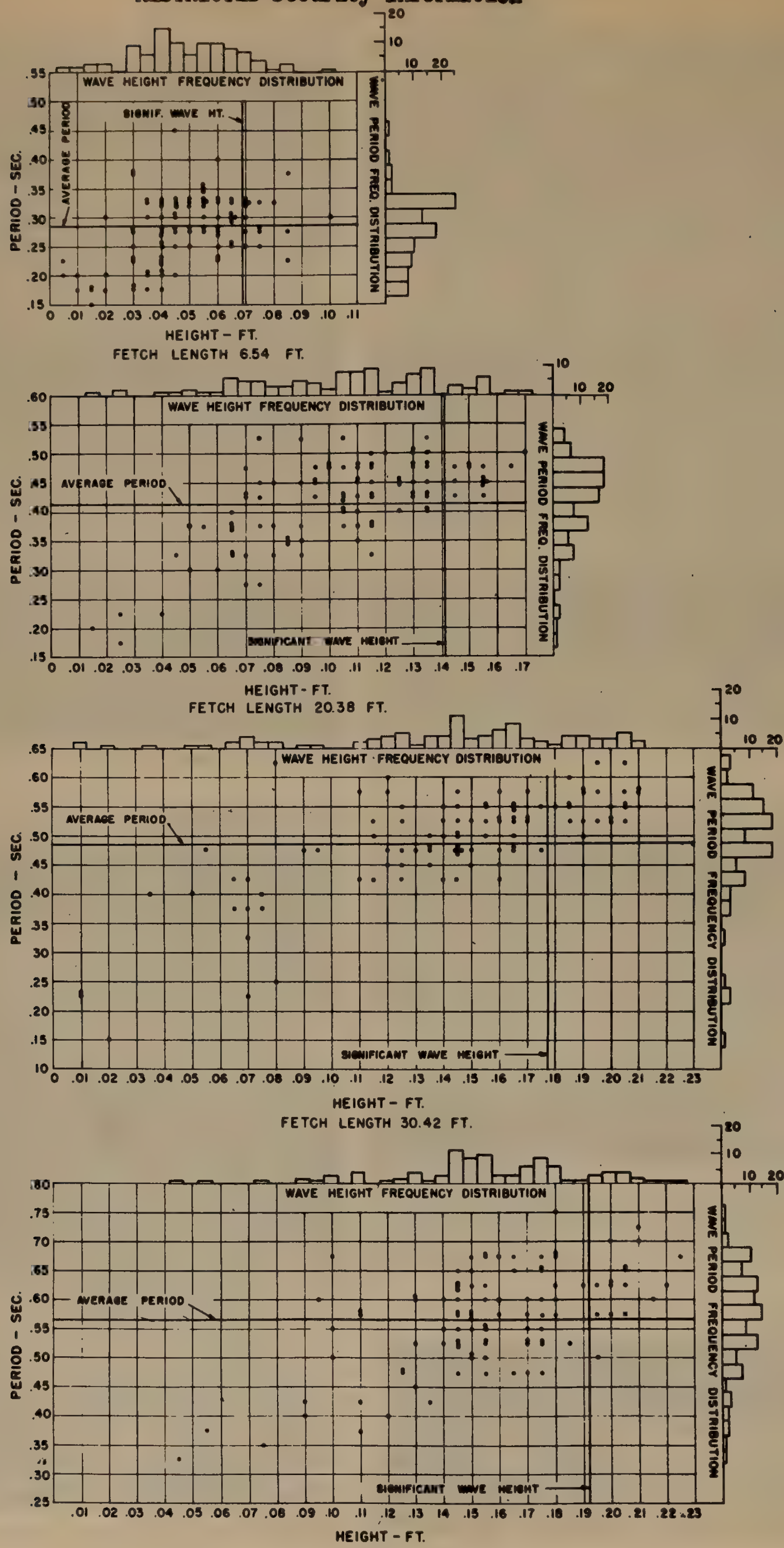


FIGURE IB-22 - JOINT FREQUENCY DISTRIBUTION OF WAVE PERIOD AND WAVE HEIGHT. (U=42.7 ft/sec)

FIGURE IB-23 - WAVE SPECTRA AT PENDEEN, 16 TO 18 MARCH 1945

SUMMARY OF SECTION I-C
TRANSFORMATION OF WAVES IN SHOALING WATER

BY

H. W. IVERSEN

Ocean waves travel from the generating areas in the open ocean to reach eventually a coastline where the energy of the waves is dissipated. The waves undergo changes during this travel; the long smooth swells of the open ocean become breakers and surf on shelving beaches, or dash against rocky headlands or cliffs. The history of the waves depends upon the "deep water" character of the waves, and angle with which they approach the shoreline, the bottom contour pattern near the shoreline, and imposed effects such as local winds or shore currents from rivers or tidal estuaries.

An individual wave may be described by a height (the vertical distance from the crest to the preceding trough) and a period (the time between passage of successive waves past a fixed point). In a series of identical waves travelling through the shoaling water as they near the coastline, the period does not change. The height may change due to the proximity of the ocean bottom. In addition, the wave has a length (the distance between successive crests), and a velocity (the speed of advance of a crest relative to the fixed ocean bottom).

The nature of the transformation of height, length, and velocity near the shore may be visualized from a consideration of a series of identical, long-crested waves that advance over a plane shoaling bottom of constant slope. The wave crests are parallel to the bottom contours. For convenience, three regimes in the wave travel are designated.

(i) Deep water, wherein the proximity of the bottom has no appreciable effect upon the wave. The waves in deep water do not change in height, length, or velocity. When the depth of water is greater than one-half the wave length, the wave is considered a deep water wave.

(ii) Shoaling Water, wherein the proximity of the bottom acts to slow the wave advance, to shorten the wave length, and to change the wave height until the wave becomes a breaker.

(iii) Very shallow water, wherein the wave velocity depends only upon the depth of water and not upon the wave period.

For practical use, the relationships of the wave variables are listed below. Magnitudes of the variables for typical ocean conditions are shown in Figure SIC-1.

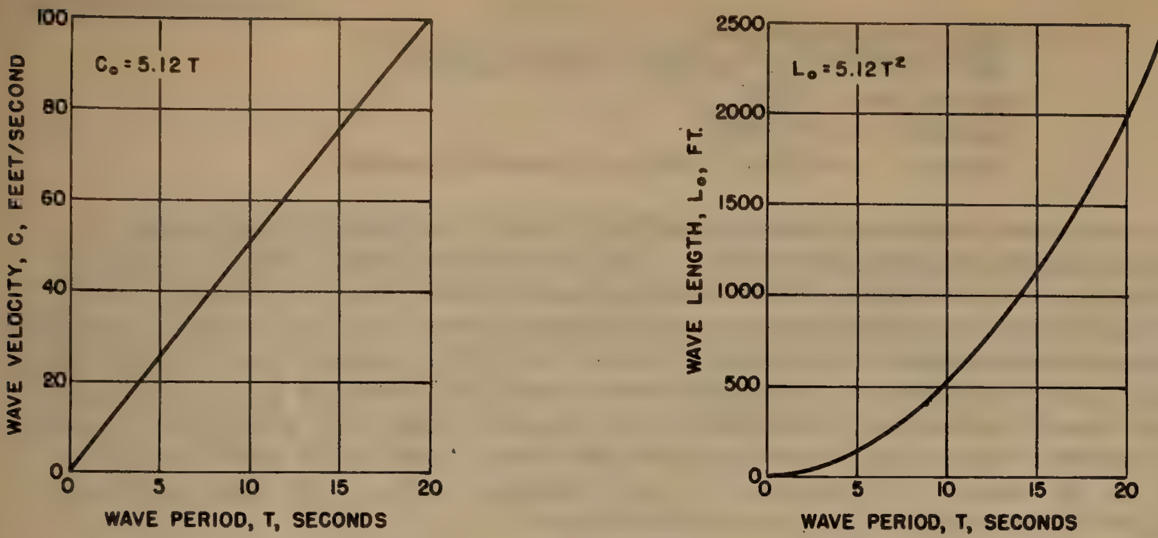
(1) Deep water.
Height = H_o = constant (as determined from forecasts)
Period = T = constant (as determined from forecast)
Length = $L_o = 5.12\ T^2$ (L_o in feet, T in seconds)
Velocity = $C_o = 5.12\ T$ (C_o in feet per second, T in seconds)

(2) Shoaling water:
Height = $H = H_o \sqrt{(1/2)(1/n)(C_o/C)}$
$n = 1/2 \left[1 + (4\pi d/L)/\sinh 4\pi d/L\right]$
Velocity $C = C_o\ (\tanh 2\pi d/L)$
Length $L = L_o\ (\tanh 2\pi d/L)$
where: d = depth of water, feet

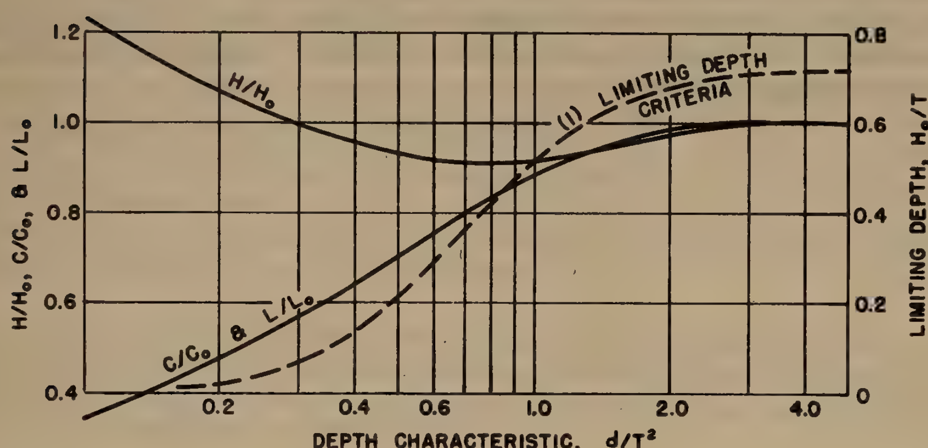
(3) Very shallow water:
Height = H = no definite relationship
Velocity = C = $\sqrt{gd}$
Length = L = T $\sqrt{gd}$
where: g = 32.2 feet per second squared.

As an ocean wave advances over a shoaling bottom the height decreases to approximately 9% at a depth equal to 16% of the deep water wave length. The height then increases until breaking occurs near the shoreline. The known transformation relationships in shoaling water are reliable to predict the wave transformation to depths of water slightly greater than the depth at which the wave breaks. When the wave nears the breaking point the symmetrical swell distorts. The front face becomes steep and white water appears at the crest or over the front face of the wave. The major portion of the energy of the wave is dissipated in the breaking action with the subsequent travel of the foam line up to the beach face.

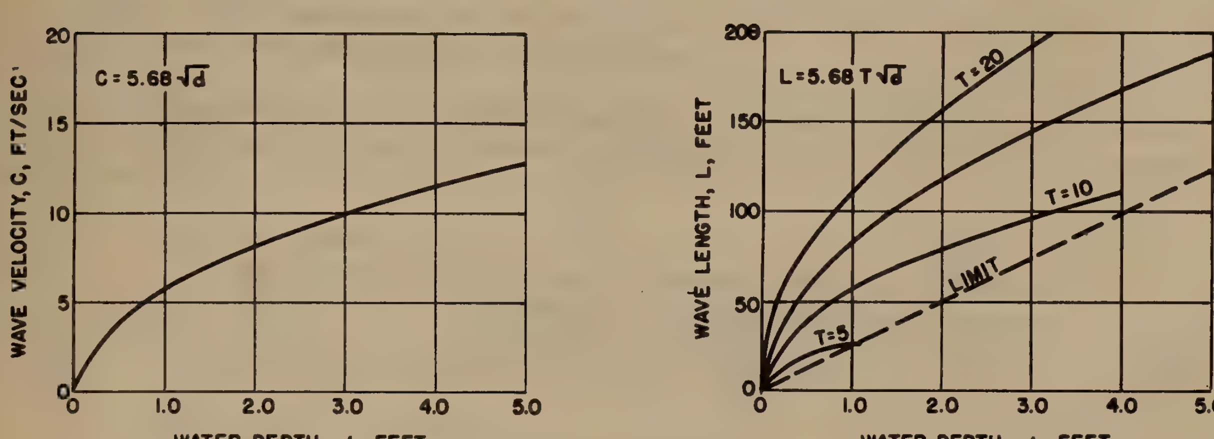
The wave velocity is a function of the depth in shoaling water. Hence, refraction occurs when waves advance over a shoaling bottom when the line of the wave crests is not parallel to the bottom contours. The portion of the wave nearest the shoreline travels slower than the portion farther offshore in deeper water. The wave front becomes curved. The described length and velocity relationships do not change, but the wave height is influenced by the refraction due to the lateral distribution of the wave energy in the curved front.



a. DEEP WATER WAVES - $d > L_o/2$



b. WAVES ON SHOALING BOTTOM - $L_o'/2 > d >$ Value from curve (1)



c. SHALLOW WATER WAVES - $d < L/30$

FIGURE SIC-1

C. WAVES IN SHOALING WATER
BY
H. W. IVERSEN

Table of Contents

C. WAVES IN SHOALING WATER

1. Introduction:

As ocean waves approach shallow water in the regions adjacent to the coastline, the proximity of the bottom affects the character of the waves. Near the shore, the wave transforms from the symmetrical offshore swell until it finally becomes unstable and breaks. When breaking occurs, the wave crest travels faster than the main body of the wave. The action continues until the breaker becomes a complete front of a mixture of agitated entrapped air in the water, or a foam line. The foam line proceeds to the shore and up the face of the beach to the "limit of uprush". During the course of the wave transformation, breaking, and the subsequent travel of the foam line, the energy of the wave is partially dissipated in internal relative motion of the water and in friction along the bottom.

The wave transformation may be visualized from a consideration of wave theory. No theory is yet available to explain the complete history of wave motion from deep water offshore of the coastline up to the limit of uprush. However, the theory as now known serves to explain many of the features of the transformation and serves as a guide for investigations to relate the variables of the transformation. Confirmation of wave theory has been established by laboratory studies for transformation of periodic waves in shoaling water to the region immediately seaward of the breaker line. The theory was checked for a regular periodic system of waves on a plane bottom of constant slope in which each wave was identical as it passed the same locale.

Natural wave systems in the ocean are never composed of identical waves. Theory on variable wave systems has not been developed to the extent of practically describing the history of a single wave in a group of mixed waves. Until such information becomes available the assumption is made that each wave acts as an entity with the transformation characteristics as described and is not influenced by preceding or succeeding waves (the results of this assumption appear to be serious). A further simplification is made in that a group of waves of variable height and period may be described by an average height and period. The transformations then apply to the average as being representative of the group.

The idealized system under consideration is one in which a series of long-crested deep-water oscillatory waves approaches a shoaling bottom. The bottom contours are parallel to the direction of travel of the wave front. A vertical section perpendicular to the wave front is diagrammed in Figure IC-1. In deep water the bottom is far enough removed from the surface motion of the wave that the motion is independent of the proximity of the bottom. The wave is characterized by a wave height, H_o, a wave length, L_o, and a wave period, T. As the wave proceeds shoreward into the region of smaller depths, the wave motion and geometry is affected by the proximity of the bottom. The wave height, velocity, and length change and the surface shape transforms. The number of waves passing a fixed point in a given length of time does not change. Hence, the period remains constant.

2. Waves of Small Amplitude (Irrotational Theory):

The manner in which the changes in height, length, and velocity take place may be developed from a simple wave theory. While not exact for all wave conditions, the results show the physical nature of the process, and the trends in the defining characteristics. The assumptions which are made in order to evaluate relationships from the mathematical formulation of wave motion are as follows:

(i) The wave height is small as compared to wave length and the water depth;

(ii) The motion is frictionless and irrotational;

(iii) The surface profile of the progressive wave is of a single cosine function form;

$$y/H = \cos(2\pi x/L) \qquad \text{(IC-2.1)}$$

(iv) The gradient of the bottom slope is small such that, at a given depth, the wave character is the same as that which would result if the wave were in a region of the same constant depth.

The solution for the system then gives the following results (Reference IC-3)

$$\text{Wave velocity, } C = (gT/2\pi)\tanh 2\pi d/L \qquad \text{(IC-2.2)}$$
$$\text{Wave length, } L = (gT^2/2\pi)\tanh 2\pi d/L \qquad \text{(IC-2.3)}$$

A fundamental relationship of any periodic motion relates the velocity, length, and period.

$$L = CT \qquad \text{(IC-2.4)}$$

In deep water, with the wave length, L_o, small as compared to the depth, d, tanh $2\pi d/L$ approaches unity and

$$C_o = gT/2\pi = 5.12T \text{ feet per second} \qquad \text{(IC-2.5)}$$
$$L_o = gT^2/2\pi = 5.12T^2 \text{ feet} \qquad \text{(IC-2.6)}$$

where T is measured in seconds. The minimum value at which tanh $2\pi d/L$ is unity, for most practical purposes, results in d/L approximately equal to 0.5. Thus, deep water, wherein the bottom proximity has no appreciable effect upon the wave characteristics, may be assigned to depths greater than one half the wave length. When the depth becomes small as compared to the wave length (about one-twenty fifth of the wave length), tanh $2\pi d/L$ approaches $2\pi d/L$, and equations (IC-2.2) and (IC-2.3) reduce to (see Figures IC-2 and 3)

$$C = \sqrt{g\,d} \qquad \text{(IC-2.7)}$$
$$L = T\sqrt{g\,d} \qquad \text{(IC-2.8)}$$

The velocity, C, becomes a function of the depth alone. All waves approach the same velocity in the shallow water near the shoreline.

The change in the wave height as a function of the water depth results from the consideration of the energy contained in a wave. The energy is of two forms, kinetic and potential. The kinetic energy is that which is present in a wave by virtue of the movement of the water. The potential energy results from the surface displacement from the mean sea surface. This theory shows that the total energy of a wave is made up of equal parts of kinetic and potential energies. Thus, the wave height is related to the potential energy of the wave and can be determined by considering the energy rate in the movement of the wave along a shoaling bottom. The wave energy per wave length per unit crest length, E, is (see Figure IC-4)

$$E = \rho g H^2 L/8 \qquad \text{(IC-2.9)}$$

while the energy rate per unit crest length, P, past a fixed point is

$$P = n E/T \qquad \text{(IC-2.10)}$$

where n is the fraction of the energy that proceeds with the wave. Under the assumption of no kinetic or potential energy loss due to frictional effects, the energy rates at two different locations in the path of the wave travel must be equal. The energy rate in deep water is equal to that in water at any depth in the wave path; hence,

$$P_o = P \qquad \text{(IC-2.11)}$$

Combining equations (IC-2.11, 2.10, 2.9, and 2.4) results in the wave height relationship

$$H/H_o = \sqrt{\left[\tfrac{1}{2}\,(1/n)(C_o/C)\right]} \qquad \text{(IC-2.12)}$$

The factor, n, is introduced since the theory shows that the wave energy travels, not with the wave velocity but with the group velocity; this ratio of the group velocity to the wave velocity is

$$C_g/C = n = \tfrac{1}{2}\left[1 + \left\{(4\pi d/L)/\sinh 4\pi d/L\right\}\right] \qquad \text{(IC-2.13)}$$

($n = \frac{1}{2}$ in deep water, where $d/L > 0.5$) .

A simplified picture of the group velocity as contrasted to the wave velocity may be visualized by considering a wave train of a fixed number of parallel waves of constant height and length being emitted by a wave generator. The total number of waves may be considered as a group. Quoting O'Brien (Reference IC-5):

> "As the first wave in the group advances one wave length, its form induces corresponding velocities in the previously undisturbed water and the kinetic energy corresponding to these velocities must be drawn from the energy flowing ahead with the form. If there is equipartition of energy in the wave, half of the potential energy which advances with the wave must be given over to the kinetic form and the wave loses height. Advancing another wave length another half of the potential energy is used to supply kinetic energy to the undisturbed liquid. The process continues until the first wave is too small to identify. The second, third, and subsequent waves move into water already disturbed and the rate at which they lose height is less than for the first wave. At the rear of the group, the potential energy might be imagined as moving ahead, leaving a flat surface and half of the total energy behind as kinetic energy. But the velocity pattern is such that flow converges toward one section thus developing a crest and diverges from another section forming a trough. Thus the kinetic energy is converted into potential and a wave develops in the rear of the group."

It is desirable to relate all the defining wave characteristics to the deep water wave characteristics. From equations (IC-2.2) and (IC-2.5),

$$C/C_o = \tanh 2\pi d/L \qquad \text{(IC-2.14)}$$

from equations (IC-2.3) and (IC-2.5),

$$L/L_o = \tanh 2\pi d/L \qquad \text{(IC-2.15)}$$

At values of L/L_0 corresponding to d/L from equation (IC-2.15), the product $(L/L_0)(d/L) = d/L_0$ is obtained. This is unique and results in d/L, C/C_0, and L/L_0 as functions of d/L_0. Likewise, since H/H_0 is a function of n and C/C_0, which are in turn functions of d/L, or d/L_0, then H/H_0 is a function of d/L_0. Thus, the wave height, length, and velocity are related to the depth and deep water length, height and velocity. Equations (IC-2.12, 2.13, 2.14 and 2.15) are shown on Figure IC-5.

The wave height, H, from equation IC-2.12, as a function of depth, can be interpreted from a consideration of the two factors $1/n$ and C_0/C. As a water wave advances along over a shoaling bottom, the wave height first decreases and then increases. C_0/C continuously increases while $1/n$ continuously decreases as the depth becomes smaller. The influence of the transformation of the energy into that moving with the wave, and hence not available to increase the height, predominates over the decrease in velocity until a minimum height is attained. The reverse predominance then occurs to increase the wave height.

3. Waves of Appreciable Amplitude:

The small amplitude irrotational wave theory with the resulting relationships serves to illustrate the changes in wave height, length, and velocity as a wave moves along a shoaling bottom. The theory is valid for the conditions under which it was developed, namely, wave height small as compared to the wave length and depth of water, long-crested waves, irrotational motion, and no frictional effects. However, observations of waves approaching a beach show a wave shape which is not sinusoidal, but which has crests that are peaked with relatively long flat troughs between the crests.

a. Reduced Trochoidal Theory:

One attempt to represent a more natural wave shape was to apply the reduced trochoidal function (Reference IC-1), even though this theory did not fulfill either the conditions of continuity or dynamical equilibrium except at the wave trough and crest. The results are included here mainly to show that, although the wave shape is closely approximated, the other phenomena (such as mass transport) are not predicted and hence, the theory should be discarded.

The surface profile is given by surface coordinates in terms of an argument θ, which varies from 0 to 2π over one wave length.

$$\begin{aligned} y/H &= \cos\theta/2 \\ x/L &= (\theta/2\pi) - (H/2L \tanh 2\pi d/L) \sin\theta \end{aligned} \qquad \text{(IC-3.1)}$$

The wave length and velocity that result from this condition of a surface profile are the same as those developed from the small amplitude irrotational theory, equations IC-2.2, 2.3, 2.14, and 2.15. The wave height transformation, as a function of depth, becomes

$$H/H_0 = \sqrt{\tfrac{1}{2}(1/n)(C_0/C)\left[\frac{1 - (\pi^2/2)(H_0/L_0)^2}{1 - M\,(H/L)^2}\right]} \qquad \text{(IC-3.2)}$$

where $M = \pi^2/(2\tanh^2 2\pi d/L)$. The wave height transformation becomes the same as that of the small amplitude irrotational theory for waves of small height as compared to the wave length and depth.

b. Irrotational Theory:

The irrotational theory for waves of appreciable height in water of uniform depth was developed by Stokes (Reference IC-10), Rayleigh (Reference IC-9), and Levi-Civita (Reference IC-4). Experimental evidence substantiates the conclusion that this is the theory which most nearly represents actual wave motion. The theory predicts a small mass transport of water in the direction of wave motion: the water particle paths are open ellipses in shallow water rather than closed ellipses as needed in both the reduced trochoidal and the small amplitude irrotational theory. In nature, the particle paths are open and there is a mass transport.

The surface profile, to the third approaimation of the irrotational theory analysis, becomes

$$\frac{y}{H} = \frac{\cos \frac{2\pi x}{L}}{2} + \frac{\pi H}{2L}\left(\cos \frac{4\pi x}{L}\right)(F) + \frac{\pi^2 H^2}{4 L^2} \cos \frac{6\pi x}{L} \; (G) \qquad \text{(IC-3.3)}$$

where,

$$F = \frac{\left(\cosh \frac{2\pi d}{L}\right)\left(\cosh \frac{4\pi d}{L} + 2\right)}{8\left(\sinh \frac{2\pi d}{L}\right)^4}$$

$$G = \frac{\cosh \frac{12\pi d}{L} + 14 \cosh \frac{8\pi d}{L} + 19 \cosh \frac{4\pi d}{L} + 16}{32\left(\sinh \frac{2\pi d}{L}\right)^6}$$

The velocity of wave propagation becomes a function of the wave height in addition to the depth of the water and wave length.

$$C = (g\,T/2\pi)(\tanh 2\pi d/L)\left[1 + B(\pi H/L)^2\right] \qquad \text{(IC-3.4)}$$

where:

$$B = \frac{2(\cosh 4\pi d/L)^2 + 2(\cosh 4\pi d/L) + 5}{8\,(\sinh 2\pi d/L)^4}$$

Combining equations (IC-3.4 and 2.4) will give the wave length in terms of the wave period, height, and depth of water. When the water depth is large as compared to the wave length, the factor B becomes unity, and tanh $2\pi d/L$ becomes unity. Combining equations (IC-3.4) and (IC-2.4), the following ratios are obtained:

$$C/C_o = L/L_o = (\tanh 2\pi d/L)(1 - \pi^2 H^2 B/L^2)/(1 + \pi^2 H_o{}^2/L_o{}^2) \qquad \text{(IC-3.5)}$$

It should be noted that this analysis reduces to the small amplitude irrotational theory when the waves are of small height compared to the wave length.

The complexity of the resulting equations prohibits a reasonable development for a wave height transformation function of the type shown in equation (IC-2.12) for the small amplitude irrotational theory.

The foregoing results of the theoretical analyses may be compared to note the orders of magnitude of the refinements in the analyses. Measurement of wave characteristics in the laboratory and on ocean waves are used to confirm the analytical results. Some conclusions are then reached as to the practical application of the results.

The theories indicate the variables, and combinations of the variables, by which correlations and comparisons may be made. From the equations which have been presented, the prime variables are the wave period, the wave height, and the depth of water at the wave location. The wave length and velocity are uniquely defined in terms of these variables.

4. Wave Surface Profiles:

Comparisons of the wave surface profiles of equations (IC-2.1) and (IC-3.1) are shown in Figure IC-6. The trochoidal wave shape of equation (IC-3.1) is steeper near the crest and flatter near the trough than the sine wave, equation (IC-2.1). Equation (IC-3.3) is in the form of a series of cosine harmonics. More terms should be included in the approximation in order to make reasonable comparisons of the wave shape of equation (IC-3.3) as compared to equation (IC-3.1). Interpretation of the equation reveals that the wave shape is steeper near the crest and flatter near the trough when compared to the trochoid.

One point should be noted in that the foregoing is based upon waves which are symmetrical about the crest. That is, the foreward half of the wave from the crest to the preceding trough is identical in shape to the rear half of the wave from the crest to the following trough. Waves which advance into shoaling water eventually reach a condition of instability and break. Observations show that the symmetrical shape of the wave is not maintained close to the breaker point, but that the front face of the wave tends to become steeper than the rear face (Figure IC-7b). Thus, the theories do not explain the wave action near and at breaking. Figures IC-7a and 7b are included as representative examples of measured wave shapes. The sine wave shape does not differ appreciably from the trochoidal wave shape to the limit of applicability of the assumption of a symmetrical wave.

A limiting deep-water stable wave height does result from the irrotational theory. A crest angle of less than 120° cannot be maintained. Under these conditions the limiting wave is shown to have a height-length ratio of 0.142.

5. Wave Velocity and Wave Length:

Equations (IC-2.2) and (2.3) represent the wave velocity and wave length as developed from the small amplitude irrotational theory. They also apply to the reduced trochoidal wave. Equations (IC-3.4) and (IC-2.4) result in the wave velocity and wave length from the irrotational theory for waves of appreciable amplitude. In deep water, with the depth large as compared to the wave length the equations reduce to,

Small amplitude irrotational theory and reduced trochoid theory:

$$C_o = gT/2\pi ; \quad L_o = gT^2/2\pi \qquad \text{(IC-2.5 and 2.6)}$$

Irrotational theory for waves of appreciable amplitude:

$$C_o = \frac{g}{2\pi} T \left[1 + \left(\frac{\pi H_o}{L_o} \right)^2 \right] ; \quad L_o = \frac{g}{2\pi} T^2 \left[1 + \left(\frac{\pi H_o}{L_o} \right)^2 \right] \qquad \text{(IC-5.1)}$$

According to the irrotational theory for waves of appreciable height a wave travels faster than expected from the small amplitude theory. The velocity depends upon the wave steepness in addition to the period. For a 50 foot wave height with a period of 10 seconds, the two results differ by 9.4%. This corresponds to an extreme wave condition that is seldom encountered except in the immediate region of a great storm. For waves of smaller height, with the same period, the differences are not significant.

The length and velocity ratios as described by equations (IC-2.14) and (IC-2.15) from the sine theory, and by equation (IC-3.5) from the irrotational theory for waves of appreciable amplitude, are the most useful for wave travel prediction from known deep water conditions to conditions at shallower depths. These are shown in Figure IC-8 as computed from the respective equations. The "height" correction to the irrotational theory becomes increasingly larger as the depth into which the wave moves becomes smaller. However, the non-symmetry of a wave as it approaches the breaking stage, the real effects of bottom friction that tend to retard the wave, and the greater inexactness of the series approximation of the irrotational theory at the smaller depths precludes comparing the theories beyond some minimum relative depth.

6. Wave Height Transformation:

The sine wave theory predicts the change in wave height as a wave advances over a shoaling bottom, equation (IC-2.12). The more complicated theories of the trochoidal wave shape and irrotational motion for waves of appreciable amplitude show that the wave height transformation depends upon the original wave height in addition to the water depth. No direct analysis has been made for this condition. However, a reasonable approximation of the wave height influence may be obtained by including the velocity ratio of equation (IC-3.5) with the trochoidal wave height relationship of equation (IC-3.2). The result is

$$H/H_o = \tfrac{1}{2}\,(1/n)(\tanh 2\pi d/L)\left[\frac{1 + B(\pi H/L)^2}{1 + (\pi H_o/L_o)^2}\right]\left[\frac{1 - (\pi^2/2)(H_o/L_o)^2}{1 - M\,(H/L)^2}\right] \qquad \text{(IC-6.1)}$$

where B and M are defined in equations (IC-3.4) and (IC-3.2).

The deep water wave steepness is a parameter to modify the small amplitude irrotational theory wave height transformation, Figure IC-9. The small amplitude irrotational theory relationship is an asymptotic condition and applies when the deep water wave steepness is small or when the depth is large.

7. Laboratory Studies:

The relationships of wave height, length, and velocity with the wave height parameter, Figure IC-8, are shown over the actual range of applicability of the theories. This limit is imposed by the deviation of the wave shape from the symmetrical shape upon which the theories are based.

Limits and usefulness of the preceding have been determined by measurements of wave characteristics. The ultimate application is to predict the behavior of ocean waves advancing over a shoaling bottom from a region of known wave character. The ocean is a difficult medium with which to operate to obtain precise data. Deep water conditions are usually far offshore at depths of water that prohibit fixed platforms. Measurement of wave heights, lengths, and velocities are difficult. In addition, the wave pattern in the ocean is never regular. Reliable formulation of any phenomena necessitates fixing certain variables and control over others. Thus, laboratory controlled experiments over wide ranges of the variables are the best means by which to obtain confirmation of the developed theories.

Velocity measurements in the laboratory are shown in Figure IC-9 as compared to the small amplitude irrotational theory computations (Reference IC-11). On the average, the measured and computed results agree. Some experimental scatter may be noted which is attributed mainly to minor surface irregularities such as ripples superimposed on the main wave shape that makes detection of the true wave crest somewhat difficult.

Wave height transformations have been obtained as a function of depth for different initial wave conditions (Reference IC-2 and 7). When damping due to side wall and bottom friction are included in the measured results, the comparisons of measured and predicted results show the sine theory to be valid, Figure IC-10.

In addition, the measured results show the limiting depth to which the theory is applicable to predict wave heights. The measured wave height transformation shows a sharp increase in wave height as the wave nears the breaking point. The ratio of increase is larger than that predicted by the theories. Observation of the wave shape shows that the wave profile at depths close to the breaking point is no longer symmetrical. Hence, the theory does not apply for depths lower than those indicated by the marked deviation from the theoretical relationships.

These minimum depths are a function of the initial wave steepness. They have been established from the laboratory measurements and are shown on Figure IC-11. The results of Figure IC-11 have been included as cut-off points on Figure IC-8.

The use of the sine theory relationships are further substantiated for wave transformation as a function of depth as shown by the limits on Figure IC-8. The deviations from sine theory in the range where any practical theory now known is applicable are not of practical significance for ocean wave predictions.

All preceding relationships of length, velocity, and wave height transformation have been confirmed to the limiting depth as determined from the experiments. Within practical accuracies of usage the sine theories are adequate at depths greater than the minimum specified depth. At depths less than the established minimae no present theory is valid.

Thus, the important relationships may be summarized, for depths greater than the minimae as shown on Figure IC-11.

$$L = C\,T \qquad \text{Eq. (IC-2.4)}$$
$$C = (gT/2\pi)\tanh 2\pi d/L \qquad \text{Eq. (IC-2.2)}$$
$$L = (gT^2/2\pi)\tanh 2\pi d/L \qquad \text{Eq. (IC-2.3)}$$
$$C/C_o = L/L_o = \tanh 2\pi d/L \qquad \text{Eq. (IC-2.14), Eq.(IC-2.15)}$$
$$H/H_o = \tfrac{1}{2}\,(1/n)(C_o/C) \qquad \text{Eq. (IC-2.12)}$$

For depths less than those specified on Figure IC-11, reliable experimental evidence on wave height transformations, particularly of height, are not predictable by theory and have not been obtained. Some laboratory evidence is available, but frictional effects become increasingly important in the limited size channels in which the measurements were made. Corrections are possible, as shown in Figure IC-11 to the limiting depth. At smaller depths, the corrections cannot be made to the desired accuracy and interpretation of the results does not lead to reliable results.

The assumption has been made that the transformations are independent of beach slope. The evidence shown on Figure IC-10 includes two beach slopes. Within the mensuration accuracies beach slope has no effect. Since the theories should apply best for the shallower slopes, with small depth gradients, the conclusion is reached that for slopes shallower than 1:15 the small amplitude irrotational theory is adequate.

Confirmation has been established for wave transformations in shoaling water for a regular periodic system of waves in which each wave is identical in character as it passes the same locale. Natural wave systems in the ocean are never composed of identical waves. A typical ocean surface profile is shown in Figure IC-12. Information on variable wave systems has not been developed to the extent of describing the history of a single wave in a group of mixed waves. Until such information becomes available the assumption is made that each wave acts as an entity with the transformation characteristics as described and is not influenced by preceding or succeeding waves. A further simplification is made in that a group of waves may be described by an average height and period. The transformations then apply to the average as being representative of the group.

8. Bottom Friction on Ocean Beaches:

Frictional effects due to the relative motion of the water and the stationary bottom as a wave moves toward the shore can be analyzed to obtain the order of magnitude of the damping effect particularly on the wave height transformation. Another effect which occurs when wave action is present over permeable bottoms such as sand beaches also produces a damping action. Part of the energy of a wave is dissipated due to both causes with a consequent effect upon the wave height.

The decrease of wave height due to the absorption of part of the wave energy in bottom friction and movement of the water in the permeable bed of the beach has been analyzed (References IC-6 and 8). The type of bottom roughness has a large influence on the frictional effects; wave movement over a bed of rocks will decrease the wave heights to a greater degree as contrasted to movement over fine sand. To the limit of the depths for which the transformation results in Figure IC-8 apply, the maximum effect of the dissipation on sand beaches is not expected to exceed 5 percent on flat beaches with slopes on the order of 1:300. The combined effects are less than 5% on steeper slopes.

9. Refraction:

The foregoing has been presented for waves which approach a plane shoaling bottom wherein the bottom contours are parallel to the crests of the waves' travel. When waves approach a sloping shoreline which is at an angle to the wave crests, the part of the wave closest to the shore is in shallower water. The velocity of this portion of the wave is slower than the more seaward portions of the wave. The wave crest thus becomes curved with the wave in shallow water near the shore tending to become parallel to the shoreline. This phenomena of refraction can be handled from a consideration of the lateral distribution of energy contained in a wave. The influence of refraction on the wave height, length, and velocity thus is amenable to analysis. Detailed coverage of this phenomenon is presented in Section I-D (Refraction) of this Manual.

10. References:

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IC-2: Iversen, H. W. - "Wave Height Transformation over a Shoaling Bottom", Series 3, Issue 331, Institute of Engineering Research, University of California, Berkeley, California, 1 September 1951 (unpublished).

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IC-5: O'Brien, M. P. - "A Summary of the Theory of Oscillatory Waves" - Beach Erosion Board (U. S. Army, Corps of Engineers), Technical Report No. 2, 1942.

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IC-8: Putnam, J. A. and Johnson, J. W. - "The Dissipation of Wave Energy by Bottom Friction" - Transactions, American Geophysical Union, Vol. 30, No. 1, February 1949, pp. 67-74.

IC-9: Rayleigh, Lord - "On Progressive Waves" - Proceedings of the London Mathematical Society, Vol. IX, 8 November, 1877, pp. 21-26.

IC-10: Stokes, G. G. - "On the Theory of Oscillatory Waves" - Transactions, Cambridge Philosophical Society, Vol. 8, 1847, p. 441, and Supplement, Scientific Papers, Vol. I, p 314.

IC-11: Wiegel, R. L. - "Some Studies of Surface Waves in Shoaling Water" - M. S. Thesis, University of California, Berkeley, California 1948, (unpublished). Also: "Experimental Study of Surface Waves in Shoaling Water" - Transactions, American Geophysical Union, Vol. 31, No. 3, June 1950, pp. 377-385.

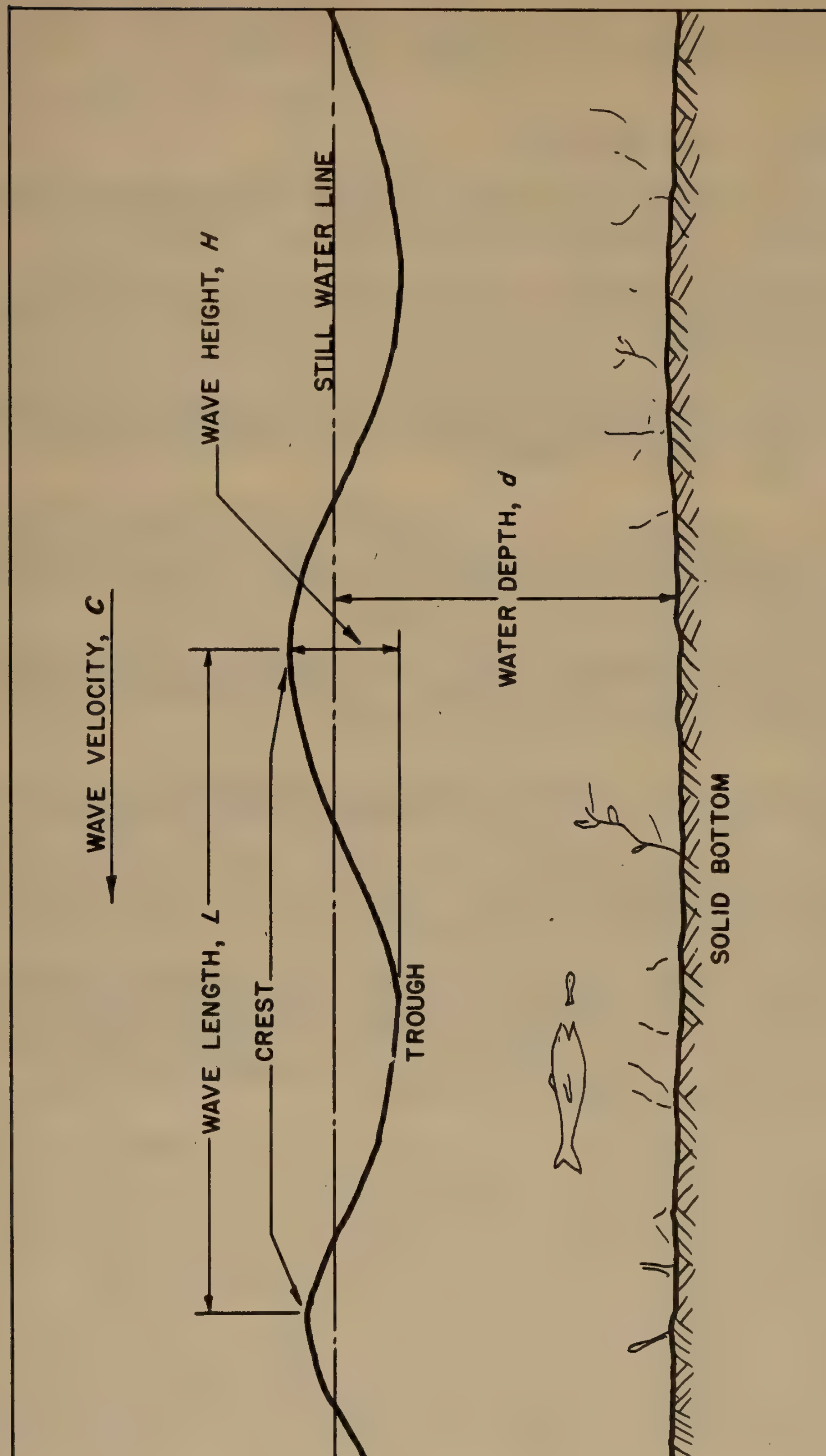


FIG. IC-1 - DIAGRAM OF A WAVE CROSS-SECTION

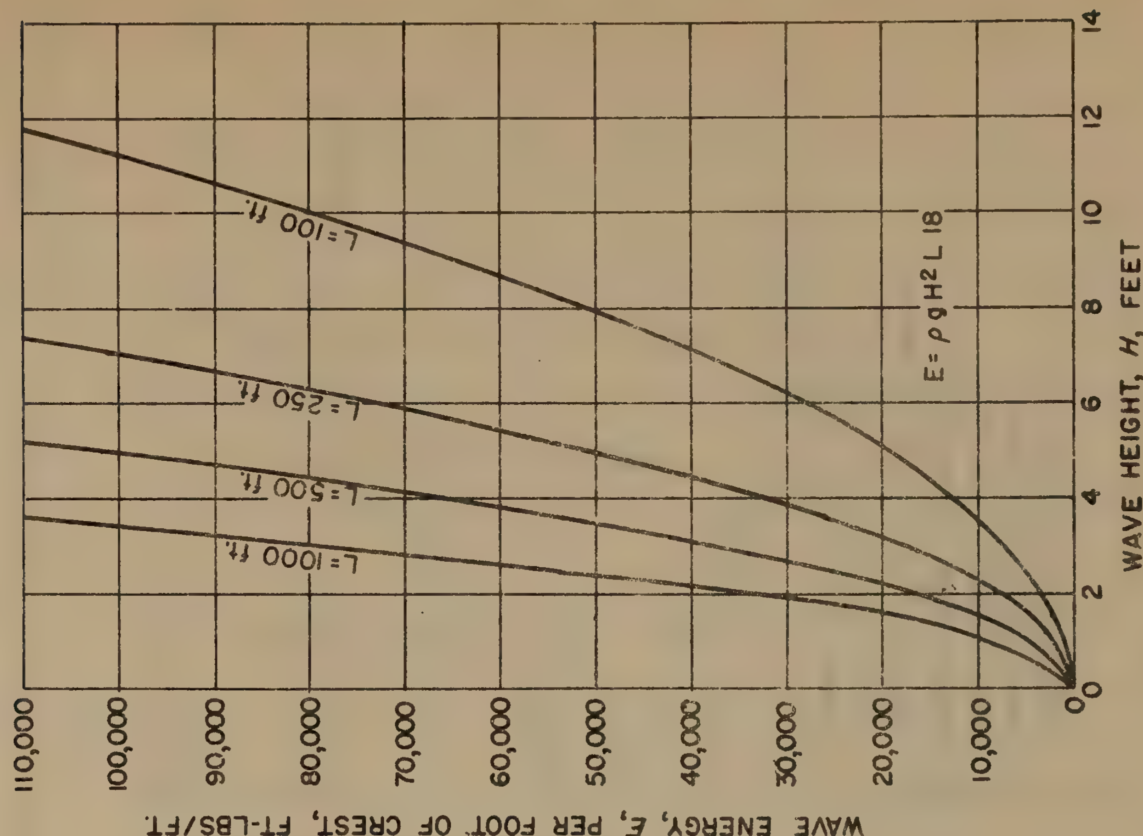


FIG. IC-4 - WAVE ENERGY

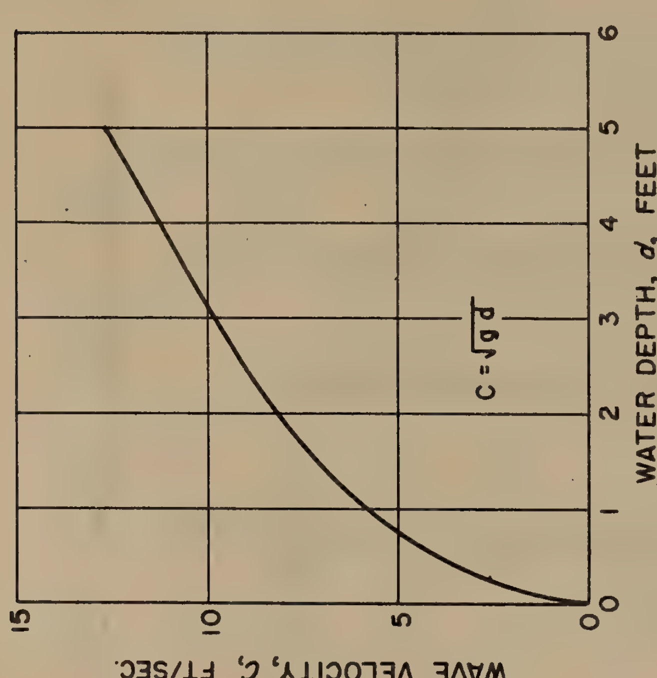


FIG. IC-2

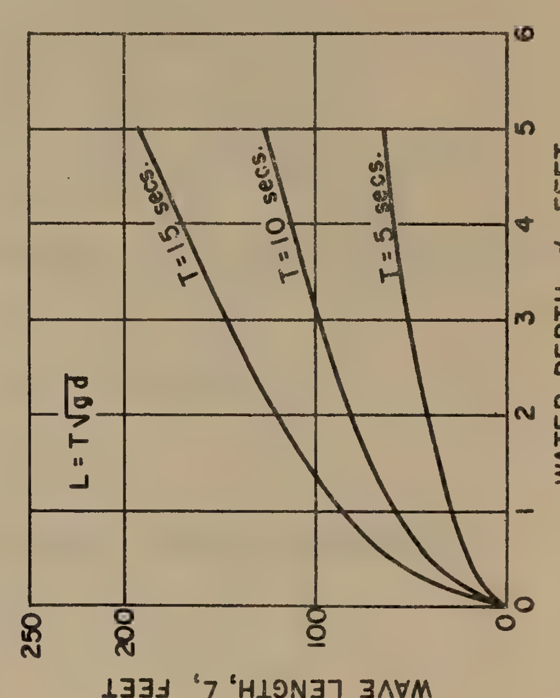


FIG. IC-3

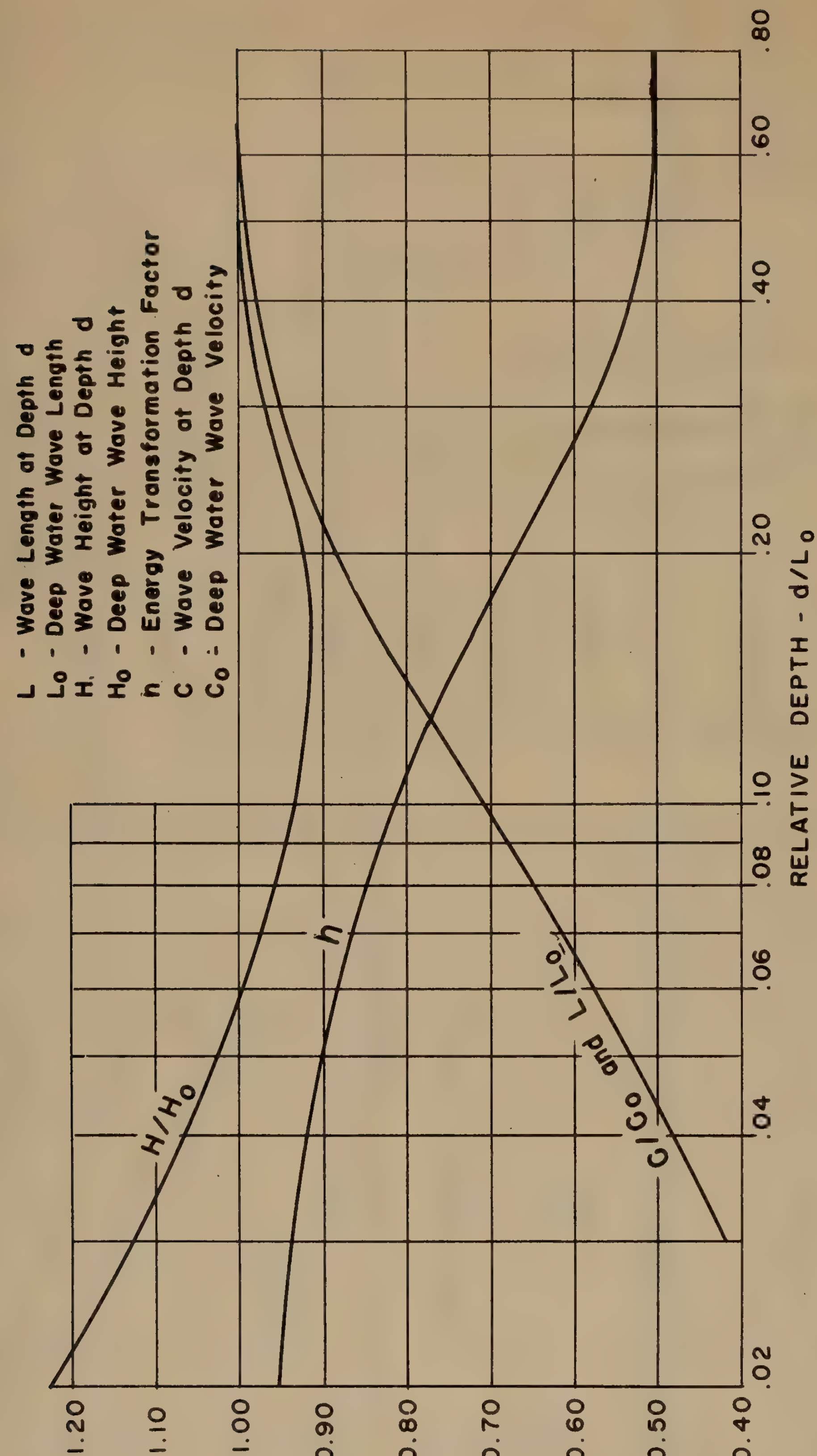
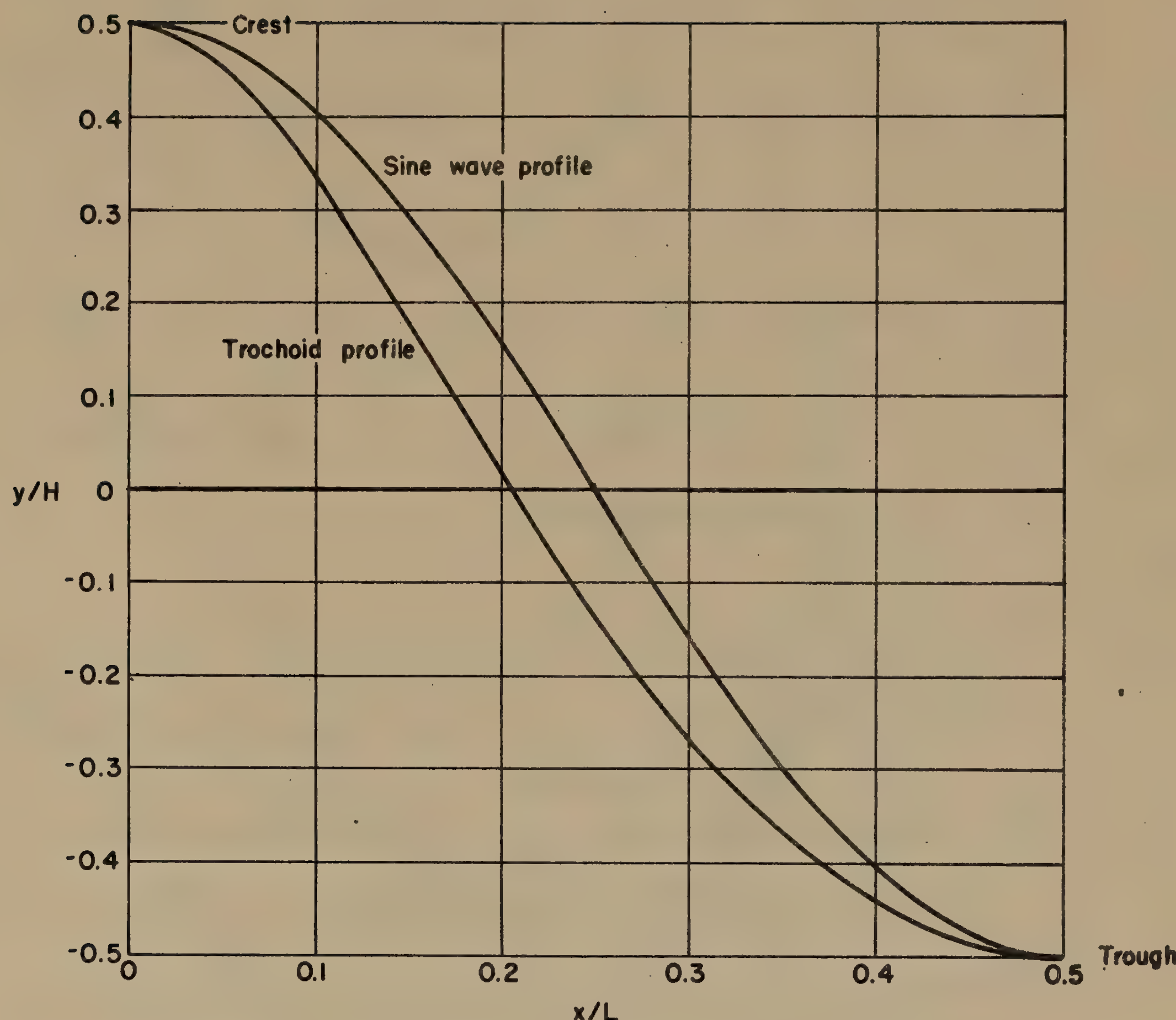
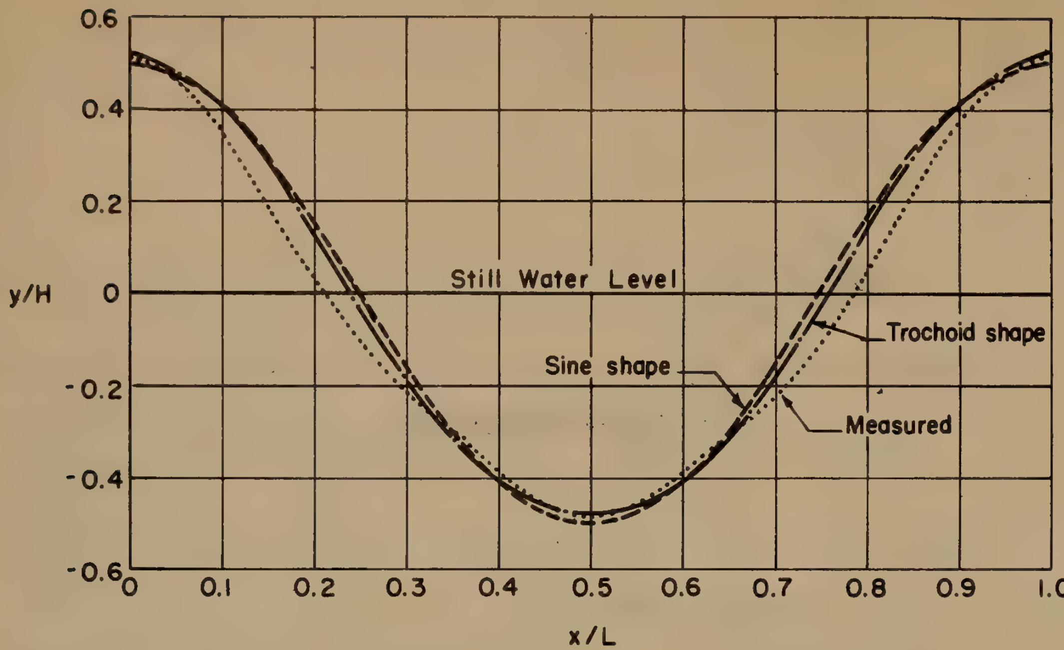


FIG. IC-5 - WAVE TRANSFORMATIONS AS FUNCTION OF DEPTH FROM SMALL AMPLITUDE THEORY



Shapes of wave profiles: $d/L = 0.10$, $H/L = 0.05$

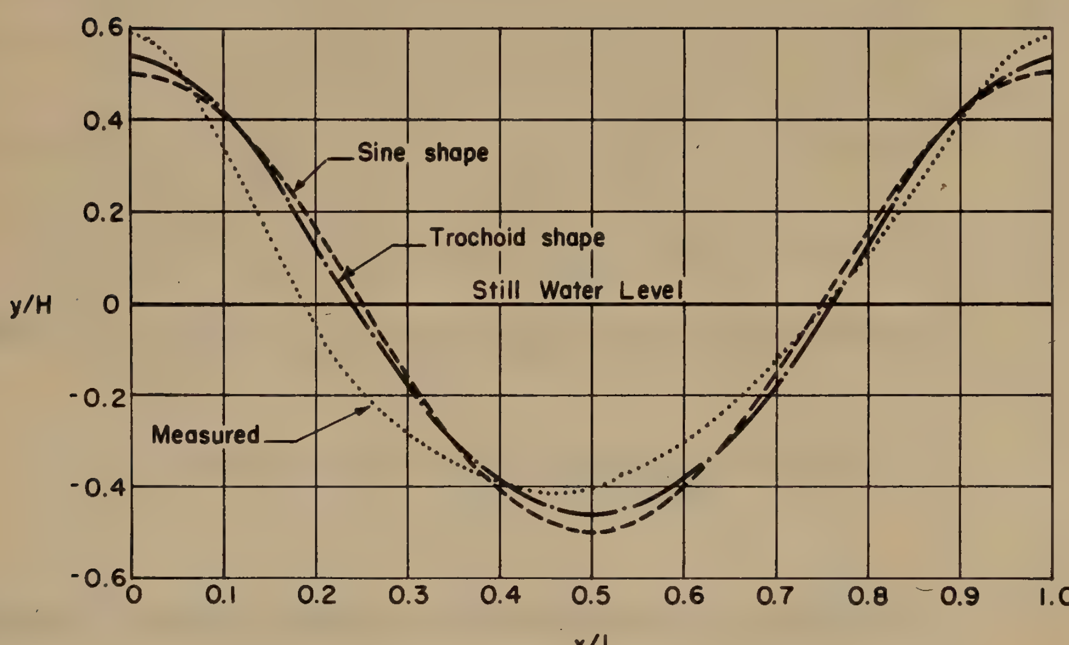
FIG. IC-6 - COMPARISON OF WAVE SURFACE PROFILES



H/L = 0.0263   Period = 1.16 secs.

d/L = 0.193   Water depth = 0.99 ft.

a. Symetrical wave shape



H/L = 0.0324   Period = 1.16 secs.

d/L = 0.118   Water depth = 0.51 ft.

b. Wave shape near breaking

FIG. IC-7 - COMPARISON OF MEASURED & PREDICTED WAVE SHAPES

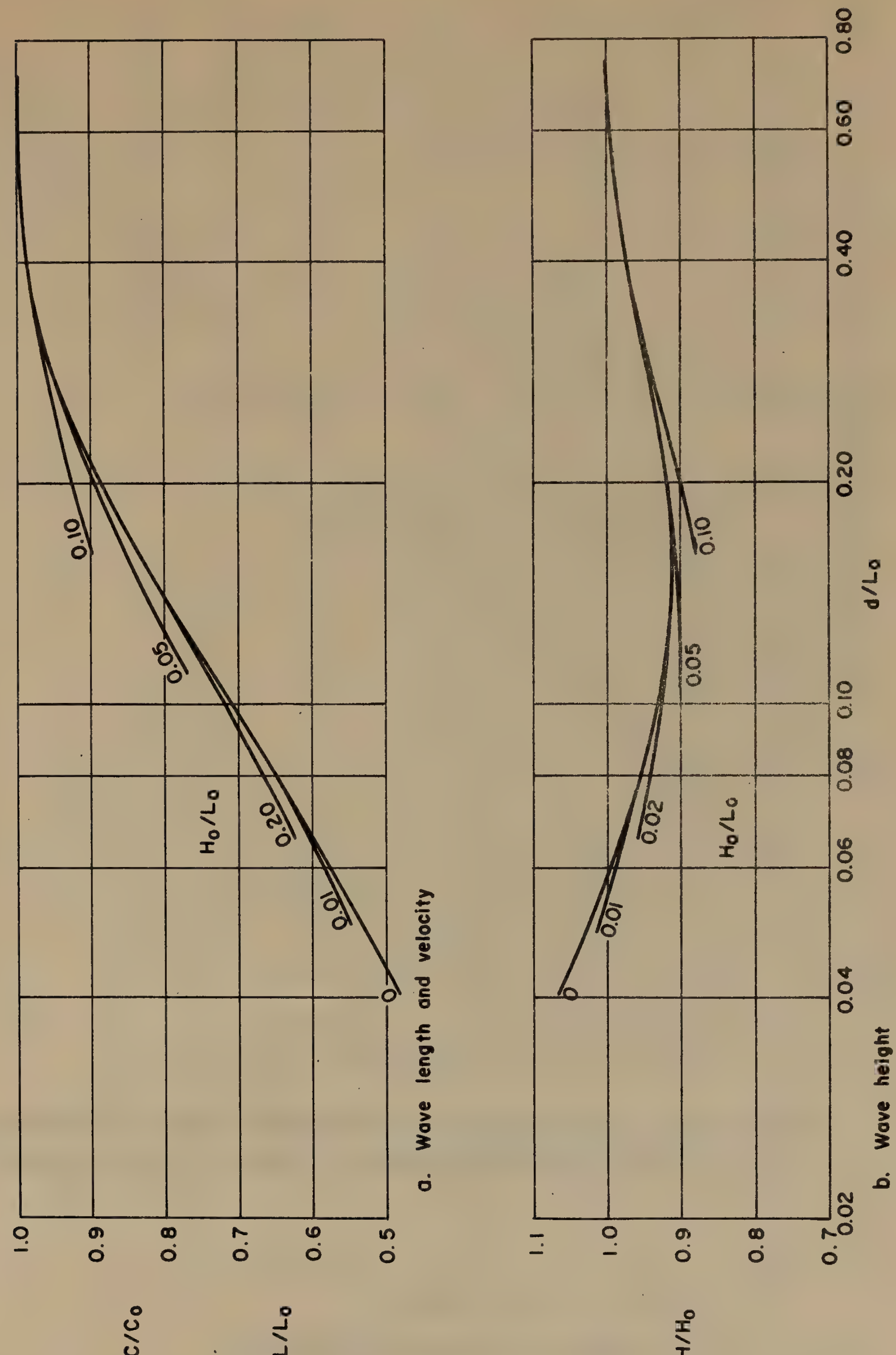
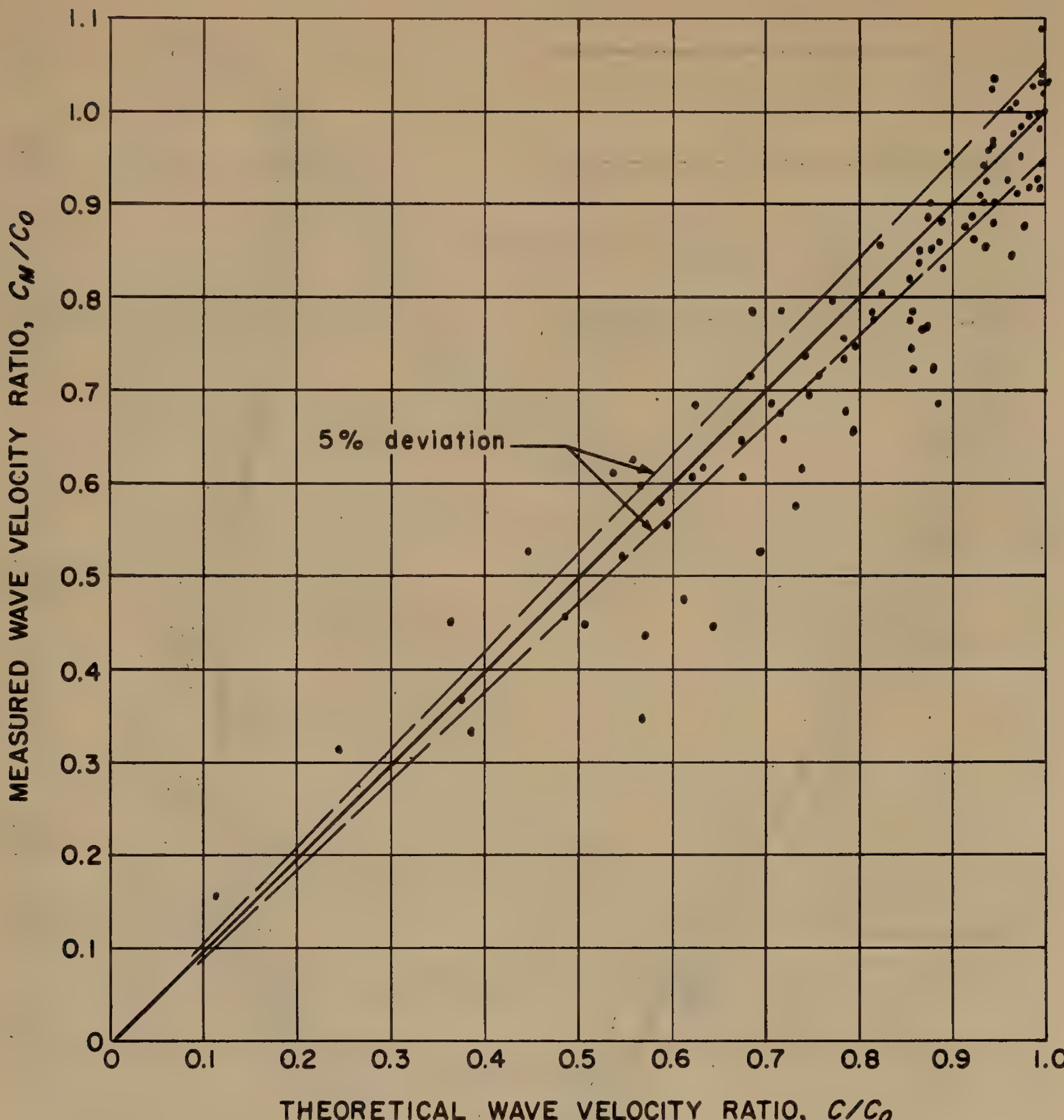


FIG. IC-8 - EFFECT OF FINITE HEIGHT ON
WAVE LENGTH, VELOCITY AND HEIGHT TRANSFORMATIONS IN SHOALING WATER



H_0/L_0 from 0.01 to 0.04

d/L_0 from 0.01 to 0.60

Theoretical velocities from sine theory

FIG. IC-9 - COMPARISON OF MEASURED AND THEORETICAL WAVE VELOCITIES FROM LABORATORY EXPERIMENTS

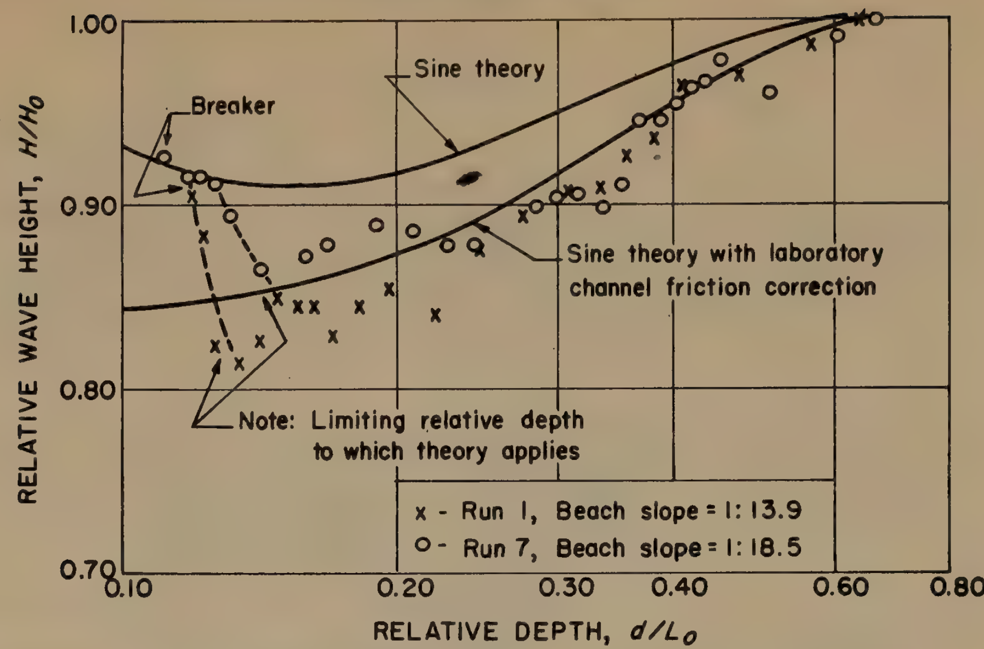
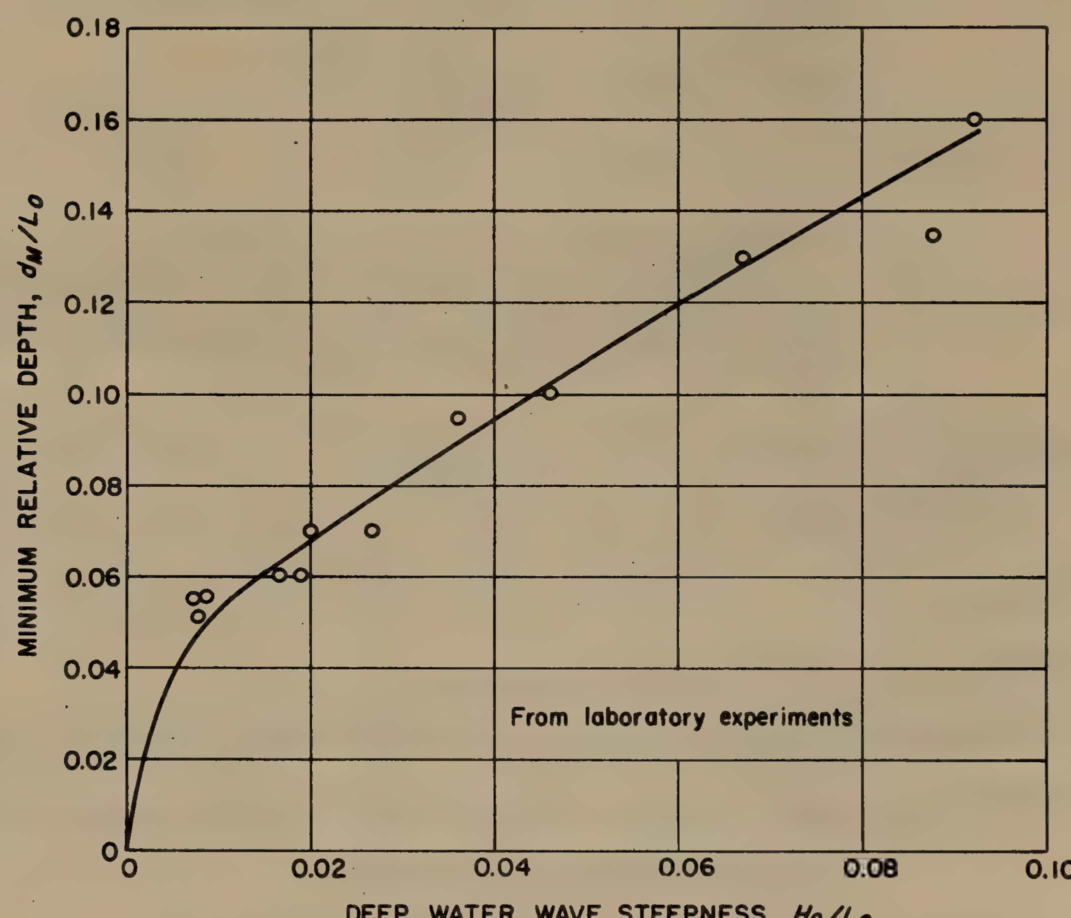


FIG. IC-10 - EXPERIMENTAL CONFIRMATION OF WAVE HEIGHT TRANSFORMATION ON A SHOALING BEACH

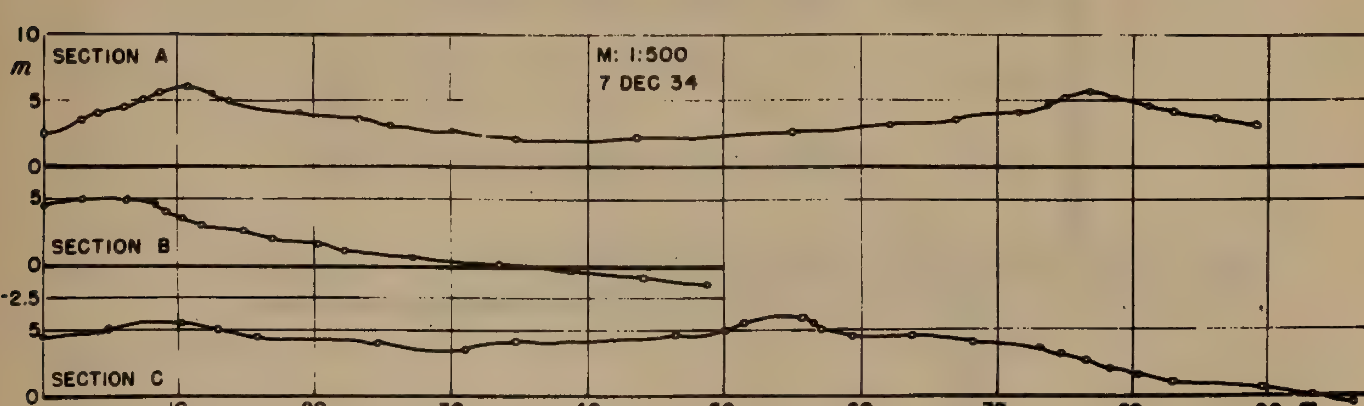


d_M - Minimum depth
L_0 - Deep water wave length
H_0 - Deep water wave height

FIG. IC-11 - MINIMUM DEPTH TO WHICH WAVE THEORIES APPLY FOR WAVE HEIGHT TRANSFORMATIONS IN SHOALING WATER

a. Stereophotogrammetric Plot of Wave Formation from set of photographs.

The photograms for the above figure were taken aboard Test Expedition on MS SAN FRANCISCO by Dr.-Ing.habil. Georg Weinblum, Prussian Research Institute for Hydraulics and Shipbuilding. Plotted with aerocartograph by Walter Block, Engineer in charge of measurements. Chair of Photogrammetry at the Technological Institute, Berlin (Prof. Dr.-Ing. Lachman).



b. Wave Profiles from set of photographs.

FIGURE IC-12

(from "Stereophotogrammetric Wave Photographs" (*Stereophotogrammetrische Wellenaufnahmen*) by Georg Weinblum and Walter Block, translated by F.A. Raven, PhD)

SUMMARY OF SECTION I-D

REFRACTION

BY

R. L. WIEGEL

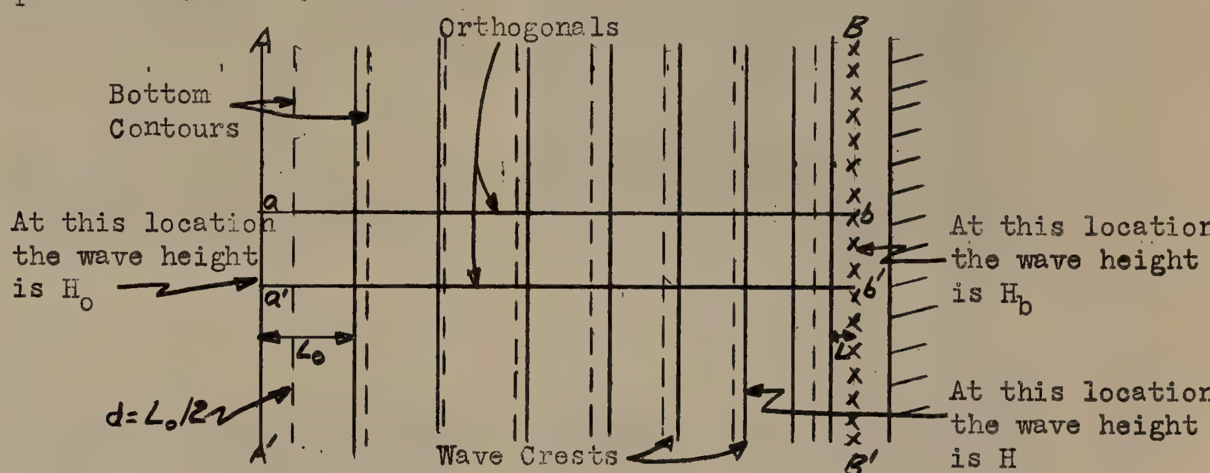
Refraction - General Discussion:

In shallow water the velocity of a wave depends upon the depth of water in which it is travelling. When a wave is travelling over a shoaling bottom at an angle to the contours different parts of the waves will be travelling at different velocities; hence the wave bends. This bending process is known as refraction. (Refraction is also caused by currents such as near the entrances to rivers, bays, etc.).

The phenomenon of wave refraction causes variations in wave and breaker heights on different beaches and over different areas of the same beach. Ocean wave refraction is analagous to light wave refraction (as through a prism). Examples are given in Figures SID - 1 through 4.

Significance of Refraction

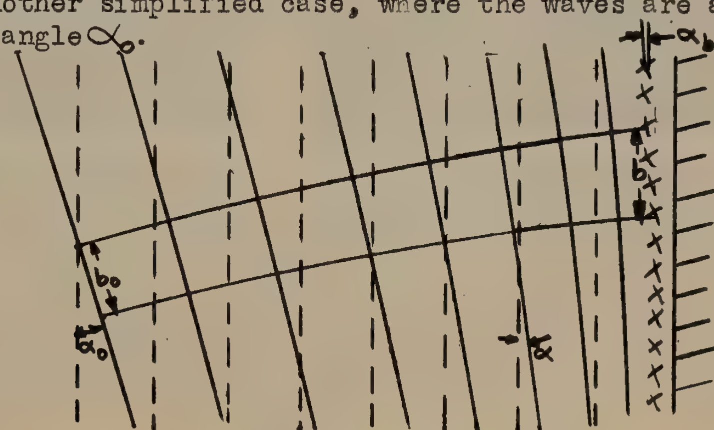
Consider first the case of waves approaching a straight shore over straight parallel contours. It can be seen



in the sketch that the wave crests do not bend. Because of this there is no divergence or convergence of the orthogonals (a line drawn at right angles to the wave crests). Since it is assumed (apparently a correct assumption for most conditions) that there is no flow of energy across orthogonals, the power being transmitted past bb' is the same as past aa'. Hence, the wave height of the wave as it assumes position B is given by the equation

$$\frac{H}{Ho'} = \sqrt{\frac{1}{2}\ \frac{1}{n}\ \frac{1}{C/C_o}}$$

Now, consider another simplified case, where the waves are approaching the same beach at an angle α_o.



As the distance between the orthogonals has increased from b_o (distance between a and a') to b (distance between b and b') the energy per foot of wave crest has decreased and hence the power transmitted per foot of wave crest has decreased. Hence, the wave height, considering refraction alone has decreased by an amount.

$$H = KH_o$$

where K (the refraction coefficient) is given by

$$K = \sqrt{\frac{b}{bo}}$$

Thus, the wave height along bb' is given by

$$H = Ho\ K\sqrt{\frac{1}{2}\ \frac{1}{n}\ \frac{1}{C/Co}}$$

Certain types of underwater contours cause a focusing effect - similar to light being focused through a lens. In these cases the orthogonals converge and the wave heights increase.

There is another effect of refraction. It is the change of angle between the waves and the shoreline. Since the velocity of littoral currents (currents set up parallel to shore due to waves breaking at an angle to the beach) is proportional to the angle the breakers made with the beach refraction drawings must be made so that this angle α, may be determined.

Determination of the Effects of Refraction:

Qualitative effects of refraction may be estimated by experienced persons from inspection of hydrographic charts or aerial photos. Such estimation by inspection is seldom sufficiently accurate, quantitatively, for practical purposes. The best estimates of effects of refraction involve construction of refraction diagrams as in Figure SID-5. Under certain ideal conditions these diagrams may be constructed from aerial photos but they are usually constructed from hydrographic charts. The diagrams should be constructed under the supervision of a person who has had experience in this field. The construction of these diagrams is laborious and time-consuming, but the results usually justify the effort expended. In general, waves of about five different periods from about seven different directions may require examinations for refraction effects on a particular section of coast, which would require some thirty-five refraction diagrams. From these diagrams refraction coefficients may be determined which express the relationship between deep-water wave height and shallow water wave height (specifically, breaker height) as affected by refraction as in Figure SID-6. In some cases, if the coast is partially sheltered by offshore islands or headlands, the number of diagrams required may be reduced by inspection of the factors involved. The refraction coefficients are used in conjunction with wave forecasts or offshore observations to help determine breaker conditions near the beach.

Reliability of Refraction Coefficients:

The methods for construction of refraction diagrams presented in this manual are reasonably accurate, but their limitations must be recognized. If the hydrographic charts are up-to-date and have been plotted from a large number of soundings, if the hydrography is fairly regular without sharp discontinuities, and if the construction of the diagrams has been done with care, refraction coefficients may be expected to be accurate within the order of five or ten per cent. A complete refraction picture should be developed even on the basis of limited information.

FIG. SID-1

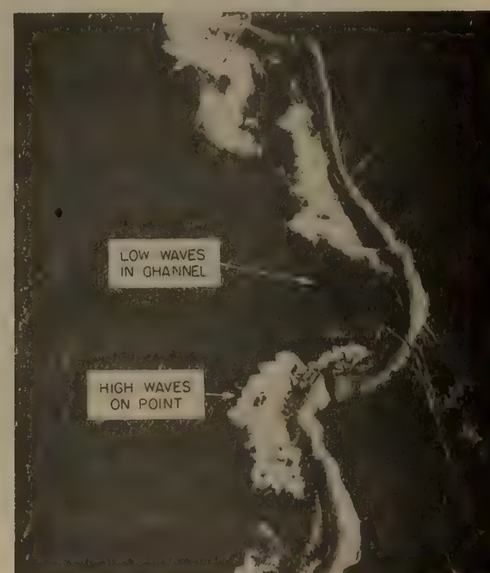
REFRACTION OF WAVES

Shoaling Bottom

As waves move into shallow water and approach the shore at an angle to the bottom contours, the waves are bent. This is known as refraction. The bending of waves may cause a converging or diverging of the crests (depending upon the hydrography) with a subsequent increase or decrease in wave or breaker height. Another effect of waves approaching a beach at an angle is to induce a longshore current inside the breaker zone. This current is effective in transporting sand along the shore and is also a hazard to landing craft operating through the surf zone.

Pt. Pinos, California

Waves moving over a submarine ridge concentrate to give large wave heights on a point.

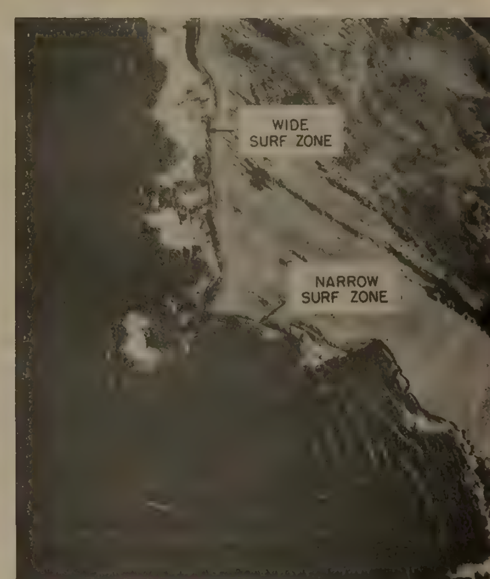


Arena Cove, California

Spreading of waves by refraction produces low wave heights at the pier.

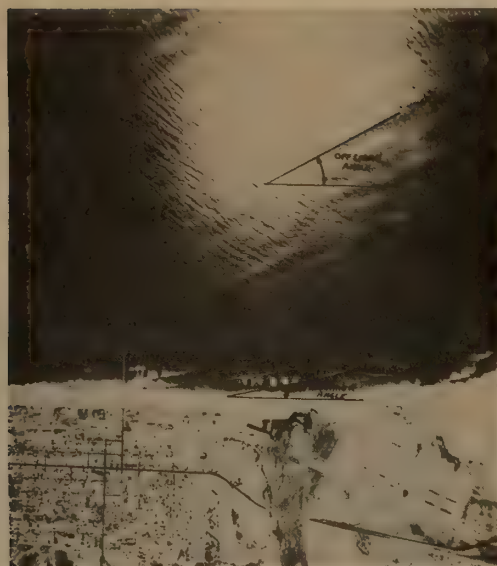
Halfmoon Bay, California

Note the increasing width of the surf zone with increasing degree of exposure to the south.



Purisima Pt., California

Refraction of waves around a headland produces low waves and a narrow surf zone where bending is greatest.



Oceanside, California

Waves tend to become parallel to the beach.

FIG. SID-2

REFRACTION OF WAVES

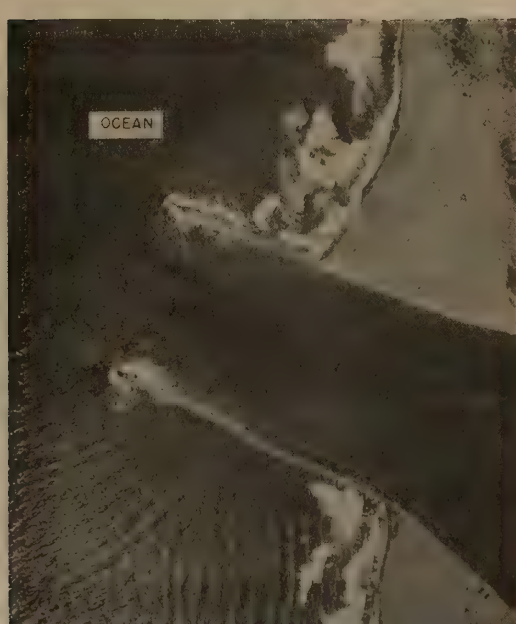
Shoaling Bottom
(Continued)

Waves in shoaling water refract in such a manner that they tend to become parallel to the underwater contours and, eventually, the shore. However, they usually break before becoming quite parallel to the beach.

REFRACTION OF WAVES

Currents

When the waves buck a current, as when they run against an ebb current at a harbor entrance, they shorten up and grow higher, producing a choppy sea and, frequently, breakers. When waves run with a current, as in flood current at a harbor entrance, they become longer and lower.

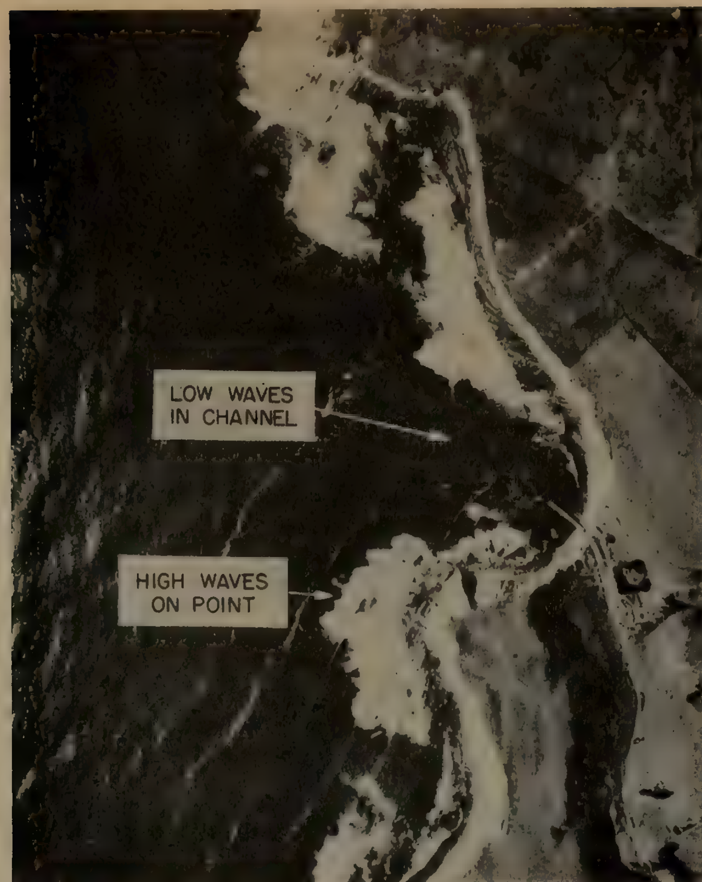


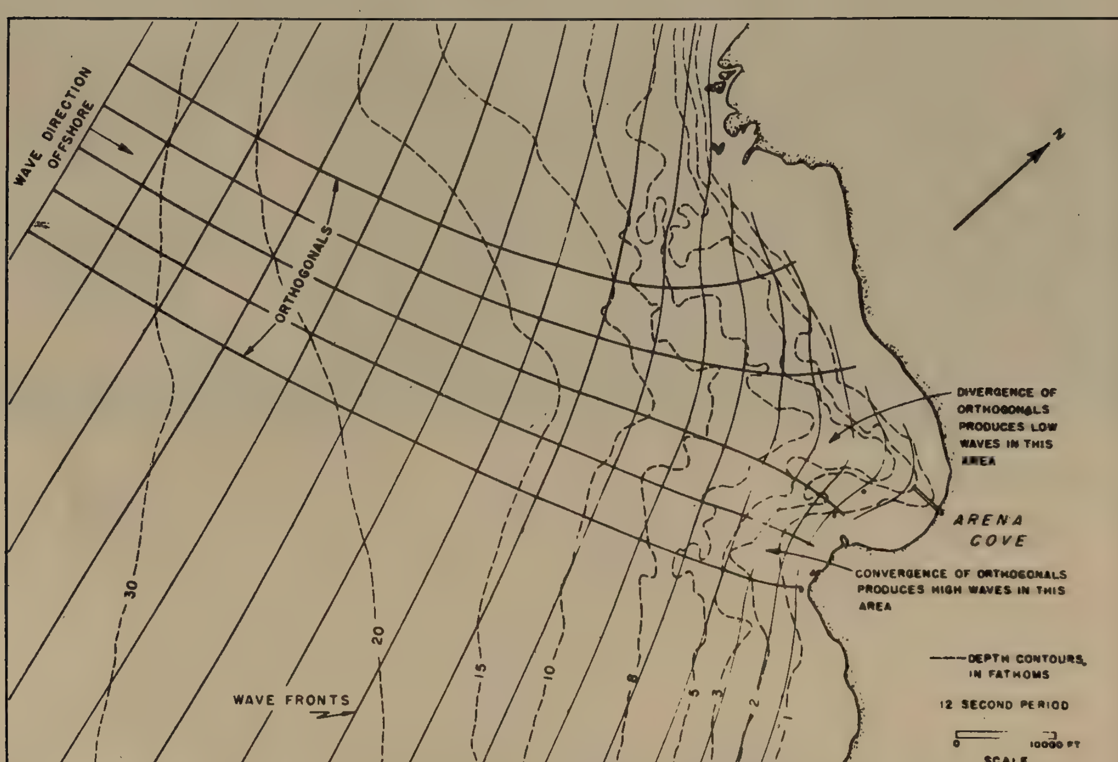
Humboldt Bay, California

FLOOD TIDE

Humboldt Bay, California

EBB TIDE





Refraction diagram for Arena Cove, California. Compare with the aerial photograph shown above for illustration of how such diagrams can be utilized to determine points where low and high wave action can be expected.

FIGURE SID-3

FIG. SID-4 - Refraction and diffraction of both swell and wind waves at Farallon Island, California. Note reflection from small island at lower left.

FIG. SID-5 - Typical example of construction of a wave refraction diagram

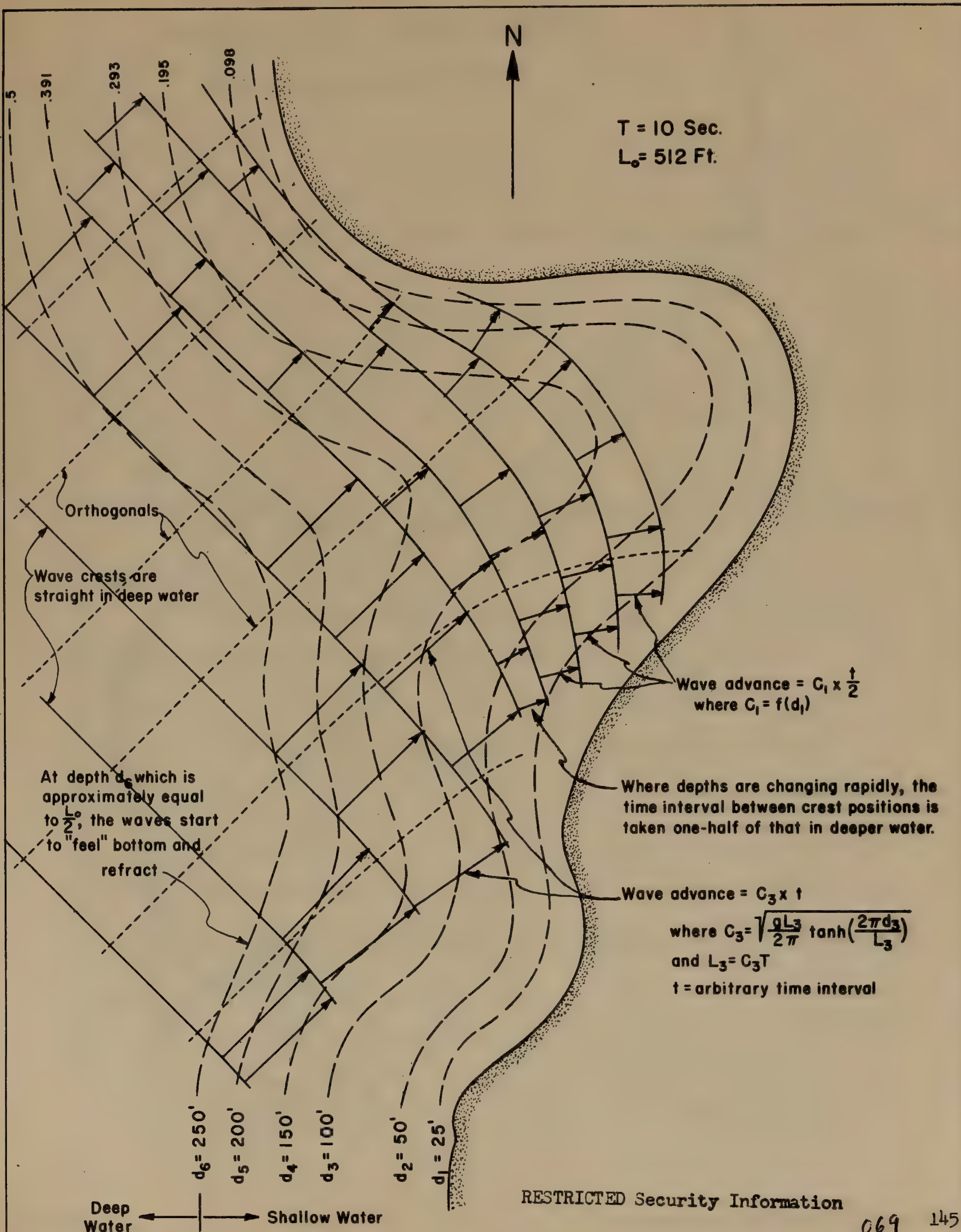


FIG. SID-6 - Refraction graph for Fort Ord, California. Concentric arcs indicate wave period in seconds. Curves are drawn for constant refraction coefficients.

RESTRICTED Security Information

D. REFRACTION

BY R. L. WIEGEL

Table of Contents

D. REFRACTION

1. Introduction

In shallow water (defined as having a depth less than one-half the length of the wave) the velocity of a wave depends upon the water depth as well as upon its length. When a wave travels over a shoaling bottom at an angle to the contours, each part of the wave travels at a velocity which depends upon the depth under it; hence, the wave bends. This bending process is known as <u>refraction</u> (Figure ID-1). Changes in wave height and in direction of travel result from refraction. Waves are also refracted by currents.

The height, period and direction of waves in deep water may be forecast either from synoptic or prognostic weather maps or by direct measurements with suitable instruments. Landing amphibious vehicles and craft successfully depends upon breaker heights and the angle between the breaker line and beach. In order to forecast their quantities the effect of refraction must be taken into account. Some simplified cases which were solved analytically gave very accurate results, but unfortunately were not applicable to most natural situations. The changes due to refraction are best estimated by use of <u>refraction diagrams</u>. Such diagrams can be prepared entirely from aerial photographs, as was done in Figure ID-2, but are generally constructed graphically.

Refraction diagrams are constructed in two ways. The first, known as the <u>wave-front method</u>, is essentially a map showing the wave crests at a given time, or the successive positions of a particular wave crest as it moves shoreward. A second set of lines, everywhere perpendicular to the "wave crests" are then constructed on the "map" (Figure ID-2). These new lines are known as <u>orthogonals</u>. As the wave energy is considered to remain constant between any two orthogonals, it is possible to estimate variations in wave height due to refraction. In the second method, known as the <u>crest-less method</u>, the orthogonals are drawn directly.

Sometimes the hydrography is such that adjacent orthogonals cross. There are two general types of crossing. The simpler of the two occurs when the wave crest is severed and the severed sections later cross each other. A simplified example of this situation, shown in Figure ID-4, illustrates the case of a wave train passing over a corner-shaped ledge, where its speed is reduced to one-half its deep water speed. As a result, the wave train breaks into two trains which cross each other. The more complex situation occurs where the orthogonals form a caustic envelope. This is illustrated in Figure ID-3 (Reference ID-9). The convergence of adjacent orthogonals would indicate on the basis of the simple theory that the waves become infinitely high which, of course, is not the actual case. Refraction theory is based upon a <u>gradual change</u> in depth and so is not expected to hold for such cases. However, in nature, conditions similar to these (bars at river mouths with gently sloping areas to the seaward, for example) do occur. The waves sometimes peak up and break; often a chaotic sea surface results.

Even though the refraction techniques discussed in this section are not applicable for all conditions they are usually valuable in obtaining

reliable quantitative information for large portions of the area under study and will give qualitative information about the complex areas.

2. Theory

a. General *

The power transmitted by a train of sinusoidal waves is,

$$P = C_g \, bH^2 \, (w/8) \qquad \text{(ID-2.1)}$$

Here, C_g is the velocity of transmission of the energy, w is the weight of water per unit volume, b is the length of crest (perpendicular to the local direction of travel) and H is the wave height (from trough to crest). Ocean waves are not sinusoidal in form, and their departure from this form increases as they approach the condition of breaking; however, this formula is sufficiently precise for estimating wave height in many cases and can be corrected by empirical results in the vicinity of the line of breakers. If no energy flows laterally along the wave crest, the same power should flow past all positions between two orthogonals in a steady state of wave motion. Indicating the conditions in deep water by the subscript zero,

$$P = P_o \qquad \text{(ID-2.2)}$$

$$H = H_o \left[\sqrt{C_{go}/C_g} \sqrt{b_o/b} \right] \qquad \text{(ID-2.3)}$$

The quantity $\sqrt{b_o/b}$ is termed the refraction coefficient. It will be designated as K_d. The quantity $\sqrt{C_{go}/C_g}$ represents the effect of a change in depth on the wave height (shoaling coefficient). It will be designated as D_d. The wave height in any depth of water then may be written as,

$$H = H_o \cdot D_d \cdot K_d \qquad \text{(ID-2.4)}$$

Methods for determining K_d, the refraction coefficient, are presented in this section. Values of the shoaling coefficient, D_d, which depends on the water depth and wave length, have been tabulated in the section of tables in this manual (where it is designated as H/H_o ' rather than D_d). For reference, some values of D_d are presented in Table ID-1. It should be cautioned that the values of D_d are valid only up to several wave lengths seaward of the breaker zone. Empirical values must be used in the vicinity of the breakers. These values have been presented in the section on breakers.

* Taken largely from Reference ID-5.

TABLE ID-1

Theoretical coefficient of shoaling, D_d

d/L_o	0.002	0.005	0.007	0.01	0.02	0.04
D_d	2.12	1.69	1.57	1.45	1.23	1.06

d/L_o	0.056	0.08	0.1	0.15	0.2	0.3	0.4
D_d	1.0	.94	.92	.91	.92	.93	.96

The graphical or analytical determination of wave refraction coefficients assume (i) that the velocity of the wave crest depends only upon the wave length and the still water depth under the crest at each point, (ii) that the elements of the wave crest advance in a direction perpendicular to the crest line, and (iii) that the wave energy is confined between orthogonals, (iv) that the waves are long-crested and (v) that the period is constant.

b. Refraction Along A Straight Shore Line With Parallel Contours

When the shore line and offshore contours are straight the parallel refraction may be treated analytically by utilizing what is known as Snell's Law:

$$\sin\alpha / \sin\alpha_o = C/C_o \qquad \text{(ID-2.5)}$$

Where α is the angle between the wave crest and the shore line, α_o is the angle between the deep water wave and the shoreline, C is the wave velocity and C_o is the deep water wave velocity. The change in angle determines the increase in crest length, and thus the value of K_d is fixed by the depth (which determines C for a particular C_o) and α_o, the angle in deep water. Referring to Figure ID-5, the value of K_d may be computed from the relationship,

$$b_o/\cos\alpha_o = b_s = b/\cos\alpha \qquad \text{(ID-2.6)}$$

or

$$K_d = \sqrt{b_o/b} = \sqrt{\cos\alpha_o/\cos\alpha} \qquad \text{(ID-2.7)}$$

where

$$\alpha = \sin^{-1}(C \sin\alpha_o/C_o) \qquad \text{(ID-2.8)}$$

For example, if $\alpha_o = 45^\circ$ and the depth and period at the point for which K_d is to be computed, are such that $C/C_o = 0.5$,

$$\alpha = \sin^{-1}(0.5 \times 0.71) = 20.8 \text{ deg.}$$

$$\cos\alpha = 0.935 \text{ and } \cos\alpha_o = 0.707$$

$$K_d = 0.87$$

For convenience, the relationships between α, α_o, depth, period, and K_d have been summarized in Figure ID-6.

A thorough understanding of the nature and magnitude of refraction effects along straight coast lines is helpful in constructing refraction diagrams for complex hydrography. The beginner should study Figure ID-6 in order to develop judgment in regard to determining which hydrographic conditions require graphical analysis as well as developing a basis for checking, approximately, the numerical values of K_d resulting from a graphical determination.

It is noteworthy that, along a straight shore line, the reduction in wave height by refraction is less than 10 percent when the initial angle in deep water is less than 36 degrees.

c. Refraction Around Islands and Shoals with Concentric Circular Contours

(1) Theory

An analytical solution for the refraction of waves around an island with concentric circular contours may be obtained by the application of Fermat's principle (ID-1) i.e., light waves will travel in a path such that the travel time is a minimum (Reference ID-8). Consequently the problem is one of determining the path between two points for which the following integral has a minimum value:

$$I = \int_{0}^{\theta} (C_o/C)\sqrt{r^2 + (dr/d\theta)^2}\, d\theta \qquad \text{(ID-2.9)}$$

where the initial conditions to be satisfied are:

(i) $C = C_o$ for $r = r_o$

(ii) A given orthogonal passes through point (r_o, θ_o)

(iii) A given orthogonal is parallel to the polar axis of the polar coordinate system at point (r_o, θ_o)

Application of the method of the calculus of variation led to the Euler-Lagrange condition which Dr. Arthur solved for this case. The solution was

$$\frac{dR}{R\left\{\sqrt{\frac{\csc^2 \theta_o \cdot R^2}{C^2/C_o^2} - 1}\right\}} = \pm\, d\theta \qquad \text{(ID-2.10)}$$

where $R = r/r_o$ and where the region $0 \leq R \leq 1$ was considered. Solution of this can be obtained when the function $C = C(R)$ is known. Certain simplified solutions were obtained (such as assuming that $R = C/C_o$, etc.). The different profiles of the "islands" studied are shown in Figure ID-10.

(2) Case 1A: Point Island

Assume $R = C/C_o$, where $0 \leq R \leq 1$. The solution is

$$R \doteq 1/e^{(\theta - \theta_o)\,\mathrm{ctn}\,\theta_o} \qquad \text{(ID-2.11)}$$

This is the equation of a logarithmic spiral and hence, all orthogonals converge towards the center, $R = 0$. This case has been shown in Figure ID-8 with only the orthogonals entering one quadrant shown.

This is the limiting case between an island and a shoal and is known as a <u>point island</u>. There is extreme convergence at the center and at the time there are no refracted waves in the lee beyond the contour $d = 0.5L_o$.

(3) Case 1B: Circular Island

Assume $(R-n)/(1-n) = C/C_o$, where $0 < n < 1$ and $n \leq R \leq 1$.

The diagram in Figure ID-9 shows the results for one quadrant. All orthogonals meet the shore of the island ($R = n$, in this case $n = 0.2$). In addition to the orthogonals, the refraction coefficient, K, has been plotted for $d = 0$. Actually the waves would break just offshore; however, the actual value of K would be nearly as given in the sketch. The zone in the lee of the island, beyond $d = 0.5\ L_o$, is free of refracted waves. The curve of K shows that the minimum value is 0.6; however, interference between wave trains refracted around each side of the island would produce waves higher than the incident waves all along the shore. Hence, such an island would have no "lee shore" as far as protection is concerned.

(4) Case 1C: Shoal

Assume $(1-n)\ R + n = C/C_o$ where $0 < n < 1$ and $0 \leq R \leq 1$.

The diagram in Figure ID-11, for $n = 0.4$, shows the results for one quadrant. This represents a shoal, with the wave velocity at the most shallow point being $C = 0.4\ C_o$ and the depth at this point being $d = 0.028L_o$.

Arthur discusses this condition as follows:

"There has been considerable discussion about the possibility of orthogonals associated with an undivided wave train intersecting, and this case clearly shows that orthogonals can intersect. Beyond the center of the shoal, two crests exist as shown in [Figure ID-11]. Crest AB' is associated with the solid portion of the orthogonals, and crest AB_2 is associated with the dashed portion. Point A is located on what is known as the envelope of the orthogonals. The envelope is not drawn, but it may be visualized as a curve tangent to each of the orthogonals corresponding to values of θ_o from zero up to a certain value. The orthogonal is drawn as a solid line to the point of tangency with the envelope. Thereafter, the orthogonal is drawn as a dashed line. At a point of intersection between two orthogonals, each of the orthogonals is associated with a different wave-crest. Consequently at such an intersection point there is not what might be called a complete 'focusing' effect and K is therefore not infinite."

"K is also never infinite along the envelope, since no two orthogonals intersect at a point on the envelope. However, (Figure ID-11) shows that a region of convergence exists along the envelope. The maximum value of K along any wave-crest occurs at a point on the envelope such as point A. Computation of the value of K at point O gives the approximate magnitude of the convergence occurring along the envelope. It can be shown that the value of K at point O is approximately 1.6."

It can be seen that if waves are refracted about both sides of the island, a very confused sea would exist in the lee due to the intersection of four sets of crests.

(5) Case 2: Point Island

Assume $R = (C/C_o)^2$, where $0 \leq R \leq 1$. The cross section in Figure ID-10 snows a moderate bottom-slope changing to a steep slope near $d = 0.5\ L_o$. Figure ID-13 shows orthogonals and a curve showing the variation in K along a circle of 5 r_o. This curve shows that the height of the refracted curves is greatest for that portion of the wave-crest turned through the largest angle.

(6) Case 3: Point Island

Assume $R\ (2-R) = (C/C_o)^2$, where $0 \leq R \leq 1$. As is shown in Figure ID-10 the bottom slope is nearly linear. The curve of K shows a decrease in height of refracted waves for increasing and hence the decrease in wave-height is rapid in the lee of the island (Figure ID-14).

(7) Conclusions of Arthur

"Type 1, bottom-slope -- Very steep slope near contour $d = 0.5\ L_o$ and very gentle slope in more shallow water (Cases 1A and 1B). Refraction effect: No refracted waves in lee of island beyond contour $d = 0.5\ L_o$."

"Type 2, bottom-slope -- Steep slope near contour $d = 0.5\ L_o$ and moderate slope in more shallow water (Case 2). Refraction effect: At a large distance refracted wave height is very low at edge of lee but increases towards center of lee. Waves refracted through greater angles are higher."

"Type 3, bottom-slope -- Approximately linear bottom-slope (Case 3). Refraction effect: At a large distance refracted wave-height rapidly decreases from incident wave-height at edge of lee to lower values towards center of lee. Waves refracted through greater angles are lower."

"Islands where the bottom slope resembles Type 1 have waves on the lee shore as high as the incident waves if the radius of the contour $d = L_o$ is large compared to the island radius. This type of bottom slope is characteristic of coral islands and volcanic islands with a shelf cut by wave-erosion. It is probable that a refraction effect of this type rather than diffraction explains the high breakers known to be frequent on the lee shore of certain islands."

"Shoals with a steep slope near the contour $d = 0.5\ L_o$ are characterized by intersecting orthogonals which indicate that a single incident wave-crest is divided into two crests. Shoals of this type exist in regions where glacial erosion has occurred. The associated convergence and interference may explain the confused sea and breaking waves frequently observed over such shoals."

d. Caustic

Analytical and experimental studies of refraction in the region of a caustic have recently been made. (Reference ID-9) A case, similar to the shoal studied by Arthur, was examined in detail. Although the two were not identical it was thought possible to represent the orthogonal pattern over the submerged clock glass (shoal) in a ripple tank schematically from Arthur's theoretical considerations.

Figure ID-15 shows the orthogonal pattern over the clock glass. The wave crests shown beyond the caustic are not as observed, but rather were constructed after the method of Arthur. Figure ID-16 shows the pattern of wave-crest positions as found by the geometric optics approximation which predicts a phase shift through the caustic. Figure ID-17 shows the actual crest-orthogonal pattern for waves passing over a clock glass. This illustration was drawn from the shadowgraph (a photograph of the light and dark shadows on a screen, caused by the focusing effect of the wave crests on a light shining up through the bottom of a transparent ripple tank) shown in Figure ID-18.

At present, no quantitative information may be obtained for the area beyond the caustic. However, the fact that one may be able to recognize areas in which the technique of refraction diagrams fail is in itself very important. It places certain known engineering limitations on the problem.

3. Graphical Construction of Refraction Diagrams by the Wave-Front Method*

a. General

Ideally, waves in deep water move forward with their crests parallel, while in shallow water the reduction in wave velocity causes the crest to swing around in the direction which will decrease the angle between the crest and the bottom contour. A quantitative definition of what is meant by "deep water" and by "shallow water" is usually given as

$$\left.\begin{array}{r} \text{deep water, } d > L_o/2 \\ \text{shallow water, } d < L_o/2 \end{array}\right. \quad L_o = gT^2/2\pi = 5.12T^2$$

where d and L_o are in feet and T in seconds.

The meaning of these terms may be examined in the light of refractive effects in shallow water by considering the velocities and angle involved. The velocity and length of a wave decreases in shallow water as is shown in the following table:

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* This section is taken largely from reference ID-5.
(Johnson, J. W., O'Brien, M. P. and Isaacs, J. D. -Graphical Construction of Wave Refraction Diagrams- U. S. Navy Hydrographic Office Publication No. 605, January 1948.)

d/L_o	0.5	0.4	0.3	0.2	0.1
L/L_o, C/C_o	0.996	0.98	0.96	0.89	0.71

The angle through which a wave crest will turn between deep water and value of d/L_o may be obtained from Figure ID-6. By selecting a few values to represent the effect, Table ID-2 shows that the limits of "shallow water" depend upon the angle α_o and the accuracy to which the diagram is to be constructed. If $\alpha_o = 70°$ and if the construction is at an accuracy of ± 1 degree, the diagram should start in depths even greater than $d = 0.5L_o$, but if α_o is only 10 degrees, a negligible error is introduced if the diagram is started from $d = 0.3\ L_o$.

As a working rule, which should be modified as circumstances indicate, refraction diagrams should start from straight wave-crests in a depth equal to half the deep water wave length or at $d = 0.5 \times 5.12T^2$ feet.

TABLE ID-2
Values of as a function of d/L_o and $_o$

α_o \ d/L_o	0.5	0.4	0.3	0.2	0.1
70	69	68	64	57	41
50	49.5	48	46	42	32
30	30	29	28	26	21
10	10	10	9.95	8.5	7

The velocity of a wave in deep water is 5.12T feet per second, where T is in seconds. As the wave moves into shallow water, its velocity decreases and, if the crest makes an angle with the bottom contours, the wave velocity will vary from point to point along the crest. Graphical construction of a refraction diagram consists simply in moving each point of the crest in a direction perpendicular to the crest by a distance equal to the wave velocity times the time interval selected. The initial form of the wave is a straight line in the deep water area, as previously defined. Figure ID-19a shows scales constructed in such a manner as to give the advance of the wave crest at any value of d/L_o on a chart of any scale S. (These scales should be printed on thin paper and made available for distribution.) See Section ID-3c for methods of construction of the scales. The two scales of Figure ID-19a differ only in that scale A gives the wave advance during an interval which is twice that of scale B.

To construct a refraction diagram, the hydrographic chart first is contoured with an interval which will represent accurately the details of the bottom topography. Each contour on the map is converted to mean sea level--or any other desired stage of the tide--adding the proper constant to the chart soundings. The most reliable chart having the most hydrographic detail should be used. For many areas, the work sheets from

which the navigation charts were prepared are available (USC&GS work sheets.) Those sheets are generally larger scale and more detailed than the charts. For the wave period selected, the deep water wave length is computed from the relationship, $L_o = 5.12\ T^2$. The contour values, in depth in feet below the tide stage selected, are then divided by L_o (in feet) to give contours in terms of d/L_o. Thus, in Figure ID-20, for example, the contours in fathoms have been re-labeled in terms of values of d/L_o. Additional contours of d/L_o may be added if considered desirable. In Figure ID-20, for example, contours of d/L_o of 0.5 and 0.4 have been added.

Generally, it is sufficient to draw every nth crest, where the value of the crest interval, n, depends upon the scale of the chart and the complexity of the bottom topography. The crest interval is determined by the scales used and may be expressed as n, a multiple value of wave length, or as a time interval, t. The crest interval does not have to be an even value, nor does it have to be the same for the entire chart, since more crests should be drawn where the bottom topography is particularly complex. The two transparent scales (Figure ID-19) for plotting the wave advance are provided so that the crest interval in one scale (scale A) is just twice that for the second scale (scale B). These scales are applicable to charts of any scale and of any wave period. The only variable between refraction diagrams prepared by the use of the scales is the crest interval, this interval being a function of the scale of the hydrographic chart. Formulas are provided for computation of the crest interval, n, or time interval, t, for any particular refraction diagram (Figure ID-19a).

It is sometimes necessary and often helpful to draw a refraction diagram for a particualr locality, in several steps: First, the over-all pattern for a long stretch of coast line is drawn on a relatively small scale chart, following the waves from deep water to within a few thousand feet from shore; then the results are transferred to larger scale charts, and a detailed pattern is constructed of the waves close to shore in bays, harbors, and other areas of particualr importance. If the refraction diagram is drawn for the purpose of determining the local angle between the shore line and the breaking crest, then it is obviously necessary to continue the diagram to the depth in which the wave breaks. This refinement is unnecessary in determining wave heights, but is necessary in estimating the strength of the littoral current (which depends upon the angle between the breaking crests and the shore line).

If the tidal range is large, it may be necessary to construct several diagrams for different stages of the tide. On the California coast, where the range of tide is approximately 5 feet, diagrams prepared for an average stage of tide will usually suffice.

b. <u>Example-Refraction Diagram, Monterey Bay, California</u>

A refraction diagram for Monterey Bay is constructed in this section as an example. Then values of K_d are determined for points along the coast from Monterey to Santa Cruz. The diagram is prepared for a mean tide condition of 2 feet above M.L.L.W. (Mean Lower Low Water), direction of advance in deep water from W.N.W., and T = 14 secs. Thus, $L_o = 5.12\ T^2 = 5.12\ (14)^2 = 1{,}000$ feet, and the depth which is the dividing contour between deep and shallow water is $L_o/2 = 500$ feet.

The available U.S.C.&G.S. charts that show bottom topography are No. 5402, scale 1:214,000 and No. 5403, scale 1:50,000.

The contours appearing on these charts are in fathoms. The equivalent values in terms of d/L_0 are as follows:

$$100 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (100)\ \ + 2}{1{,}000} = 0.602$$

$$50 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (50)\ \ + 2}{1{,}000} = 0.302$$

$$40 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (40)\ \ + 2}{1{,}000} = 0.242$$

$$30 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (30)\ \ + 2}{1{,}000} = 0.182$$

$$20 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (20)\ \ + 2}{1{,}000} = 0.122$$

$$15 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (15)\ \ + 2}{1{,}000} = 0.092$$

$$10 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (10)\ \ + 2}{1{,}000} = 0.062$$

$$5 \text{ fathoms}; \frac{d}{L_0} = \frac{(6)\ (5)\ \ + 2}{1{,}000} = 0.032$$

For accuracy in preparing the refraction diagram, it is necessary to use the chart with the largest scale (chart 5403). This chart does not extend to deep water (that is, beyond a depth of 500 feet); hence, chart No. 5402 must be used, necessitating carrying the waves part way on this smaller scale chart and then transferring the front to the larger scale chart (No. 5403) near to the shore line. (Usually it is desirable to draw refraction diagrams on tracing paper overlays placed over the hydrographic chart; however, for illustrative purposes in this report, the diagrams are drawn directly on the charts.)

Figure ID-20 shows a portion of USC&GS Chart 5402 with 14 wave-crests marked 1, 2, ... 14. Crest 1 lies in deep water and is drawn as a straight line. The southerly portion of crests 1-14 remain in deep water (that is, where the contours of d/L_0 have values greater than 0.5), and the distances between them are equal to a constant multiple of L_0. The northerly portions of the crests advance into shallow water with the result that the distances between the crests decrease. The position of each crest is determined from that of the crest behind it by locating a few points on the new crest and drawing a smooth curve through them. These points are shown as small circles in Figure ID-20. The points are located by means of scale B in Figure ID-19, which has been cut out and placed on the refraction diagram as illustrated in Figure ID-20. For example, to locate point c on crest 12:

(i) Lay the scale on the chart so that the dashed center line and the line of $d/L_0 = 0.302$ on the scale intersect with the contour $d/L_0 = 0.302$ on the chart (point a).

(ii) Move the scale so that condition (1) remains satisfied and the lower side of the scale is tangent to crest 11 on the diagram at the end of the d/L_o = 0.302 line on the scale (point b).

(iii) Mark point c where the d/L_o = 0.302 line on the scale reaches the upper side of the scale.

(iv) Thus, other points as d, e, and f are found by making use of the d/L_o contours of 0.242, 0.182, and 0.122, respectively. The process is repeated until a sufficient number of points are found to determine the position of crest 12.

In a similar manner to that outlined in items 1-4, above, other crests are located until the diagram is carried into a locality which is within the limits of a larger scale chart. On Figure ID-20 it is noted that crest 14 is within the limits of the area shown on chart 5403. This crest is then transferred from chart 5402 by taking offsets from a convenient longitude (122°), correcting for scale ratio, and replotting on chart 5403 (Figure ID-21). Thus, the offsets in inches at each minute of latitude on chart 5402 are multiplied by the ratio 214,000/-50,000 = 4.28 and are shown plotted on chart 5403 with a smooth curve drawn to give the position of crest 14. (Note that charts 5402 and 5403 have been reduced by photostat for inclusion in this report.) Changing from one chart to another must be done with special care as slight errors here may cause large errors in K_d.

Starting with wave crest 14 on chart 5403, crests 15 to 23 are plotted by the method illustrated in steps 1 to 4 above. Because the bottom slope in the area covered by this chart is relatively uniform, crests can be plotted using a larger interval between crests; thus, crests 15 to 23 were plotted using scale A of Figure ID-19.

On chart 5402 the spacing between crests was determined by means of scale B and the corresponding values of n and t are:

$$n = 0.0163 \; \frac{214,000}{(14)^2} = 17.8 \text{ wave lengths}$$

$$t = 0.0163 \; \frac{(214,000)}{14} = 249 \text{ seconds}$$

Similarly, on chart 5403 scale A was used, and the values of n and t are:

$$n = 0.0326 \; \frac{(50,000)}{(14)^2} = 8.3 \text{ wave lengths}$$

$$t = 0.0326 \; \frac{(50,000)}{14} = 117 \text{ seconds}$$

At a few localities where the bottom configurations are irregular, intermediate crests on chart 5403 (Figure ID-21) have been added by the use of scale B. These localities are between crests 16 and 17 near a side canyon of the Monterey Canyon and shoreward from crests 21, 22, and 23. Even this spacing is inadequate to describe the refraction which probably occurs at such locations as Santa Cruz Harbor, Monterey Harbor, and at the

head of Monterey Canyon at Moss Landing. For greater detail in these areas, charts with a larger scale should be used. Photostat enlargements could be made of the areas from chart No. 5403, but for more accuracy the original USC&GS work sheets should be used. Thus, Figure ID-22, shows contours from Hydrographic Chart 5415, which covers Monterey Harbor and vicinity with a scale of 1:5000. Chart No. 5415 shows that the bottom contours extend seaward only to the 17-fathom contour, which is about the location of crest 23 on chart 5403. Consequently, crest 23 has been transferred to chart 5415 by the offset method described above. Crests 24 to 33 have been constructed by the use of scale A. The crest interval on this diagram is as follows:

$$n = 0.0326 \frac{(5,000)}{(14)^2} = 0.83 \text{ wave lengths}$$

$$t = 0.0326 \frac{(5,000)}{14} = 11.7 \text{ seconds}$$

Further detail near the Monterey breakwater and within the harbor could be obtained by enlarging that area from chart 5415.

Upon completion of a plot showing the wave-crests, the orthogonals are drawn on the diagrams. The orthogonals are started at the shore, or at some specified depth contour, and carried seaward as perpendiculars to the wave-crests until deep water is reached. Orthogonals are curves in shallow water and straight lines in deep water.

In constructing the orthogonals, a small triangle and a short straight edge are convenient. The triangle is adjusted so as to be tangent to a wave crest (Figure ID-23a) where the orthogonal is to be started (crest 1). The straight edge is held against the triangle and the triangle is then slid along the straight edge so that a perpendicular can be drawn through point a and to point b half way to crest 2. The process is repeated for crest 2 with the perpendicular drawn shoreward to point b and then extended seaward to a point c midway between crests 2 and 3. The procedure is repeated until the wave crest in deep water is reached. If desired a smooth curve could be drawn through the points where the perpendiculars cross the crests.

Another method of constructing orthogonals is to cut about a four inch square from acetate or other thin transparent drafting plastic (Figure ID-23b). A straight line is then drawn diagonally across the square and two narrow slots cut perpendicular to the line to accommodate a pencil point. Then the line is held tangent to the first wave front and a light line drawn in the forward slot from a point halfway to the next wave front, back toward the first wave front. The square is then moved foreward so that the inked line is tangent to the second wave front with the pencil line from the first wave front terminating in the rear slot. A light line is then drawn from a point halfway to the third wave front and back through the rear slot to the first line. The positions of these light lines are usually satisfactory for orthogonals. If the wave front curvature is great the orthogonals can be drawn through their intersections with wave fronts just as in the triangle method. This method has been found to be a more rapid method than the triangle method.

In Figure ID-21 orthogonals have been drawn every two nautical miles starting at the Monterey Municipal Pier. With the exception of the vicinity of Moss Landing and northward, this spacing appears suitable for most of the coast between the orthogonals A-F. As mentioned above, if the refraction coefficients are desired for the Moss Landing shore line, a refraction diagram should be prepared on a larger scale map, and the orthogonals then be drawn with smaller spacings. Additional orthogonals should be constructed northward from Moss Landing to better define the refraction coefficients in this area.

The orthogonals shown on chart 5403 (Figure ID-21) ended at crest 14, and then had to be transferred to chart 5402 (Figure ID-20). In all probability, not all the orthogonals can be carried to deep water on the smaller scale chart. Actually, only a few of the orthogonals are required to give a measure of refraction between deep water and crest 14. (Thus, the points on crest 14 which are midway between orthogonals A-L (Figure ID-21) have been transferred to Figure ID-20.) It has not been necessary to carry orthogonal I, seaward from crest 14 as orthogonals H and J give a measure of the refraction for this portion of the wave.

Consider one example of the computation of the refraction coefficient, K_d. It is desired to obtain K_d for a point on the 5-fathom contour midway between orthogonals K and L. The procedure is as follows:

On chart 5403 (Figure ID-21) the distance between orthogonals K and L at the 5-fathom line is 2.9 inches and at crest 14 the distance is 1.72 inches.

On Figure ID-20 (chart 5402) the distance between orthogonals K' and L' is 0.49 inches at crest 14 and 0.12 inches at crest 1. Therefore, the refraction coefficient for the point midway between orthogonals K and L at the 5-fathom line is:

$$K_d = \sqrt{\frac{1.72}{2.9} \text{ X } \frac{0.12}{0.49}} = 0.38$$

Computations for refraction coefficients at other points along the 5 fathom contour are summarized in Table ID-3. To obtain a refraction coefficient from a point nearer to the Municipal Pier than obtained by orthogonals A and B, additional orthogonals have been drawn on charts 5403 and 5415 (Figure ID-22). Point M on crest 23 was selected as a starting point. On chart 5403 two orthogonals (0.3 inch on either side of point M) have been carried seaward to crest 17 in deep water, where the distance between orthogonals is 0.13 inch. On chart 5415 the two orthogonals (1 1/2 inch on either side of point M) have been carried to the 5 fathom contour, where the distance between orthogonals is 4.72 inches. The value of K_d at this location of the 5 fathom contour, which is 0.8 nautical mile from the Municipal Pier is

$$K_d = \sqrt{\frac{0.13}{0.6} \text{ X } \frac{3}{4.72}} = 0.37$$

(On some refraction diagrams, as in Figure ID-36, the crests may divide and intermingle shoreward of an offshore island or shoal, or underwater ridge, etc. The problem should then be considered as one of two separate wave trains.)

c. Presentation of Refraction Coefficients

Refraction diagrams should be prepared for various periods and several deep water wave directions. The coefficients should be summarized in convenient table or graph form. If forecasts are being made for only one point on shore, a plot of K_d as a function of wave direction and period is sufficient. One of the easiest ways to plot a complex refraction coefficient pattern is similar to drawing logical contours on topographic maps from spot elevations. A range of periods may be tested as ordinates and a range of different directions as abscissas. The refraction coefficients are then placed on the diagram in appropriate places and the contours drawn by linear interpolation and inspection of the overall refraction diagrams. (See Figure ID-25).

This information may also be plotted in a polar plot instead of rectangularly (which may be a little more realistic to some people) with the origin at the point on shore, the various wave periods as concentric circles and the different directions as radial lines in their true directions (Figure ID-24b). Contours are interpolated as for the rectangular plot. If forecasts are being made for several points along a coast, a convenient means of summarizing the data is a graph which shows K_d factors plotted versus distance along the coast for various wave periods (Figure ID-24a). A separate graph for each wave direction is necessary. Another possible method of summarizing data is to show a map of an area with contours of equal K_d values indicated. A map must be prepared for each wave direction and period. Figure ID-26 shows such a map for Monterey Bay as prepared from the refraction diagrams shown in Figures ID-20 and ID-21.

It is usually necessary, where the tidal range is large, to construct separate diagrams for different stages of the tide. It is important to note that the assumption of constant wave energy between orthogonals does not apply after a wave breaks. If waves pass over a submerged reef, it is necessary to examine this area critically to determine whether the waves break at some, or all, stages of the tide. Should breaking occur, wave heights beyond the reef would be lower than that determined by use of K_d factors from a refraction diagram. As a wave passes over a reef, whether or not breaking occurs, the crest may break into several crests. Thus, the further refraction of the wave may not be simple.

The refraction coefficient, K_d, is a function of depth, of period, and of the initial angle of the wave crest. In the preceding examples, representation of the results was simplified by reporting the coefficient of refraction at a constant depth of 5 fathoms, thus eliminating one variable. The question now arises as to the magnitude of the difference in this coefficient if the wave breaks in a lesser depth, say, 10 feet. For this purpose, the 30-foot contour may be assumed as parallel to the shore line and the effect worked out approximately from Figure ID-6. Using the example of Monterey Bay, with a period of 14 seconds and waves from W.N.W., the refraction coefficient between stations E and F is around 0.60 at d/L_o = 0.03. On a straight shore line, from Figure ID-6, this coefficient and depth would correspond to $\alpha_o = 71^\circ$ and $\alpha = 24^\circ$. At $d/L_o = 0.01$, $\alpha = 13^\circ$ so that further change in α between $d/L_o = 0.03$ and $d/L_o = 0.01$ would be about 11 degrees. The change in coefficient is then determined from

$$K_{30} = \sqrt{b_o/b_{30}} \qquad\qquad K_{10} = \sqrt{b_o/b_{10}}$$

$$K_{10}/K_{30} = \sqrt{b_{30}/b_{10}} = \sqrt{\cos\alpha_{30}/\cos\alpha_{10}} = 0.97$$

TABLE ID-3

Computation of refraction coefficients for Monterey Bay

K_d values apply to the 5-fathom contour for waves of 14-second period from W. N. W.

Data from chart 5403					Data from chart 5402					K_d	Distance at mid-point between orthogonals (nautical miles from municipal pier)
Segment at 5-fathom line		Segment at crest 14		Col. (4) Col. (2)	Segment at Crest 14		Segment in deep water		Col. (9) Col. (7)	Col. (10) x Col. (5)	
No.	Length (inches)	No.	Length (inches)		No.	Length (inches)	No.	Length (inches)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A-B	2.9	A'-B'	1.87	0.65	A'-B'	0.44	A"-B"	0.44	1.0	0.81	3
B-C	2.9	B'-C'	2.61	.9	B'-C'	.61	B"-C"	.61	1.0	.95	5
C-D	2.9	C'-D'	2.78	.96	C'-D'	.65	C"-D"	.64	.99	.98	7
D-E	2.9	D'-E'	1.73	.60	D'-E'	.41	D"-E"	.40	.98	.77	9
E-F	2.9	E'-F'	1.46	.50	E'-F'	.34	E"-F"	.24	.71	.59	11
F-G	[1] 1.42	F'-G'	.90	.63	F'-G'	.20	F"-G"	.11	.55	2.59	13
G-H	[1] 1.88	G'-H'	1.15	.61	G'-H'	.26	G"-H"	.17	.65	2.63	15
H-I	2.9	H'-I'	1.02	.35	H'-I'	.34	H"-I"	.18	.53	3.43	17
I-J	2.9	I'-J'	.42	.15	I'-J'	.34	I"-J"	.18	.53	3.28	19
J-K	2.9	J'-K'	2.0	.69	J'-K'	.43	J"-K"	.14	.33	.48	21
K-L	2.9	K'-L'	1.72	.59	K'-L'	.49	K"-L"	.12	.25	.38	23

1. Distance between orthogonals measured at crest 20. Refraction shoreward of this crest should be determined from larger scale chart.
2. Coefficients apply to crest 20.
3. Refraction seaward from crest 14 is based on distance between orthogonals H and J.

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With the change in angle from 24 degrees at d = 30 feet to 13 degrees at d = 10 feet, the value of K_d at the 10-foot contour is only 97 percent of the value at the 30-foot contour. Evidently, the exact depth to which the refraction diagram is carried does not greatly affect the value. As a working rule (to be modified as special circumstances indicate) carry refraction diagrams shoreward to the depth of water in which the wave breaks.

d. <u>Theory and Plotting Data for Refraction Scales</u>

The theory involved in the construction of the scales shown in Figure ID-19a for plotting refraction diagrams by the wave-advance method is briefly as follows:

It is necessary to plot values of wave advance as a function of the ratio d/L_o. By proper spacing of values of d/L_o, the upper plotting edge of the scale can be made a straight line, hence it can be constructed with considerable accuracy.

Refering to the sketch (Figure ID-19b) x represents the base length of the scale, and the ordinate at the right hand side represents the wave advance in deep water; that is, the advance is some multiple n of the deep water wave length, L_o.

For any particular value of the depth-length ratio, such as d_n/L_o, the distance from the left hand end of the scale to the point where the wave advance is nL_n is, by similar triangles, given by the relationship,

$$X_n/X = nL_n/nL_o$$

or

$$X_n = XL_n/L_o \qquad \text{(ID-3.1)}$$

For any chosen length of scale (8 inches for scales A and B, Figure ID-19b), values of X_n, for various assumed values of d/L_o, are calculated by the following procedure:

For shallow water, the wave velocity is

$$C^2 = (gL/2\pi) \tanh (2\pi d/L) \qquad \text{(ID-3.2)}$$

and

$$C = L/T \qquad \text{(ID-3.3)}$$

or equation (ID-3.3) may be written

$$C^2 = (L^2/T^2) = \left[(L^2/(L_o/5.12)\right] \qquad \text{(ID-3.4)}$$

hence, a combination of equations (ID-3.2) and (ID-3.4) gives

$$L/L_o = \tanh (2\pi d/L) \qquad \text{(ID-3.5)}$$

Table ID-4 gives the steps in computing values of X_n by use of equation (ID-3.1) and (ID-3.5). Column (1) shows various values of d/L_o. From the tables in another section of the manual, values of L/L_o have been obtained for the corresponding values of d/L_o and tabulated in column (2). Column (3) shows values of d/L which were obtained by dividing column (1) by column (2). Column (4) is the product of 2π and the values in column (3). Column (5) is the hyperbolic tangent of the values in column (4). Column

(6) gives values of X_n in inches and is obtained by multiplying values in column (5) by 8 inches, the base length of the scale.

In plotting the scales, the ordinate at the right hand side (where $d/L_o = 0.5$) can be constructed with any convenient scale. Scale A was made with a height of 2 inches, whereas, scale B was made only 1 inch. This selection of heights seems suitable for the usual hydrographic charts.

The equations for determining the time interval, t, between wave crests or the number of wave lengths, n, between crests are determined as follows:

If the chart scale is in the form 1/S and y represents the ordinate in inches where $d/L_o = 0.5$, then

$$y/12 = nL_o/S$$

$$n = yS/12L_o = yS/12(5.12T^2) = yS/61.44T^2 \quad \text{(ID-3.6)}$$

The time interval, t, between crests is the distance advanced by the wave divided by the wave velocity; that is, at $d/L_o = 0.5$

$$t = nL_o/C_o = nL_o/(L_o/T) = nT \quad \text{(ID-3.7)}$$

or

$$t = yS/61.44T \quad \text{(ID-3.8)}$$

Thus, for scale A (Figure ID-19), where y = 12 inches

$$n = (0.0326)\ S/T^2 \quad \text{(ID-3.9)}$$

and

$$t = (0.0326)\ S/T \quad \text{(ID-3.10)}$$

For scale B (Figure ID-19), where y = 1 inch

$$n = (0.0163)\ S/T^2 \quad \text{(ID-3.11)}$$

and

$$t = (0.0163)\ S/T \quad \text{(ID-3.12)}$$

4. Construction of Refraction Diagrams From Aerial Photographs*

a. Method

The graphical method of preparing refraction diagrams may be replaced by a purely photographic method, using accurately timed aerial photographs. Various components of the method have been described in other reports and the procedure will be summarized only briefly here. Steps in the analysis are:

(i) Obtain aerial photographs of the shore line and offshore area, preferably verticals, taken at an accurately timed interval of approximately 3 seconds and with about 85 percent overlap.

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* Taken largely from Reference ID-5.

TABLE ID- 4

Computations and plotting data for wave refraction scales

$\frac{d}{L_o}$ 1	$\frac{L}{L_o}$ 2	$\frac{d}{L}$ 3	$2\pi\frac{d}{L}$ 4	$\tanh\frac{2\pi d}{L}$ 5	X_n (inches) 6
0.0020	0.1120	0.1079	0.1125	0.1120	0.896
.0025	.1250	.0200	.1257	.1250	1.000
.0030	.1368	.0219	.1376	.1367	1.094
.0035	.1478	.0237	.1489	.1478	1.182
.0040	.1578	.0253	.1590	.1577	1.262
.0050	.1764	.0284	.1784	.1765	1.412
.0060	.1930	.0311	.1954	.1930	1.544
.0070	.2082	.0336	.2111	.2082	1.666
.0080	.2224	.0360	.2262	.2224	1.779
.0090	.2355	.0382	.2400	.2355	1.884
.010	.2479	.0403	.2532	.2479	1.983
.012	.2712	.0443	.2790	.2714	2.171
.014	.2922	.0479	.3010	.2922	2.388
.016	.3116	.0513	.3223	.3116	2.493
.018	.3300	.0546	.3431	.3302	2.642
.020	.3469	.0576	.3619	.3469	2.775
.022	.3633	.0606	.3808	.3634	2.907
.024	.3786	.0634	.3984	.3786	3.029
.026	.3931	.0661	.4153	.3930	3.144
.028	.4072	.0688	.4323	.4072	3.258
.030	.4199	.0714	.4486	.4205	3.364
.035	.4518	.0775	.4870	.4518	3.614
.040	.4803	.0833	.5234	.4803	3.842
.045	.5065	.0888	.5580	.5065	4.052
.050	.5310	.0942	.5919	.5313	4.250
.055	.5538	.0993	.6239	.5538	4.430
.060	.5752	.1043	.6553	.5752	4.602
.065	.5954	.1092	.6861	.5954	4.763
.070	.6143	.1139	.7157	.6142	4.914
.075	.6323	.1186	.7452	.6323	5.058
.080	.6493	.1232	.7741	.6493	5.194
.085	.6654	.1277	.8024	.6654	5.323
.090	.6808	.1322	.8306	.6808	5.446
.095	.6954	.1366	.8583	.6954	5.563
.100	.7094	.1410	.8859	.7094	5.675
.110	.7352	.1496	.9400	.7352	5.882
.120	.7588	.1581	.9934	.7588	6.070
.130	.7805	.1666	1.0468	.7806	6.245
.140	.8002	.1750	1.0996	.8004	6.403
.150	.8183	.1833	1.1517	.8183	6.546
.160	.8349	.1917	1.2045	.8350	6.680
.170	.8501	.2000	1.2566	.8501	6.801

TABLE ID-4 (Continued)

$\frac{d}{L_o}$ 1	$\frac{L}{L_o}$ 2	$\frac{d}{L}$ 3	$\frac{2\pi d}{L}$ 4	$\tanh \frac{2\pi d}{L}$ 5	X_n (inches) 6
.180	.8639	.2084	1.3094	.8639	6.911
.190	.8768	.2167	1.3616	.8768	7.014
.200	.8884	.2251	1.4143	.8884	7.107
.220	.9089	.2421	1.5212	.9089	7.271
.240	.9259	.2592	1.6286	.9259	7.407
.260	.9400	.2766	1.7379	.9400	7.520
.280	.9516	.2942	1.8485	.9516	7.613
.300	.9612	.3121	1.9610	.9612	7.690
.350	.9780	.3579	2.2488	.9780	7.824
.400	.9878	.4049	2.5441	.9878	7.902

(ii) Check photographs for altitude and tilt and determine ground scale. Enlargements corrected for tilt are desirable.

(iii) Trace crest of major wave train from selected photographs in the set and transfer to overlay of hydrographic chart. As the set of photographs will show different crest angles at the same position, averaged curves should be drawn.

(iv) Draw the orthogonals and from the spacing of the orthogonals determine K_d at a selected depth contour.

(v) Determine the average wave period of the major wave train by (a) measuring from the photographs the time interval between breakers or between the instants at which crests pass identifiable points such as rocks or small patches of foam or (b) measuring the wave length at points where the depth is known and compute the period (or obtain it from available graphs).

Except for (v b), this procedure may be followed even when the hydrography is unknown.

b. Advantages

This aerial method of preparing a refraction diagram has the practical advantage that it deals with real waves, which vary in period and direction, and it truly represents the effect of local irregularities in the bottom.

c. Disadvantages

The method has a number of disadvantages among which may be mentioned:

(i) Obtaining photographs representing a range of periods and directions, and possible low and high tide, will require considerable flying time and, almost invariably, a long period of waiting for the desired wave conditions to occur in good photographic weather.

(ii) Swells, which dominate the breaker zone, are frequently obscured at a relatively short distance from shore by small steep waves and it is usually the long period waves which are of interest.

If the refractive effect occurs in a limited area (as in a small bay or near a harbor entrance) direct use of aerial photographs as outlined here is feasible. If the refractive effect takes place gradually over large areas, the aerial photographs become difficult to analyze because of the problem of ground control. A judicious combination of the graphical method with measurements of aerial photographs will yield the most reliable results at reasonable cost.

5. Graphical Construction of Refraction Diagrams Directly by Orthogonals*

a. Introduction

A system has been devised whereby orthogonals may be constructed

* Taken from reference ID-5.

directly without first drawing the wave fronts. This method has the advantage of eliminating an entire graphical step and its attendant inaccuracies. In trained hands, a refraction diagram can be constructed by this method in much less time than is required for its construction by the wave front method. Personnel with a higher degree of training are necessary if this method is used, however, and the operation does not become so nearly automatic as does the wave front method. The method is thus suitable for use by specialist draftsmen or engineers.

Physically the method is carried out by a special protractor which incorporates the requisite scales. The protractor is manipulated in steps from contour to contour and at each step indicates the direction of the orthogonal. One orthogonal is drawn from deep water to shore in each series of operations. The device has been made in the form of a protractor so that it would be entirely adequate for the construction of refraction diagrams. A drafting machine can be employed to advantage, however, and the protractor used merely for the graph and tables on it, for obtaining the required values for the manipulation of the drafting machine.

b. Development of the Method

A detailed description of the development of the method and its application follows:

(1) Assumptions

(i) Contours can be drawn at every abrupt change in slope of the chart.

(ii) The depth at a point between contours in a linear function of its distance from the contours.

(iii) Between contours, wave length and velocity may be considered to vary linearly (the usual assumption).

(iv) The radius of curvature of the orthogonal between contours may be considered constant (a circular arc).

(v) The angle of the arc is equal to the change of angle of the orthogonal.

(vi) That the undulations of small magnitude extant in most contours are more likely to be a measure of observational inaccuracies magnified by rigorous drafting than an indication of the direction of level bottom. Also that bottom features with dimensions small compared with the wave length do not influence the motion of the wave to any appreciable extent. If the contours are smoothed out and only those features preserved that are obviously characteristics of the hydrography, the result is more nearly accurate.

(vii) That the angle of convergence or divergence of orthogonals at differential intervals is small compared to the angle of refraction. (Please note that convergence or divergence is not considered non existent.)

(viii) That a line drawn through any point midway between two contours and making equal angles with the adjacent contours is approximately the direction of a level line at that point (provided the requirement implied in assumption (i) is accomplished).

(2) Derivation

(See Figure ID-27)

considering $\Delta\alpha < 13^\circ$ for two-place accuracy, or $\Delta\alpha < 6^\circ$ for three-place accuracy, $\tan\Delta\alpha = \sin\Delta\alpha = \Delta\alpha = (R_{C_2} - R_{C_1})/BB'$. $AA' = OO' = BB' = d$ (a differential distance).

Assumption No. 4.
and: $\Delta\alpha = (R_{C_2} - R_{C_1})/d$
let C' be the velocity from A to B (effective)
let C'' be the velocity from A' to B' (effective)
let t be the time required for wave front to move from AA' to BB'
$R_{C_2} = C''t$; $R_{C_1} = C't$; then $\Delta\alpha = (C't)/d$

$C'' = C' + \Delta C$ and $\Delta C/(C_1 - C_2) = d\tan\alpha/(J/\cos\alpha) = d\sin\alpha/J$

$C'' = C' \left[(C_1 - C_2)/J\right] d\sin\alpha$

then $\Delta\alpha = (t\sin\alpha)(C_1 - C_2)/J$; but $t = R_{C_1}/C'$

and $C' = (C_1 + C_2)/2$

$= (R_{C_1}/J)(C_1 - C_2)\sin\alpha / \left[(C_1 + C_2)/2\right]$; or $\Delta\alpha = (R_C/J)(\Delta C/C_{ave})\sin\alpha$

(ID-5.1)

or $\Delta\alpha = (R_C/J)(\Delta L/L_{ave})\sin\alpha$

but for the general case $R_C/J = \sec\alpha$

therefore $\Delta\alpha = (\Delta L/L_{ave})\tan\alpha$ (ID-5.2)

(3) Utilization

These two equations are thus seen to be independent of the scale of the chart, and they have been made independent of the wave period by the reduction of this factor to the dimensionless ratio $\alpha L/L_{ave}$.

Equation (ID-5.1) is applicable to all cases where $\Delta\alpha$ is less than some predetermined limit, depending upon the accuracy desired. In general, good results are obtained when $\Delta\alpha$ is less than 13°. In practice $\Delta\alpha$ rarely approaches this limit.

Equation (ID-5.2) is more readily applied under ordinary conditions than is equation (ID-5.1). The limitation of equation (ID-5.2) stems from the fact that as α approaches 90°, $\tan\alpha$ becomes infinite. Physically this situation may occur but is instantaneously altered by refraction so that α becomes less than 90°. However, the application of equation (ID-5.2) normally necessitates crossing an entire contour interval at each step.

The value of $\Delta\alpha$ changes very rapidly in the region when α approaches 90°. Thus the instantaneous refraction over a small distance results in a great change in the rate of refraction, and the interval must be crossed in a series of shorter steps. It is therefore desirable to employ equation (ID-5.1) whenever α exceeds about 80°. Equation (ID-5.1) readily lends itself to crossing a contour interval in partial steps, by the judicious selection of R_C (the distance of wave advance).

The protractor has thus been constructed with graphs for equation (ID-5.2) and a special table for use when α exceeds 80°, which adapts equation (ID-5.1) to the graph of equation (ID-5.2).

The use of the graph suffices for the great majority of diagrams and, for all ordinary cases, there is never any necessity to refer to this table. The table therefore has been made very simple. The graph requires the measurement of one factor only (α) and thus is much faster to use than the table, which requires the measurement of three factors (α, R_C and J).

The uses of these two component operations are summarized in the following:

TABLE ID-5

Equation	Measured Quantities	Use
Graph $\Delta\alpha = (\Delta L/L_{ave})\tan$	α - - - - -	For values of α less than 80°.
Table $\Delta\alpha = (R_C/J)(\Delta L/L_{ave}) \sin\alpha$ or $= (\Delta L/L_{ave})\tan\alpha_c$ where $\alpha_c = \tan^{-1}(R_C{}^*/J)$	, R and J	For values of α greater than 80°.

* Where the angle made by the orthogonal and the contours is greater than 80°, the graph is still used by crossing the contour interval in a series of steps and progressing a distance R_C which has some definite relation to J such as 1, 0.5, etc. Equivalent α for such cases comprise the table. Here α_c (equivalent α) = $\tan^{-1}(R_C/J)\sin\alpha$, but, as α is between 80° and 90° for these cases, $\sin\alpha$ may be given the value of 1 and the equation becomes

$$\alpha_c = \tan^{-1}(R_C/J) \qquad \text{(ID-5.3)}$$

Figure ID-28 is a reproduction of the type I protractor, showing the component scales and their functions. The protractor is 14 inches in diameter.

Figure ID-35 shows the type II protractor. This protractor has a moveable arm. When this arm is aligned along the direction of level bottom, $\Delta\alpha$ is read directly along the pointer at the appropriate values of $\Delta L/L_{ave}$. α and $\Delta\alpha$ are entered on the graphs in their actual dimensions. It is thus unnecessary to determine their numerical values. The factors R_C/J are entered on a circular scale. In this way α_c is indicated directly on the graph for a value of R_C/J. The type II protractor facilitates the operation but is more difficult to construct.

c. Application

(1) Preparation

Contours are drawn upon the chart at intervals that will adequately represent the details of the bottom topography and which are consistent with assumption (i) above. Normally (a rule of thumb) about as many contours will be required as the period in seconds of the longest period wave expected. These contours must extend to a condition of deep water for the longest period wave. That is, to $d = 2.56T^2$, where d is the depth of the deepest contour required and T is the period of the longest period wave to be studied. A table is then prepared for each wave period to be studied as shown in Table ID-6 following:

TABLE ID-6

Computation for use of protractor in example

Period: 10 seconds $L_o = 512$ feet

1	2	3	4	5	6	7
d fathom	d feet	d/L_o	L/L_o	$\Delta L/L_o$	$(L/L_o)ave$	$\Delta L/L_{ave}$
1	6	0.0117	0.26	- - -	- - -	- - -
				0.11	0.31	0.35
2	12	.0235	.37			
				.20	.47	.43
5	30	.0587	.57			
				.18	.66	.27
10	60	.117	.75			
				.18	.84	.21
20	120	.235	.93			
				.05	.95	.05
30	180	.352	.98			
				.02	.99	.02
50	300	.587	1.00			

EXPLANATION OF TABLE

The first column, d, fathom, is a list of the contours on the chart. Should the chart be in feet, this column naturally is eliminated. In practical problems, it would be advisable to have contours at 3, 4, and 7 fathom, as the values of $\Delta L/L_{ave}$ should not exceed about 0.20. For brevity in the following figures, however, fewer contour intervals have been used.

Column 2, d, feet, is the result of multiplying column 1 by the factor 6. If a stage of tide is used other than that of the datum of the chart, a certain constant must be added or subtracted from this column.

Column 3, d/L_o is the ratio of the depth to the deep water wave length for the wave period. In this case (i.e., 10-second period) the wave length is 512 feet and is computed from the equation $L_o = 5.12T^2$, where T is the period in seconds, and L_o is the deep water wave length in feet.

Column 4, L/L_o, is taken from a graph of this function which is on the tables section of this manual, or they can be obtained by interpolation in Table ID-4.

Column 5, $\Delta L/L_o$, is the change in the L/L_o ratio and is written between the lines of the depths between which the change occurred.

Column 6, $(L/L_o)ave$, is the average of the L/L_o ratio between any two depths.

This is also written between the lines. This figure can be obtained simply by adding one-half of the $\Delta(L/L_o)$ figure to the previous value for L/L_o.

Column 7, $\Delta L/L_{ave}$, is obtained by dividing column 5 by column 6.

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(2) General Procedure

The general method of drawing a refraction diagram by use of Type I protractor is shown below for a case where contours are simple and α is always less than 80°. The steps are fully explained under each drawing in Figures ID-29 and 30.

(3) Special Procedure

A special method is shown for two difficult cases. First, when the angle of approach is greater than 80° to the contours and, second, where complex hydrography is encountered. The steps are again explained under each drawing in Figure ID-31.

6. Discussion of the Methods

Explanations of two graphical methods, the wave front and the orthogonal, for constructing refraction diagrams have been presented. Also, several methods of summarizing the refraction coefficients have been illustrated. The presentation for a particular circumstance is largely a choice based upon several factors. If time permits, the supervisor should try

both methods to determine which is preferable. In any case a few diagrams should be checked by both methods to see if they are in reasonable agreement. As an aid in the choosing of the basic method to be used for the refraction study the comparative advantages and disadvantages of each method are presented, although as stated before, circumstances of the specific job will dictate the final choice.

a. Wave Front Method

(1) Advantages

(i) Contours need not be drawn over the base hydrography, but they are desirable as an aid to visual interpretation of the results. The wave fronts may be drawn by knowing only the average depth over which the front passes and this depth can be determined from the plotted soundings.

(ii) Wave crests are shown. Thus the action of the wave in the ocean becomes easier to visualize. In certain cases they may be correlated directly with aerial photographs.

(iii) The method is easy to understand and apply. A person need not understand the complicated wave theory and mathematics involved in the development of the wave front method to be able to draw the wave fronts as it can be done by following normal drafting and graphical construction methods.

(iv) When wave fronts are properly drawn crossed orthogonals are easier to interpret. In fact they can usually be predicted on the basis of the wave fronts.

(v) The method is easy to supervise. A trained supervisor, by close scrutiny of the wave fronts and hydrography, can usually perform a satisfactory detailed visual check of the diagram. If any irregularities are indicated, he can then easily check them with the scales.

(vi) Wave front refraction diagrams are easy to explain to non-technical people. Progressive positions of a wave front have been indicated and from their shape and curvature most people can visualize whether they are converging or diverging.

(vii) In certain cases insignificant wave trains may be eliminated before the wave fronts are carried all the way to shore.

(2) Disadvantages

(i) Wave crests are often smoothed in cases when the crest should be severed to indicate crossed orthogonals. Many published reports show that even experienced people have not severed the crests on the refraction diagrams when the hydrography clearly indicated such severance and aerial photos have shown that such severance existed. Nothing in the foregoing presentation of the wave front method has been said which would prohibit the drawing of severed crests. In fact one example of severed crests has been shown, (Figure ID-36).

(ii) Different scales or different chart markings must be used for each wave period considered. Because the scales are not readily changeable, an unreasonable scale might be used. Thus regardless of the scale of the chart, if the hydrography is regualr and slopes are gradual, larger wave advances are permissable than if the hydrography is irregular or slopes are steep.

b. Orthogonal Method

(1) Advantages

(i) Orthogonals are constructed in a single operation in much shorter time. For the majority of cases only the refraction coefficients are desired and so only orthogonals would be needed.

(ii) The dimensionless protractor which is used is independent of the scale of the chart and the period of the wave. Thus only one scale need be used for an entire series of refraction diagrams and the rate of advance of the orthogonals may be adjusted to suit the hydrography.

(iii) Wave fronts may be drawn if needed. Since by definition orthogonals are everywhere perpendicular to wave fronts, the fronts may be easily drawn in wherever needed for illustration.

(2) Disadvantages

(i) A higher degree of training is required. This method is less straightforward in the graphical stages and requires not only a draftsman to apply it, but also someone with a working knowledge of fundamental mathematics.

(ii) Orthogonals can not be drawn from shore seaward for a given deep water direction. On cursory examination this limitation may not appear to be a drawback since in the wave front method the wave fronts must first be drawn. In some cases (over a particularly deceiving hydrography) not only will the first pair of orthogonals miss the desired area, but the first five or six may miss the area.

7. Examples of Refraction

Various aerial photographs and refraction diagrams are of interest in providing typical examples. Figure ID-36A shows a refraction diagram for Little Placentia Harbor, Newfoundland, for waves of 10 second period from NW carried from deep water, on Hydrographic Chart 2376, to the mouth of the harbor. Wave fronts then were transferred to a larger scale chart (Chart 5621) and carried into the harbor as shown in Figure ID-36B.

Aerial photographs which show examples of wave refraction often are of value in the preparation of refraction diagrams. With the exception of Figure ID-38B such examples are shown in Figures ID-37-44.

Figure ID-37 shows a mosaic prepared from aerial photographs of refraction effects at Half Moon Bay, California. Note that the waves are breaking over a submerged reef offshore.

Figure ID-38A shows waves which have passed through the entrance of Humboldt Bay into the bay. Note that waves are breaking inside the inlet. This breaking is probably the result of a combination of shoal water and a tidal current running opposite to the direction of wave travel. Very often in the preparation of a refraction diagram, when waves are carried over a shoal of limited area, the wave crests appear to cross each other. That such a condition can occur is illustrated in the upper right hand corner of Figure ID-38A where the waves do cross. That a shoal area exists at this locality is shown by the hydrographic chart in Figure ID-38B which covers the section of the bay appearing in Figure ID-39A. Note that where the waves cross, they augment each other and breaking results.

The important features of refraction illustrated by the photos in Figures ID-37-44 inclusive, are indicated in the caption under each photo.

8. Refraction by Currents*

When waves moving through still water encounter a current, moving with, against, or at an angle with the wave direction, the waves undergo a change in length and steepness. In the case where the waves meet a current at an angle the waves also change their direction. In the forecasting of wave conditions there are two situations in which the refraction of waves by currents may be of practical importance. At tidal entrances, ebb currents run counter to the waves and increase the wave height and steepness, thereby adding to the hazards of navigation, while flood currents flatten out the waves. Large scale ocean currents, such as the Gulf Stream, may have a great effect on the height, length, and direction of waves approaching the current discontinuity. In some instances almost complete reflection of waves of certain periods will occur. In other instances the waves have been forced by refraction to exceed their critical steepness and then to break.

The principal application of refraction methods is to predict the occurence and height of breakers around a tidal entrance. The direction, height and period of the waves in deep water are variable, the bottom contours usually are irregular and the currents are variable throughout the tidal cycle. Under such conditions quantitative forecasting by the purely analytical approach probably is not reliable, but a combination of theory and observation would be adequate for making forecasts of probable wave conditions at harbor entrances. In other instances the engineer might use the general principles of refraction by currents to design the shape of dredged tidal channels such that wave action within the channel will be reduced in magnitude (Reference ID-4).

Surface waves in deep or shallow water are refracted by currents to an extent which depends upon the initial wave velocity and direction and the strength of the current. Two common conditions are treated below, which for simplicity sake are concerned with only deep-water waves.

a. Waves Meeting or Following a Current

When waves proceed from still, deep water into a deep sound where a current runs directly with or against the advancing waves, the wave period remains constant but the wave length, velocity, and height changes (Reference

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* Taken from Reference ID-7.

ID-10). Thus, (see Figure ID-45) if L_o and C_o represent the deep water wave length and velocity, the wave length in the sound will be changed to a new value L. The wave velocity (relative to the water) corresponding to this wave length is $C^2 = g\ L/2\pi$; thus,

$$L/L_o = (C/C_o)^2 \qquad \text{(ID-7.1)}$$

The wave velocity over the ground is equal to (C + U), where U is the velocity of the current in the sound. For constant period

$$T = L_o/C_o = L/(C+U) \qquad \text{(ID-7.2)}$$

From the above equations

$$C/C_o = 1/2 \left[1 \pm \sqrt{1 + 4\ (U/C_o)}\right] \qquad \text{(ID-7.3)}$$

Here the plus sign must be taken since C must equal C_o where U = 0. Thus if,

$$\sqrt{1 + 4\ (U/C_o)} = a$$

$$C/C_o = (1/2)\ (1 + a)$$

and

$$L/L_o = (1 + a/2)^2 \qquad \text{(ID-7.4)}$$

Considering equations (ID-7.1) and (ID-7.3) it is seen that the effect of a following current is to increase the wave length, whereas the opposite effect results from an opposing current.

From a consideration of the advance of wave energy it can be shown that the ratio of the wave heights is

$$H/H_o = \sqrt{2/\left[a\ (1+a)\right]} \qquad \text{(ID-7.5)}$$

A plot of this equation to show the change in wave height in an opposing or following current is presented in Figure ID-47. The effect on the wave steepness is obtained from a combination of equations (ID-7.4) and (ID-7.5), thus

$$H/L = H_o/L_o \sqrt{2/\left[a\ (1+a)\right]}\ \cdot \left\{2/1+a\right\}^2 \qquad \text{(ID-7.6)}$$

where the wave steepness H/L in the sound is seen to depend upon the initial steepness H_o/L_o and upon the ratio U/C_o. The waves in the sound will break when the value of H/L reaches the critical value of 1/7. As is apparent from Figure ID-47 this condition takes place only when the waves meet an opposing current; that is, it is in this type of current that the height in the sound is increased over the deep-water height H_o.

b. Waves at an Angle to a Current

Referring to Figure ID-46, when a wave crest progresses from the position A B C in deep-still water across a current discontinuity to a position A' B' C' changes in the wave length, height and steepness occur. This refraction by currents has two effects on the wave steepness. One effect is on the change in the wave length and the other is on the stretching of compressing of the wave crest. These effects may oppose or add depending on whether the current direction is plus or minus. A treatment (Reference ID-6) of the general conditions sketched in Figure ID-46 shows that the effect on wave length is

$$L/L_o = 1/(1-U/C_o \sin\alpha)^2 \qquad \text{(ID-7.7)}$$

and the effect on wave steepness is

$$H/L = H_o/L_o \sqrt{\frac{\cos\alpha}{\cos\beta}\left[\frac{(1-U/C_o \sin\alpha)^6}{(1+U/C_o \sin\alpha)}\right]} \qquad \text{(ID-7.8)}$$

or

$$H/L = H_o/L_o \,(M)$$

This can be rewritten in the form

$$H/H_o = (L/L_o)\, M$$

This combination of equations ID-7.7 and ID-7.8 permit the preparation of a diagram (Figure ID-48) which gives the change in wave height as a function of the strength and direction of the current compared to that of the incident wave. The most interesting feature of the curves presented in Figure ID-48 is that whether the waves enter an oncoming or a following current they may increase in height (and therefore steepness) and then break. As an example, examination of Figure ID-48 shows that when the value of $U/C_o = +0.1$ (a following current) all waves will rapidly increase in height (and therefore break) for which the incident angle is larger than about 58 degrees. For higher plus values of U/C_o, the waves will break at much smaller incident angles.

The above analyses for waves meeting a current either directly or at an angle apply to deep water conditions. A similar but more tedious analysis would result from an analysis of waves refracted by currents in shallow water.

9. Difficulties Encountered in the Construction of Refraction Diagrams

One should constantly refer to the basic assumptions in the methods for constructing refraction diagrams to see if they agree reasonably with the practical factors. One should also use every means to check the refraction picture which has been developed graphically. Aerial photos and visual observations are two of the best practical checks of the diagrams. Some of the irregularities which may be encountered are discussed below.

a. Short Crested Waves

Refraction theory is based upon the assumption that waves are long crested. If the waves are short crested, velocities are up to 40% higher than predicted, the refraction effect is larger and if the angle of incidence of the wave to the contours is large, the refraction coefficients will likely be too small for extreme cases of diverging orthogonals or too large for similarly extreme cases of convergence.

b. Theoretical Wave Velocity

Wave velocities are computed using the formula based on the irrotational theory for waves of infinitely small height as follows:

$$C^2 = (gL/2\pi)\tanh(2\pi d/L) \qquad \text{(ID-6.1)}$$

These velocities are somewhat too small for waves of finite height and the formula to the third approximation should be as follows:

$$C^2 = (gL/2\pi) \tanh (2\pi d/L) \left\{ 1 + (\pi H/L)^2 \left[\frac{2\,[\cosh (4\pi d/L)]^2 + 2 \cosh (4\pi d/L) + 5}{8\,[\sinh (2\pi d/L)]^4} \right] \right\} \qquad \text{(ID-6.2)}$$

These two formulae are seen to be the same except for the terms in brackets in the latter. In general, the greater the deep water steepness of the wave and the shallower the depth, the greater the variation of velocities computed from the two formulae. Thus if the wave front is convex and orthogonals are diverging, the measured refraction coefficient will be too small; if the wave front is concave (convergence), the refraction coefficient will be too large.

c. Wave Period

Refraction diagrams are drawn for wave trains of constant period, while in nature the period of the waves vary in a statistical manner. Thus if the refraction picture changes markedly for some of the periods investigated from a given direction, some doubt might arise as to the validity of the coefficients.

d. Variation of Wave Direction

Refraction diagrams are drawn for waves in deep water from specific directions. Small changes in direction from the directions investigated should be examined visually to determine if more diagrams are required from slightly different direction in the case of say, offshore islands (Reference ID-3).

e. Refraction by Unknown Currents

Refraction by unknown currents of transverse set may occur in the case of a train of waves progressing into a current at an angle. Very few ocean currents are known which have stable boundaries and known velocity distributions across them. Idealized theoretical studies have been made of the additional refraction effect of currents and many observations of the effect of currents on waves have been made (References ID-2 and 6).

f. Diffraction

Diffraction along the wave crest may occur. An extreme convergence of orthogonals indicates an excessive wave height in that area, but if some of the wave energy flows laterally along the crest, the wave height will not be as high as indicated by the refraction study. Similarly a zone of extreme divergence may indicate wave heights lower than actual.

g. Crossed Orthogonals

Crossed orthogonals are a phenomemon which have occupied much attention in published reports (See Reference ID-9, especially). They probably represent either a severance of the wave crest and subsequent intermingling of the resulting pair of wave trains or a peaking up and breaking of the waves. The only difficulties which should arise over the question of wave crest severance are the relative sizes of underwater obstacles, rapidly changing hydrography, or the relative size of islands which will cause an appreciable intermingling of crests. Bottom discontinuities such as breakwaters, ledges, abrupt changes in slope etc., if of appreciable size and extent may have a large effect on the refraction of the waves and should be examined carefully. Quantitative information as to the relationships of the size of these discontinuities to the size of the waves is not now available and these factors are at present largely a matter of reasonable judgment.

h. Hydrography may not be as Indicated

The hydrographic charts must of course be examined carefully to determine their reliability. One should be suspicious of each chart until he has checked its source, date of survey, and general character to see if it rings true. He should also check it against every other known bit of information available. Sometimes spot checks can be made by limited surveys, reconnaisance or aerial photos. Adjacent areas to the coast under consideration should be examined to see if rivers are dumping sediments into the ocean or nearby areas are being eroded rapidly in such a manner as to change the hydrography.

The irregularities in the developed refraction patterns which have been described above may seem formidable. They may seem to provide reason for not developing the refraction picture at all. But remember that they will probably not ever be all combining at once to distort the refraction picture. Whether or not they are known or suggested by the conditions in a particular case can help to establish the reliability of the refraction picture. They should always be investigated and evaluated as closely as possible. Even if all the above factors combine to form the most unfortunate conditions, and even if hydrographic information is scant, A REFRACTION PICTURE OF SOME SORT SHOULD ALWAYS BE DEVELOPED.

10. References

ID-1. Arthur, R.S. - "Refraction of Water Waves by Islands and Shoals with Circular Bottom Contours". Transactions, American Geophysical Union, Vol. 27, No.2, April 1946, pp. 168-177

ID-2. Arthur, R.S. - "Refraction of Shallow Water Waves: The Combined Effects of Currents and Underwater Topography". Transactions, American Geophysical Union, Vol. 31, No. 4, August 1950, pp. 549-552

ID-3. Arthur, R.S. - "The Effect of Islands on Surface Waves". Bulletin, Scripps Institution of Oceanography, La Jolla, California, Vol. 6, No. 1, 1951, pp. 1-26

ID-4. Isaacs, J.D. - "The Refraction of Surface Waves by Currents: A Discussion". Transactions, American Geophysical Union, Vol. 29, No. 5, October, 1948, pp. 739-742

ID-5. Johnson, J.W., O'Brien, M.P. and Isaacs, J.D. - "Graphical Construction of Wave Refraction Diagrams". U.S. Navy Hydrographic Office Publication No. 605, January 1948

ID-6. Johnson, J.W. - "The Refraction of Surface Waves by Currents". Transactions, American Geophysical Union, Vol. 28, No. 6, December 1947, pp. 867-874

ID-7. Johnson, J.W. - "Engineering Aspects of Diffraction and Refraction". Proceedings, Separate, American Society of Civil Engineers (in press)

ID-8. Joos, G. - "Theoretical Physics". pp. 381-382

ID-9. Pierson, Willard J., Jr. - "The Interpretation of Crossed Orthogonals in Wave Refraction Phenomena". U. S. Army, Corps of Engineers, Beach Erosion Board, Washington, D. C., Technical Report No. 21, January 1951.

ID-10. Scripps Institution of Oceanography - "On Wave Heights in Straits and Sounds When Incoming Waves Meet a Strong Tidal Current". Scripps Institution of Oceanography, La Jolla, California, April 20, 1944 (Unpublished)

FIG. ID-1. AERIAL PHOTOGRAPH OF SWELL, BREAKERS AND SURF NORTH OF OCEANSIDE, CALIF., 17 AUG 1945 (UTILITY SQUADRON 12)

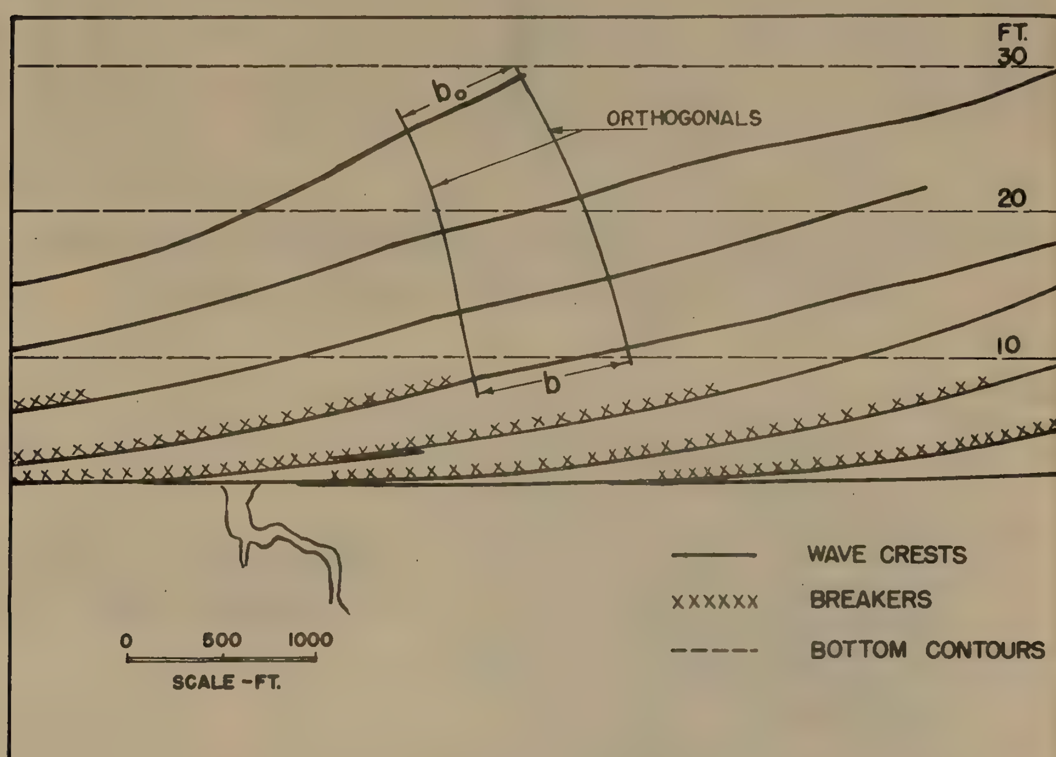


FIG. ID-2. WAVE PATTERN FROM AERIAL PHOTOGRAPH, FIG. ID-1.

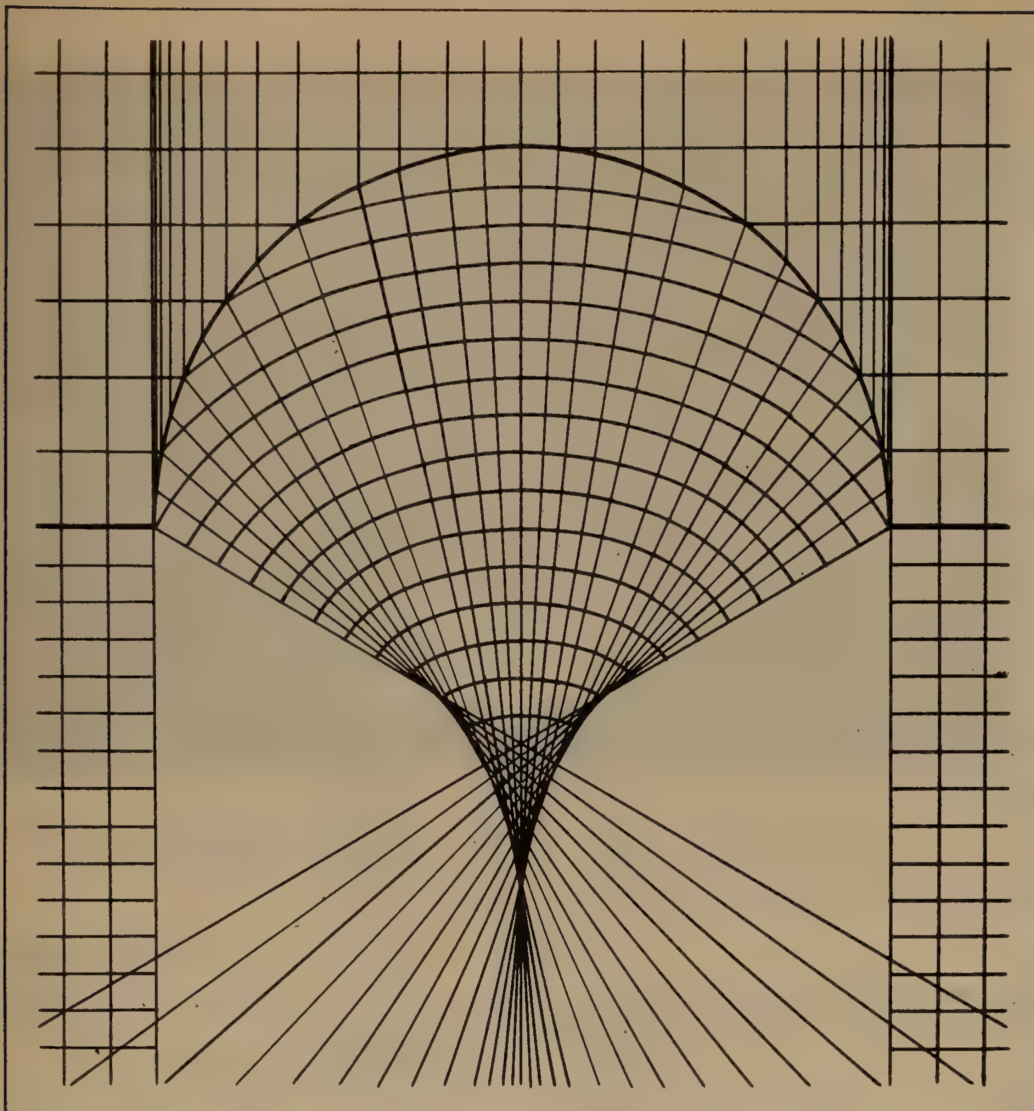


FIGURE ID-3. WAVE CREST-ORTHOGONAL PATTERN FOR THE EXAMPLE OF A CAUSTIC CURVE

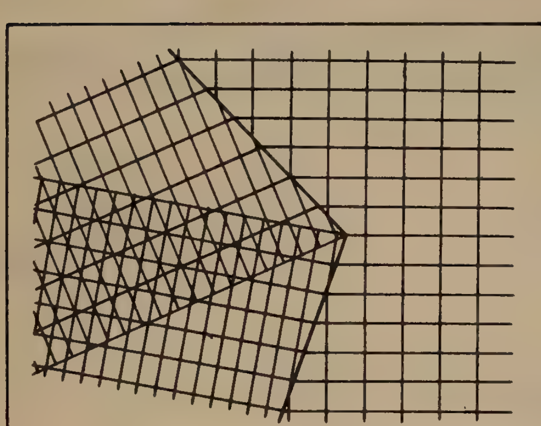


FIGURE ID-4. EXAMPLE OF CROSSED WAVE TRAINS

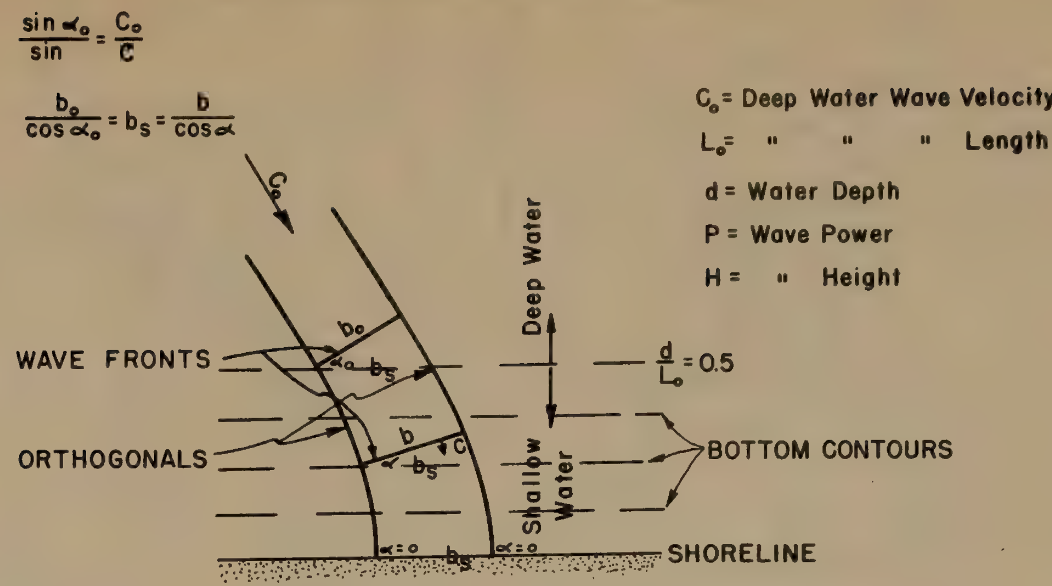


FIG. ID-5-- Wave refraction assuming a gradual change in wave velocity.

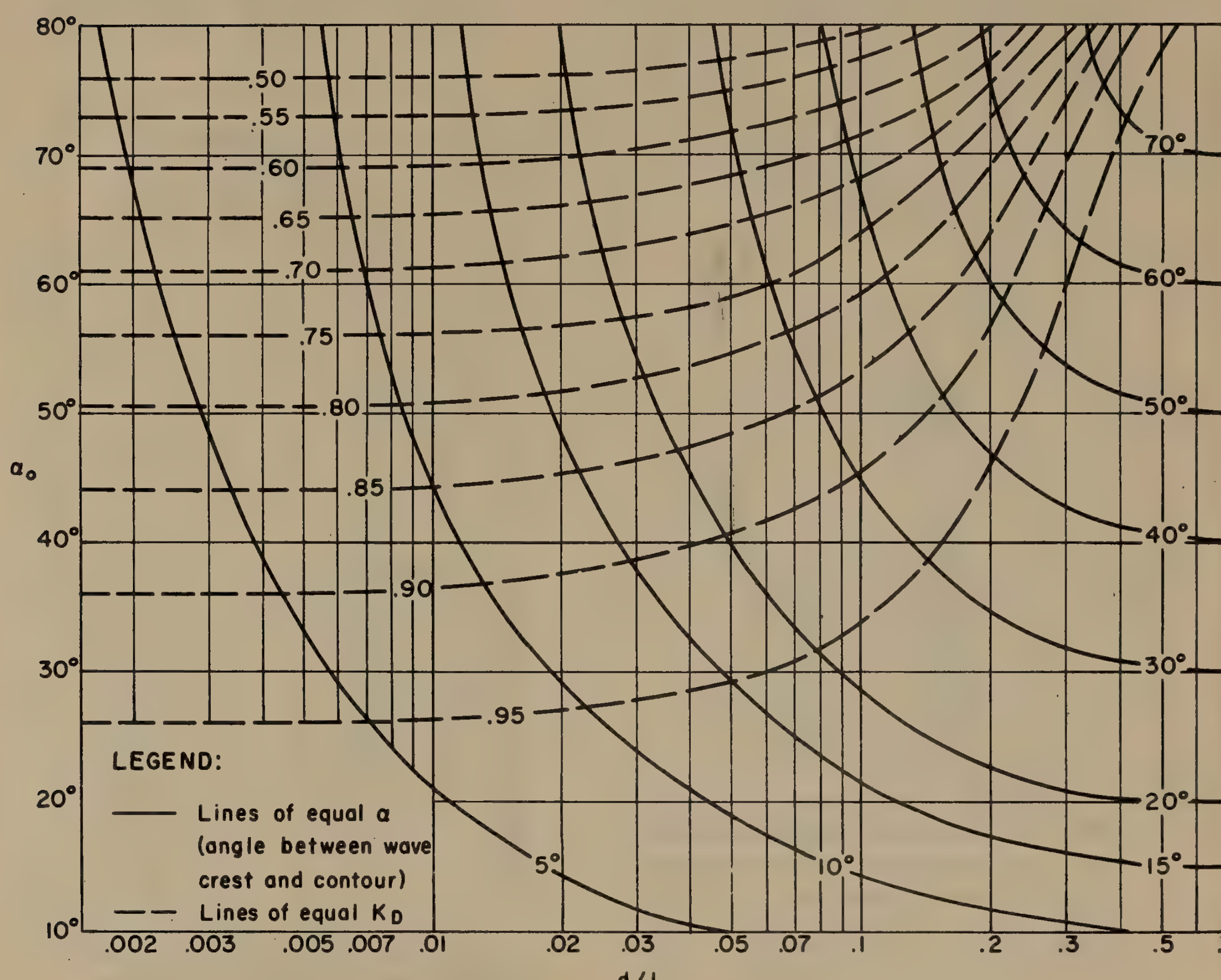


FIG. ID-6 -- Change in wave direction and height due to refraction on beaches with straight, parallel depth contours.

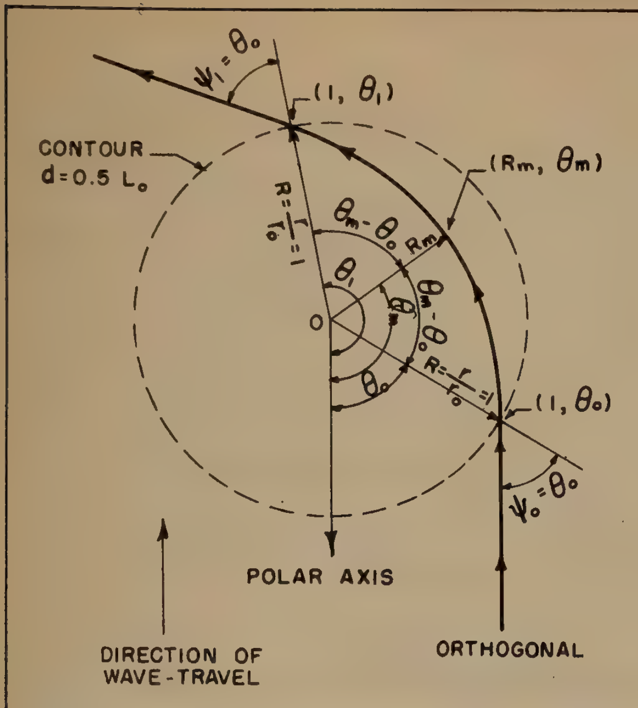


Fig.ID-7. Polar-coordinate representation of an orthogonal associated with wave-crests.

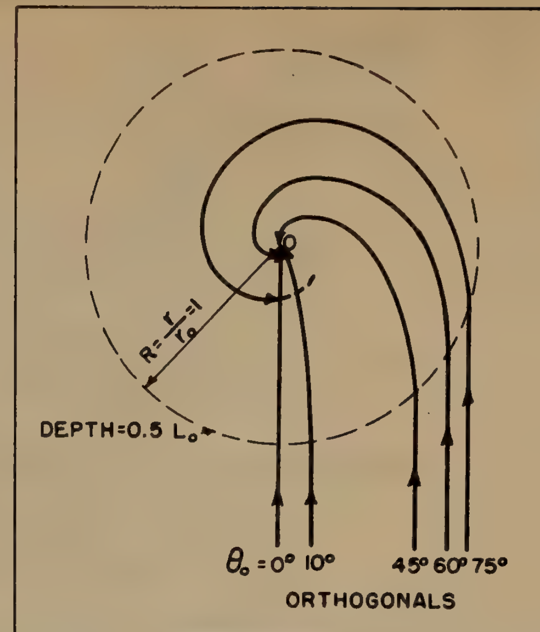


Fig.ID-8. Orthogonals for a "point island" off which the slope changes from very gentle to very steep, Case IA.

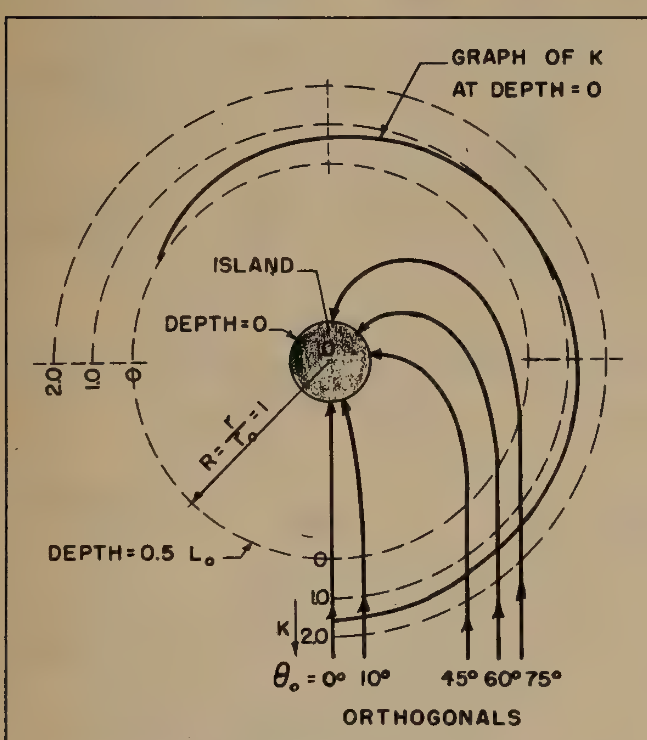


Fig.ID-9. Orthogonals and K-variation at shore for a circular island off which the slope changes from very gentle to very steep, Case IB.

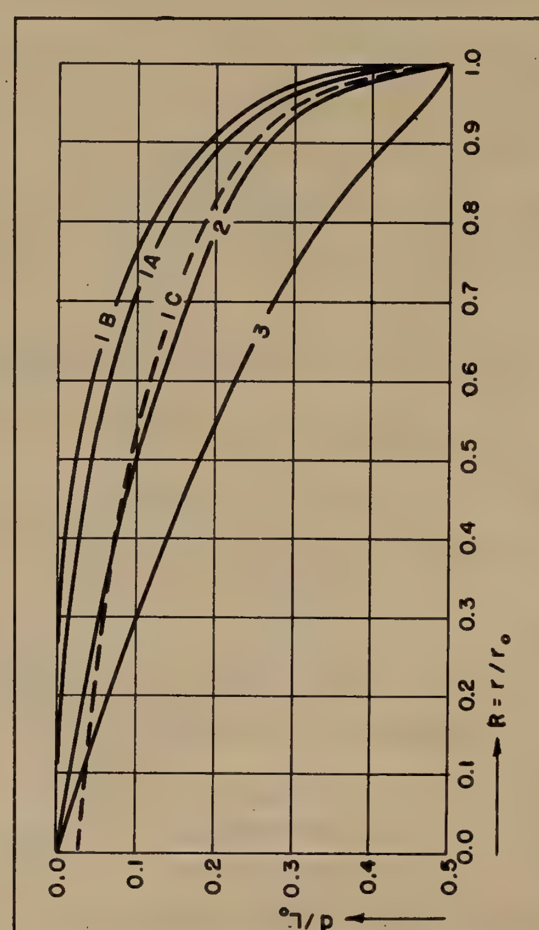


Fig.ID-10. Cross-sections of bottom-slopes for 5 cases.

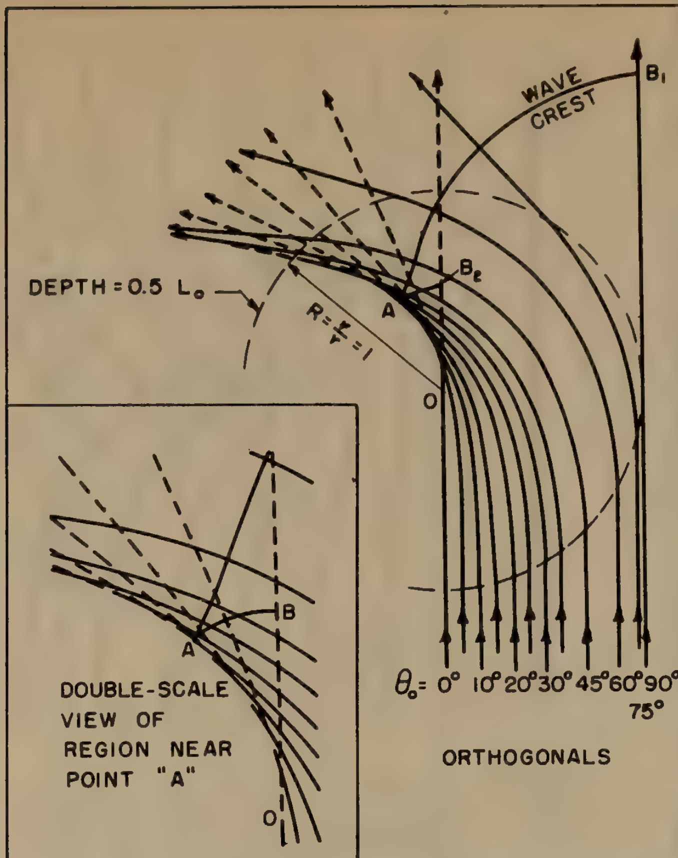


Fig. ID-11 Intersecting orthogonals associated with wave-crests passing over a shoal, Case 1C.

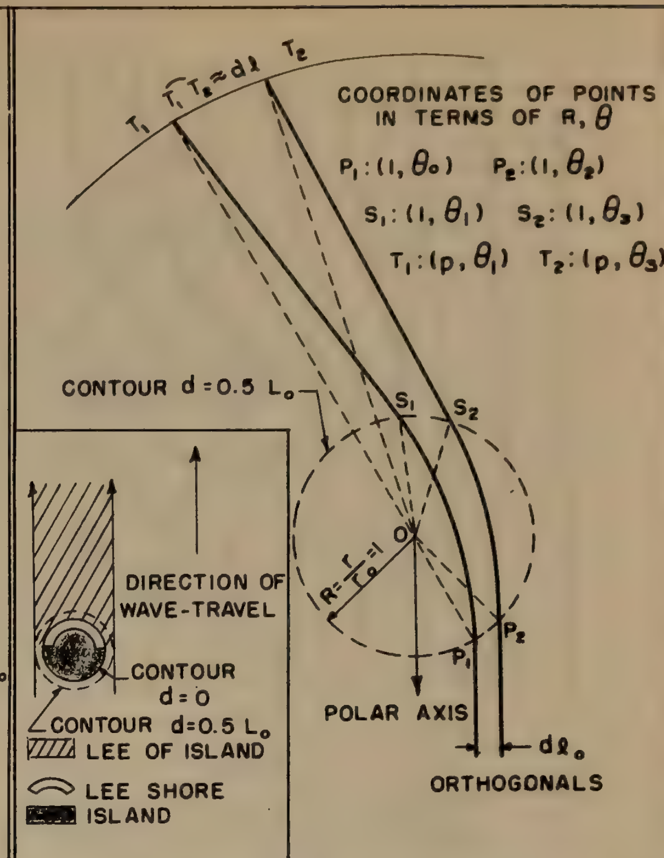


Fig. ID-12 Diagram for determination of K at a distance from an island (inset illustrates meaning of term "lee")

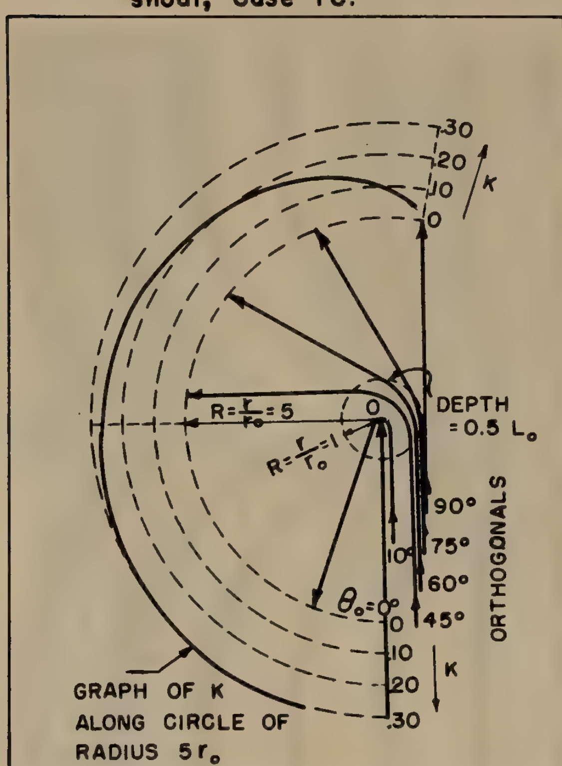


Fig. ID-13 Orthogonals and K-variation (at distance R=5) for a "point island" off which slope changes from moderate to steep, Case 2.

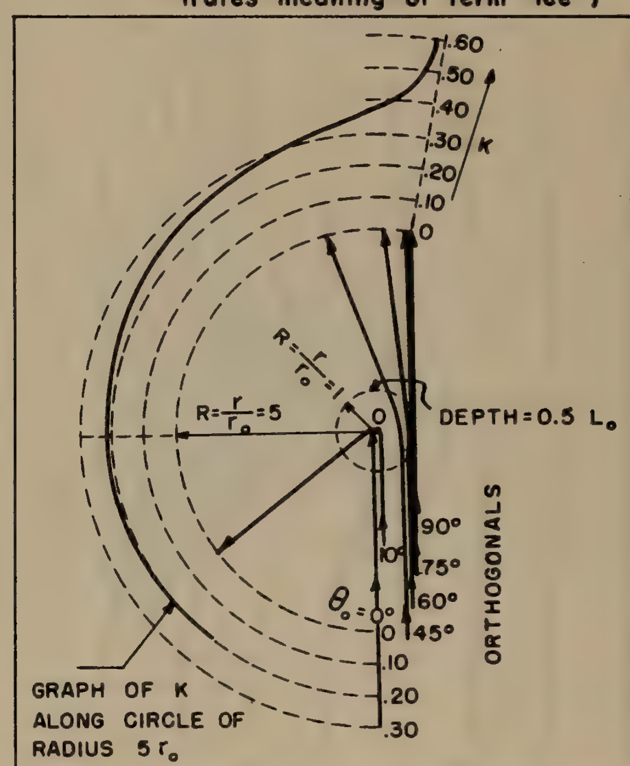


Fig. ID-14 Orthogonals and K-variation (at distance R=5) for a "point island" off which the slope is approximately linear, Case 3.

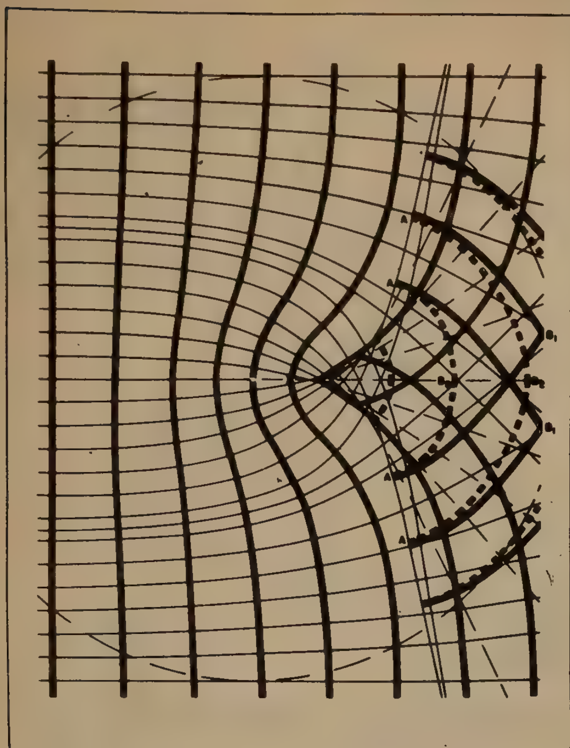


FIG. ID-15. THEORETICAL WAVE CREST-ORTHOGONAL PATTERN FOR WAVES PASSING OVER A CLOCK GLASS. NO PHASE SHIFT.

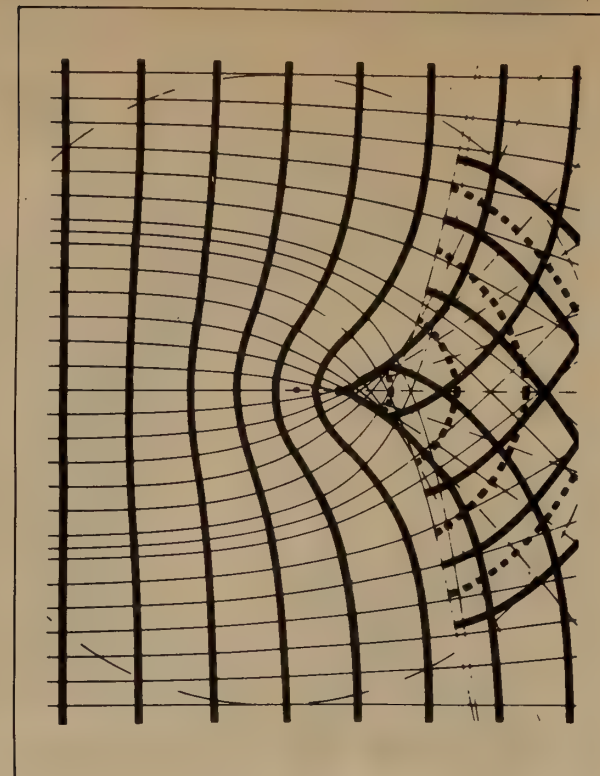


FIG. ID-16. THEORETICAL WAVE CREST-ORTHOGONAL PATTERN FOR WAVES PASSING OVER A CLOCK GLASS. PHASE SHIFT.

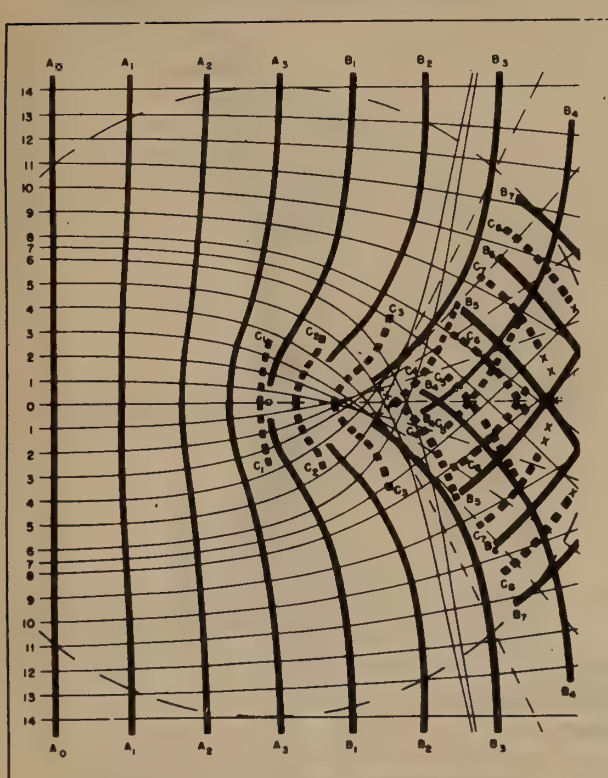


FIG. ID-17. ACTUAL WAVE CREST-ORTHOGONAL PATTERN FOR WAVES PASSING OVER A CLOCK GLASS.

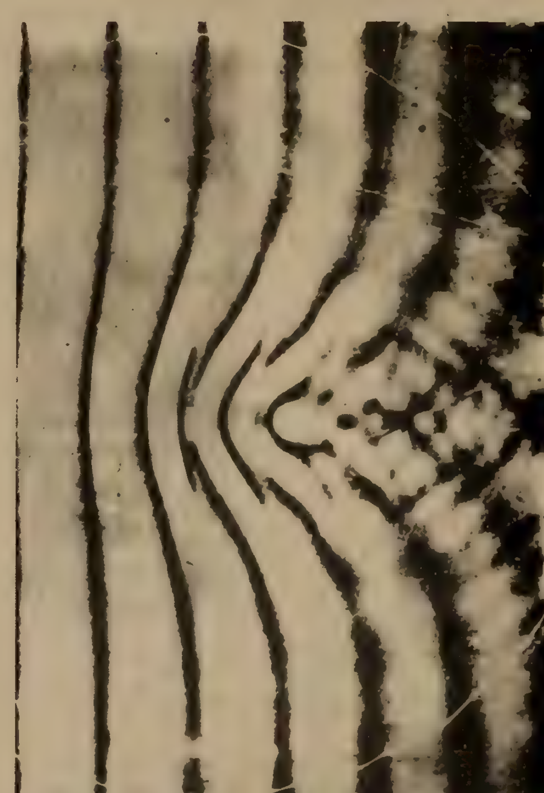


FIG. ID-18. SHADOWGRAPH FOR WAVES OF MODERATE LENGTH PASSING OVER A CLOCK GLASS.

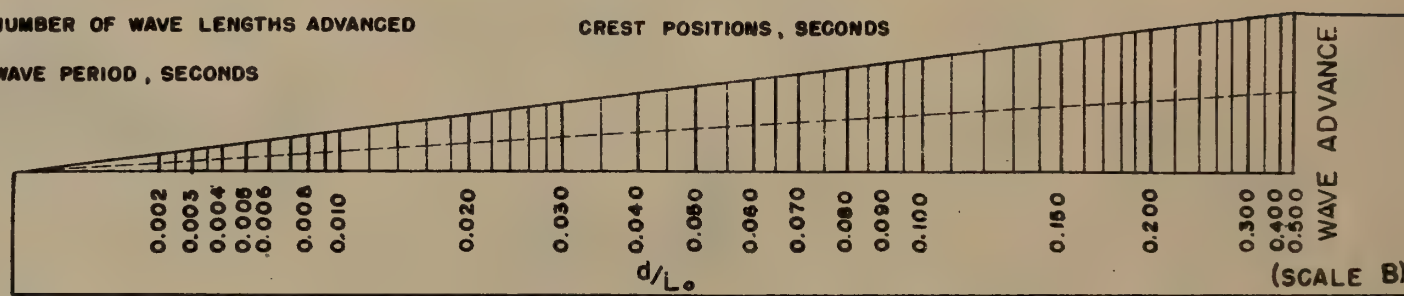
FOR SCALE A; $n = 0.0326 \frac{S}{T^2}$ AND $t = 0.0326 \frac{S}{T}$ FOR SCALE B; $n = 0.0163 \frac{S}{T^2}$ AND $t = 0.0163 \frac{S}{T}$

<u>WHERE</u> S = MAP SCALE IN THE FORM $1/S$

n = NUMBER OF WAVE LENGTHS ADVANCED

T = WAVE PERIOD, SECONDS

t = TIME INTERVAL BETWEEN CREST POSITIONS, SECONDS



EXAMPLE

FOR A MAP SCALE OF $\frac{1}{50,000}$ AND T = 10 SEC.; WHEN USING SCALE B, THE DISTANCE BETWEEN SUCCESSIVE CREST POSITIONS IS -

$$n = \frac{(0.0163)(50,000)}{(10)^2} = 8.14 \text{ WAVE LENGTHS}$$

AND

$$t = \frac{(0.0163)(50,000)}{(10)} = 81.4 \text{ SECONDS}$$

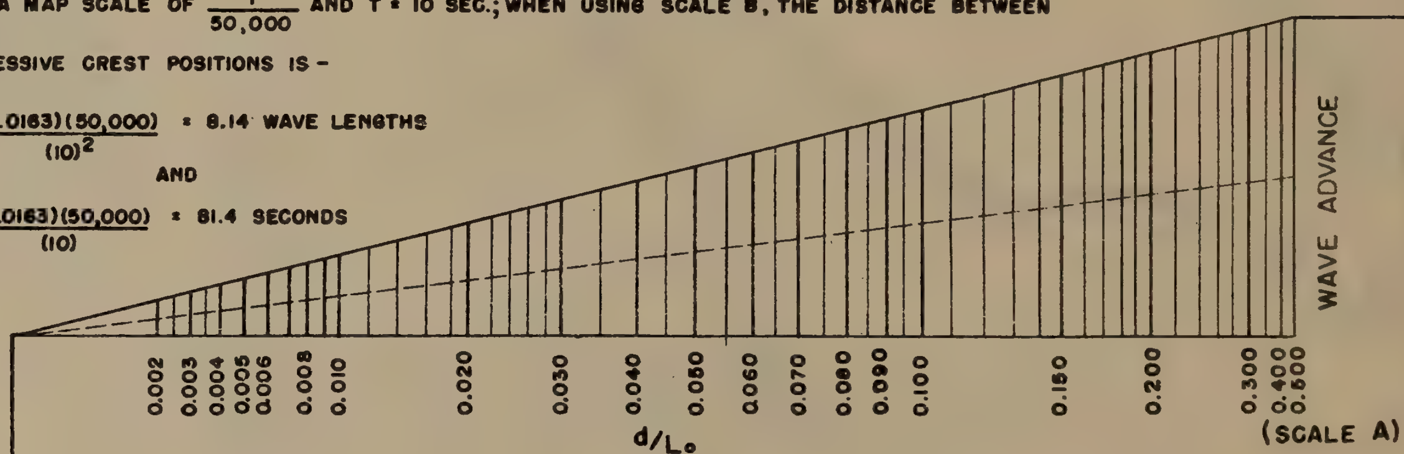


FIG. ID-19a. SCALES FOR PREPARATION OF WAVE REFRACTION DIAGRAMS

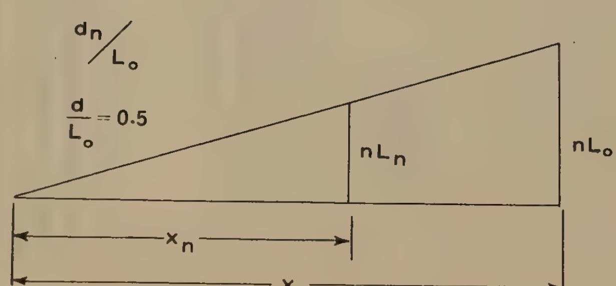


FIG. ID-19b.

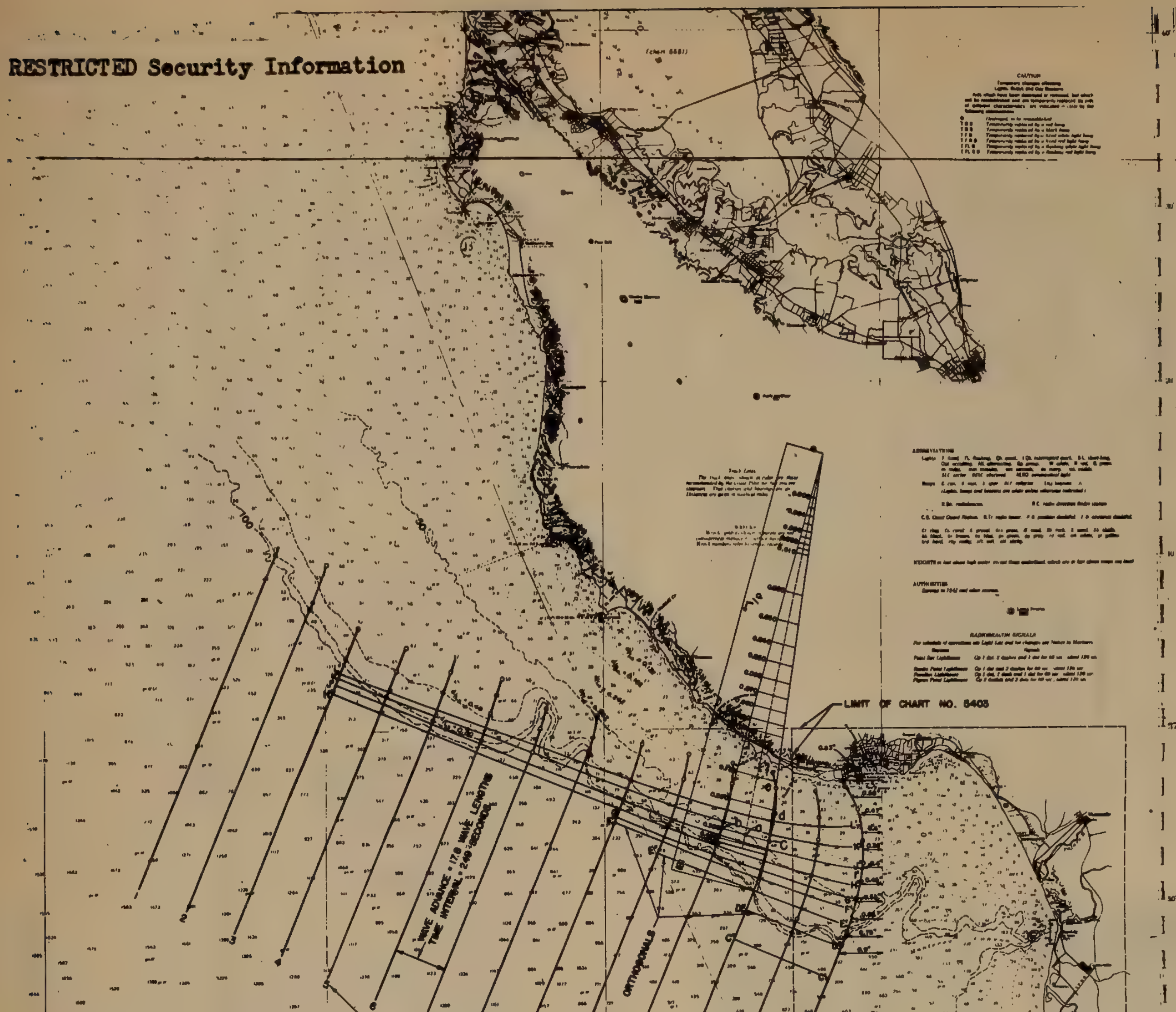

LIMIT OF CHART NO. 5403
ORTHOGONALS
WAVE CREST NUMBERS
WAVE PERIOD = 14 SECS.
WAVE DIRECTION = WNW
UNITED STATES—WEST COAST
CALIFORNIA
POINT SUR TO SAN FRANCISCO
SOUNDINGS IN FATHOMS
AT MEAN LOWER LOW WATER
(Point Sur to San Francisco)
USC&GS
5402

FIG. ID-20

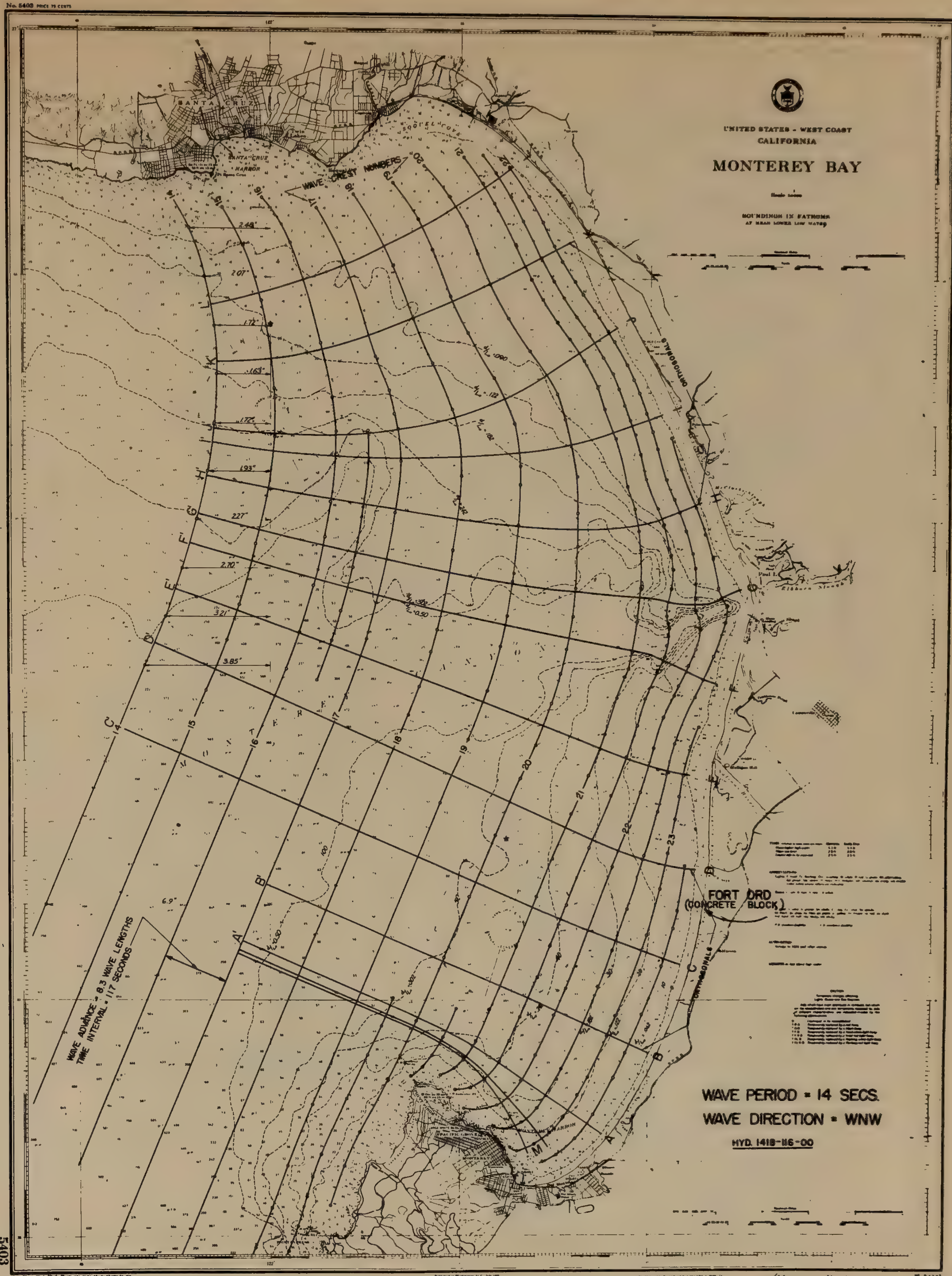

No. 5403 PRICE 75 CENTS
UNITED STATES - WEST COAST
CALIFORNIA
MONTEREY BAY
SOUNDINGS IN FATHOMS
AT MEAN LOWER LOW WATER
SANTA CRUZ
SANTA CRUZ HARBOR
WAVE CREST NUMBERS
ORTHOGONALS
FORT ORD
(CONCRETE BLOCK)
WAVE ADVANCE = 8.3 WAVE LENGTHS
TIME INTERVAL = 117 SECONDS
WAVE PERIOD = 14 SECS.
WAVE DIRECTION = WNW
HYD. 1418-116-00
5403
(Monterey Bay)

FIG. ID-21

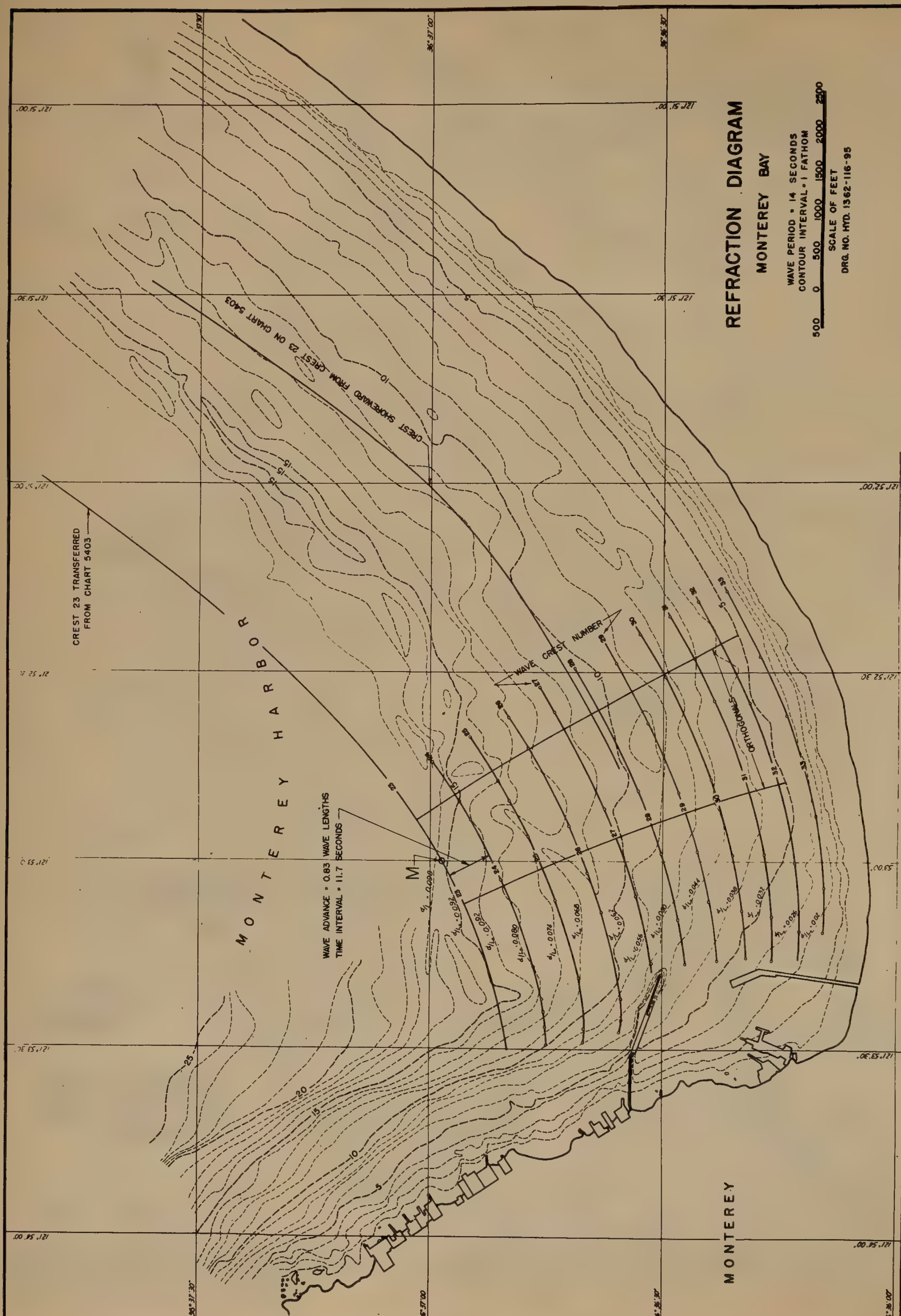

REFRACTION DIAGRAM
MONTEREY BAY
WAVE PERIOD = 14 SECONDS
CONTOUR INTERVAL = 1 FATHOM
500 0 500 1000 1500 2000 2500
SCALE OF FEET
DRG. NO. HYD. 1362-116-95
CREST SHOREWARD FROM CREST 23 ON CHART 5403
CREST 23 TRANSFERRED FROM CHART 5403
WAVE CREST NUMBER
ORTHOGONALS
WAVE ADVANCE = 0.83 WAVE LENGTHS
TIME INTERVAL = 11.7 SECONDS
MONTEREY HARBOR
MONTEREY

FIG. ID-22

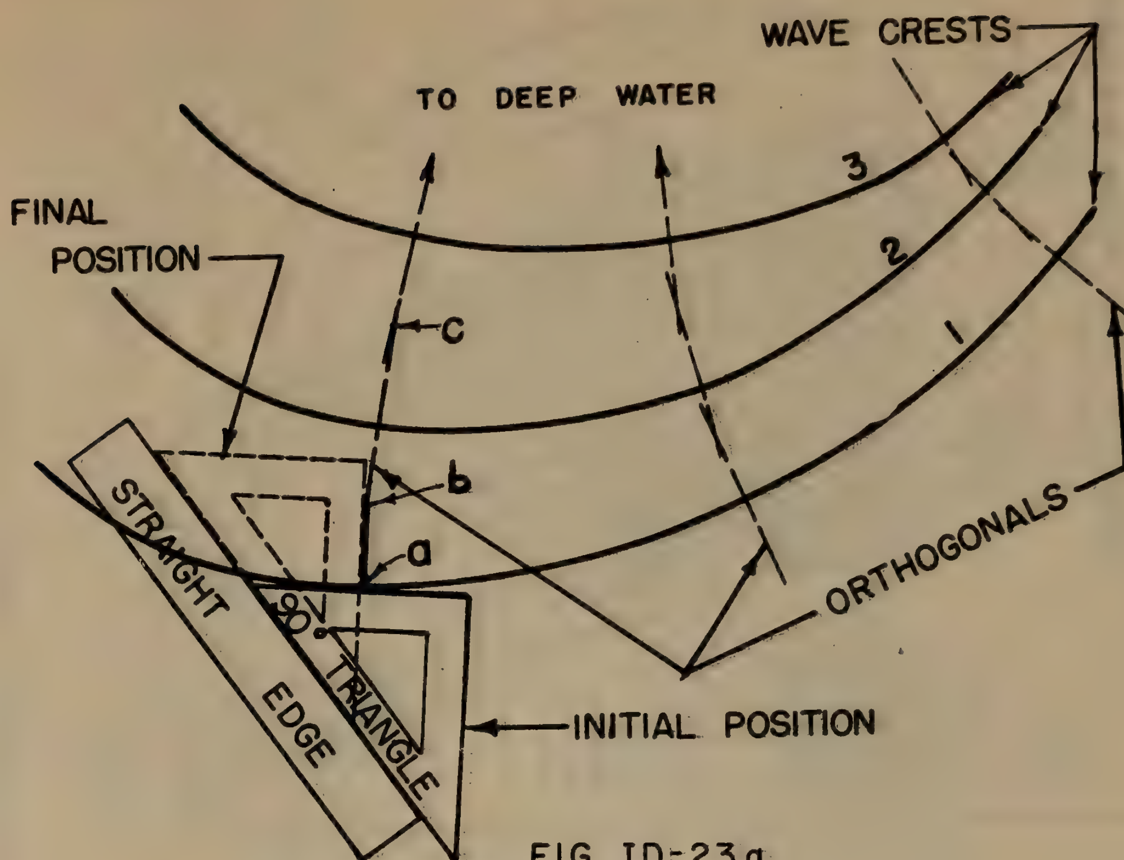


FIG. ID-23a

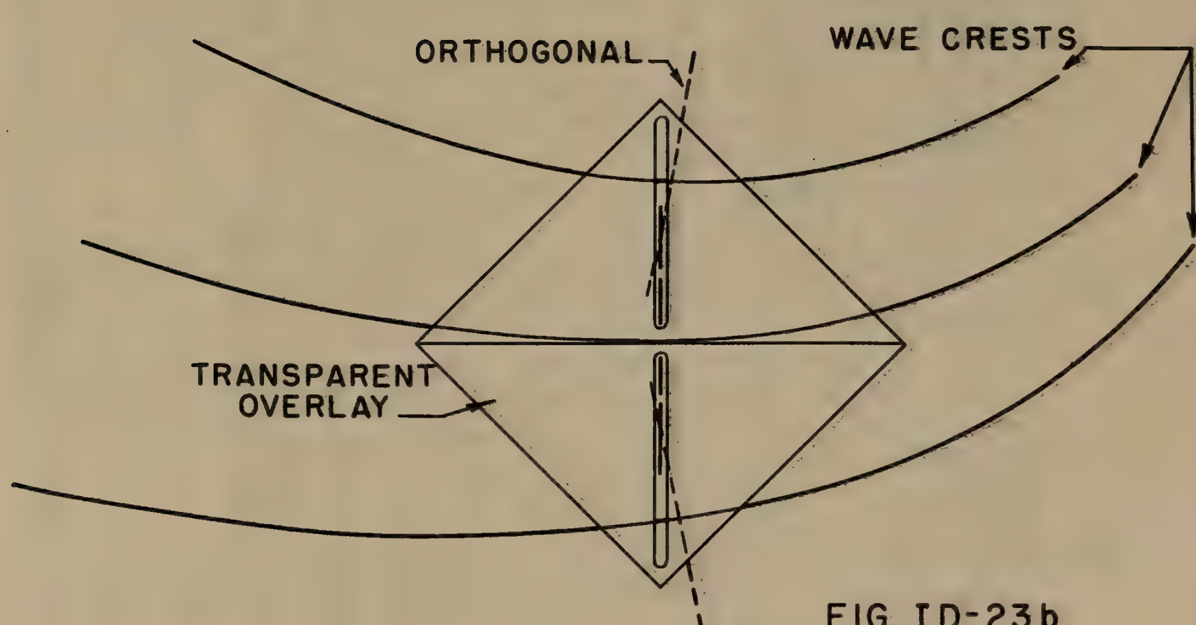
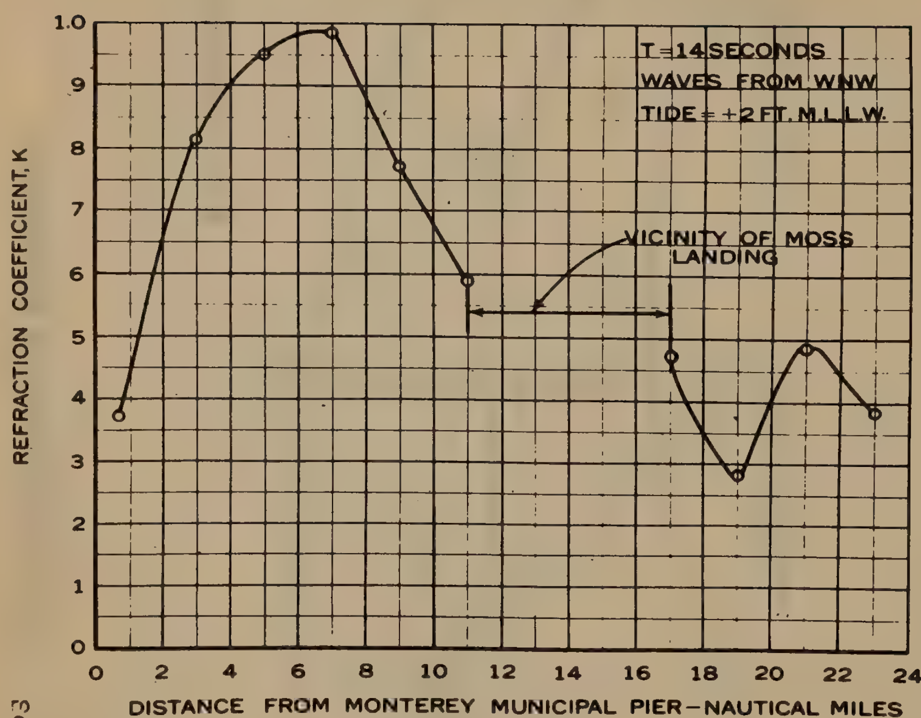
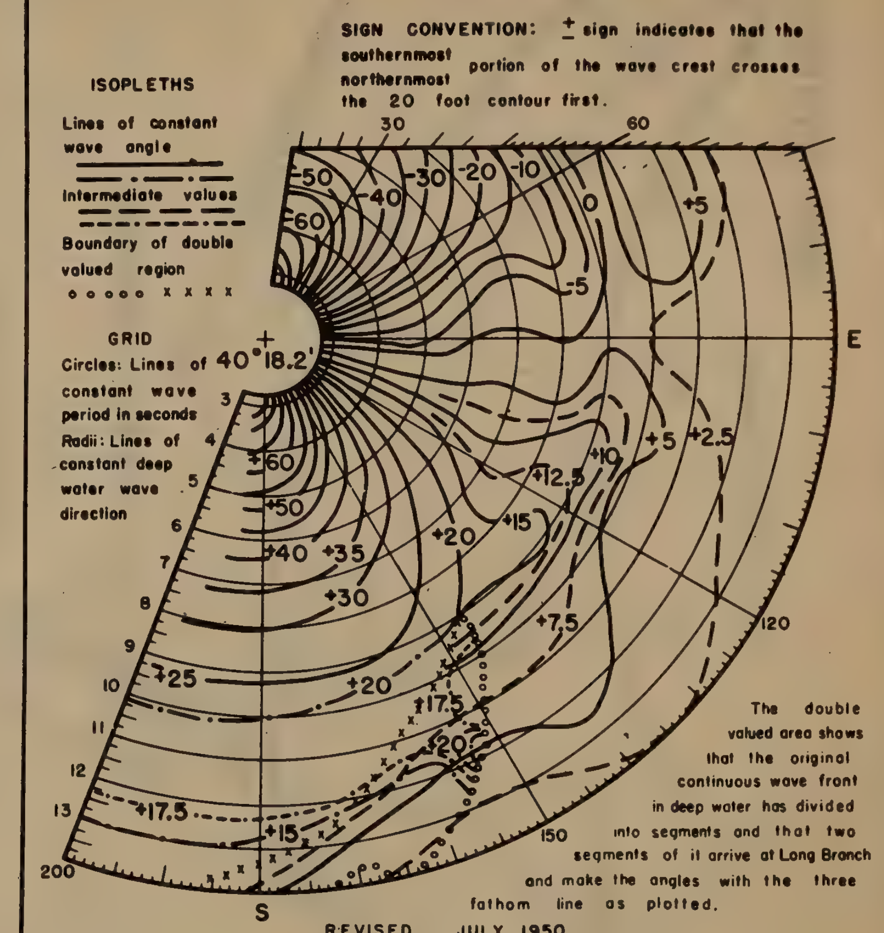


FIG. ID-23b



a. REFRACTION COEFFICIENT AT 5-FATHOM LINE, MONTEREY BAY



b. ANGLE OF WAVE CREST WITH 20 FOOT CONTOUR AT LONG BRANCH, NEW JERSEY FOR MEAN SEA LEVEL

FIG. ID-24. EXAMPLES OF REFRACTION COEFFICIENT PRESENTATION

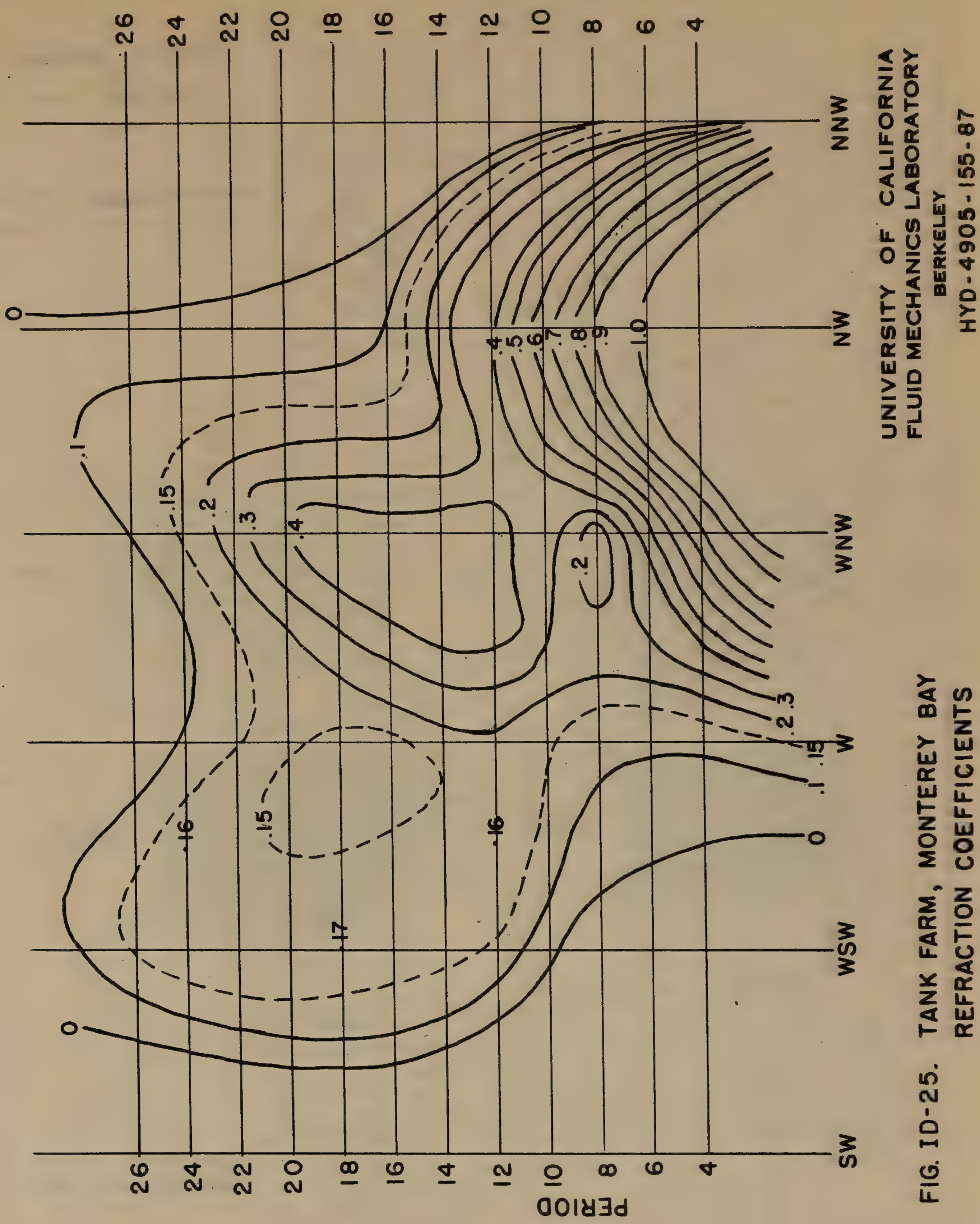


FIG. ID-25. TANK FARM, MONTEREY BAY
REFRACTION COEFFICIENTS

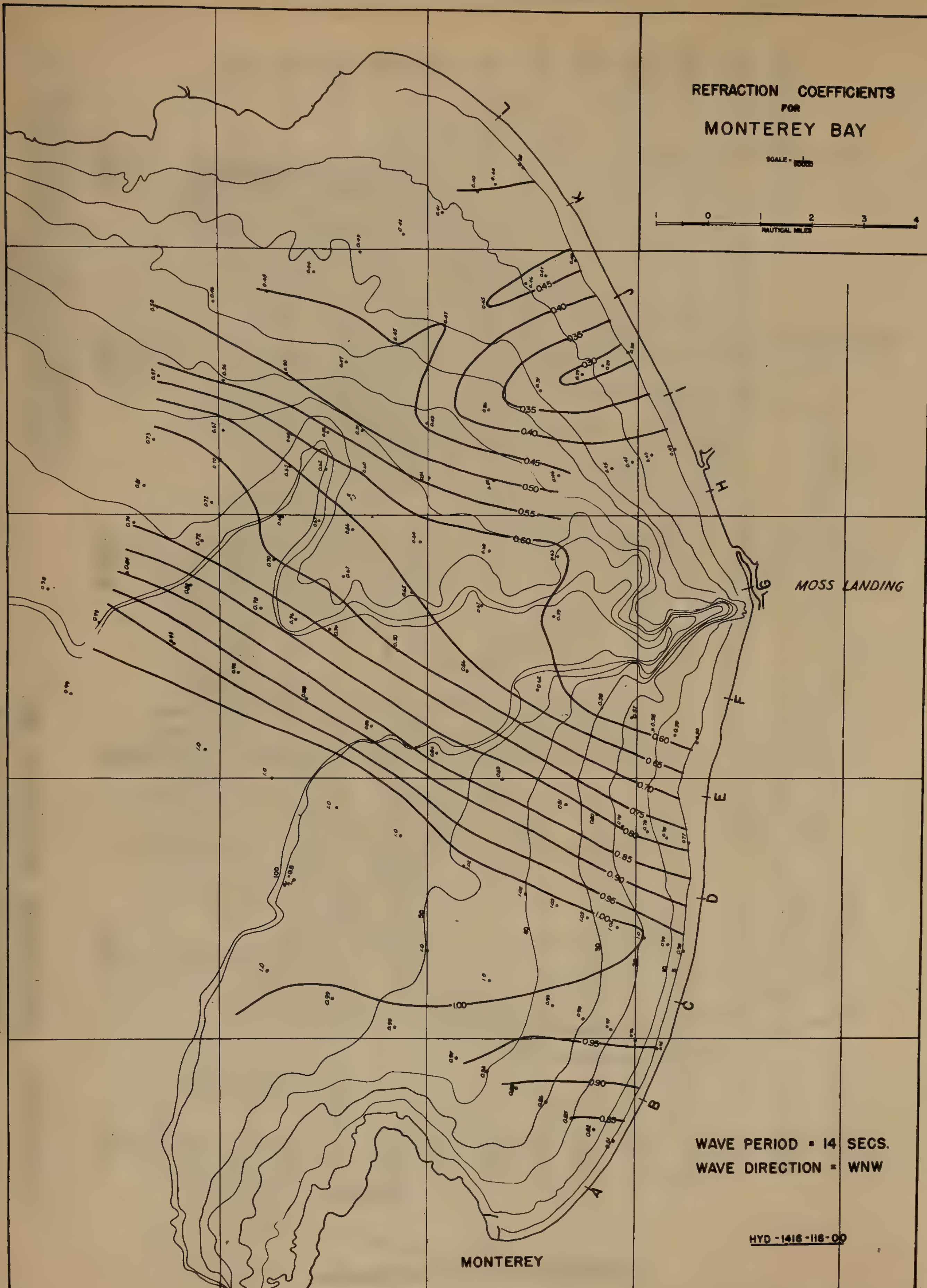

REFRACTION COEFFICIENTS
FOR
MONTEREY BAY
NAUTICAL MILES
MOSS LANDING
MONTEREY
WAVE PERIOD = 14 SECS.
WAVE DIRECTION = WNW
HYD-1416-116-00

FIGURE ID-26

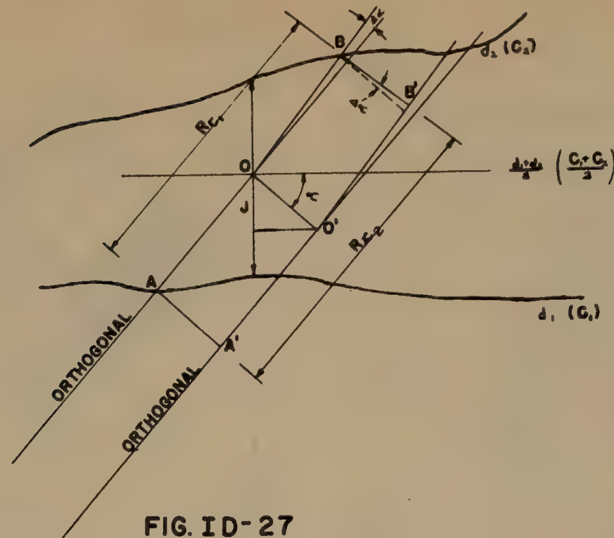
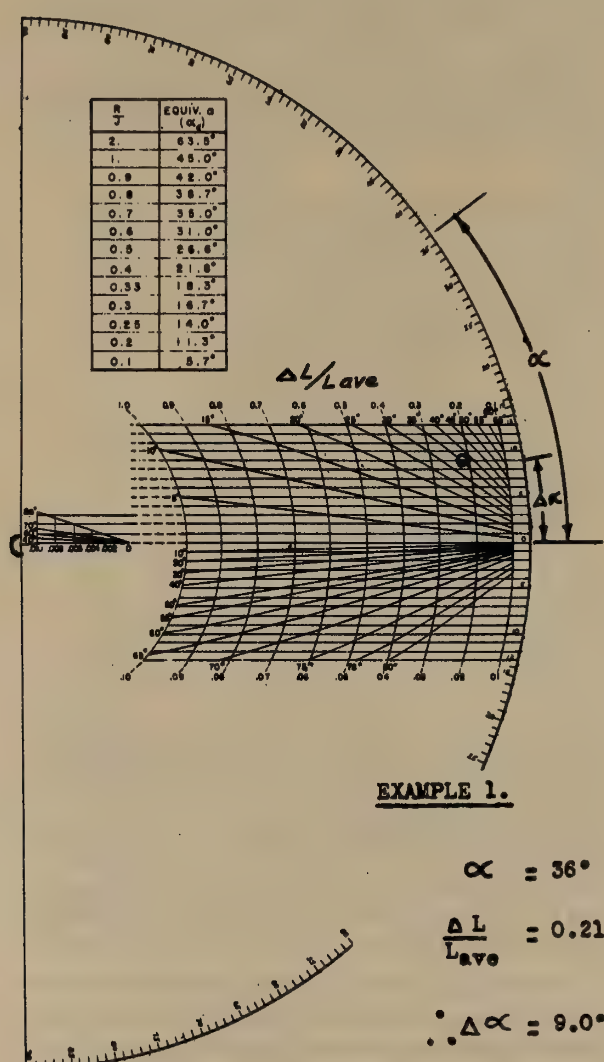


FIG. ID-27

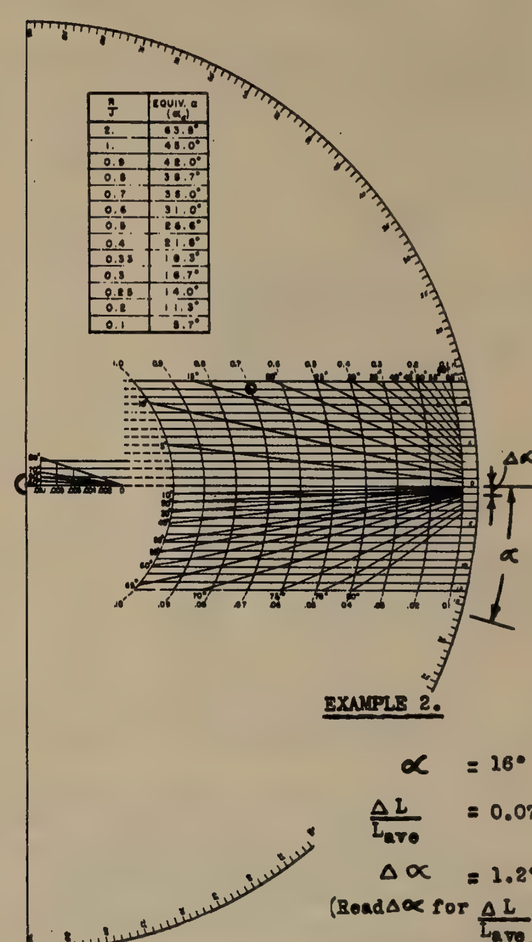


EXAMPLE 1.

$\alpha = 36°$

$\frac{\Delta L}{L_{ave}} = 0.21$

$\therefore \Delta\alpha = 9.0°$



EXAMPLE 2.

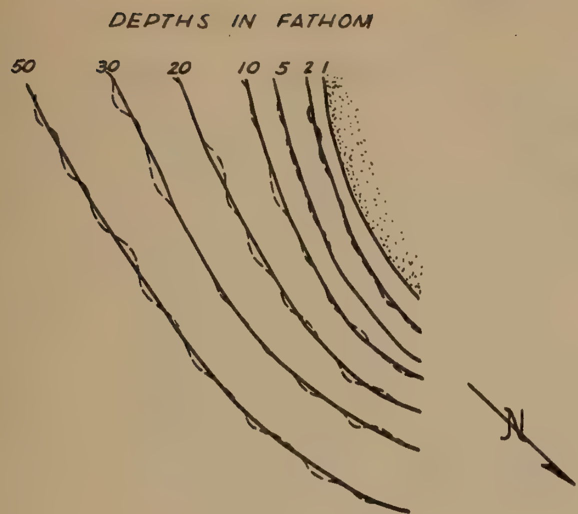
$\alpha = 16°$

$\frac{\Delta L}{L_{ave}} = 0.07$

$\Delta\alpha = 1.2°$

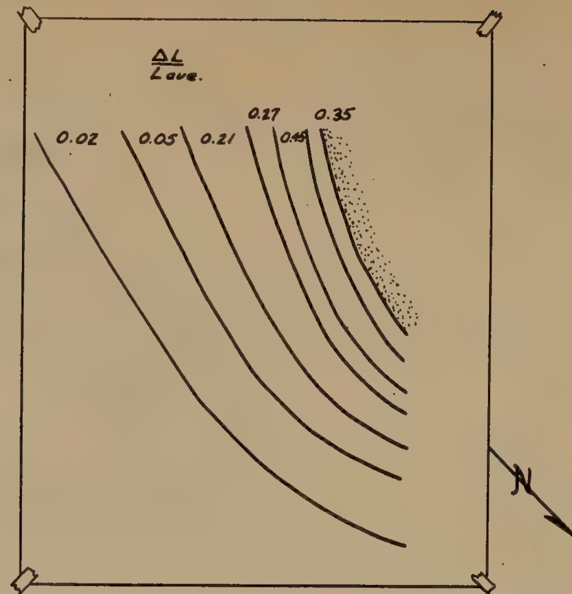
(Read $\Delta\alpha$ for $\frac{\Delta L}{L_{ave}}$ of 0.7 and divide by 10).

FIG. ID-28

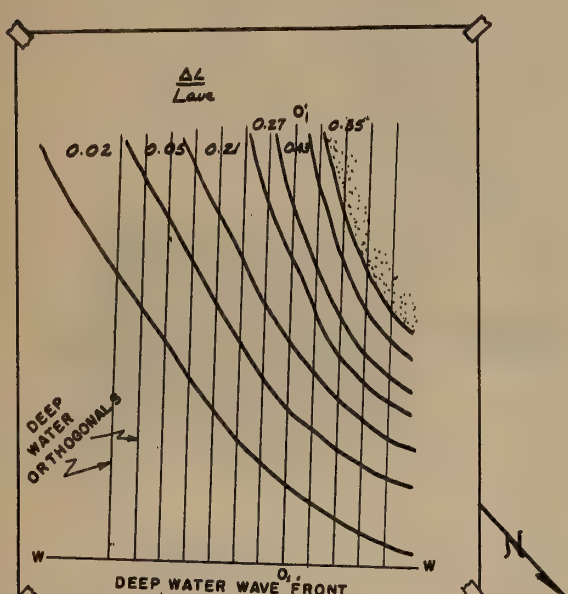


Step 1.—The chart has been checked carefully for changes in slope and the requisite contours drawn in heavily in ink smoothing them off as is consistent with assumption No. 3.

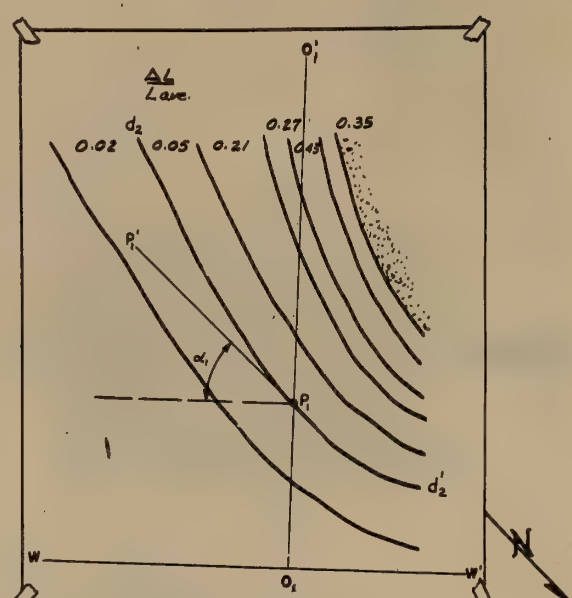
This chart is then used for all periods and directions without further work.



Step 2.—A tracing paper overlay is placed on the contour map and the $\frac{\Delta L}{L_{ave}}$ is written between each of the contours as computed in Table II. This overlay is then good for all directions of approach for a particular period, and can be prepared in a very few minutes. All further work is performed on the overlay.



Step 3.—A deep water wave front (WW′) is drawn in for the direction to be studied. A suitable interval for orthogonals is stepped off and the directions of the deep water orthogonals are drawn in. These are all straight lines, of course. The direction selected was from the NW. Deep water refers to depths greater than $L_o/2$ as described in the text, in this case 50 fathoms.

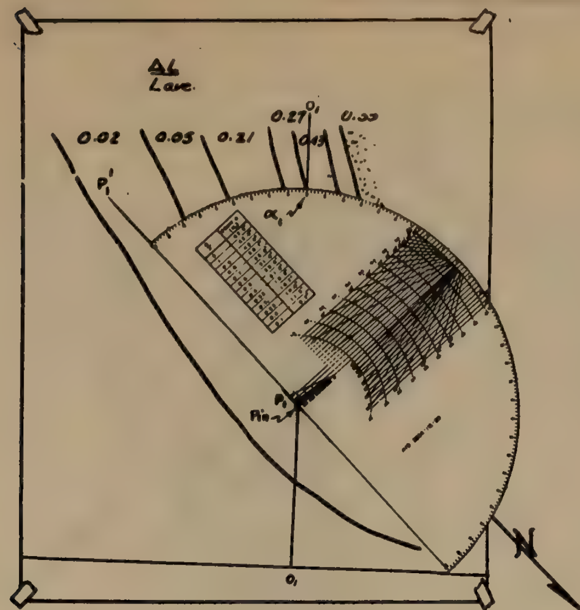


Case 1.—Regular, simple hydrography, no angles in excess of 80°.

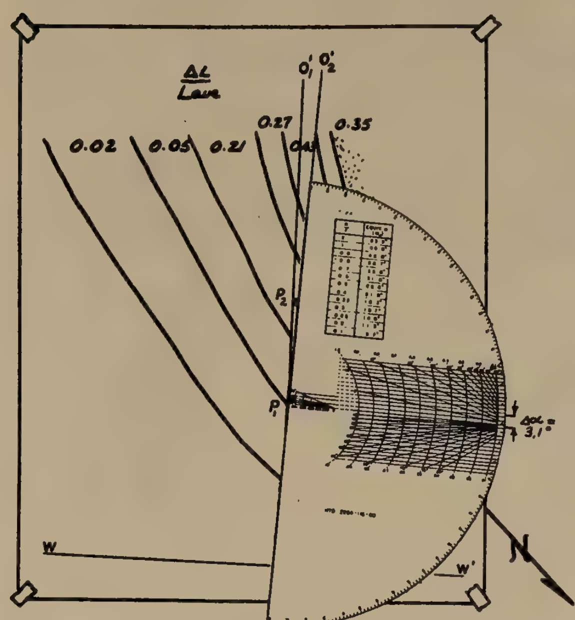
Step 4.—The diagram is started on any orthogonal. If the K value is required for a partiular point on the beach, orthogonals can be selected which will reach the point.

Where the $\frac{\Delta L}{L_{ave}}$ values are small, two contour intervals can be crossed simultaneously. Thus, the first point selected is P_1, where the orthogonal intersects the second contour. α is measured to an estimated line P_1P_1' which is the direction of level bottom at the point P_1. In this case it is a line tangent to d_2d_2'.

FIG. TD-29



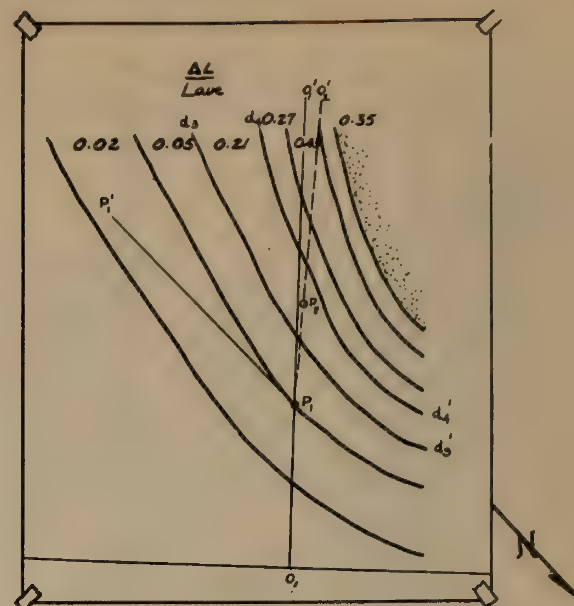
Step 4.—Measuring α_1 with protractor pinned at P_1, $\Delta\alpha$ is then turned from the line P_1O_1' by use of the degree scale. When a drafting machine is used, α_1 is measured with an ordinary protractor.



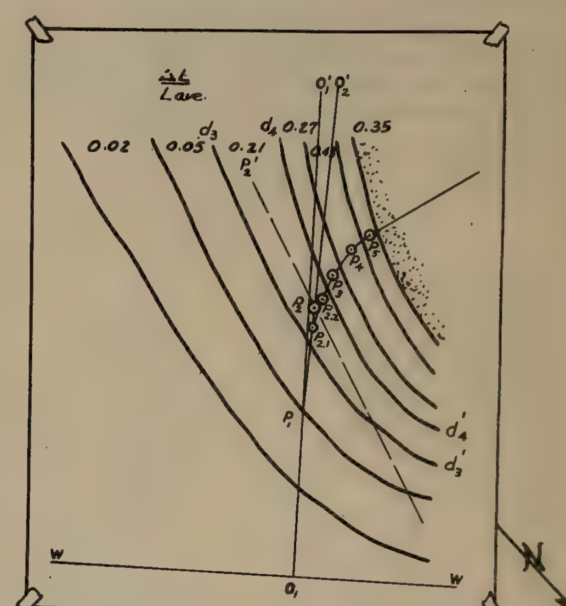
Step 5.—Measuring $\Delta\alpha_1$ with protractor pinned at P_1, the line P_2O_2' is the new direction of the orthogonal.

When a drafting machine is used, $\Delta\alpha$ is turned on the protractor head. $\Delta\alpha$ is always turned in a direction so that α is *decreased* except in the rare case when the wave is progressing from shallow to deeper water. In this case $\Delta\alpha$ is turned so that α is *increased.*

Step 6.—At P_2, α_2 is measured in respect to a line P_2P_2' which is drawn in by eye. P_2P_2' has a direction midway between the directions of the contours d_3d_3' and d_4d_4' at the points where they intersect the line P_1O_1. $\Delta\alpha_2$ is turned

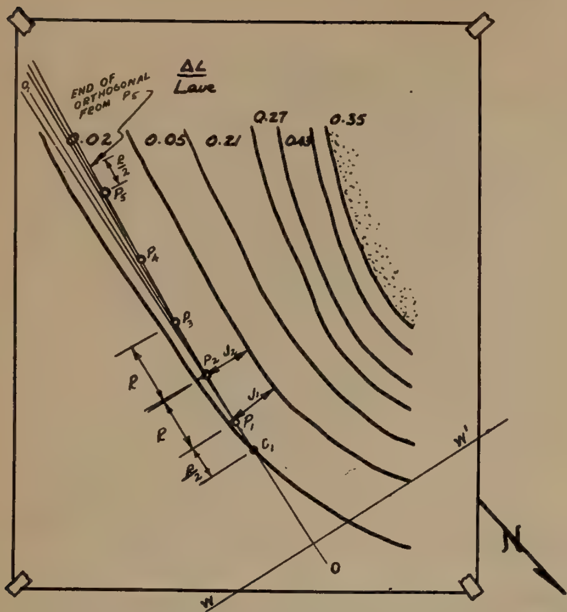


Step 5.—$\Delta\alpha_1$ at P_1 is determined from the graph on the protractor. Since two contour intervals are being crossed, the $\frac{\Delta L}{L_{ave}}$ values for the two are added (i. e., 0.07). $\Delta\alpha$ is turned at P_1 to the right or *so that α_1 is decreased.* The orthogonal is carried into P_2, midway between d_3d_3' and d_4d_4'. Two contour intervals cannot be crossed simultaneously from P_2 shoreward as the refraction is too great. If it were not so great to shoreward, P_2, could be established at the intersection of P_1O_1', and the contour d_4d_4'.



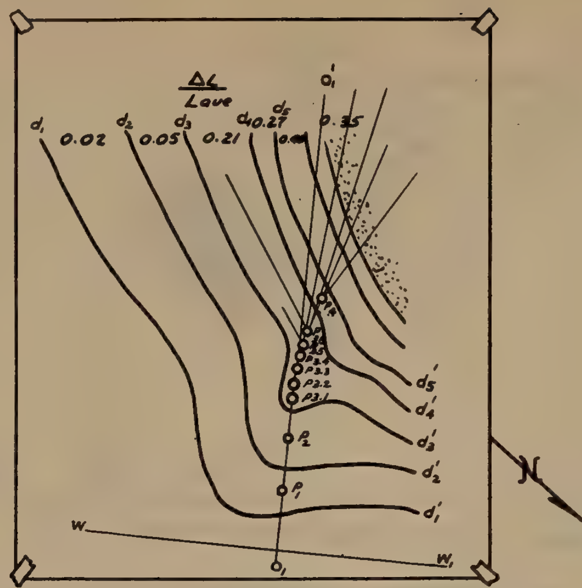
off at P_2 and P_3 is established in the same manner as P_2. This is continued to the beach. In the example $\Delta\alpha$ at P_2 was greater than 13°. Thus P_2' and P_2'' were established using the line P_2P_2' as an intermediate contour with $\frac{\Delta L}{L_{ave}}$ values of 10.5 on each side. This step is rarely necessary in practice. The true orthogonal corresponds with the line $P_1 \ldots P_5$ only at the intersections with contours. For all ordinary purposes, however, the line $P_1 \ldots P_5$ is an orthogonal.

FIG. ID-30



Case II. α greater than 80°.

Step 4.—(Steps 1 to 3 are the same as in Case I). P_1 is established along the deep water orthogonal at a distance from C_1 equal to half of one of the proportions of J shown on the protractor. In this case R was selected equal to J. The $\Delta\alpha$ for $R/J=1$ and $\frac{\Delta L}{L_{ave}}=0.02$ is 1.15°. Thus P_2P_3, etc., can be established by turning 1.15° at each point and progressing a distance R_1, R_2 . . . R_5, etc., equal to J_1, J_2 . . . J_5 until α becomes less than 80°. Any other proportion of R/J can be selected at any time. It is to be remembered that the angle turned at any P carries the orthogonal a distance of ½R beyond the point.

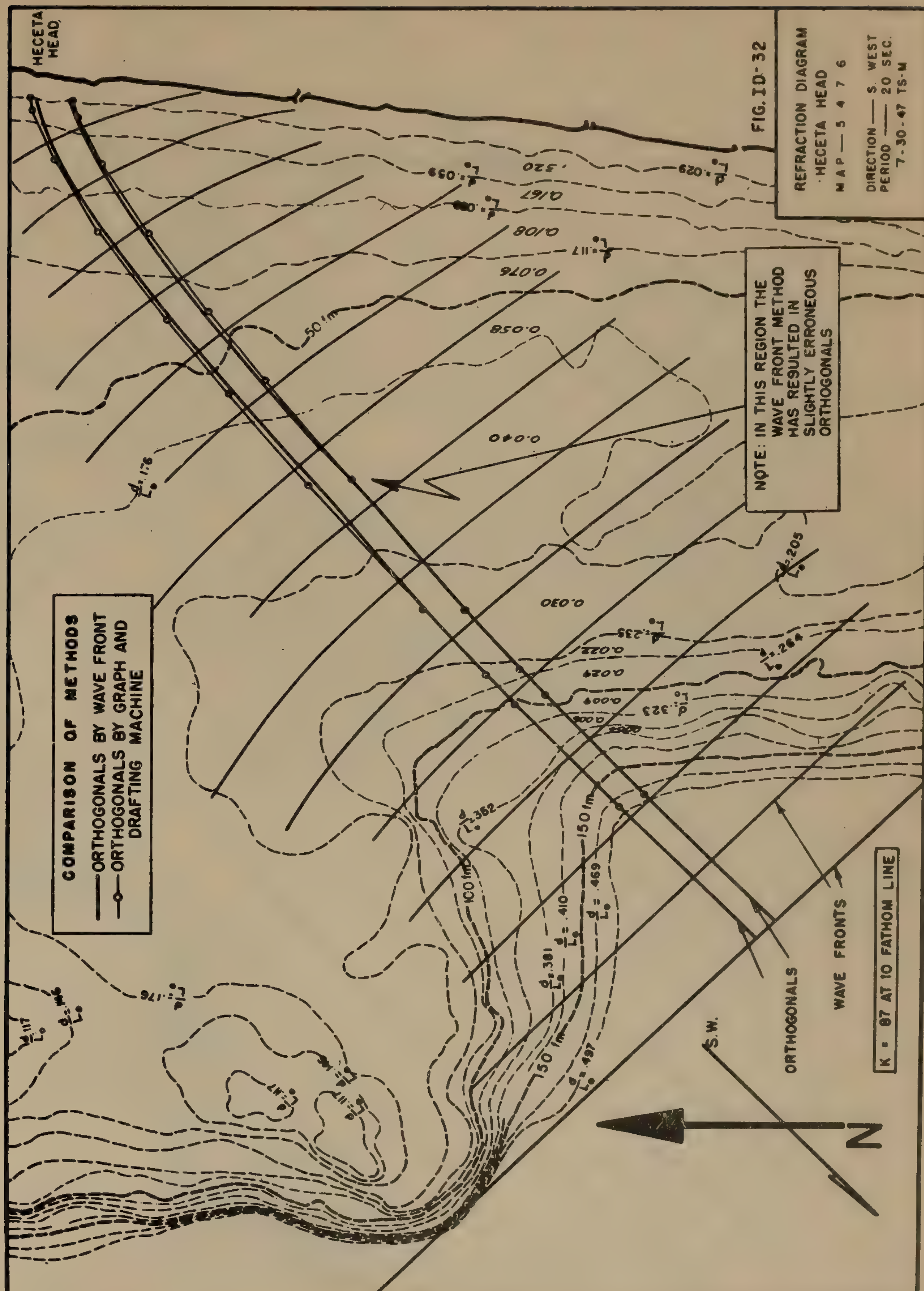


Case III.—Contours are not simple.

Step 4.—(Steps 1 to 3 are the same as for Case I.) As α_1 and α_2 are zero no refraction occurs between d_1d_1' and d_3d_3'. $P_{3.1}$ can be established on the line O_1O_1' as the limit of no refraction. $\alpha_{3.2}$ and $\alpha_{3.4}$ are approximately 90° so $P_{3.2}$ and $P_{3.4}$ are drawn in using $R/J=0.5$. At $P_{3.5}$, conditions change and $P_{3.6}$ is established as though a contour existed at $P_{3.5}$ with a $\frac{\Delta L}{L_{ave}}$ between it and d_4d_4' of ⅝× 0.21=0.13.

That is, $\frac{\Delta L}{L_{ave}}$ is reduced by a simple proportion to establish an intermediate contour interval. The diagram then progresses as usual to P_4, P_5, etc.

FIG. ID-31


HECETA HEAD
FIG. ID-32
REFRACTION DIAGRAM
HECETA HEAD
MAP — 5 4 7 6
DIRECTION —— S. WEST
PERIOD —— 20 SEC.
7-30-47 TS-M
NOTE: IN THIS REGION THE WAVE FRONT METHOD HAS RESULTED IN SLIGHTLY ERRONEOUS ORTHOGONALS
COMPARISON OF METHODS
ORTHOGONALS BY WAVE FRONT
ORTHOGONALS BY GRAPH AND DRAFTING MACHINE
ORTHOGONALS
WAVE FRONTS
K = 87 AT 10 FATHOM LINE
S.W.
N

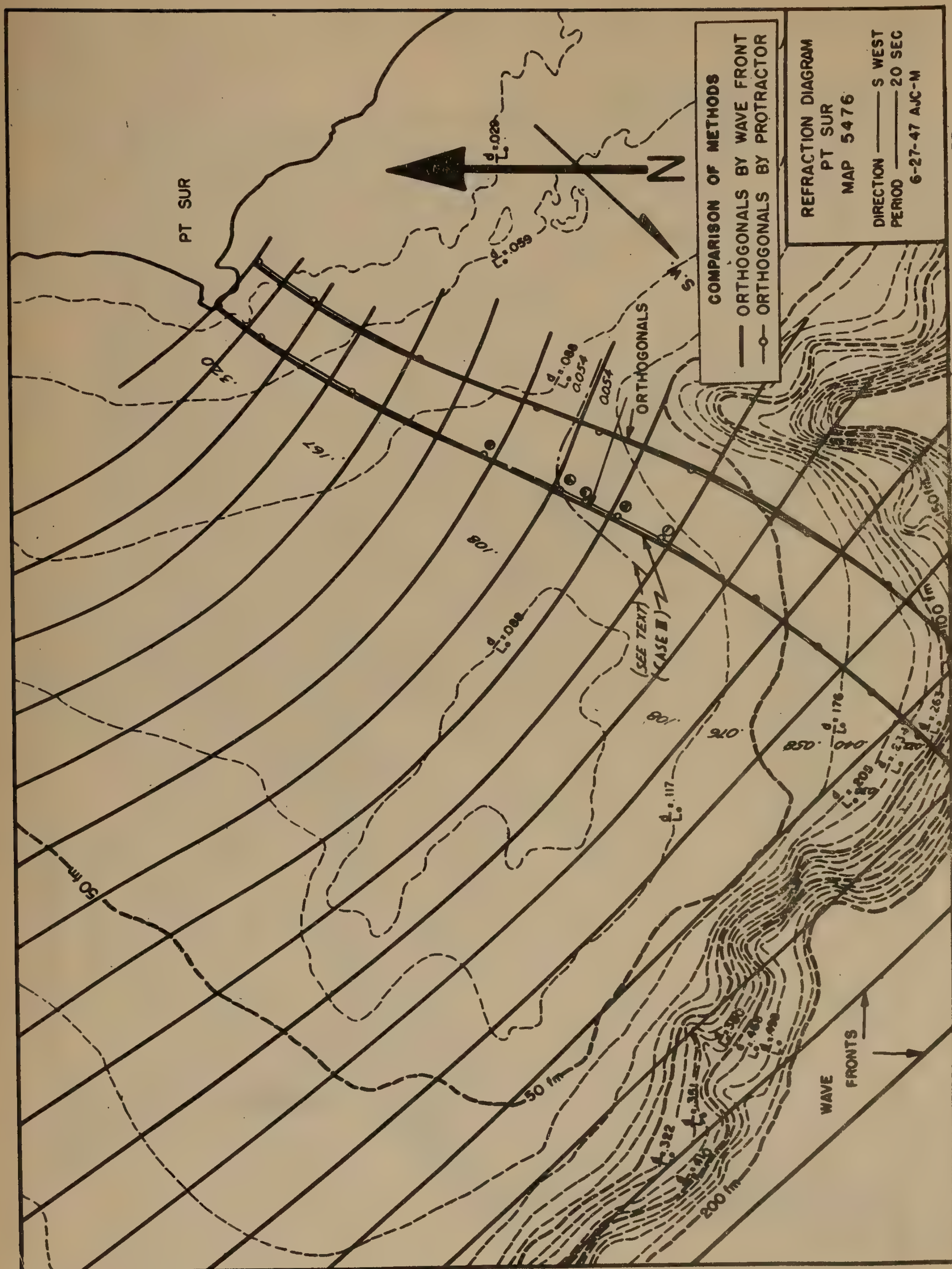
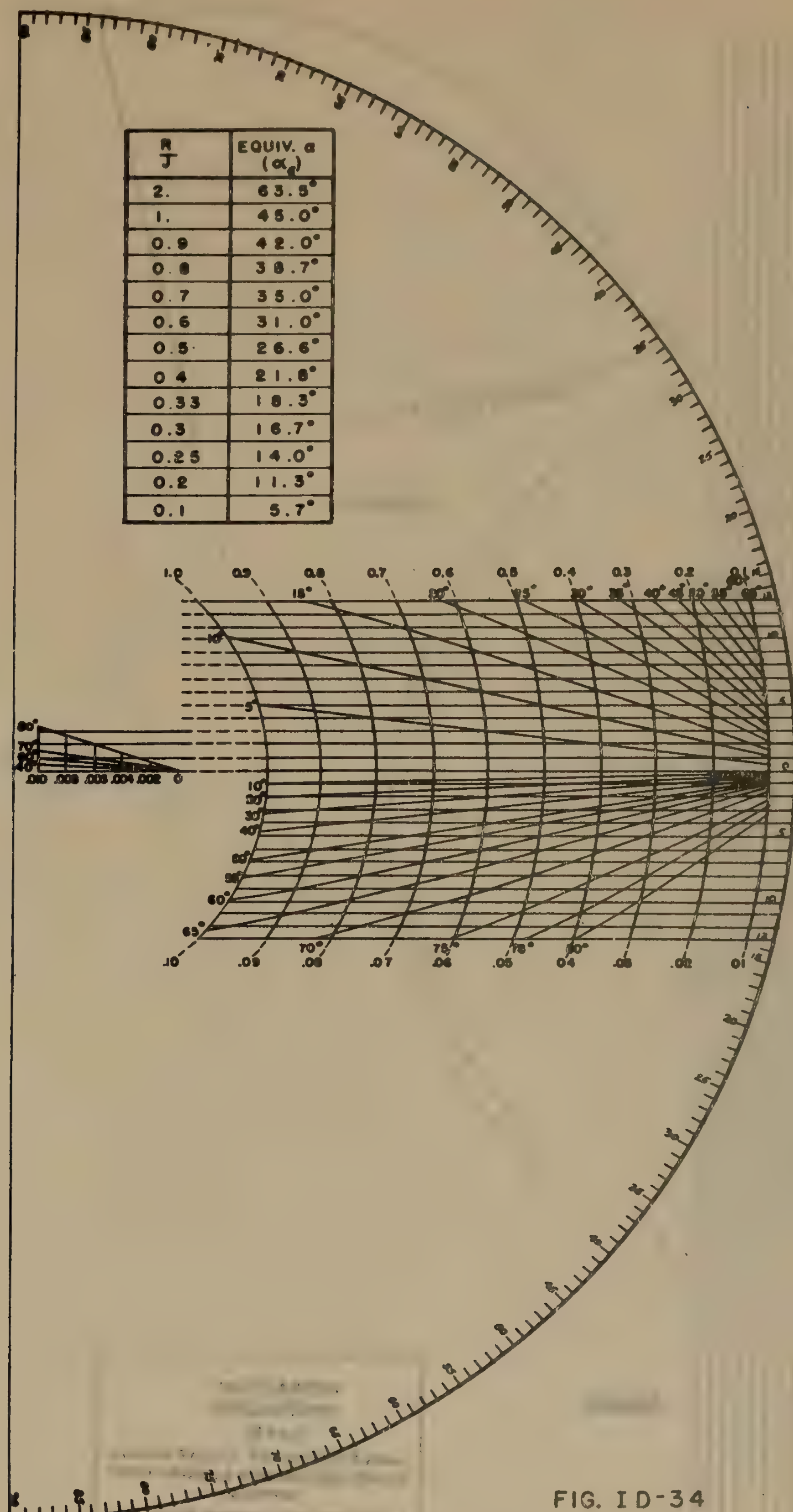

COMPARISON OF METHODS
ORTHOGONALS BY WAVE FRONT
ORTHOGONALS BY PROTRACTOR
REFRACTION DIAGRAM
PT SUR
MAP 5476
DIRECTION —— S WEST
PERIOD —— 20 SEC
6-27-47 AJC-M
PT SUR
N
S W
ORTHOGONALS
(SEE TEXT)
(CASE II)
WAVE
FRONTS
50 fm
100 fm
200 fm

FIG. ID-33

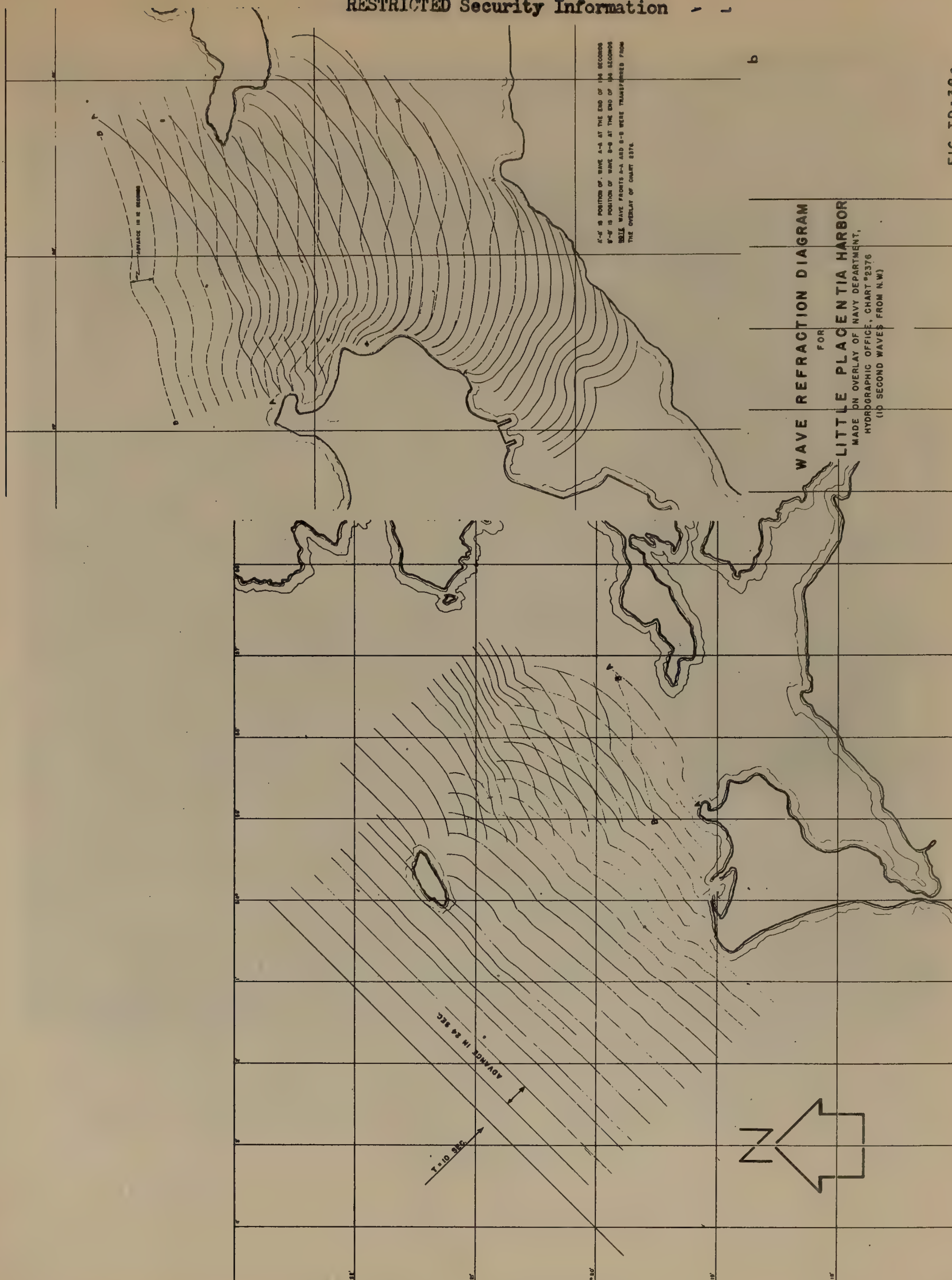


$\frac{R}{J}$	EQUIV. α (α_e)
2.	63.5°
1.	45.0°
0.9	42.0°
0.8	38.7°
0.7	35.0°
0.6	31.0°
0.5	26.6°
0.4	21.8°
0.33	18.3°
0.3	16.7°
0.25	14.0°
0.2	11.3°
0.1	5.7°

FIG. ID-34

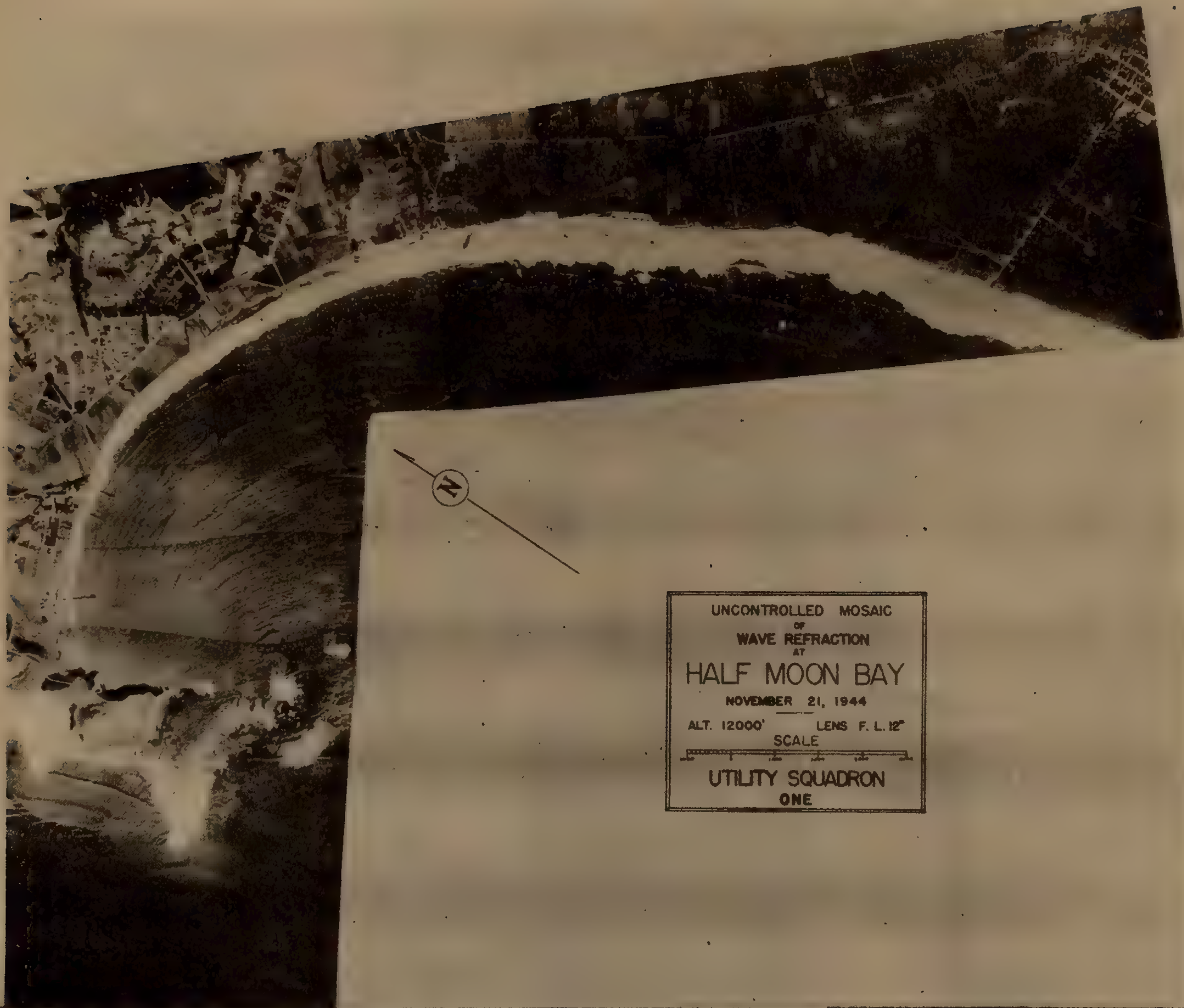

INDICATOR LINE
Protractor Arm
Protractor
REFRACTION
PROTRACTOR
Type II
UNIVERSITY OF CALIFORNIA
FLUID MECHANICS LABORATORY
BERKELEY

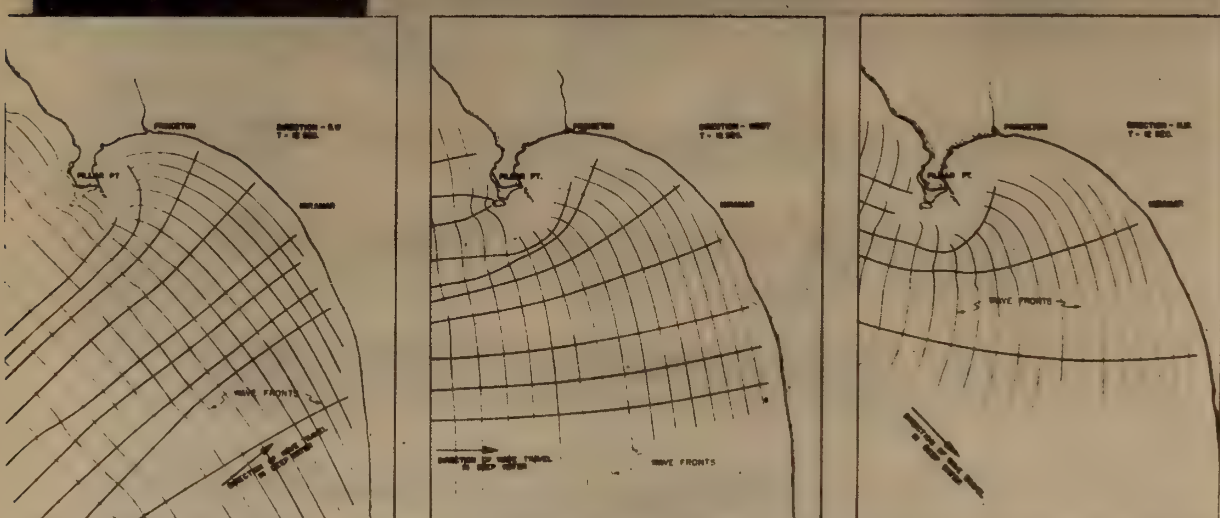
FIG. ID-38



WAVE REFRACTION DIAGRAM
FOR
LITTLE PLACENTIA HARBOR
MADE ON OVERLAY OF NAVY DEPARTMENT, HYDROGRAPHIC OFFICE, CHART #2376
(10 SECOND WAVES FROM N.W.)
ADVANCE IN 24 SEC.
T=10 SEC.

FIG. ID-38a





WAVE REFRACTION DIAGRAMS

FIG. ID-37.

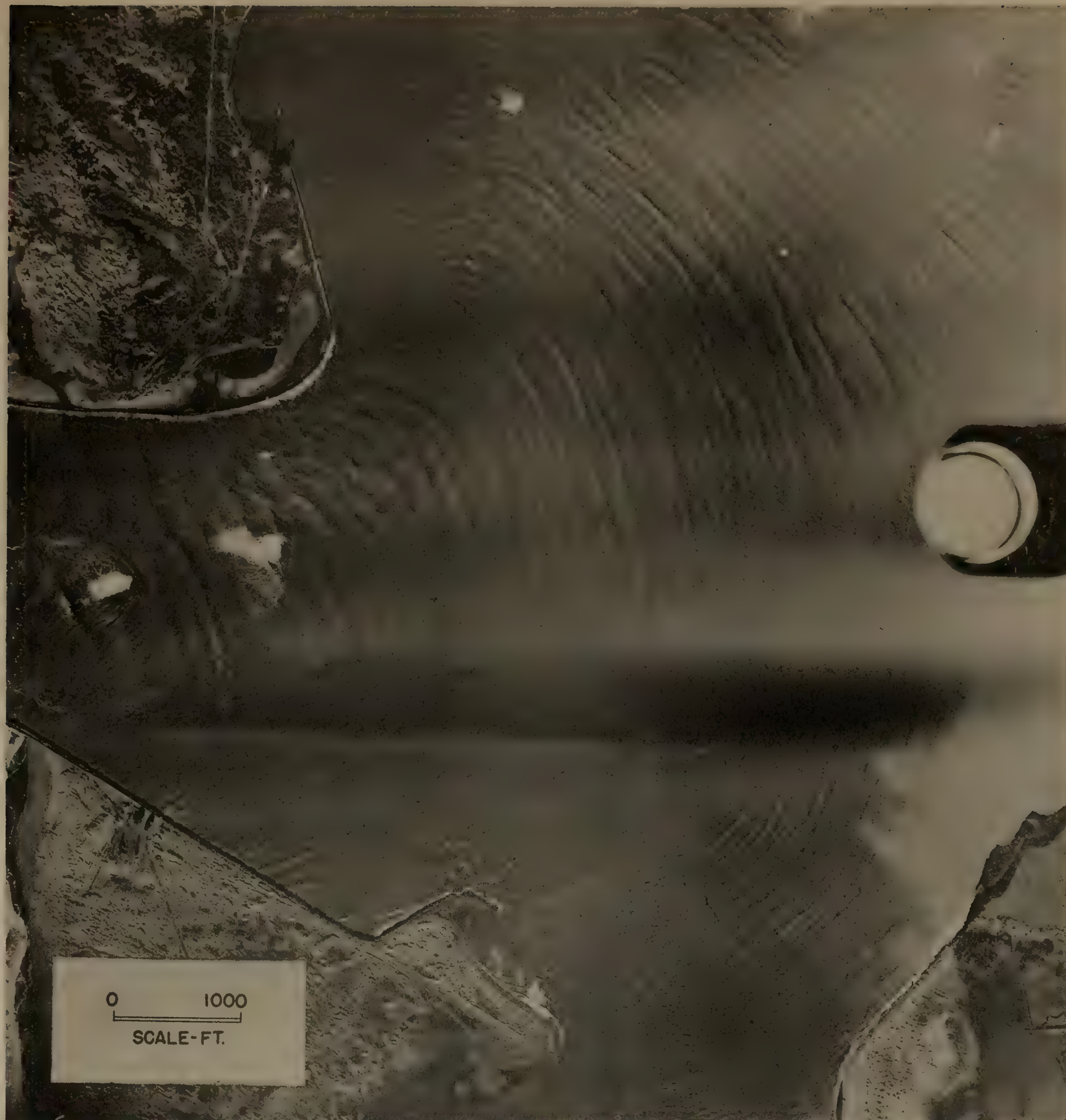


Fig. ID-38. Wave refraction inside Humboldt Bay. Note wave fronts crossing at shoal in upper right-hand corner. At this point the waves augment each other and break.

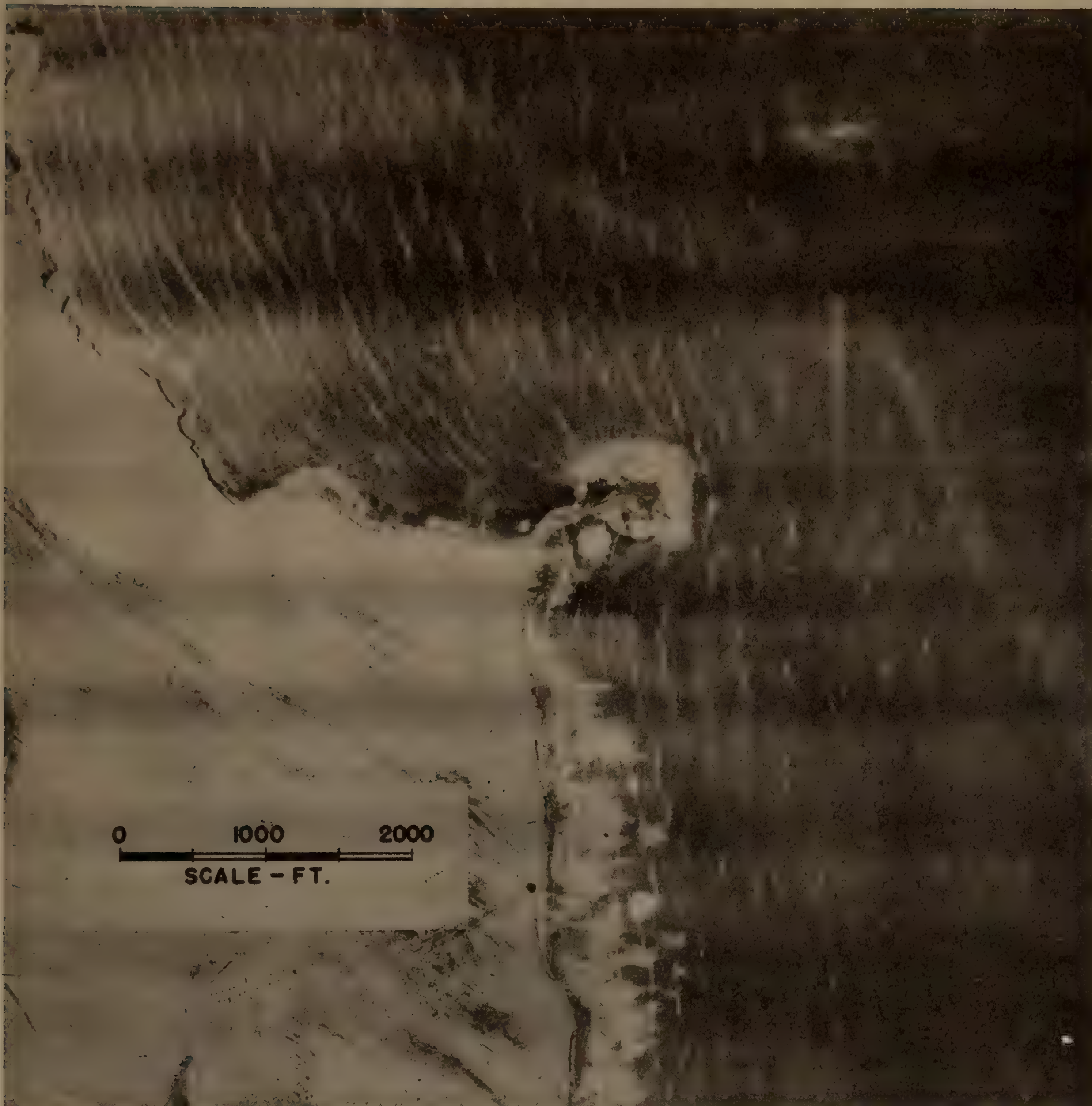


FIG. ID-39. Aerial photograph showing refraction of waves at Purisima Point, Calif. The waves shown here are from winds blowing directly on to the coast. Note that the waves in the upper portion of the picture are almost parallel to the shore line as a result of refraction. The deep water wave length is about 600 feet; the wave height 6 feet; and the breaker height 9 feet.

FIG. ID-40. Aerial photograph of surf at Oceanside, Calif. Two wave systems are present in this picture. There are small wind waves coming from the upper right, and a long, low swell from the upper left. The swell is almost invisible in deep water, but peaks up near the shore to form the predominate breakers. Note how the waves break in short segments, where crests of the wind waves are superimposed on the crest of the swell. The wind waves have a deep water wave length of about 50 feet and a height of 1 to 2 feet. The swell has a deep water wave length of about 1,000 feet, with a height of 2 to 3 feet. The breaker height is about 5 feet.

FIG. ID-41. Wave refraction at Moss Landing. Note the "flat" water at the end of the pier where the Monterey Canyon approaches the shore. Compare complicated refraction pattern with hydrography in Fig. ID-21.

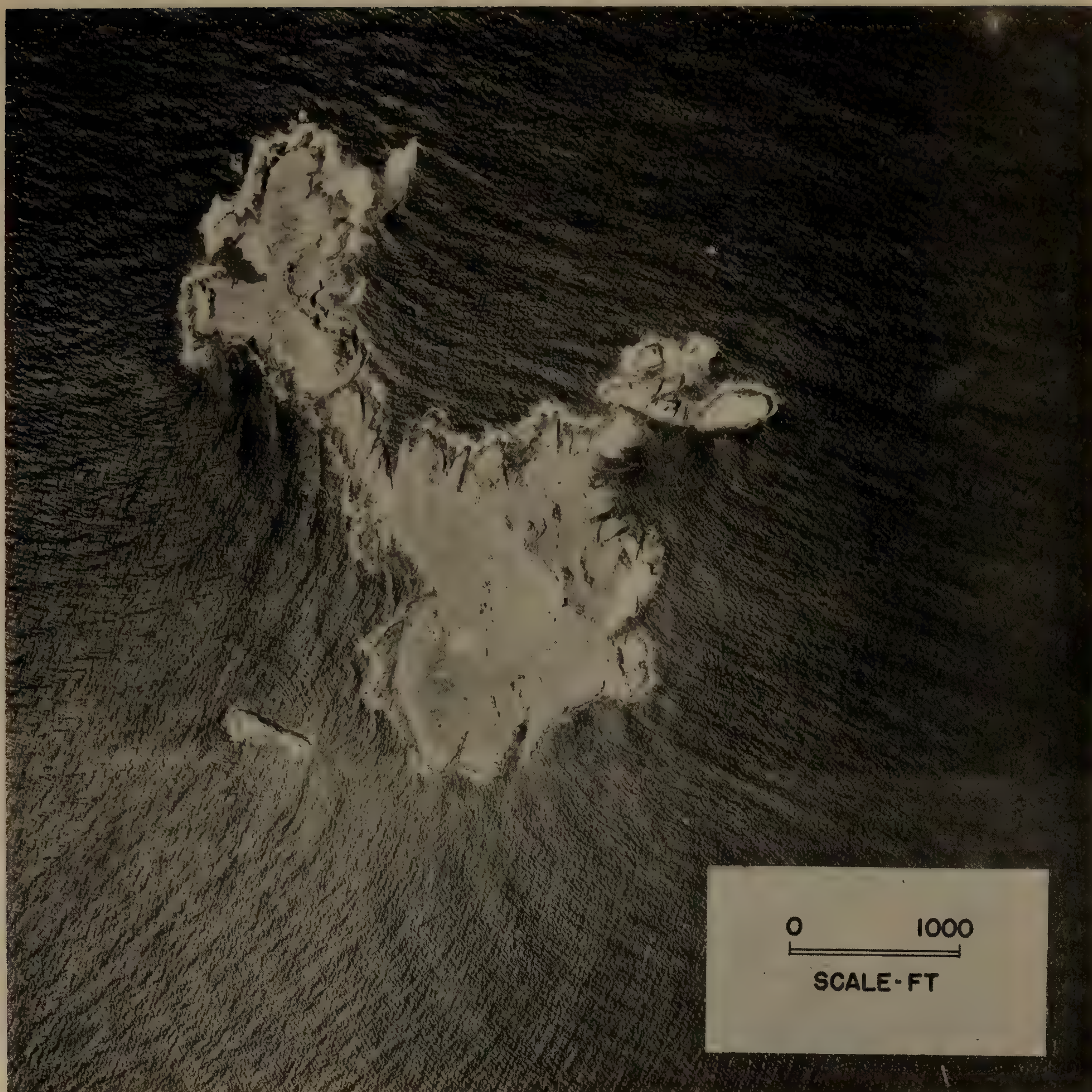


FIG. ID-42. Refraction and diffraction of both swell and wind waves at Farallon Island, Calif. Note reflection from small island at lower left.

FIG. ID-43. Wave refraction in inlet to Morro Bay. Note reflections from Morro Rock.

FIG. ID-44. Wave refraction at Duxbury reef near Bolinas, Calif. Note waves breaking on reef.

Fig. ID-45. Change in wave length and velocity over ground when waves enter a deep sound with a following current.

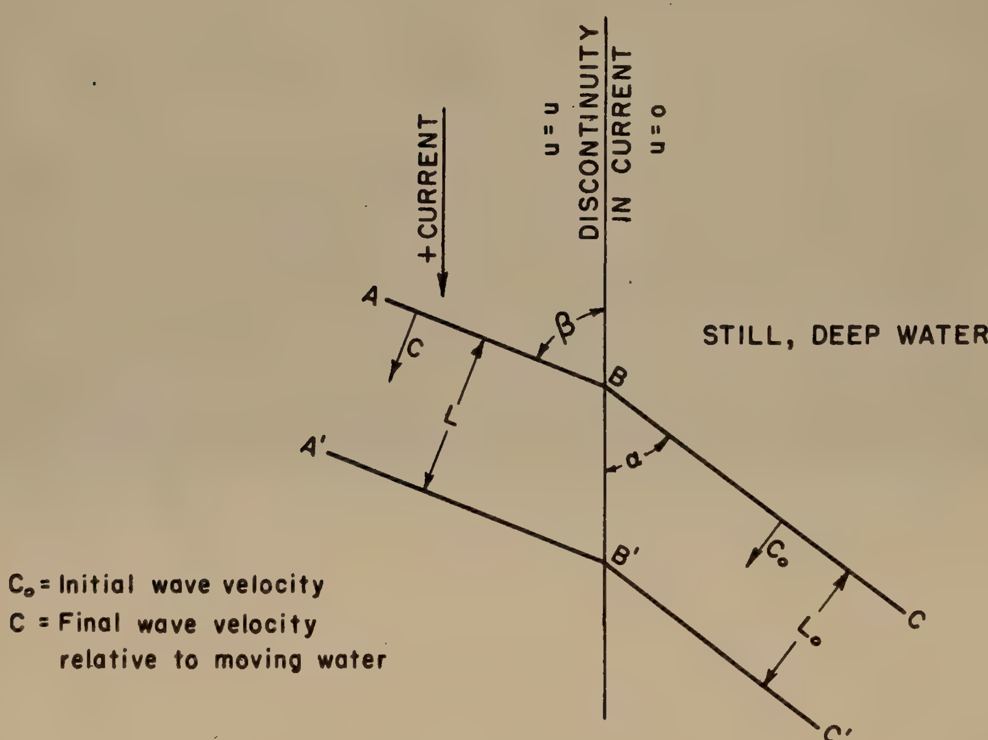


Fig. I D-46. Relative positions of positions of wave crests before and after refraction by a current.

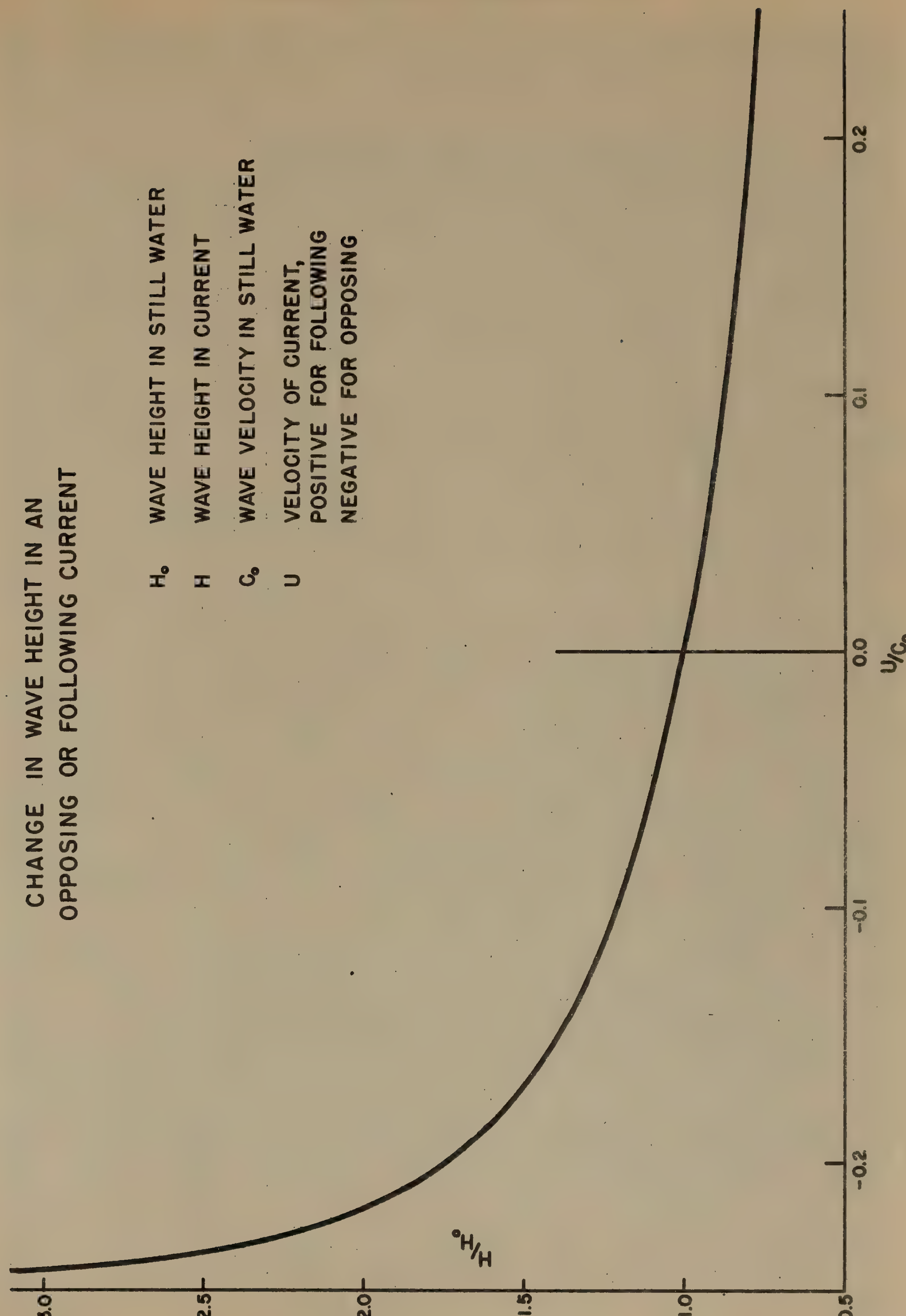
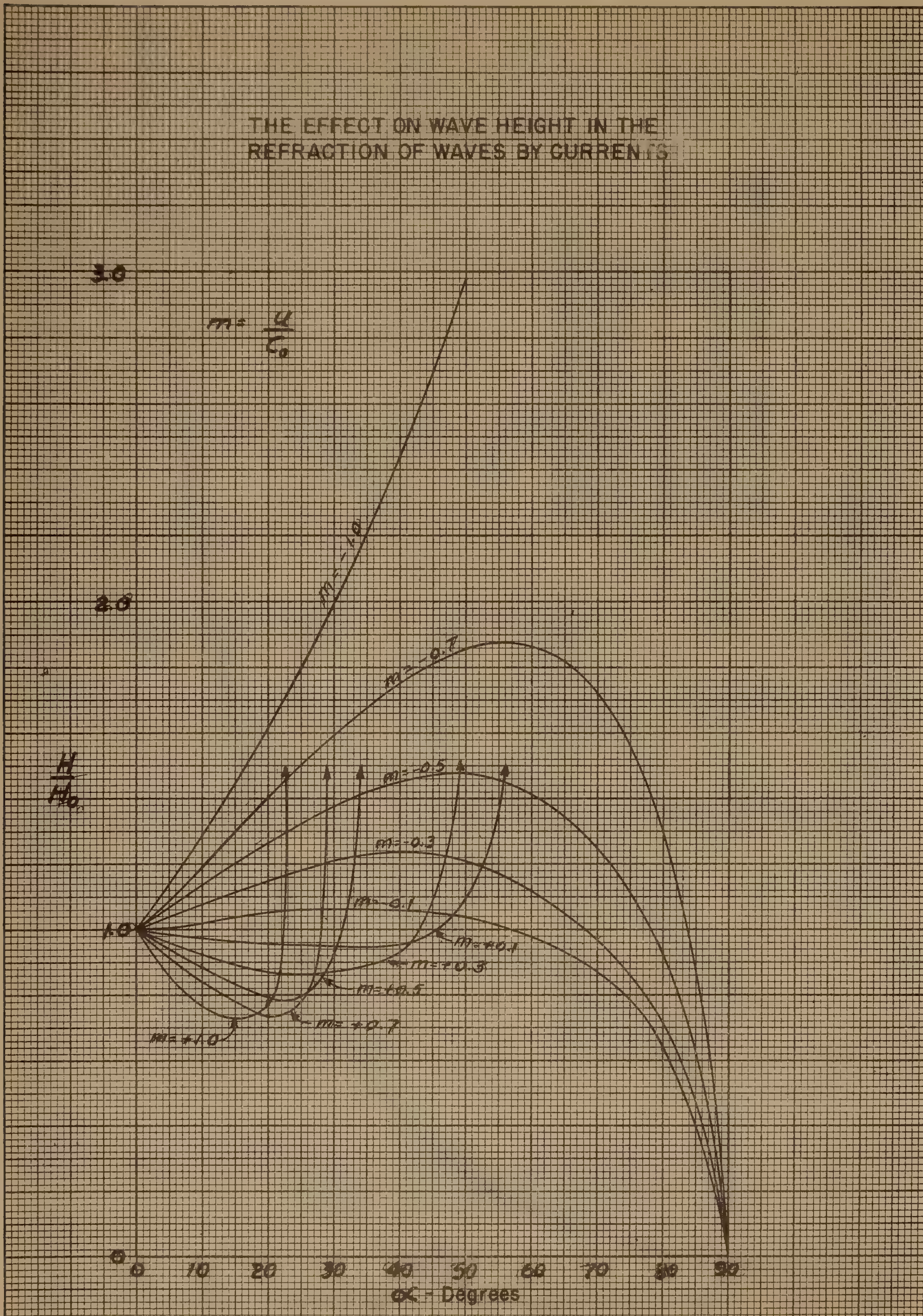

CHANGE IN WAVE HEIGHT IN AN
OPPOSING OR FOLLOWING CURRENT
H_o WAVE HEIGHT IN STILL WATER
H WAVE HEIGHT IN CURRENT
C_o WAVE VELOCITY IN STILL WATER
U VELOCITY OF CURRENT,
POSITIVE FOR FOLLOWING
NEGATIVE FOR OPPOSING
H/H_o
3.0
2.5
2.0
1.5
1.0
0.5
-0.2
-0.1
0.0
0.1
0.2
U/C_o

FIGURE ID-47



THE EFFECT ON WAVE HEIGHT IN THE
REFRACTION OF WAVES BY CURRENTS
m = u/C₀
3.0
2.0
1.0
0
H/H₀
m = -1.0
m = -0.7
m = -0.5
m = -0.3
m = -0.1
m = +0.1
m = +0.3
m = +0.5
m = +0.7
m = +1.0
0
10
20
30
40
50
60
70
80
90
α - Degrees

FIGURE ID-48

SUMMARY OF SECTION I-E

WAVE DIFFRACTION

BY
J. W. JOHNSON

Introduction: Wave diffraction is the phenomenon in which water waves are propagated into a sheltered region formed by a breakwater or similar barrier which interrupts a portion of a regular wave train (Figure SIE-1). The principles of diffraction have considerable practical application in connection with the design of breakwaters. The phenomenon is analagous to the diffraction of light, sound, and electromognetic waves. Two general types of diffraction problems usually are encountered; one, the passage of waves around the end of a semi-infinite impermeable breakwater, and, second, the passage of waves through a gap in a breakwater. In general, the theoretical solutions have been found to apply with conservative results, that is, the predicted wave heights in the lee of a breakwater are found to be slightly larger than the height of waves that may be expected under actual conditions. The use of the diffraction theory in breakwater design is made convenient when summarized in a diagram with curves of equal values of diffraction coefficients on a coordinate system in which the origin of the system is at the tip of a single breakwater (Figure SIE-2) or at the center of a gap (Figure SIE-4). The diffraction coefficient in this instance is defined as the ratio of the diffracted wave height to the incident wave height and usually is designated by the symbol K'.

Semi-infinite Breakwater:

The generalized diffraction diagram shown in Figure SIE-2 can be applied to a particular breakwater problem once the characteristics of the design wave has been selected; that is, the height, period and direction of the incident wave from which protection is to be provided. For example, Figure SIE-3 shows a map of a harbor for which protection is desired for a specified reach of the shoreline for waves approaching from the critical direction. For the given wave period (or length) a diagram similar to Figure SIE-2 is plotted on transparent paper to the same length scale as the map of the harbor area. This transparent overlay then is moved over the map (keeping the geometric shadow parallel to the direction of travel) until the desired degree of protection for the selected reach of the shoreline is obtained. The location of the tip of the breakwater thus is obtained as illustrated by the final location of the overlay shown in Figure SIE-3.

Diffraction at a Breakwater Gap:

The treatment of diffraction problems, as discussed above, is concerned with waves moving past a breakwater tip with an infinite expanse of water existing away from the tip. In many harbors, however, waves move through a relatively narrow gap in a breakwater; hence, diffraction occurs at the two sides of the gap and changes in wave height in the lee of the breakwater will be different than if a single tip existed. A theory for this condition also has been developed. An experimental study has verified the general form of the theoretical expressions for breakwater gap openings as small as a half wave length. As long as the water depth in the lee of the structure remains constant the diffraction pattern is independent of the actual depth. In natural harbors, however, rarely is this condition of uniform depth encountered. Instead a shoaling bottom usually exists - in which case the waves are not only diffracted but refraction also results as the waves move further to the lee of the structure. At a considerable distance from the breakwater, it is probable that the refraction effects predominate over the diffraction effects.

As an illustration of a generalized diagram which gives diffraction coefficients to the lee of a breakwater gap, Figure SIE-4 shows a diagram for the case where the gap is two wave lengths in width. The method of making the necessary computations of these diffraction coefficients as well as the computations for the position of the wave fronts is shown in the technical section. These generalized diagrams, when used as transparent overlays, can be moved over a map of an area to obtain the most desirable protection, similar to the procedure described above for the single breakwater.

Generalized diagrams may be prepared for other gap widths. The diagrams which have been prepared for the technical manual, cover a wide range of gap openings with a sufficiently small spacing of values that one of the diagrams can be selected and applied with reasonable accuracy to a specific problem. For some specific gap width it may be desirable to obtain the diffraction pattern by interpolation between two diagrams; however, the accuracy with which the design wave data are known usually does not justify such a refinement.

The diffraction diagrams for the breakwater gap problem were arbitrarily terminated at a distance of 20 wave lengths in the lee of a gap. It is believed that in most applications the effects of refraction, as discussed above, would predominate by the time the waves had traveled a distance of 20 wave lengths beyond a breakwater; therefore, the extension of the diffraction patterns to greater distances is unnecessary.

When a gap width is in excess of about five wave lengths, the diffraction patterns at each side of the opening are more or less independent of each other. In such cases the pattern given by Figure SIE-2 for a semi-infinite breakwater can be used to estimate the height and direction of waves on the leeward side. For these relatively large gap openings the direction of the incident waves with respect to the breakwater alignment can lie anywhere within the zone indicated in Figure SIE-2 without the diffraction pattern being appreciably affected. For relatively narrow gaps (gap openings of about 2 wave lengths and less) the diffraction pattern can be computed easily for almost any angle between the incident wave and the breakwater. For wider gap openings where oblique approaches make computations of diffraction patterns relatively difficult useful approximations can be made by drawing a line through the gap center and normal to the incident wave direction, and then computing diffraction coefficients as though the breakwater were along this line--the end of the imaginary gap being at the projections on this line of the true gap ends.

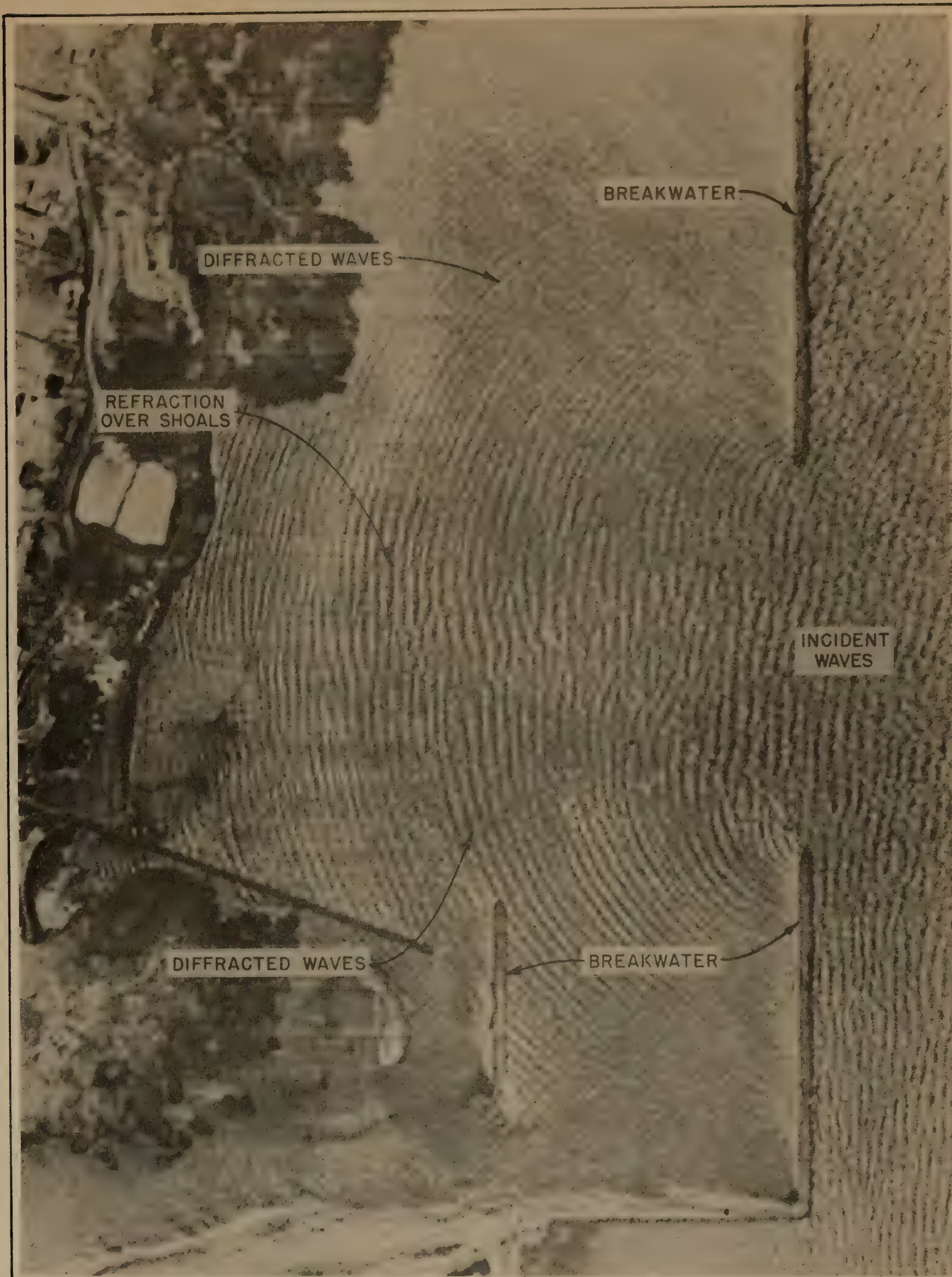
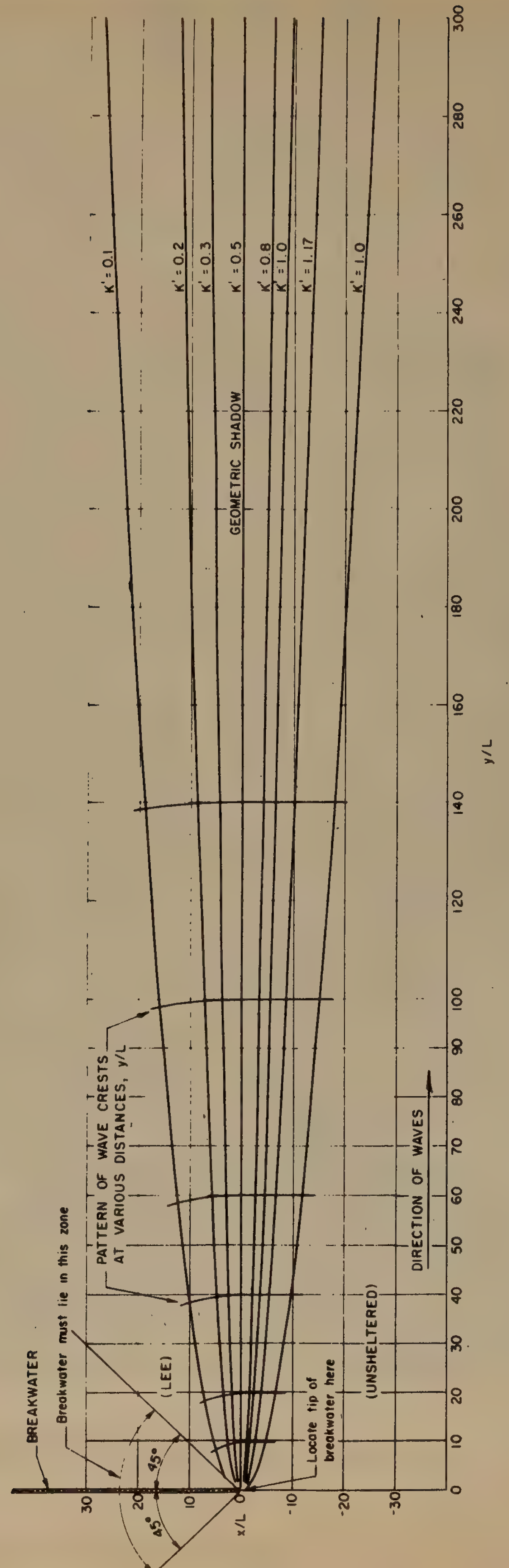


FIG. SIE-1 -- DIFFRACTION OF WAVES AT A BREAKWATER.

FIG. SIE-2 - DIMENSIONLESS PLOT OF CURVES OF EQUAL K'
FOR DIFFRACTION OF WAVES AT TIP OF BREAKWATER

L = Wave length

K' = Diffraction coefficient = $\frac{\text{Diffracted wave height}}{\text{Incident wave height}}$



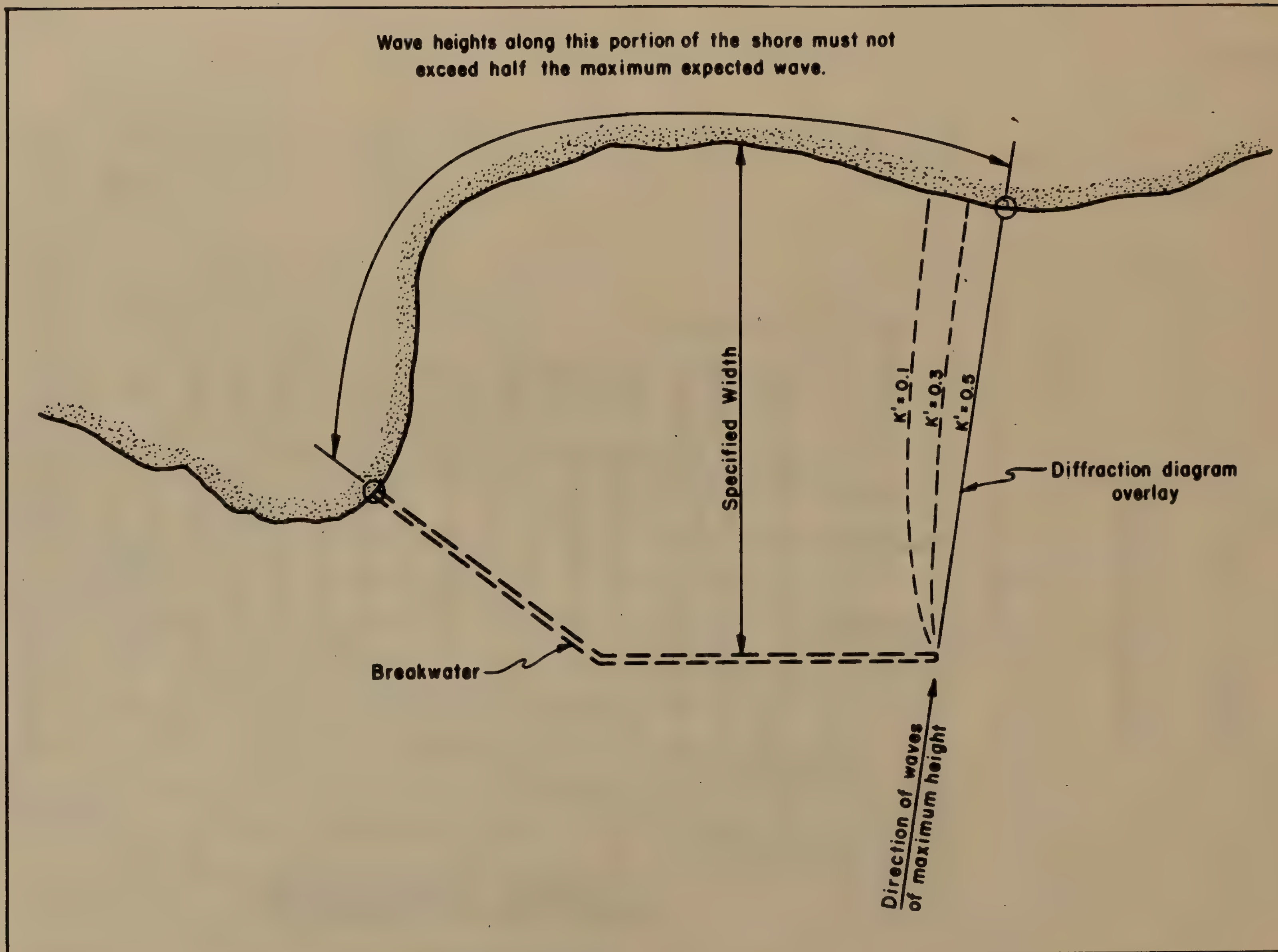


FIG. SIE-3 -- Illustration of the use of a diffraction diagram in determining the location of a breakwater tip to provide a specified degree of protection against wave action.

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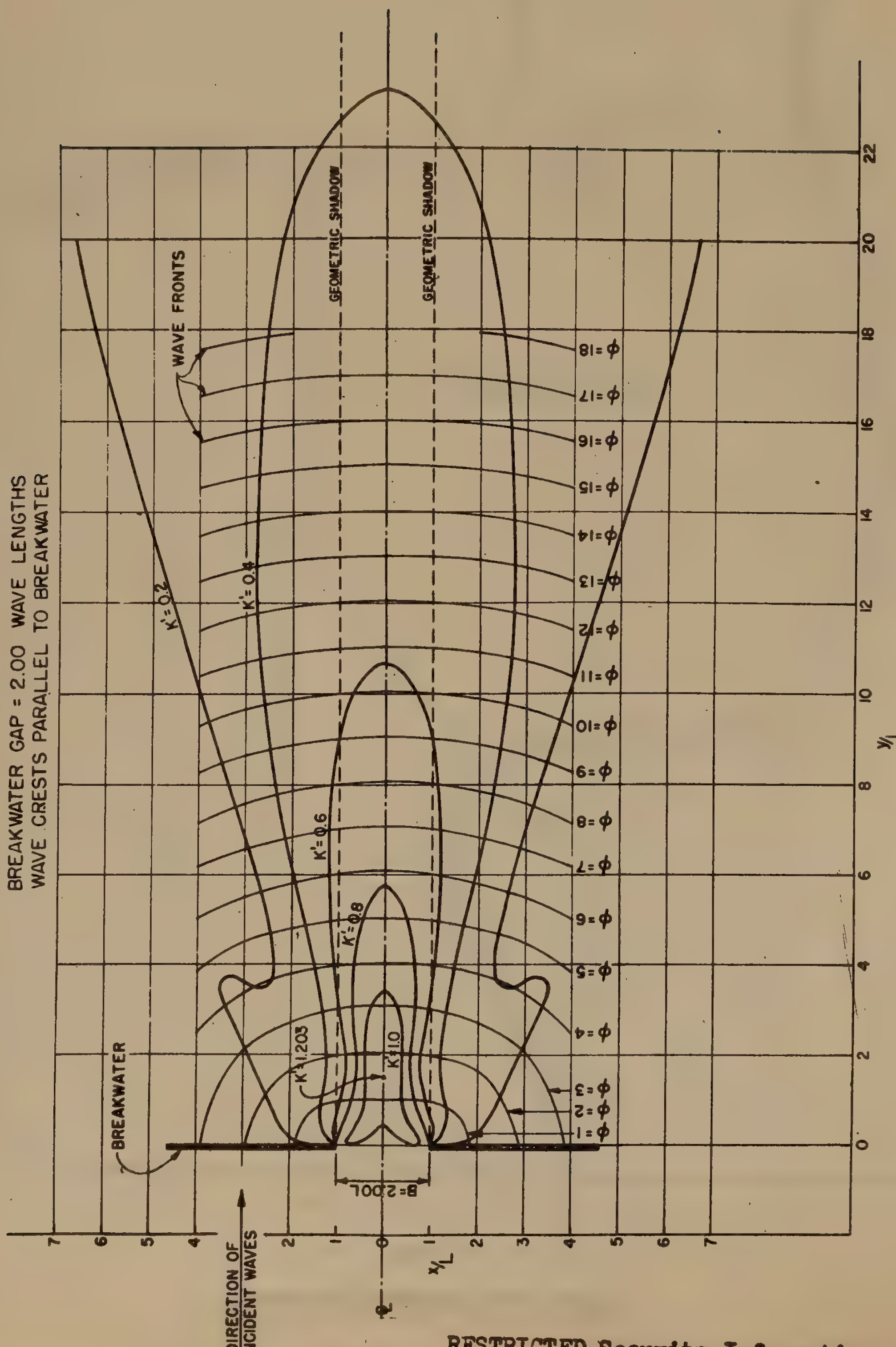
FIG. SIE-4 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}} \right)$



E. WAVE DIFFRACTION

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E. WAVE DIFFRACTION

1. Introduction:

Wave diffraction is the phenomenon in which water waves are propagated into a sheltered region formed by a breakwater or similar barrier which interrupts a portion of a regular wave train (Figure IE-1). The principles of diffraction have considerable practical application in connection with the design of breakwaters. The phenomenon is analagous to the diffraction of light, sound, and electromagnetic waves. Two general types of diffraction problems usually are encountered; one, the passage of waves around the end of a semi-infinite impermeable breakwater, and, second, the passage of waves through a gap in a breakwater. In general, the theoretical solutions have been found to apply with conservative results, that is, the predicted wave heights in the lee of a breakwater are found to be slightly larger than the height of waves that may be expected under actual conditions. The use of the diffraction theory in breakwater design is made convenient when summarized in a diagram with curves of equal values of diffraction coefficients on a coordinate system in which the origin of the system is at the tip of a single breakwater (Figure IE-2) or at the center of a gap (Figure IE-4). The diffraction coefficient in this instance is defined as the ratio of the diffracted wave height to the incident wave height and usually is designated by the symbol K'. The procedure in preparing diffraction diagrams appears below.

2. Semi-infinite Breakwater:

The generalized diffraction diagram shown in Figure IE-2 can be applied to a particular breakwater problem once the characteristics of the design wave have been selected; that is, the height, period and direction of the incident wave from which protection is to be provided. For example, Figure IE-3 shows a map of a harbor for which protection is desired for a specified reach of the shoreline for waves approaching from the critical direction. For the given wave period (or length) a diagram similar to Figure IE-2 is plotted on transparent paper to the same length scale as the map of the harbor area. This transparent overlay is then moved over the map (keeping the geometric shadow parallel to the direction of travel) until the desired degree of protection for the selected reach of the shoreline is obtained. The location of the tip of the breakwater thus is obtained as illustrated by the final location of the overlay shown in Figure IE-3.

3. Diffraction at a Breakwater Gap:

The treatment of diffraction problems, as discussed above, is concerned with waves moving past a breakwater tip with an infinite expanse of water existing away from the tip. In many harbors, however, waves move through a relatively narrow gap in a breakwater; hence, diffraction occurs at the two sides of the gap and changes in wave height in the lee of the breakwater will be different than if a single tip existed. A theory for this condition also has been developed. An experimental study has verified the general form of the theoretical expressions for breakwater gap openings as small as a half wave length. As long as the water depth in the lee of the structure remains constant the diffraction pattern is independent of the actual depth. In natural harbors, however, rarely is this condition of uniform depth encountered. Instead a shoaling bottom usually exists - in which case the waves are not only diffracted but refraction also results as the waves move further to the lee of the structure. At a considerable distance from the breakwater, it is probable that the refraction effects predominate over the diffraction effects.

As an illustration of a generalized diagram which gives diffraction coefficients to the lee of a breakwater gap, Figure IE-4 shows a diagram for the case where the gap is two wave lengths in width. The method of making the necessary computation of these diffraction coefficients as well as the computations for the position of the wave fronts is presented below. These generalized diagrams, when used as transparent overlays, can be moved over a map of an area to obtain the most desirable protection, similar to the procedure described above for the single breakwater.

4. Generalized Diagrams:

For semi-infinite breakwaters the single diagram shown in Figure IE-2 is sufficient for all such breakwaters with waves approaching from any direction within the limits indicated. In the case of breakwater gaps however, a different diagram is required for each combination of gap width and direction of wave approach. A number of representative generalized diagrams for gaps are shown in Figures IE-4 to 12, inclusive, and Figure 19. These diagrams pertain to gaps which range in width from $\frac{1}{2}$ to 5 wave lengths with the direction of wave approach being normal to the gap. These diagrams cover a wide range of gap openings with a sufficiently small spacing of values that one of the diagrams can be selected and applied with reasonable accuracy to a specific problem. For some specific gap width it may be desirable to obtain the diffraction pattern by interpolation between two diagrams; however, the accuracy with which the design wave data are known usually does not justify such a refinement. It is to be noted that in Figures IE-4-12, inclusive, the diffraction diagrams have been terminated arbitrarily at a distance of 20 wave lengths in the lee of a gap. It is believed that in most applications the effects of refraction, as discussed above, would predominate by the time the waves had travelled a distance of 20 wave lengths beyond a breakwater; therefore, the extension of the diffraction patterns to greater distances is unnecessary.

When a gap width is in excess of about five wave lengths, the diffraction patterns at each side of the opening are more or less independent of each other. In such cases the pattern given by Figure IE-2 for a semi-infinite breakwater can be used to estimate the height and direction of waves on the leeward side. For these relatively large gap openings the direction of the incident waves with respect to the breakwater alignment can lie anywhere within the zone indicated in Figure IE-2 without the diffraction pattern being appreciably affected. For relatively narrow gaps (gap openings of about 2 wave lengths and less) the diffraction pattern can be computed easily for almost any angle between the incident wave and the breakwater. As an example, Figures IE-13-19, inclusive, show diffraction patterns for waves approaching a breakwater gap with a width of one wave length from directions which range from 0 to 90 degrees.

For wider gap openings where oblique approaches make computations of diffraction patterns relatively difficult useful approximations can be made by drawing a line through the gap center and normal to the incident wave direction, and then computing diffraction coefficients as though the breakwater were along this line--the end of the imaginary gap being at the projections on this line of the true gap ends (Figure IE-20).

5. Computations for Diffraction Diagrams:

The basic theory of diffraction has been presented elsewhere (References IE-2, IE-4) and need not be discussed in detail in this section. Basically the theory assumes (i) that the waves are of small amplitude compared to the wave length and (ii) that the water is of uniform depth. The first assumption covers the range of wave steepness up to that of storm waves; that is, $H_o/L_o = 0.03$.

a. Diffraction by a Vertical Impermeable Semi-infinite Breakwater:

As mentioned above, the diffraction coefficient is defined as follows:

$$K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}}$$

The value of K' is a function of position with respect to the breakwater, that is

$$K' = f\,(u_1)$$

where

$$u_1 = \sqrt{(4/L)\left\{\sqrt{(x^2 + y^2)} - y\right\}} \qquad \text{(IE-5.1)}$$

or

$$x/L = \pm \left[u_1 \big/ \sqrt{2}\right] \sqrt{(y/L) + u_1^2/8} \qquad \text{(IE-5.2)}$$

Referring to Figure IE-21 the plus sign in equation (IE-5.2) applies to the region of $-(x/L)$ and the minus sign applies to the $+(x/L)$ region. The evaluation of $K' = f(u_1)$ is obtained from a projection of the Cornu spiral. The values of u_1 for various values of K' are given in the following table:

TABLE IE-1

Values of the term u_1

K'	u_1		Crest Lag
0.1	-2.25		1.4L
0.15	-1.44		0.60L
0.2	-1.02	$+\frac{x}{L}$ (Lee)	0.33L
0.3	-0.528		0.13L
0.4	-0.225		0.06L
0.5	0.000	Geo. Shadow	0.00L
0.6	0.184		-0.03L
0.7	0.341		-0.05L
0.8	0.486		-0.06L
0.9	0.631		-0.05L
1.0	0.779	$-\frac{x}{L}$	-0.05L
1.17	1.218		0.00L
1.0	1.610	(Unsheltered)	0.00L
0.88	1.878		-0.03L
1.0	2.124		

Equation (IE-5.2), used in conjunction with Table IE-1, permits the construction of a diffraction diagram which shows parabolas of constant K'. For example, to plot the curve of K' = 0.2, the following calculations are made:

From Table IE-1

$$u_1 = -1.02$$

and from equation (IE-5.2)

$$x/L = (-)\ (0.707)\ (-1.02)\ \sqrt{(y/L) + (0.125)\ (-1.02)^2}$$

$$= + 0.722\sqrt{y/L + 0.13}$$

The plus sign indicates the region in the lee of the breakwater. The values of x/L now can be computed for various assumed values of relative distance (y/L); thus, the computation form is:

TABLE IE-2

Computation Form for Diffraction Coefficients

K' = 0.2

(y/L)	(y/L + 0.13)	$\sqrt{(y/L + 0.13)}$	x/L
5	5.13	2.27	1.64
10	10.13	3.18	2.30
15	15.13	3.89	2.80
etc.			

Complete plotting data for various constant values of diffraction coefficients are summarized in Table IE-3. A plot of these data are shown in Figure IE-2. As previously discussed this figure is a generalized diagram which shows curves of equal values of diffraction coefficients on a coordinate system in which the origin is at the breakwater tip. It is of interest to note on this diagram that along the geometric shadow the wave heights are one-half of the height of the incident waves, and that waves slightly greater in height than the incident waves are possible beyond the breakwater. This diagram is applicable for a uniform depth with either deep-water or shallow-water waves, provided that the proper wave length is used in the particular case. It also applies for conditions where the angle between the incident wave and the breakwater is other than 90 degrees as shown by Figure IE-2.

TABLE IE-3

Dimensionless Coordinates of Curves of Constant K'
for Diffraction at the end of a Semi-infinite Breakwater

y/L	Values of x/L						
	Lee or + values		Geometric Shadow	Unsheltered or - values			
	Diffraction Coefficients K'						
	0.1	0.2	0.5	0.8	1.0	1.17	1.0
5	3.78	1.64	0.00	0.77	1.24	1.96	3.54
10	5.19	2.30	0.00	1.09	1.75	2.75	4.89
15	6.29	2.80	0.00	1.33	2.14	3.36	5.93
20	7.23	3.23	0.00	1.54	2.47	3.87	6.82
30	8.81	3.95	0.00	1.88	3.02	4.74	8.31
40	10.13	4.56	0.00	2.17	3.49	5.46	9.58
50	11.31	5.10	0.00	2.43	3.90	6.11	10.68
60	12.38	5.59	0.00	2.66	4.27	6.68	11.70
70	13.37	6.04	0.00	2.87	4.61	7.22	12.62
80	14.28	6.46	0.00	3.07	4.93	7.71	13.48
90	15.14	6.85	0.00	3.26	5.23	8.18	14.30
100	15.91	7.22	0.00	3.44	5.51	8.62	15.03
120	17.43	7.90	0.00	3.76	6.04	9.44	16.47
140	18.83	8.53	0.00	4.07	6.52	10.20	17.79
160	20.13	9.13	0.00	4.34	6.97	10.90	19.03
180	21.35	9.68	0.00	4.61	7.40	11.55	20.16
200	22.50	10.20	0.00	4.86	7.80	12.18	21.24
220	23.59	10.70	0.00	5.10	8.18	12.78	22.27
240	24.64	11.18	0.00	5.32	8.54	13.34	23.27
260	25.66	11.63	0.00	5.54	8.89	13.89	24.21
280	26.62	12.07	0.00	5.75	9.23	14.42	25.13
300	27.56	12.50	0.00	5.95	9.55	14.92	26.01

In addition to the plot of diffraction coefficients shown in Figure IE-2, a plot of wave patterns is desirable in certain investigations. Data for plotting wave patterns are presented in the last column of Table IE-1 where the lag, or lead, of the wave front is given. Thus, along the parabola $K' =$ constant, at the corresponding value of (y/L) and (x/L) the wave front will lag (or lead) by the given fraction of the wave-length that portion of the front of the same wave which is at the geometric shadow. It should be recognized that the wave patterns plotted by this method apply only if the water is of uniform depth. Should these waves, after passing the breakwater, move into shoaling water where the bottom contours are not normal to the wave direction, refraction as well as diffraction must be considered; in fact, after a few wave lengths beyond the barrier, refraction may be more important than diffraction if the depth is changing rapidly.

b. Diffraction at a Breakwater Gap:

(1) Theory: The complete theory for this problem has been discussed elsewhere (References IE-1 and IE-2) and only a summary and an illustrated example need be presented herein.

There are two theoretical methods by which the general problem of water wave diffraction through a breakwater gap may be most directly approached. The first mode of attack (Reference IE-4) involves a solution for diffraction of light waves by a semi-infinite screen, or a half plane. The procedure is extended by superposition to a breakwater with a gap. The resulting solution is reasonably accurate, however, only for gap widths of over two wave lengths.

The second method of approach may be credited largely to Morse and Rubenstein (Reference IE-3). This analysis, based on elliptic-cylinder coordinates and the associated Mathieu functions, was originally developed for the diffraction of sound and electromagnetic waves.

The latter method of calculation is presented first. This theory is an exact solution for small gaps and possesses the feature of leading to direct expressions for angular distribution of energy transmission through the opening, and for the total of such transmitted energy. The diffraction coefficient K' at any radius, r, from the center of the gap is given by the expression

$$K' = \sqrt{I_{r,\phi} \cdot \frac{L}{r}} \qquad \text{(IE-5.3)}$$

where L is the wave length and $I_{r,\phi}$ is the intensity factor. Polar plots of this intensity factor for various gap openings and directions of wave approach are shown in Figure IE-22. The diffraction coefficient at any point (r, θ) located at a relative distance L/r in the lee of a gap (Figure IE-23a) can be calculated by first determining the value of $I_{r,\phi}$ (as illustrated in Figure IE-23b) and then substituting in equation IE-5.3. Systematic calculation of other points in the lee of the breakwater will permit contours of equal diffraction coefficients to be drawn, thus yielding a generalized diffraction diagram, for the particular gap opening and approach direction.

For relatively wide gaps the method of computation is much more complicated and laborious than that described above for the narrow gaps. For any position (x, y) illustrated in Figure IE-24 the diffraction coefficient is given by the expression:

$$K' = \left[F\ (x,\ y) \right] \qquad \text{(IE-5.4)}$$

and the phase difference is,

$$\text{phase difference} = \arg\ (F \text{ for diffracted wave}) - ky \qquad \text{(IE-5.5)}$$

In these expressions

$$F(x,y) = e^{-iky} - f_1 + g_1 + f_2 + g_2 \text{ for } x \leqq B/2 \qquad \text{(IE-5.6)}$$

and

$$F(x,y) = f_1 + g_1 - f_2 + g_2 \text{ for } \begin{matrix} x \geqq B/2 \\ y \leqq 0 \end{matrix} \qquad \text{(IE-5.7)}$$

At x = B/2 (geometric shadow) these expressions of course are identical. The terms in equations IE-5.6 and IE-5.7 are computed in terms of real and imaginary components, each of which are added arithmetically and combined into a resultant F (x,y). The terms f and g in equations IE-5.6 and IE-5.7 are defined as follows:

$$f = f\ (r,y) = e^{-iky} \cdot f(-u) \qquad \text{(IE-5.8)}$$

and

$$g = f\ (r,-y) = e^{iky} \cdot f(-u) \qquad \text{(IE-5.9)}$$

where

$$f(-u) = S + iw$$

in which S and w are the real and imaginary components of f (-u). The term u is defined by the expression

$$u = \sqrt{4(r-y)/L} \qquad \text{(IE-5.10)}$$

Values of S and w as functions of the quantity u^2 are shown in Figures IE-25 and 26.

The diffraction coefficient and phase difference at any point (x,y) for a particular breakwater gap of width B are computed by using equations IE-5.6 and IE-5.7 in the following steps. Values of r_1 and r_2 are first obtained from the following relationships which are readily derived from the geometry of the system shown in Figure IE-24.

$$r_1 = \sqrt{y^2 + (x - B/2)^2} \qquad x \geqq B/2 \qquad \text{(IE-5.11)}$$

$$r_1 = \sqrt{y^2 + (B/2 - x)^2} \qquad x \leqq B/2$$

and

$$r_2 = \sqrt{y^2 + (x + B/2)^2} \qquad \text{(IE-5.12)}$$

Values of r_1 and r_2 from these equations are used in equation IE-5.10 to determine values of u_1 and u_2, respectively. The values of S_1 and w_1 corresponding to u_1, and S_2 and w_2 corresponding to u_2, next are obtained from Figure IE-25 and 26. These functions are used in determining f_1 and f_2 by equation IE-5.8.

A similar procedure is used in determining the value of g_1 and g_2 from equation IE-5.9. For a given value of x and y values of u_3 and u_4 are first determined from equation IE-2.8 in which the value of r_1 and the negative value of y yields u_3, and the value of r_2 and the negative value of y yields u_4. The pair of values S_3 and w_3 and S_4 and w_4 corresponding to u_3 and u_4, respectively, are determined from Figures IE-25 and 26.

In adding the components, f_1, f_2, g_1 and g_2, the difference in phase between f_1 and f_2 (as given by equation IE-5.8), and g_1 and g_2 (given by equation IE-5.9) has to be considered; thus,

$$f_1 = e^{-iky} \cdot f(-u_1)$$

and

$$g_1 = e^{iky} \cdot f(-u_2) = e^{2iky} \left[e^{-iky} \cdot f(-u_2) \right]$$

That is, there is a phase difference corresponding to the factor e^{2iky}, or 2ky. Therefore, in obtaining $\pm f_1 + g_1$ (disregarding the factor e^{-iky}) from $f(-u_1)$ and $f(-u_2)$, the real and imaginary components of $f(-u_2)$ must be rotated through the angle $2ky = 4\pi y/L = 720\ y/L$ degrees and then resolved into components parallel to the real and imaginary components of $f(-u_1)$ before addition. Similarly, $f(-u_4)$ must be rotated through the same angle α. To perform this operation numerically, the factors $\cos\alpha$ and $-\sin\alpha$ are applied to the real and imaginary parts, respectively, of $f(-u_2)$ and $f(-u_4)$, which then are added to obtain the components to be added to the real parts of $\pm f(-u_1)$ and $-f(-u_3)$; while the factors $\sin\alpha$ and $\cos\alpha$ are applied to the real and imaginary parts of $f(-u_2)$ and $f(-u_4)$ to obtain the components to be added to the imaginary parts of $\pm f(-u_1)$ and $f(-u_3)$. In the same manner, the term e^{-iky}, takes the value $1 + i$ in the addition.

Designating the sum of the real components S, and that of the imaginary components w, the value of the diffraction coefficient $K' = \left[F(x,y)\right]$ is obtained from

$$K' = \sqrt{w^2 + S^2} \qquad \text{(IE-5.13)}$$

and the phase difference due to diffraction is,

$$\text{phase difference} = \tan^{-1} w/S \qquad \text{(IE-5.14)}$$

In the practical case, the diffraction coefficients and phase differences are calculated for a breakwater gap of width B with waves of wave length L approaching parallel to the breakwater. Instead of making computations for specific values of B and L the results can be generalized by considering values of B, x, and y as multiples of the wave length L. Thus, computations of diffraction coefficients and phase differences are made for a given ratio of gap-width to wave length at various relative positions x/L and y/L in the lee of the breakwater. For

convenience in computations, equations IE-5.11 and IE-5.12 are written:

$$r_1/L = \sqrt{(y/L)^2 + (x/L - B/2L)^2}$$

for $x \geqq B/2$

$$r_2/L = \sqrt{(y/L)^2 + (x/L + B/2L)^2}$$

Values u_1^2, u_2^2, u_3^2 and u_4^2 therefore are computed for various values of x/L and y/L from equations (IE-5.10 transformed as follows:

$$u_1^2 = 4\ (r_1/L - y/L)$$

$$u_2^2 = 4\ (r_2/L - y/L)$$

$$u_3^2 = 4\ (r_1/L + y/L)$$

$$u_4^2 = 4\ (r_2/L + y/L)$$

From Figures IE-25 and 26 values of S and w are obtained and the diffraction coefficients and phase differences then computed. The mechanics of performing the above computations are illustrated by the following numerical example.

(2) Numerical Example: Required: a generalized diagram for diffraction at a breakwater gap whose width is two wave lengths. Thus

$$B/L = 2$$

As an illustration, diffraction coefficients and phase differences are calculated in Table IE-4 at various positions y/L along x/L ranges 0, 1, 2, and 3 from the gap. The positions y/L have been arbitrarily assumed to be multiples "n" of an initial values of y/L = 0.6. It is believed that the sequence of the steps in the calculation process in Table IE-4 is self-explanatory. A plot of the diffraction coefficients can be made directly from the data shown in Table IE-4, for which contours of equal diffraction coefficients then can be constructed as shown in Figure IE-4. A plot of the wave patterns in the lee of the breakwater gap involves additional computations, to those presented in Table IE-4, and are discussed as follows.

The "phase difference" as computed in Table IE-4 is that of equation IE-5.5 of the appendix with complete cycles omitted. To express this phase difference in terms of complete cycles, it must be divided by 360°. To obtain the complete phase the phase of the incident wave must be added to the phase difference. The phase of the incident wave also is expressed in cycles, that is $-y/L+m$, where m is an integer. The whole cycles are omitted (both phases are expressed as negative angles, thus indicating a decrease in phase as y increases). In the computations for complete phase for various positions (x/L, y/L) it is usually convenient to make a plot of the phase, ϕ, against values of y/L as shown in Figure IE-27. Since the curve for x/L = 0 rises at slightly less than 45°, a 45° line is first drawn lightly on the plot, thus making clear the value of the integer m (or the whole number of cycles) to be added to the phase difference. The computation procedure in arriving at the complete phase is illustrated in Table IE-5. Thus, for various values of y/L and x/L the corresponding values of phase difference (PD), as computed and shown in the last line of Table IE-4, were first tabulated (minus values of phase difference were subtracted from 360° before tabulation in Table IE-5).

TABLE II-4

CALCULATION OF DIFFRACTION COEFFICIENTS FOR BREAKWATER GAP WIDTH b = 2L (L = WAVE LENGTH)

$K' = f_1 + g_1 - f_2 + g_2$

x/L	2							
n (integer)	1	3	5	7	10	20	30	50
y/L	0.6	1.8	3.0	4.2	6.0	12	18	30
y/L - n/2	0.1	0.3	0.5	0.7	1.0	2	3	5
$\alpha = 720(y/L-n/2)$	72°	216°	360°	504° 144°	720°	—	—	—
Sin α	+0.951	-0.588	0	+0.588	0	0	0	0
Cos α	+0.309	-0.809	+1.00	-0.809	+1.00	+1.00	+1.00	+1.00
r_1/L	1.166	2.06	3.16	4.32	6.08	12.04	18.028	30.017
u_1^2	2.26	1.04	0.64	0.48	0.32	0.16	0.112	0.068
s_1 } f_1	-0.069	-.116	+.013	+.086	+.175	+.282	+.327	+.365
w_1 } f_1	+0.131	-.158	-.235	-.245	-.230	-.184	-.156	-.126
u_3^2	7.06	15.44	24.6	34.1	48.3	96.2	144.1	240.1
s_3 } g_1	+.062	+.054	-.006	-.022	+.010	+.010	+.011	+.008
w_3 } g_1	+.062	+.008	-.043	+.032	-.030	-.021	-.014	-.012
r_2/L	3.06	3.50	4.24	5.16	6.71	12.37	18.25	30.15
u_2^2	9.84	6.80	4.96	3.84	2.84	1.38	1.00	0.60
s_2 } f_2	-.065	+.038	-.061	+.103	+.055	-.169	-.106	+.031
w_2 } f_2	+.035	+.080	-.076	-.049	+.120	-.034	-.167	-.239
u_4^2	14.64	21.2	29.0	37.4	50.8	97.5	145.0	240.6
s_4 } g_2	+.015	-.042	-.028	-.033	+.015	-.021	-.013	-.002
w_4 } g_2	+.057	-.025	-.029	-.006	+.030	-.001	-.012	-.014
$M = s_3 + s_4$	+.077	+.012	-.034	-.055	+.025	-.011	-.002	+.006
$N = w_3 + w_4$	+.119	-.017	-.072	+.026	.000	-.022	-.026	-.026
M cos α	+.024	-.009	-.034	+.045	+.025	-.011	-.002	+.006
N sin α	-.113	-.010	.000	-.015	.000	.000	.000	.000
$s_1 - s_2$	-.004	-.154	+.074	-.017	+.120	+.451	+.433	+.334
Σs	-.093	-.173	+.040	+.013	+.145	+.440	+.431	+.340
M sin α	+.073	-.007	.000	-.032	.000	.000	.000	.000
N cos α	+.037	+.014	-.072	-.021	.000	-.022	-.026	-.026
$w_1 - w_2$	+.096	-.238	-.159	-.196	-.350	-.150	+.011	+.113
Σw	+.206	-.231	-.231	-.249	-.350	-.172	-.015	+.087
w^2	.0424	.0534	.0534	.0620	.1225	.0296	.0002	.0076
s^2	.0086	.0299	.0016	.0002	.0210	.1936	.1858	.1156
$K' = \sqrt{w^2 + s^2}$	0.226	0.289	0.235	0.249	0.379	0.472	.431	.351
w/s	-2.215	+1.335	-5.775	-19.15	-2.414	-0.3909	-0.03480	+0.2559
ϕ Diff.	114.3°	-126.8°	-80.2°	-87.0°	-67.5°	-21.4°	-2.0°	+14.4°

x/L	1 = b/2 (Geometric Shadow)							
n (integer)	1	3	5	7	10	20	30	50
y/L	0.6	1.8	3.0	4.2	6.0	12	18	30
y/L - n/2	0.1	0.3	0.5	0.7	1.0	2	3	5
$\alpha = 720(y/L-n/2)$	72°	216°	360°	144°	—	—	—	—
Sin α	+0.951	-0.588	0	+0.588	0	0	0	0
Cos α	+0.309	-0.809	+1.00	-0.809	+1.00	+1.00	+1.00	+1.00
r_1/L	0.600	1.800	3.00	4.20	6.00	12.00	18.00	30.00
u_1^2	.000	.000	.000	.000	.000	.000	.000	.000
s_1 } f_1	+.500	+.500	+.500	+.500	+.500	+.500	+.500	+.500
w_1 } f_1	.000	.000	.000	.000	.000	.000	.000	.000
u_3^2	4.80	14.40	24.0	33.6	48.0	96.0	144.0	240.0
s_3 } g_1	-.042	-.011	+.034	-.088	+.023	+.017	+.013	-.011
w_3 } g_1	-.091	+.057	-.032	+.006	-.024	-.017	-.012	-.011
r_2/L	2.09	2.69	3.61	4.65	6.33	12.17	18.11	30.07
u_2^2	5.96	3.56	2.44	1.80	1.32	0.66	0.44	0.28
s_2 } f_2	-.074	+.119	-.028	-.149	-.161	-.012	+.106	+.197
w_2 } f_2	+.059	.000	+.140	+.054	-.081	-.230	-.244	-.223
u_4^2	10.76	17.96	26.44	35.4	49.3	96.7	144.4	240.3
s_4 } g_2	+.026	-.037	-.005	+.032	-.029	-.006	+.003	+.005
w_4 } g_2	+.053	+.034	+.040	+.007	-.008	-.022	-.018	-.015
$M = s_3 + s_4$	-.016	-.048	+.029	-.006	-.006	+.011	+.016	+.016
$N = w_3 + w_4$	-.038	+.091	+.008	+.013	-.032	-.039	-.030	-.026
M cos α	-.005	+.039	+.029	+.005	-.006	+.011	+.016	+.016
N sin α	+.036	+.053	.000	-.000	.000	.000	.000	.000
$s_1 - s_2$	+.574	+.381	+.528	+.649	+.661	+.512	+.394	+.303
Σs	+.605	+.473	+.557	+.646	+.655	+.523	+.410	+.319
M sin α	-.015	+.028	.000	-.004	.000	.000	.000	.000
N cos α	-.012	-.074	+.008	-.011	-.032	-.039	-.030	-.026
$w_1 - w_2$	-.059	.000	-.140	-.054	+.081	+.230	+.244	+.223
Σw	-.086	-.046	-.132	-.069	+.049	+.191	+.214	+.197
w^2	.0074	.0021	.0174	.0048	.0024	.0365	.0458	.0388
s^2	.3660	.2237	.3102	.4173	.4290	.2735	.1681	.1018
$K' = \sqrt{w^2 + s^2}$	0.611	0.475	0.572	0.650	0.657	0.557	0.463	0.375
w/s	-0.1421	-0.09725	-0.2370	-0.1068	+0.07481	+0.3652	+0.5220	+0.6176
ϕ Diff.	-8.1°	-5.6°	-13.3°	-6.1°	+4.3°	+20.1°	+27.6°	+31.7°

$K' = e^{-iky} - f_1 + g_1 - f_2 + g_2$

x/L	0							
n (integer)	1	3	5	7	10	20	30	50
y/L	0.6	1.8	3.0	4.2	6.0	12	18	30
y/L-n/2	0.1	0.3	0.5	0.7	1.0	2	3	5
$\alpha = 720(y/L-n/2)$	72°	216°	360°	144°	—	—	—	—
Sin α	+0.951	-0.588	0	+0.588	0	0	0	0
Cos α	+0.309	-0.809	+1.00	-0.809	+1.00	+1.00	+1.00	+1.00
r_1/L	1.166	2.06	3.16	4.32	6.08	12.04	18.028	30.017
u_1^2	2.26	1.04	0.64	0.48	0.32	0.16	0.112	0.068
s_1 } f_1	-.069	-.116	+.013	+.086	+.175	+.282	+.327	+.365
w_1 } f_1	+.131	-.158	-.235	-.245	-.230	-.184	-.156	-.126
u_3^2	7.06	15.44	24.6	34.1	48.3	96.2	144.1	240.1
s_3 } g_1	+.062	+.054	-.006	-.022	+.010	+.010	+.011	+.008
w_3 } g_1	+.062	+.008	-.043	+.032	-.030	-.021	-.014	-.012
r_2/L	1.166	2.06	3.16	4.32	6.08	12.04	18.028	30.017
u_3^2	2.26	1.04	0.64	0.48	0.32	0.16	0.112	0.068
s_2 } f_2	-.069	-.116	+.013	+.086	+.175	+.282	+.327	+.365
w_2 } f_2	+.131	-.158	-.235	-.245	-.230	-.184	-.156	-.126
u_4^2	7.06	15.44	24.6	34.1	48.3	96.2	144.1	240.1
s_4 } g_2	+.062	+.054	-.006	-.022	+.010	+.010	+.011	+.008
w_4 } g_2	+.062	+.008	-.043	+.032	-.030	-.021	-.014	-.012
$M = s_3 + s_4$	+.124	+.108	-.012	-.044	+.020	+.020	+.022	+.016
$N = w_3 + w_4$	+.124	+.016	-.086	+.064	-.060	-.042	-.028	-.024
M cos α	+.038	-.087	-.012	+.036	+.020	+.020	+.022	+.016
N sin α	-.118	+.009	.000	-.038	.000	.000	.000	.000
$1 - s_1 - s_2$	+1.138	+1.232	+0.974	+0.828	+0.650	+0.436	+0.346	+0.270
Σs	+1.058	+1.154	+.962	+.826	+.670	+.456	+.368	+.286
M sin α	+.118	-.063	.000	-.026	.000	.000	.000	.000
N cos α	+.038	-.013	-.086	-.052	-.060	-.042	-.028	-.024
$-w_1 - w_2$	-.262	+.316	+.470	+.490	+.460	+.368	+.312	+.252
Σw	-.106	+.240	+.384	+.412	+.400	+.326	+.284	+.228
w^2	.0112	.0576	.1475	.1697	.1600	.1063	.0807	.0520
s^2	1.1194	1.3317	.9254	.6823	.4489	[illegible]	.1354	.0818
$K' = \sqrt{w^2 + s^2}$	1.063	1.179	1.036	0.923	0.780	0.561	0.465	0.366
w/s	-0.10019	+0.2080	+0.3992	+0.4988	+0.5970	+0.7149	+0.7717	+0.7972
ϕ Diff.	-5.7°	+11.8°	+21.8°	+26.5°	+30.8°	+35.6°	+37.7°	+38.6°

$K' = f_1 + g_1 - f_2 + g_2$

x/L		3							
n (integer)		1	3	5	7	10	20	30	50
y/L		0.6	1.8	3.0	4.2	6.0	12	18	30
y/L-n/2		0.1	0.3	0.5	0.7	1.0	2	3	5
$\alpha = 720(y/L-n/2)$		72°	216°	360°	144°	—	—	—	—
Sin α		+0.951	-0.588	0	+0.588	0	0	0	0
Cos α		+0.309	-0.809	+1.00	-0.809	+1.00	+1.00	+1.00	+1.00
r_1/L		2.09	2.69	3.61	4.65	6.33	12.166	18.111	30.067
u_1^2		5.96	3.56	2.44	1.80	1.32	0.66	0.44	0.27
s_1 } f_1		-.074	+.119	.028	-.149	-.161	+.004	+.106	+.202
w_1 } f_1		+.059	.000	+.140	+.054	-.081	-.233	-.244	-.220
u_3^2		10.76	17.96	26.44	35.4	49.3	96.7	144.4	240.3
s_3 } g_1		+.025	-.058	-.005	+.032	-.029	-.005	+.003	+.005
w_3 } g_1		+.063	+.034	+.040	+.007	-.008	-.022	-.018	-.015
r_2/L		4.045	4.39	5.00	5.80	7.21	12.65	18.44	30.266
u_2^2		13.78	10.36	8.00	6.40	4.34	2.60	1.76	1.044
s_2 } f_2		-.057	-.018	+.060	-.019	-.043	+.006	-.153	-.123
w_2 } f_2		+.025	+.068	-.055	+.086	-.089	+.138	+.044	-.160
u_4^2		18.88	24.76	32.00	40.0	52.8	98.6	145.8	241.1
s_4 } g_2		+.007	-.015	+.027	+.026	-.014	+.003	-.016	-.011
w_4 } g_2		+.024	-.041	-.026	-.023	-.028	+.023	+.009	-.007
$M = s_3 + s_4$		+.032	-.065	+.022	+.058	-.043	-.002	-.013	-.006
$N = w_3 + w_4$		+.087	-.007	+.014	-.016	-.036	+.001	-.009	-.022
M cos α		+.010	+.043	+.022	-.047	-.043	-.002	-.013	-.006
N sin α		-.063	-.004	.000	+.009	.000	.000	.000	.000
$1 - s_1 - s_2$	$s_1 - s_2$	-.017	+.137	-.088	-.130	-.118	-.002	+.259	+.325
Σs		-.090	+.176	-.066	-.168	-.161	-.004	+.246	+.319
M sin α		+.030	+.031	.000	+.064	.000	.000	.000	.000
N cos α		+.027	+.006	+.014	+.013	-.036	+.001	-.009	-.022
$-w_1 - w_2$	$w_1 - w_2$	+.054	-.066	+.195	-.082	+.008	-.371	-.288	-.070
Σw		+.091	-.029	+.207	+.015	-.028	-.370	-.297	-.092
w^2		.0083	.0008	.0428	.0002	.0008	.1369	.0882	.0085
s^2		.0081	.0310	.0044	.0282	.0259	.0000	.0606	.1018
$K' = \sqrt{w^2 + s^2}$		0.128	0.178	0.217	0.169	0.163	0.370	0.386	0.332
w/s		-1.0111	-0.1648	-3.1364	[illegible]	+0.1739	[illegible]	-1.2073	[illegible]
ϕ Diff.		+134.7°	-9.4°	+107.7°	+174.9°	-170.1°	-80.6°	-60.4°	-16.1°

These values of PD were next divided by 360°. Referring to Figure IE-16 the integer m was obtained for each value of y/L (for example, for y/L = 3, the value of m is equal to the first integer below the x/L = 0 line which is 2). Values of m for the various values of y/L are shown in Table IE-5. The remaining steps in Table IE-5 consist of the summation of the terms -PD/360, y/L, -m, for each value of x/L as follows:

$$\frac{-PD}{360} + \frac{y}{L} - m = \text{complete phase}$$

The remaining curves of phase for x/L = constant now may be drawn in (Figure IE-27) by starting at x/L = 0 with phases taken from Table IE-5, due allowance being made for the values of m or the complete cycles. Each curve of x/L = constant lies above and approximately parallel to the preceding curve as higher values of y/L are used.

Points for the wave pattern are computed by noting from the curves shown in Figure IE-27, the values of y/L at which the phases for x/L = constant reached integral values. These points are tabulated in Table IE-6. Wave patterns now may be drawn as curves joining the points having the same integral phases. The patterns for the gap width of 2L is shown in Figure IE-4, along with the contours of equal diffraction coefficients.

TABLE IE-5

Breakwater Diffraction
Tabulation for Complete Phase
Gap Width B/L = 2

m		-1	-1	1	2	4	5.0	8.0	11	14	17
y/L		0	0.6	1.8	3.0	4.2	6.0	9.0	12	15	18
$\frac{x}{L} = 0$	PD	360	354.3	11.8	21.8	26.5	30.8	33.9	35.6	36.9	37.7
	PD/360	1	.984	.033	.061	.074	.086	.094	.099	.102	.105
	$\frac{-PD}{360} + \frac{y}{L} - m$	1	.616	.767	.939	.126	.914	.906	.901	.898	.895
$\frac{x}{L} = 0.5$	PD	360	351.5	7.0	12.7	18.6	24.1	29.3	31.1	34.9	35.2
	PD/360	1	.979	.019	.035	.052	.067	.081	.086	.097	.098
	$\frac{-PD}{360} + \frac{y}{L} - m$	0	.621	.781	.965	.148	.933	.919	.914	.903	.902
$\frac{x}{L} = 1$	PD	360	352.2	354.4	346.7	353.9	4.3	15.3	20.1	23.4	27.6
	PD/360	1	.978	.984	.963	.983	.012	.042	.056	.065	.077
	$\frac{-PD}{360} + \frac{y}{L} - m$	0	.622	.816	.037	.217	.988	.958	.944	.935	.923
$\frac{x}{L} = 1.5$	PD	142.1	257.7	317.0	317.0	313.3	333.3	352.5	4.2	11.0	15.3
	PD/360	.395	.716	.881	.881	.870	.926	.979	.012	.031	.043
	$\frac{-PD}{360} + \frac{y}{L} - m$	.605	.884	.919	.119	.329	.074	.021	.988	.969	.957
$\frac{x}{L} = 2$	PD	319.7	114.3	232.9	279.8	273.0	292.5	320.5	338.6	352.0	358.0
	PD/360	.888	.318	.647	.777	.758	.813	.890	.941	.978	.994
	$\frac{-PD}{360} + \frac{y}{L} - m$	.112	.282	.153	.223	.442	.187	.110	.059	.022	.006
$\frac{x}{L} = 2.5$	PD	137.2	312.0	124.9	208.6	235.4	242.9	281.1	307.3	326.0	338.7
	PD/360	.381	.867	.347	.579	.654	.675	.781	.853	.906	.941
	$\frac{-PD}{360} + \frac{y}{L} - m$	.619	.733	.453	.421	.546	.325	.219	.147	.094	.059
$\frac{x}{L} = 3$	PD	316.5	134.7	350.6	107.7	174.9	189.9	233.5	279.4	293.3	309.6
	PD/360	.879	.374	.974	.299	.486	.528	.649	.776	.815	.860
	$\frac{-PD}{360} + \frac{y}{L} - m$	.121	.225	.826	.701	.714	.472	.351	.224	.185	.140
$\frac{x}{L} = 3.5$	PD	136.1	313.8	208.4	1.2	87.5	142.0	179.8	225.3	257.0	279.4
	PD/360	.378	.872	.579	.003	.243	.394	.499	.626	.714	.776
	$\frac{-PD}{360} + \frac{y}{L} - m$	.622	.728	.221	.997	.956	.606	.501	.374	.286	.224
$\frac{x}{L} = 4$	PD	315.6	124.2	56.8	225.5	343.3	79.1	106.6	175.5	214.8	243.7
	PD/360	.877	.345	.158	.626	.954	.220	.296	.488	.597	.677
	$\frac{-PD}{360} + \frac{y}{L} - m$	.123	.255	.642	.374	.246	.780	.704	.512	.403	.323

TABLE IE-6

Breakwater Diffraction
Tabulation for Wave Patterns
Gap Width B/L = 2

Values of y/L

ϕ \ y/L	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0	0	0	0	-	-	-	-	-	-
1	1.00	1.00	0.95	0.75	-	-	-	-	-
2	2.05	2.05	1.95	1.90	1.65	1.10	-	-	-
3	3.10	3.10	3.00	2.90	2.75	2.50	2.05	1.30	-
4	4.05	4.05	4.00	3.90	3.75	3.65	3.35	3.00	2.45
5	5.05	5.05	5.00	4.85	4.75	4.70	4.50	4.25	3.85
6	6.10	6.10	6.00	5.90	5.80	5.70	5.50	5.35	5.05
7	7.10	7.10	7.00	6.95	6.85	6.70	6.55	6.40	6.20
8	8.10	8.10	8.05	7.95	7.85	7.75	7.60	7.45	7.20
9	9.10	9.10	9.05	8.95	8.90	8.80	8.65	8.50	8.25
10	10.10	10.10	10.05	10.00	9.90	9.80	9.70	9.55	9.30
11	11.10	11.10	11.05	11.00	10.95	10.85	10.75	10.60	10.40
12	12.10	12.10	12.05	12.00	11.95	11.85	11.75	11.60	11.45
13	13.10	13.10	13.05	13.00	12.95	12.90	12.80	12.65	12.50
14	14.10	14.10	14.05	14.00	13.95	13.90	13.80	13.70	13.55
15	15.10	15.10	15.05	15.00	15.00	14.90	14.80	14.70	14.60
16	16.10	16.10	16.05	16.05	16.00	15.95	15.85	15.75	15.60
17	17.10	17.10	17.05	17.05	17.00	16.95	16.85	16.75	16.65
18	-	-	-	-	18.00	17.95	17.85	17.75	17.65

6. References:

IE-1: Blue, Lt. Col. F. L., Jr. and Johnson, J. W. - "Diffraction of Water Waves Passing Through a Breakwater Gap" - Transactions, American Geophysical Union, Vol. 30, No. 5, October 1949, pp. 705-718.

IE-2: Carr, J. H. and Stelzriede, M. E. - "Diffraction of Water Waves by Breakwaters" - U. S. National Bureau of Standards Symposium on Gravity Waves, Washington, D. C., June 18-20, 1951.

IE-3: Morse, P. M. and Rubenstein, P. J. - "The Diffraction of Waves by Ribbons and Slits " - Physical Review, Vol. 54, 1948, pp. 895-898.

IE-4: Putnam, J. A. and Arthur, R. S. - "Diffraction of Water Waves by Breakwaters " - Transactions, American Geophysical Union, Vol. 29, No. 4, August 1948.

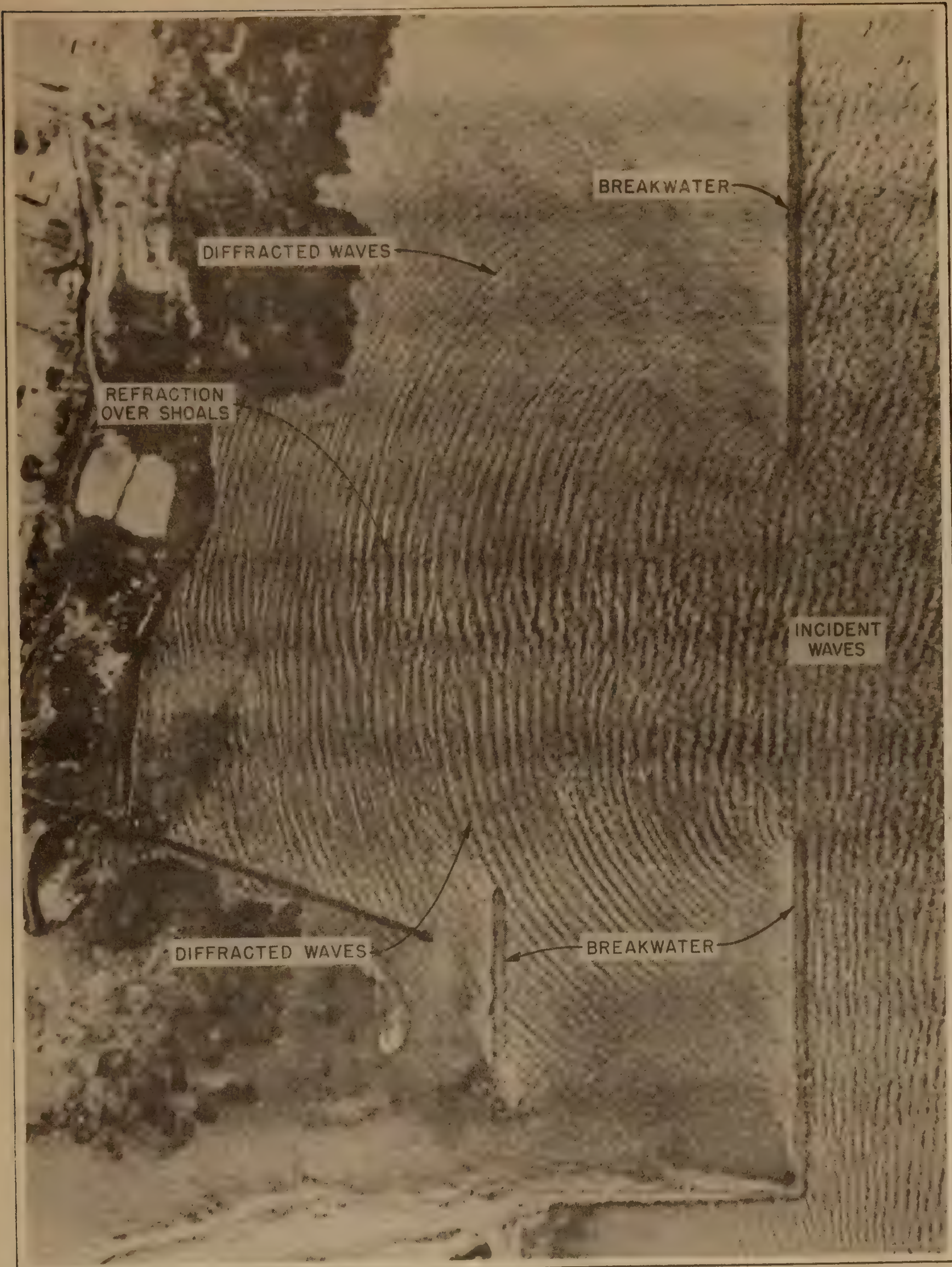

FIGURE IE-1 - DIFFRACTION OF WAVES AT A BREAKWATER

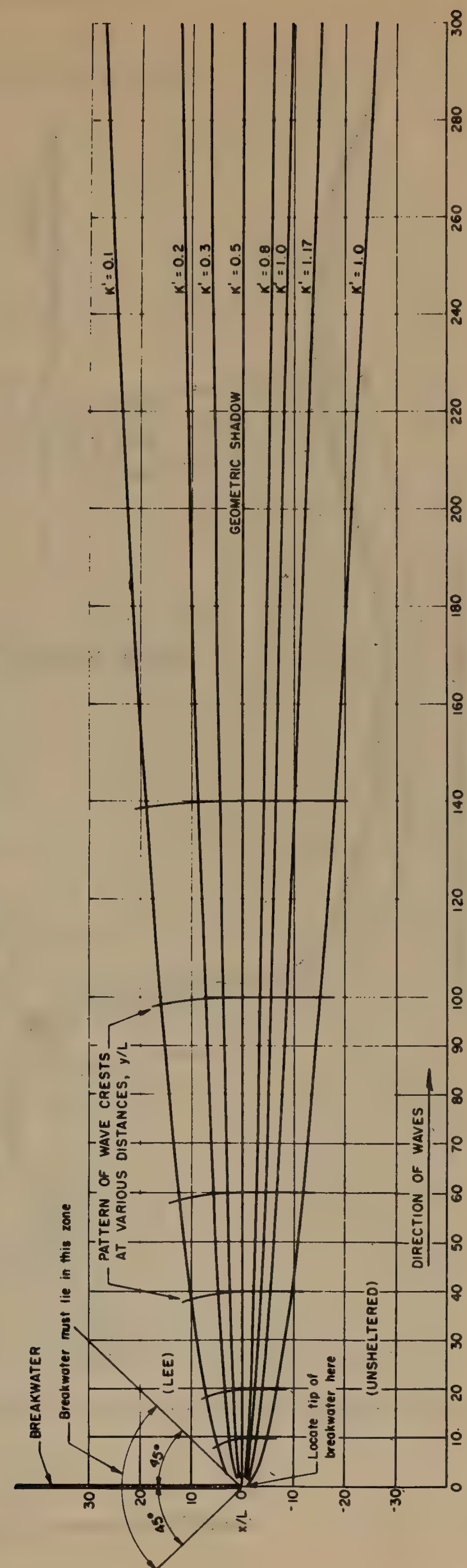

FIG. IE-2 - DIMENSIONLESS PLOT OF CURVES OF EQUAL K FOR DIFFRACTION OF WAVES AT TIP OF BREAKWATER

L = Wave length

$K' = \text{Diffraction coefficient} = \dfrac{\text{Diffracted wave height}}{\text{Incident wave height}}$

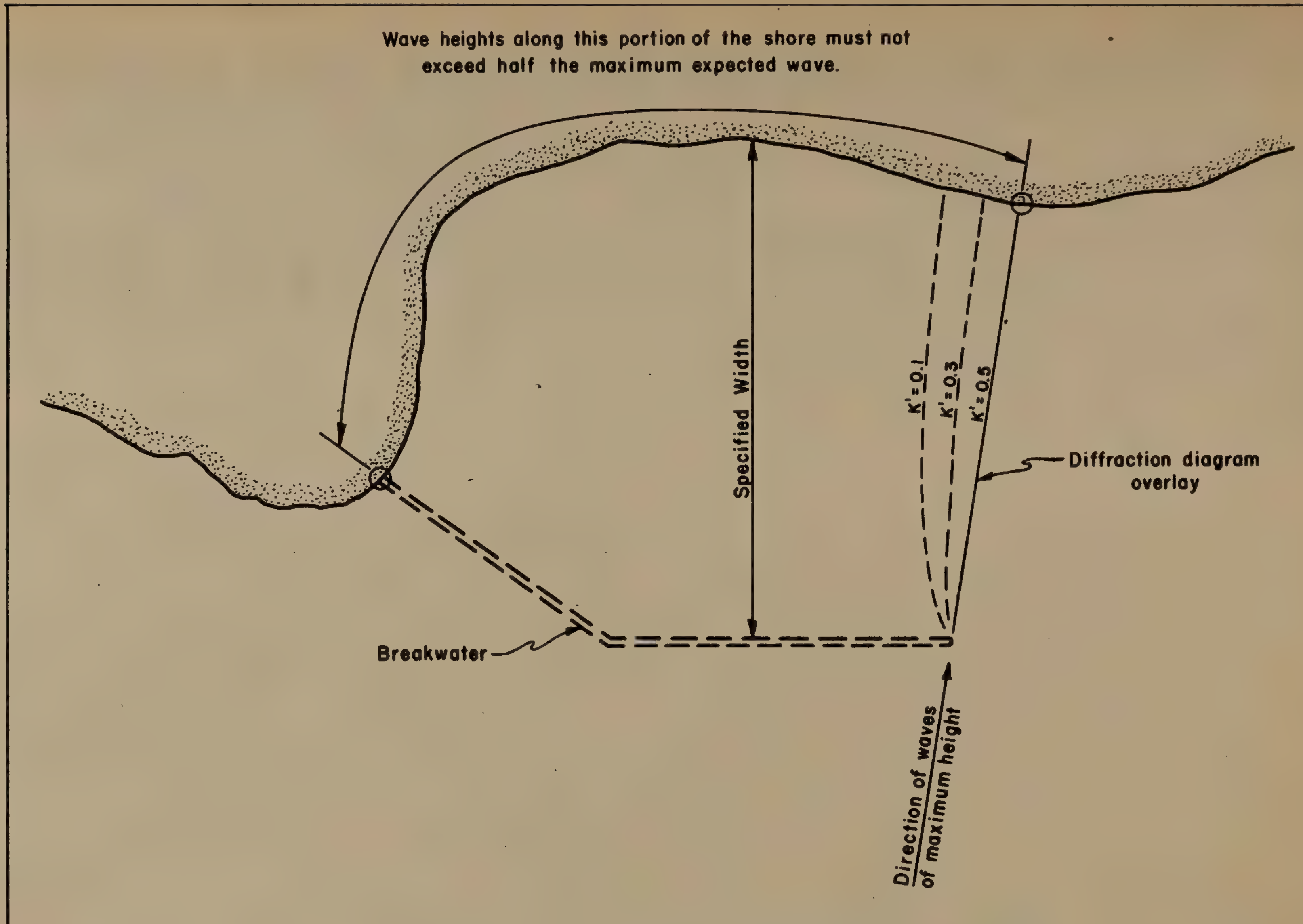

FIG. IE-3 -- Illustration of the use of a diffraction diagram in determining the location of a breakwater tip to provide a specified degree of protection against wave action.

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FIG. IE-4 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}} \right)$

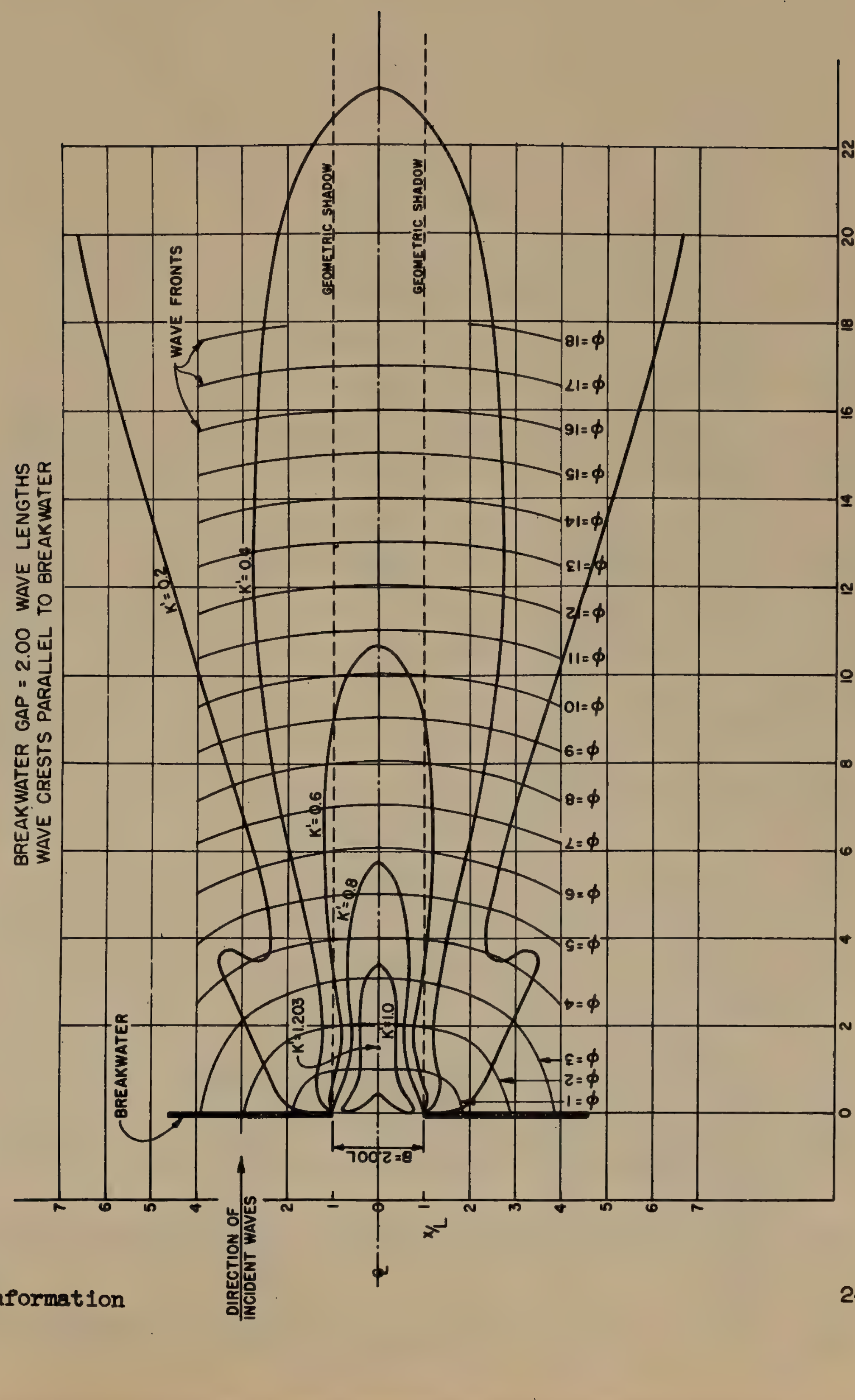

FIG. IE-5 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

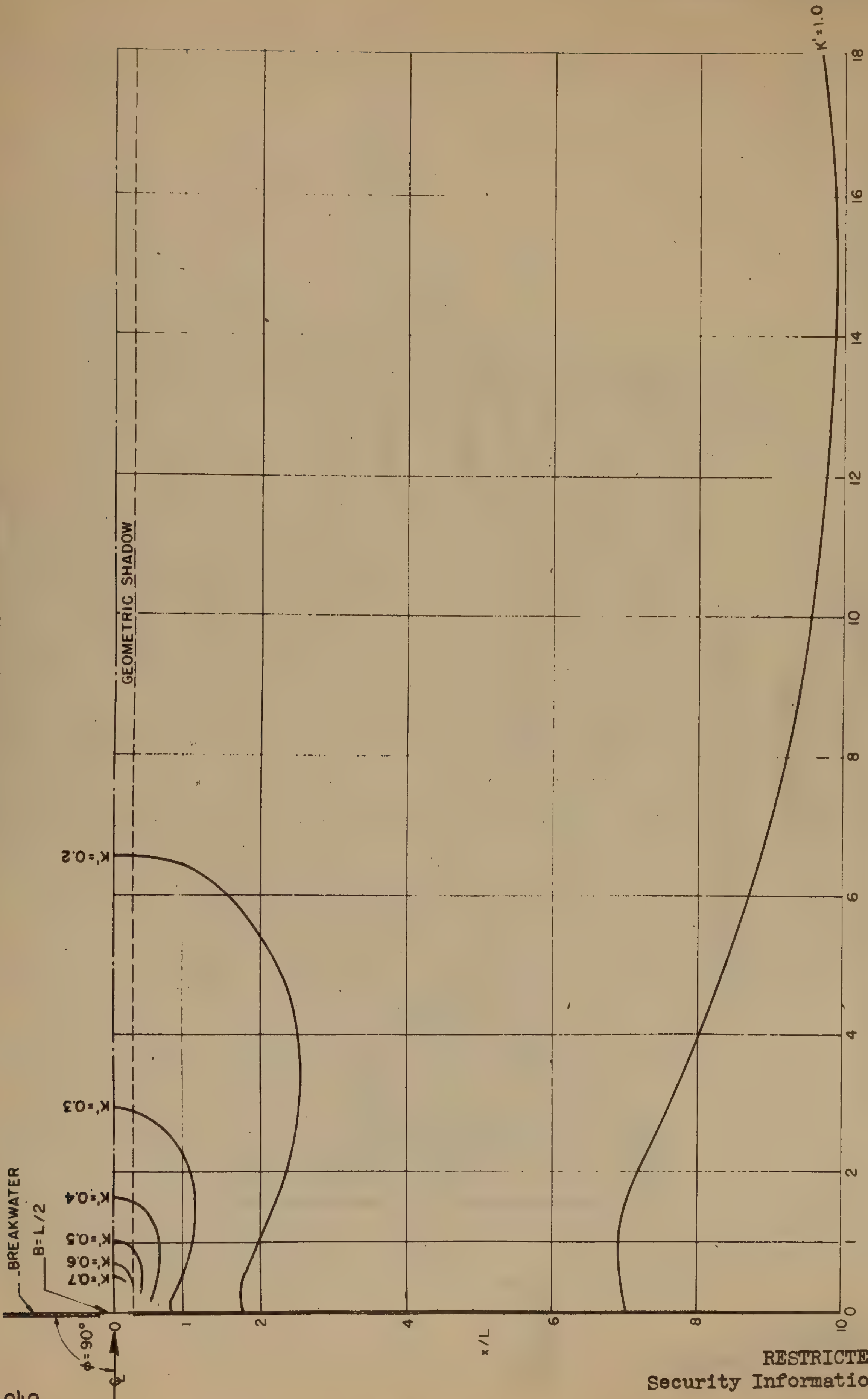

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FIG. IE-6 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}} \right)$

BREAKWATER GAP = 1.41 WAVE LENGTHS
WAVE CRESTS PARALLEL TO BREAKWATER

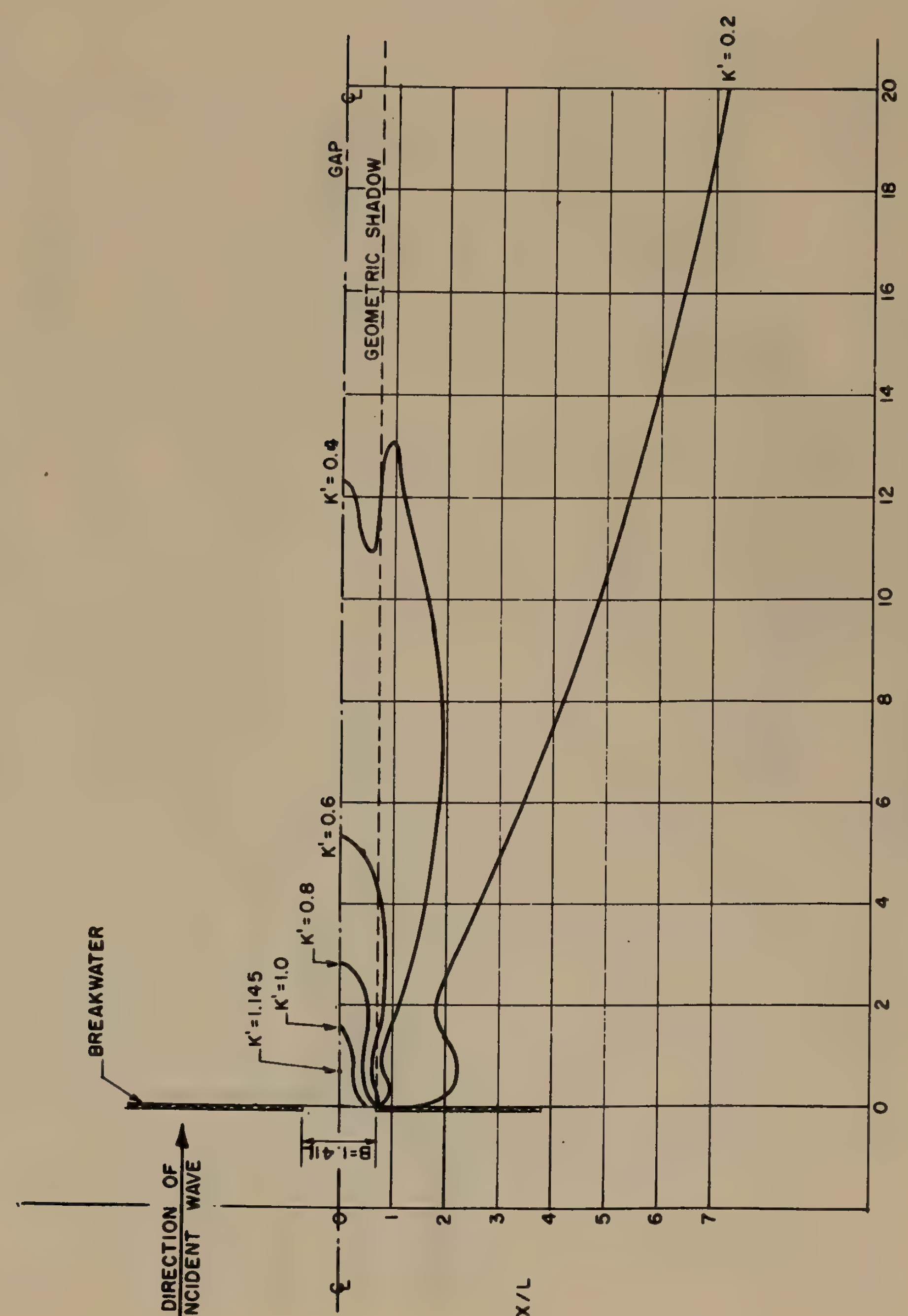

FIG. IE-7 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \dfrac{\text{Diffracted wave height}}{\text{Incident wave height}} \right)$

BREAKWATER GAP = 1.64 WAVE LENGTHS
WAVE CRESTS PARALLEL TO BREAKWATER

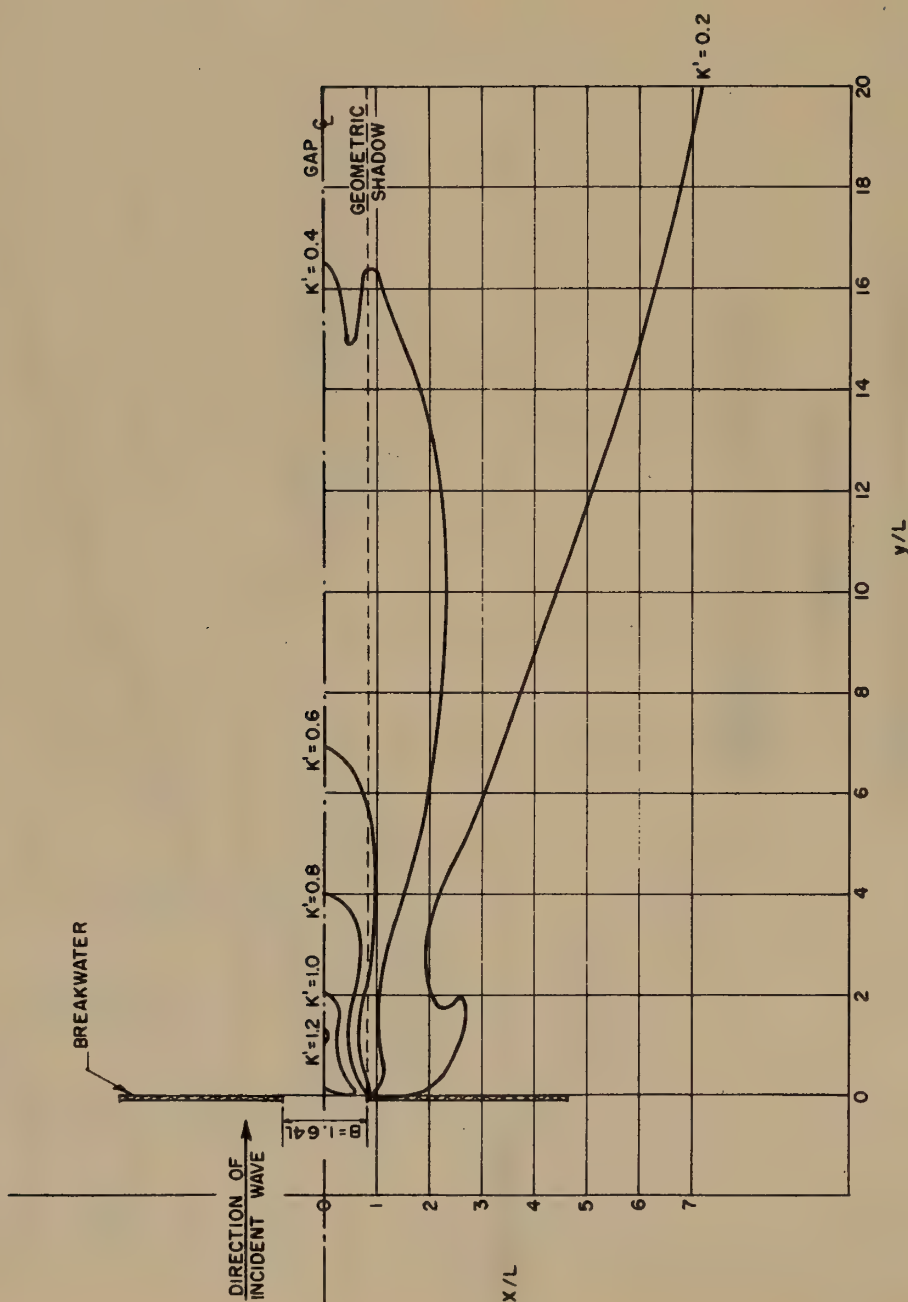

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FIG. IE-8 - DIFFRACTION OF WAVES AT A BREAKWATER GAP

CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}} \right)$

BREAKWATER GAP = 1.78 WAVE LENGTHS

WAVE CRESTS PARALLEL TO BREAKWATER

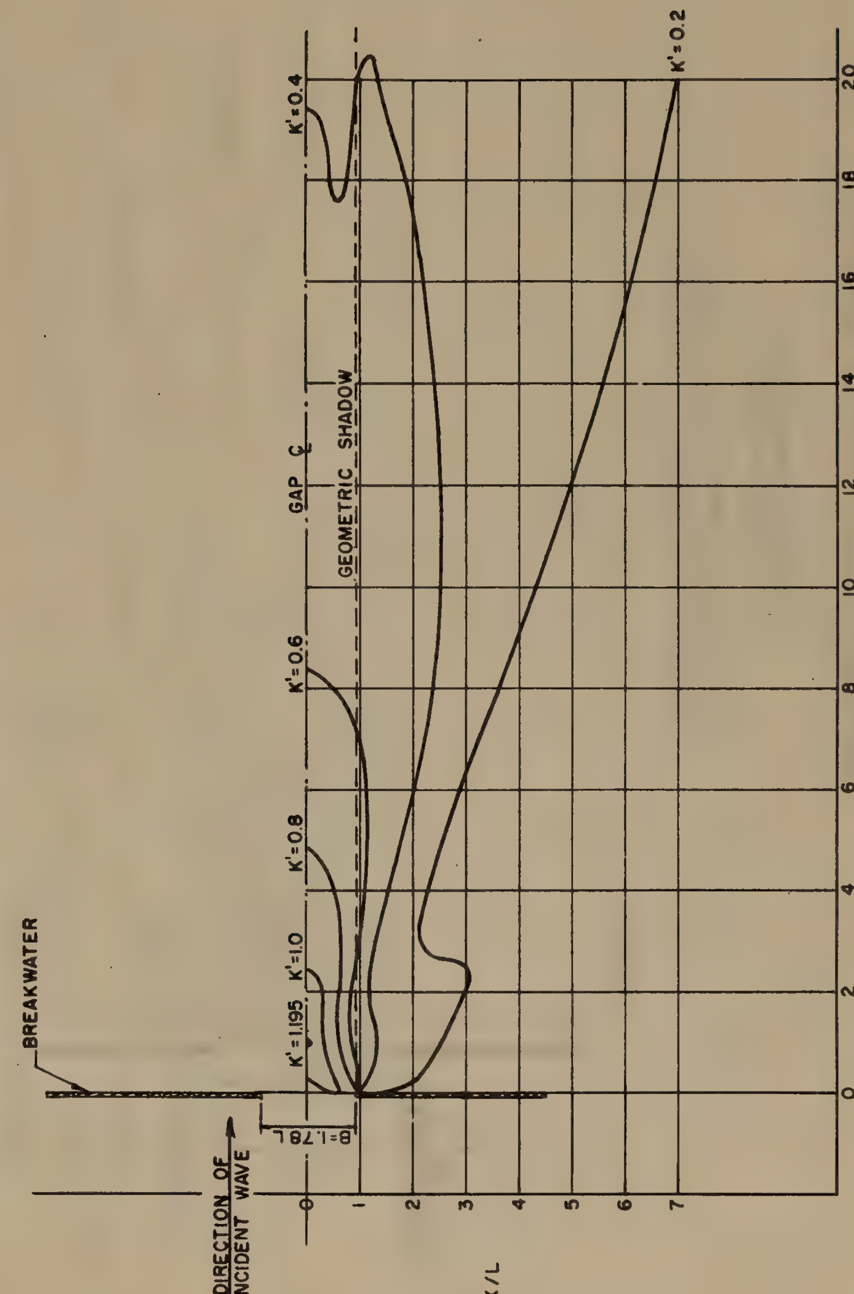

FIG. IE-9 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}}\right)$

BREAKWATER GAP = 2.50 WAVE LENGTHS
WAVE CRESTS PARALLEL TO BREAKWATER

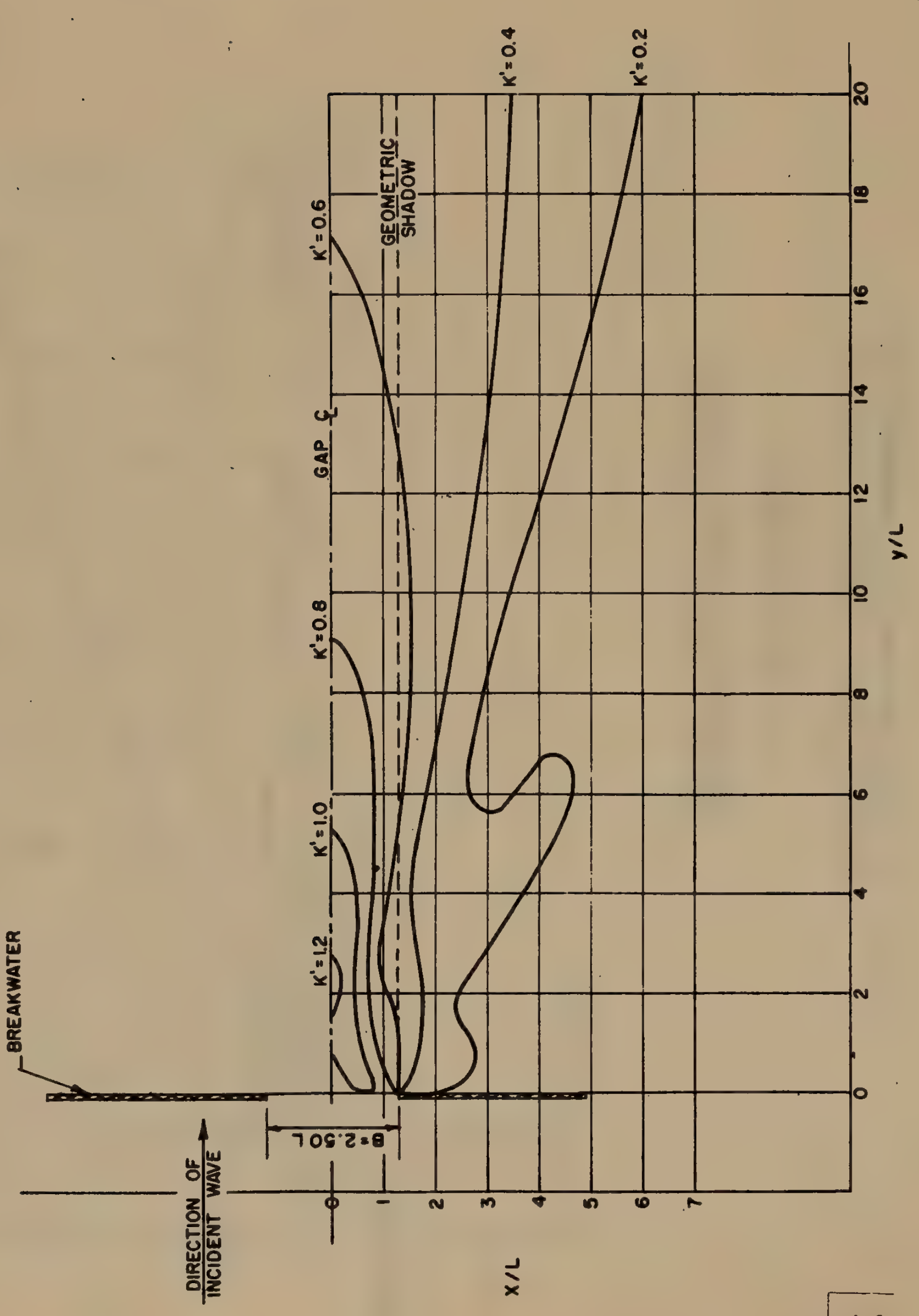

FIG. IE-10 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}}\right)$

BREAKWATER GAP = 2.95 WAVE LENGTHS
WAVE CRESTS PARALLEL TO BREAKWATER

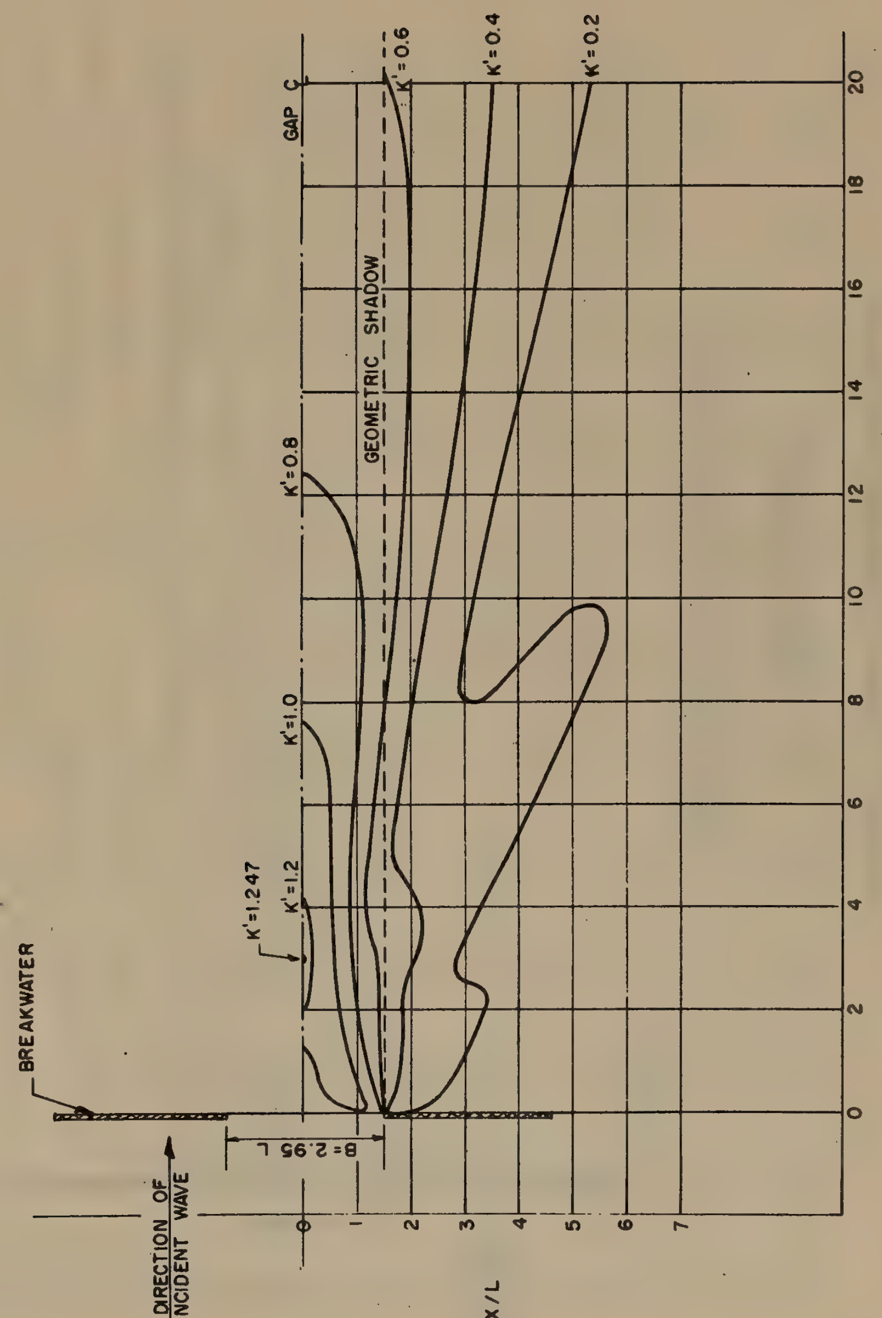

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FIG. IE-11 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}} \right)$

BREAKWATER GAP = 3.82 WAVE LENGTHS
WAVE CRESTS PARALLEL TO BREAKWATER

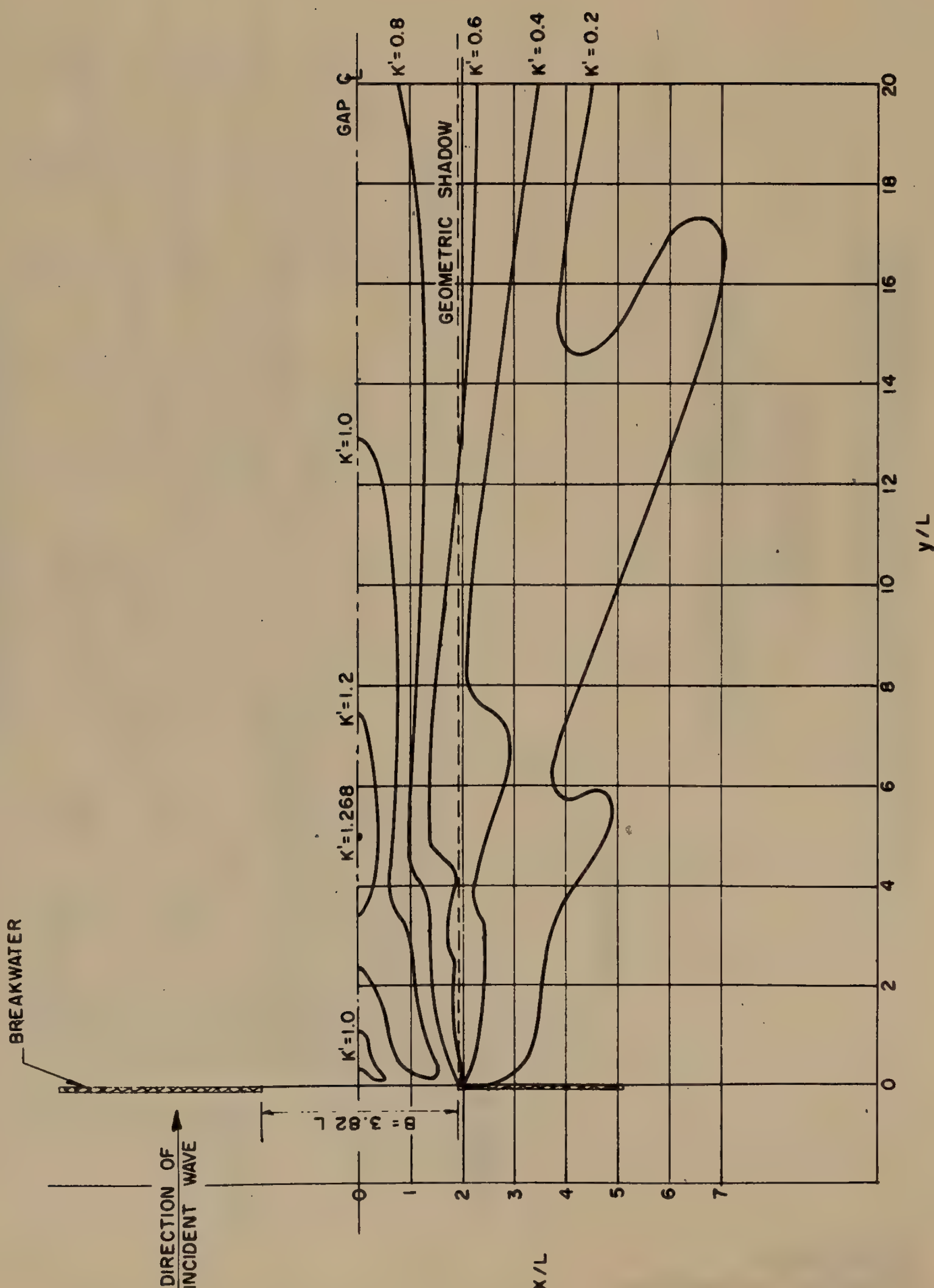


FIG. IE-12 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

$\left(K' = \frac{\text{Diffracted wave height}}{\text{Incident wave height}} \right)$

BREAKWATER GAP = 5.00 WAVE LENGTHS
WAVE CRESTS PARALLEL TO BREAKWATER

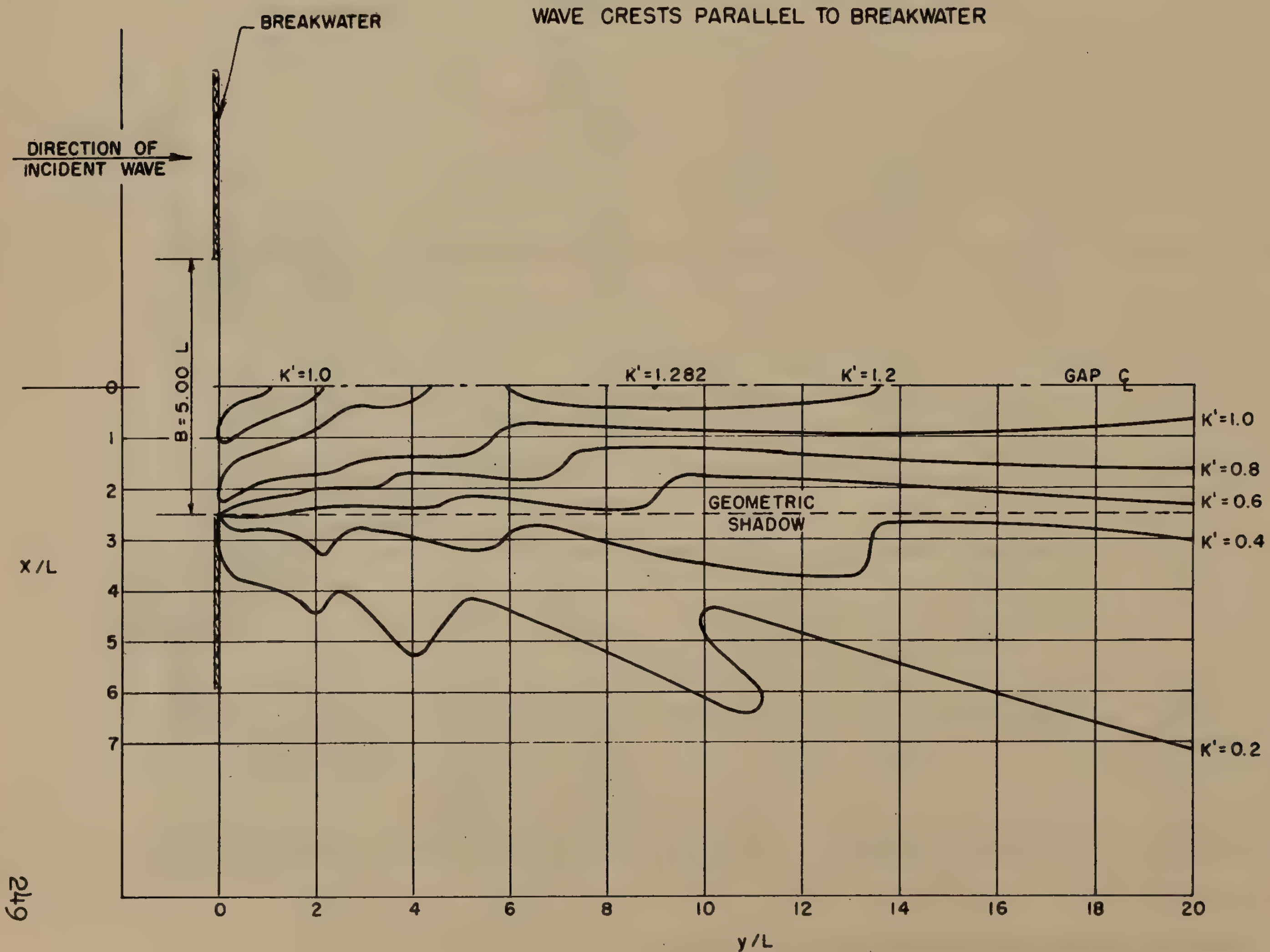


FIG. IE-13 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

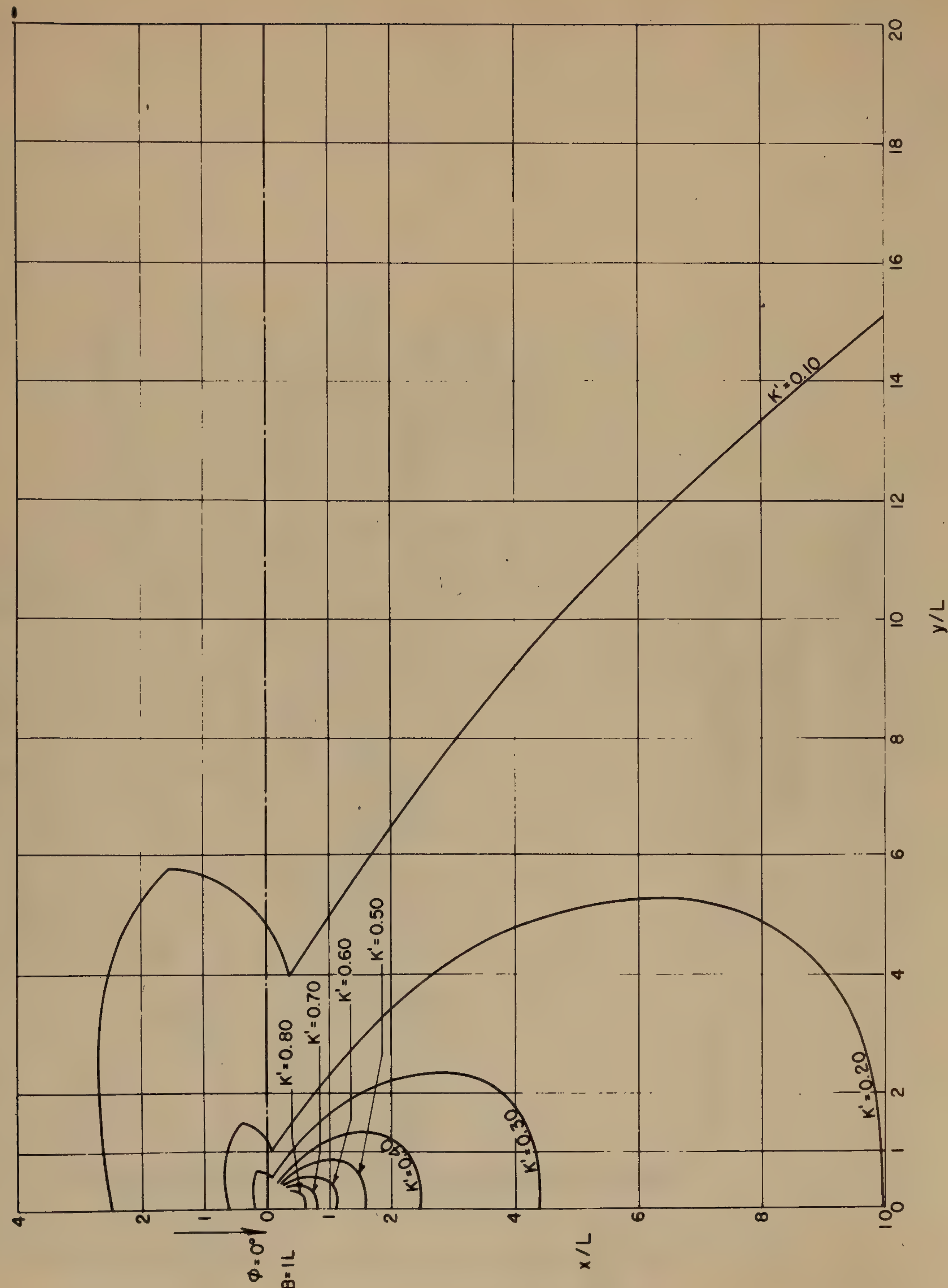


FIG. IE-14 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

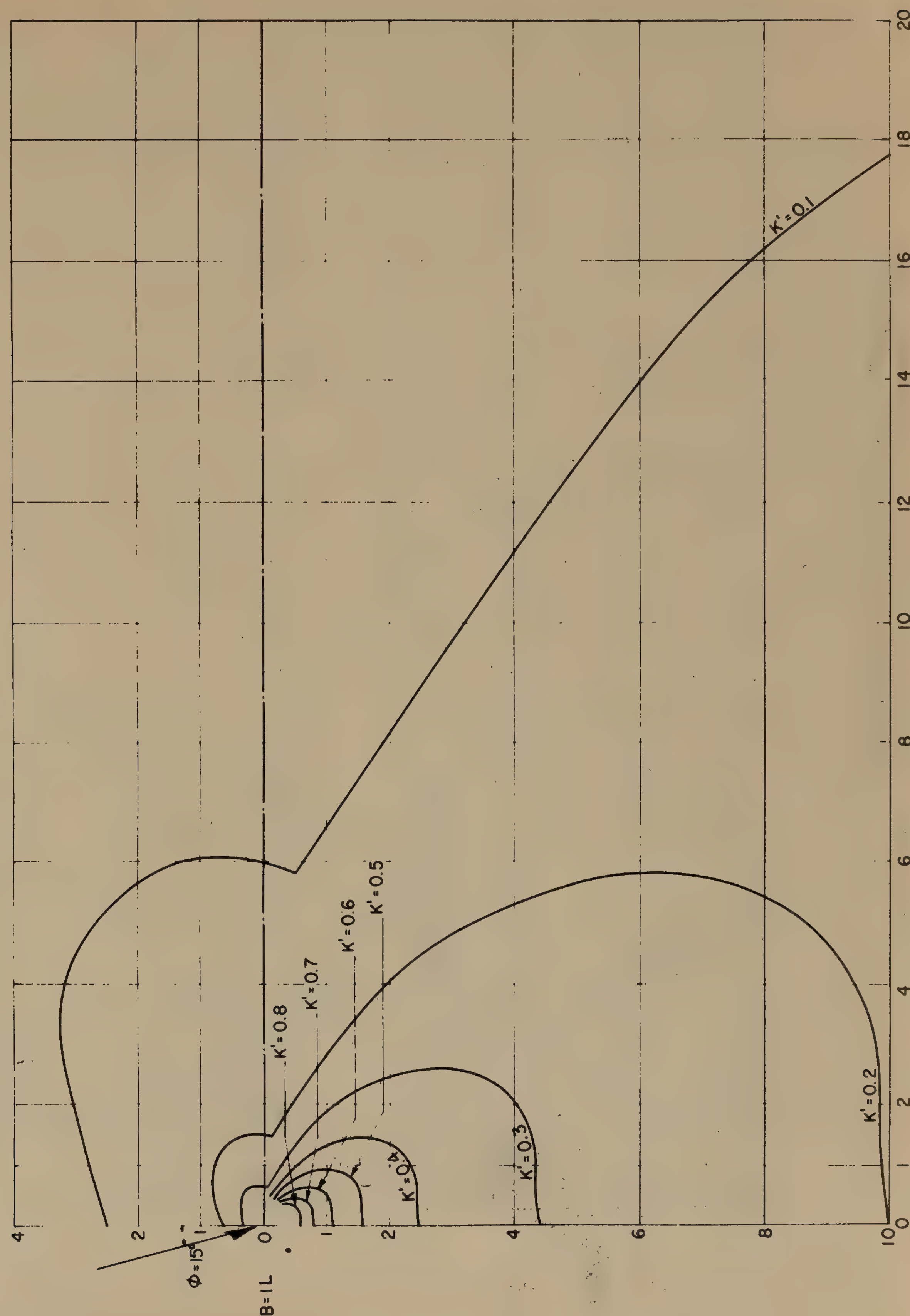


FIG. IE-15 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

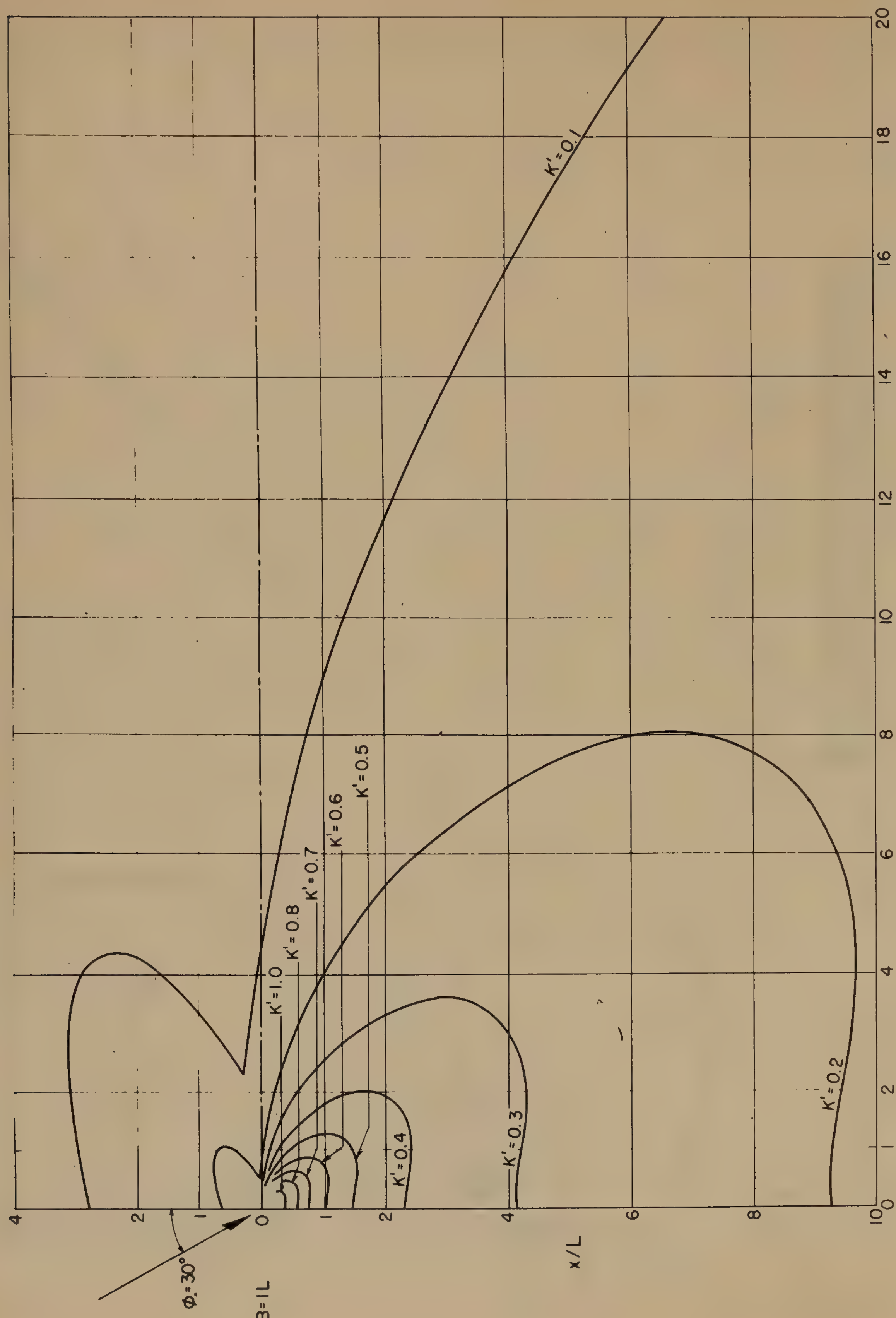


FIG. IE-16 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

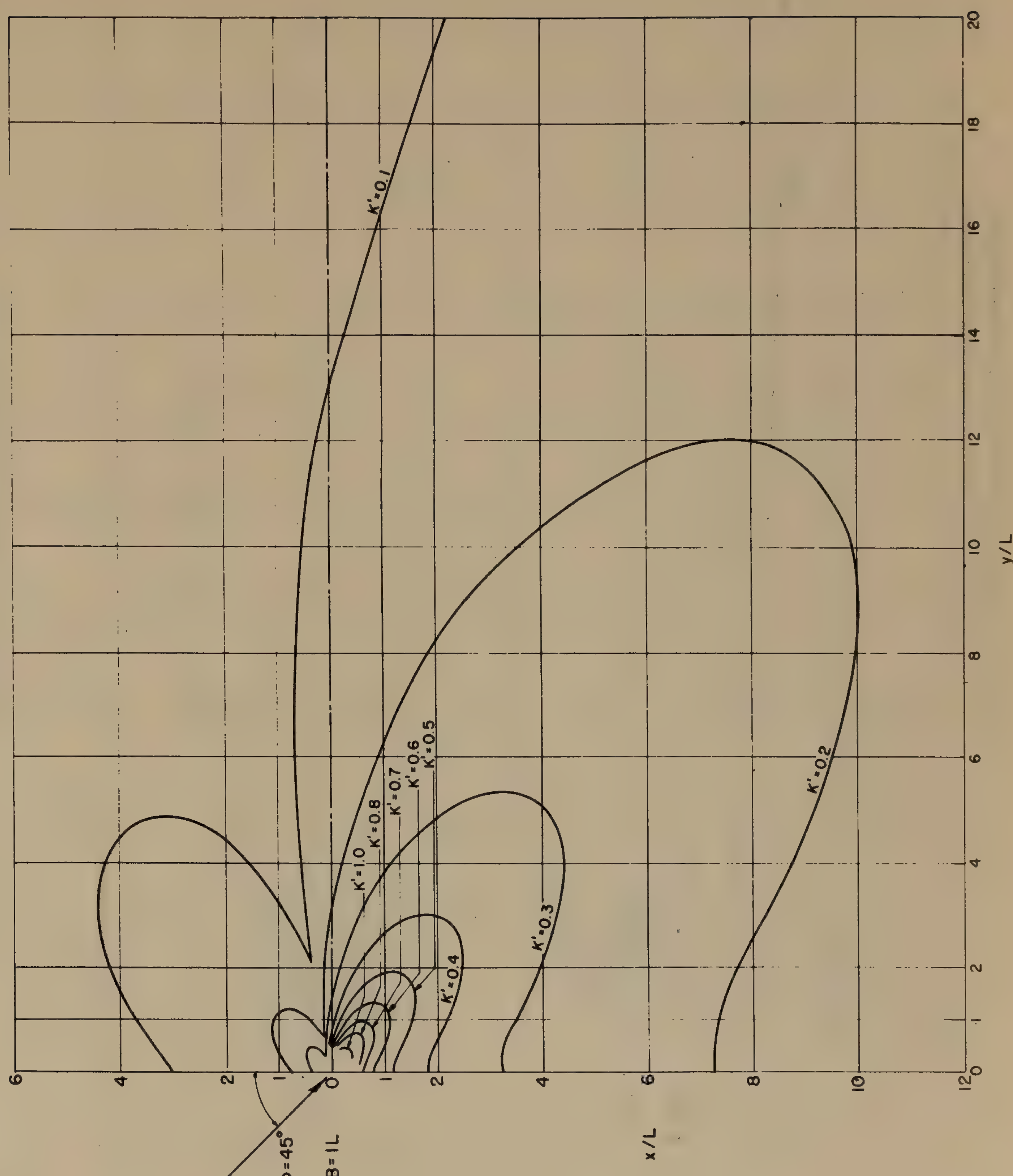
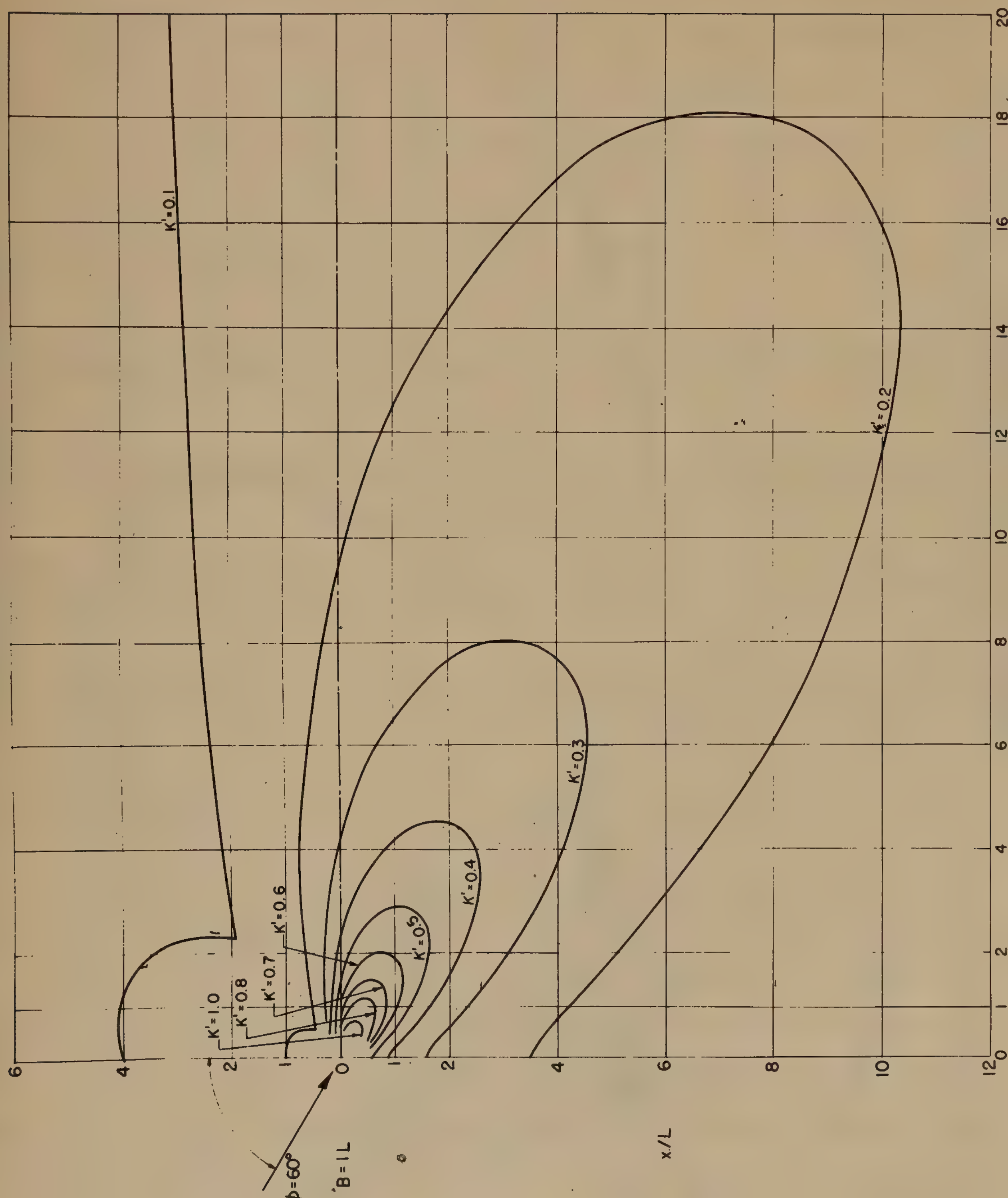


FIG. IE-17 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT



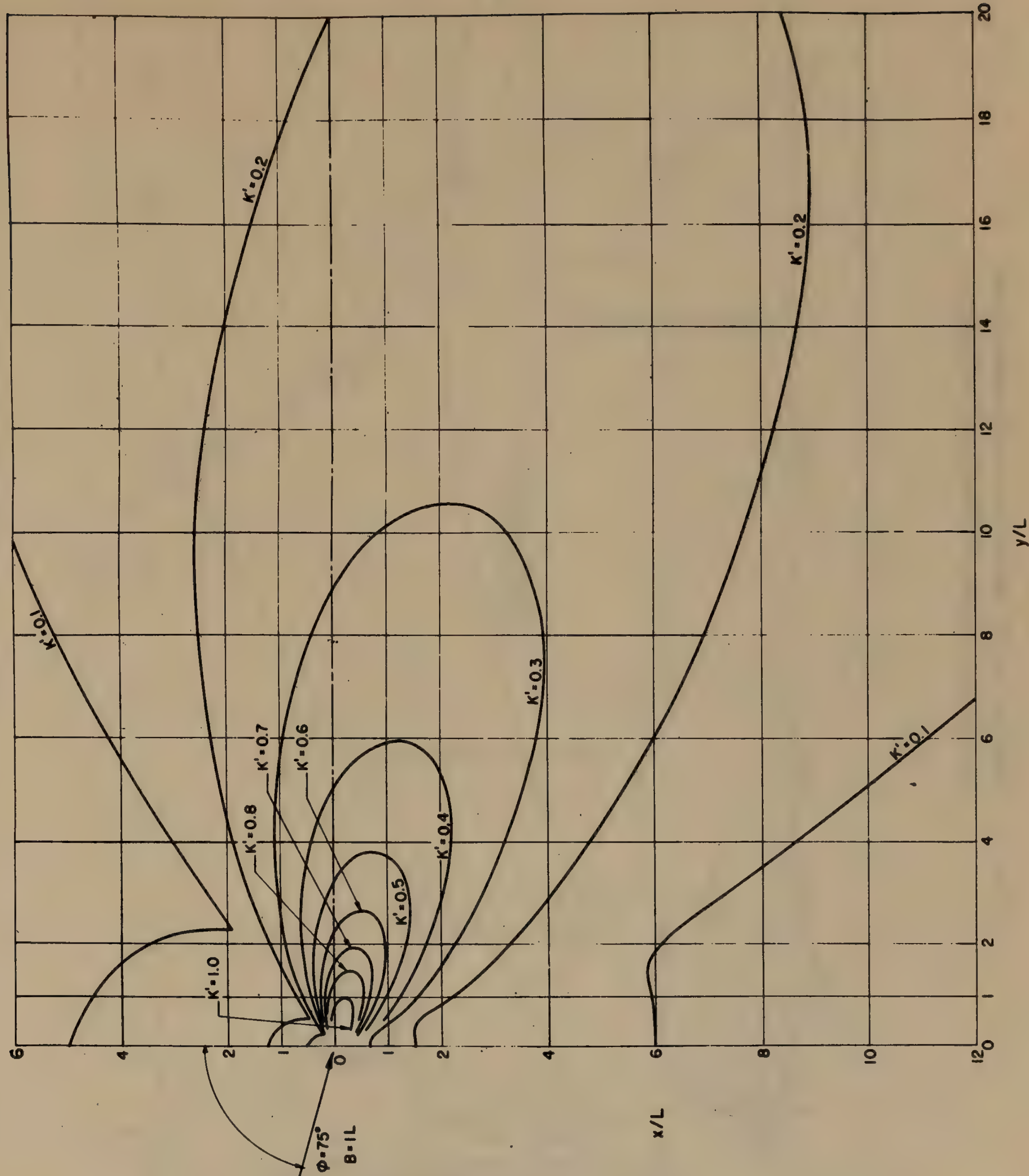


FIG. IE-18 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

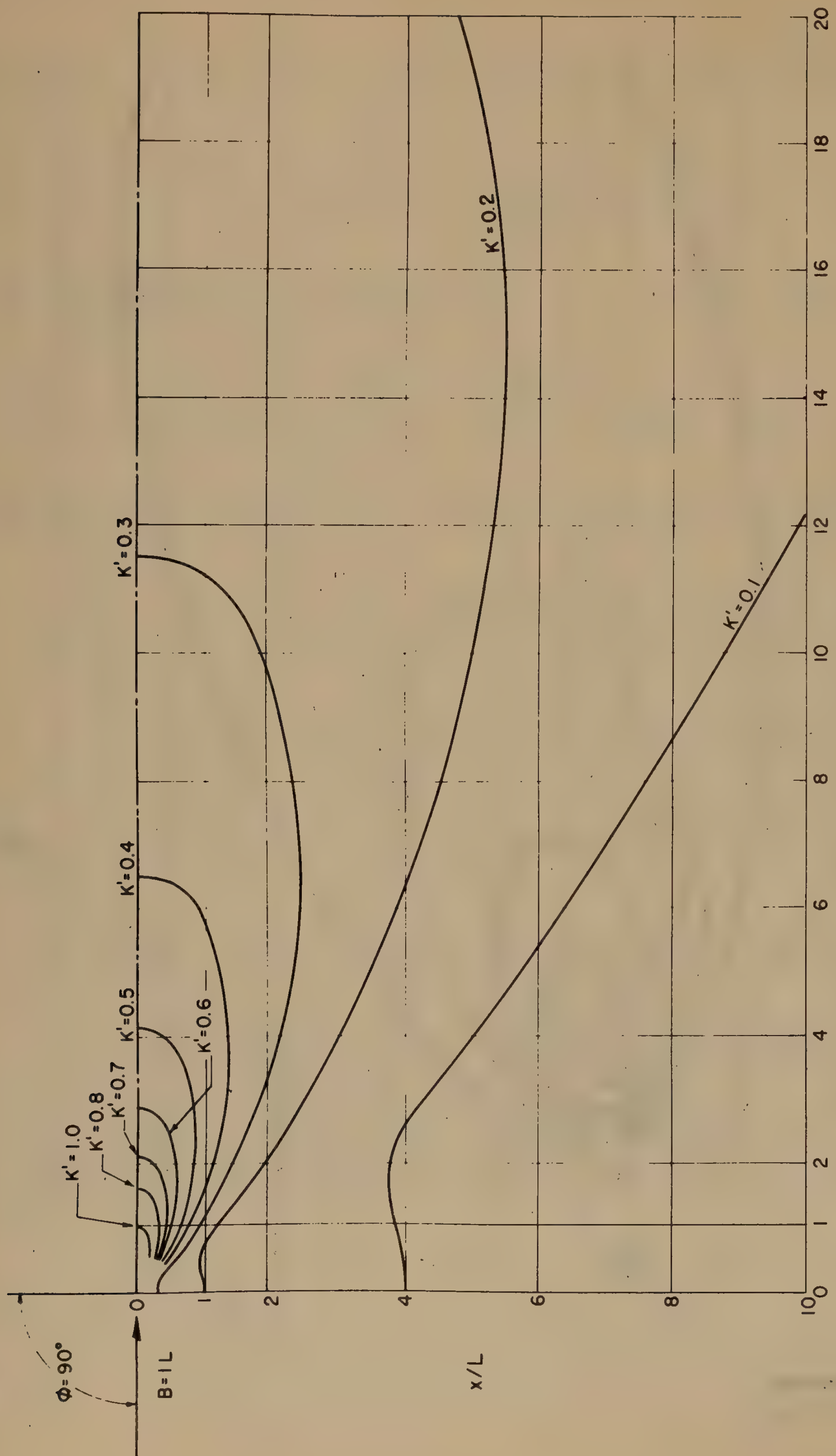


FIG. IE-19 - DIFFRACTION OF WAVES AT A BREAKWATER GAP
CONTOURS OF EQUAL DIFFRACTION COEFFICIENT

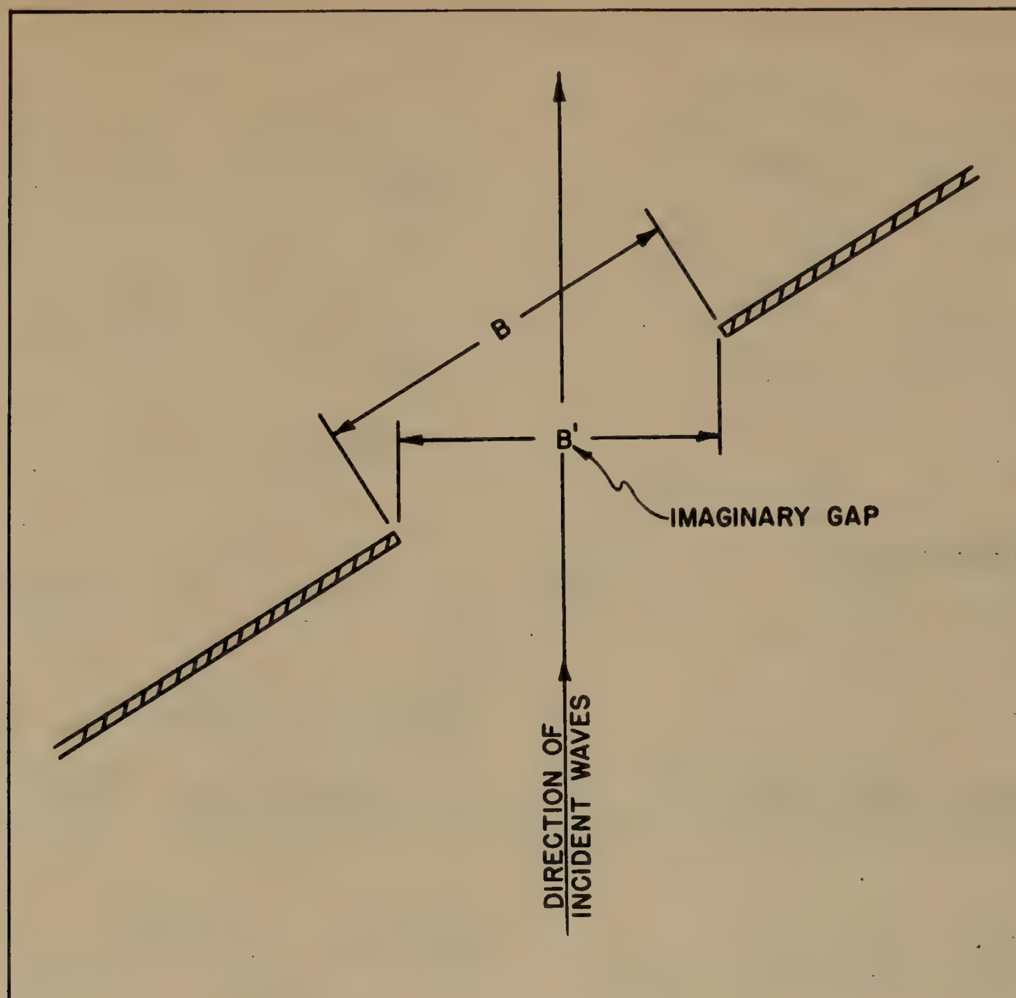


FIGURE IE-20

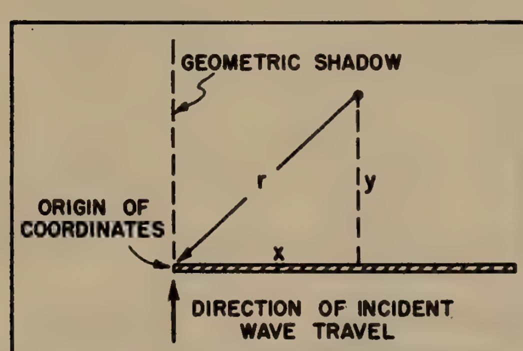


Fig. IE-21- Nomenclature for wave diffraction analysis at breakwater tip

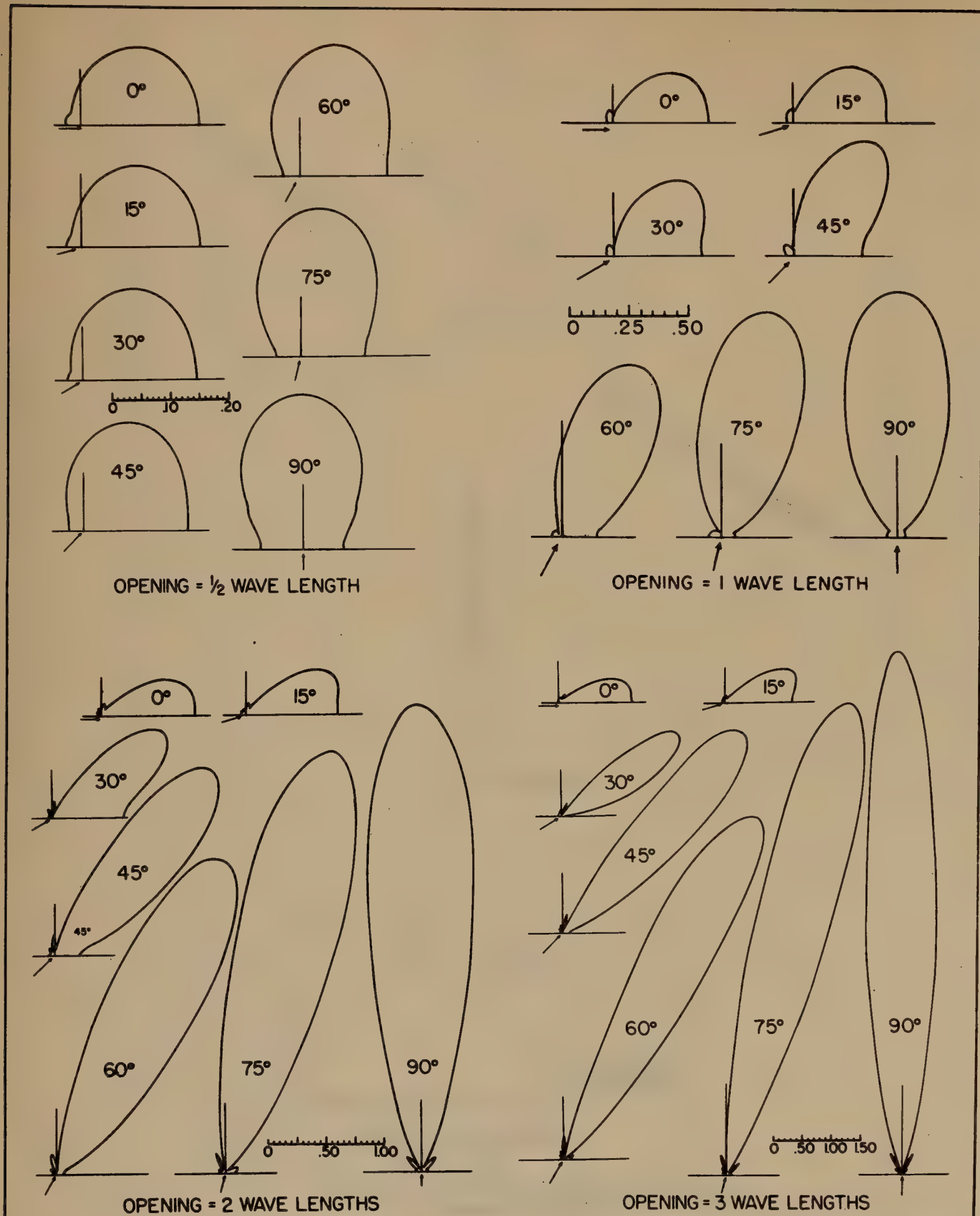


FIG. IE-22 - POLAR PLOTS OF INTENSITY FACTORS
(Vertical face straight breakwaters)

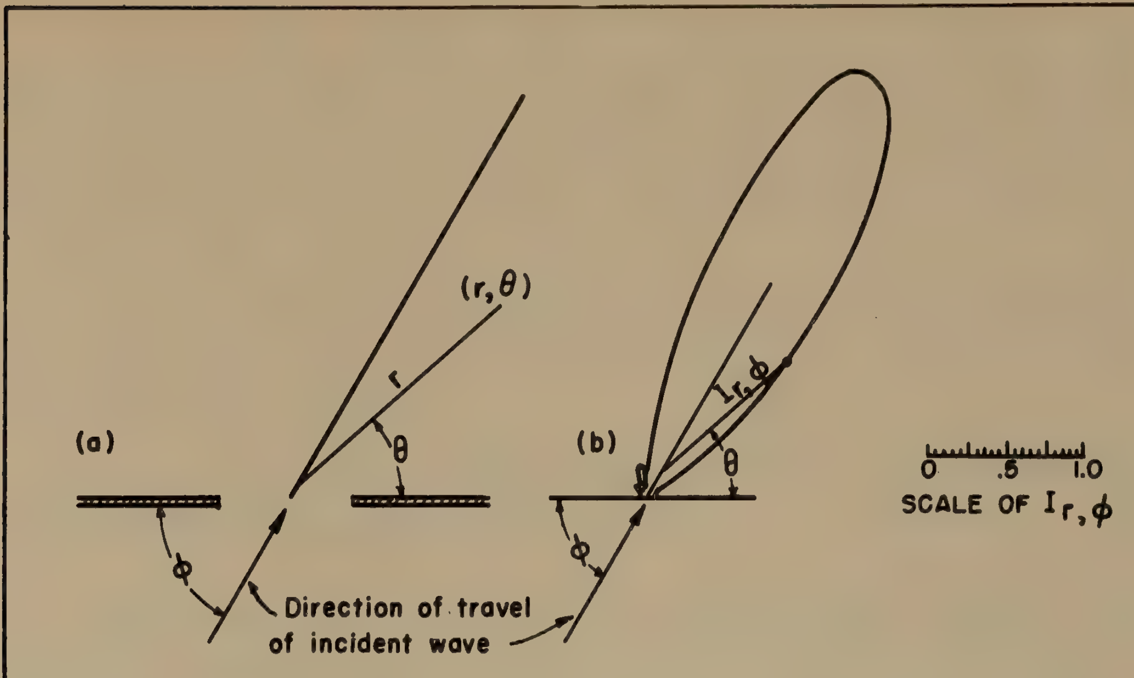


FIG. IE-23 - NOMENCLATURE FOR NARROW BREAKWATER GAPS

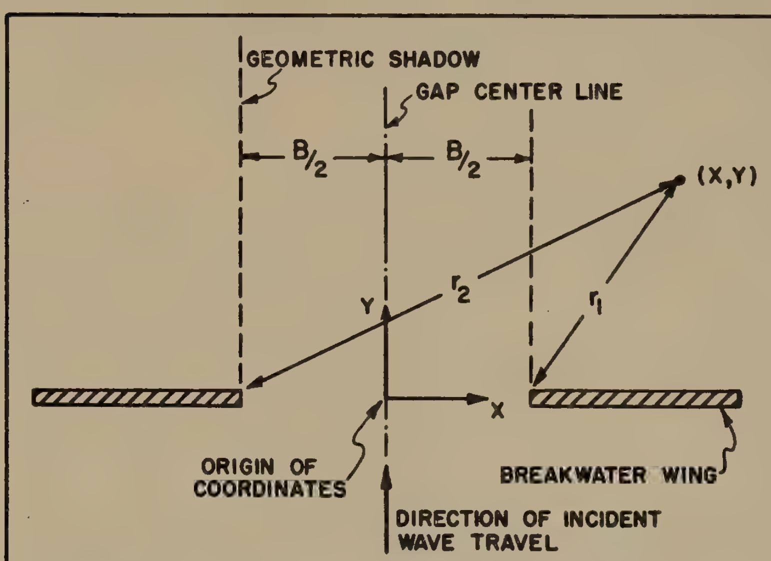


Fig. IE-24 - Nomenclature for breakwater-gap problem

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BREAKERS

BY
H. W. IVERSEN

When a wave nears the shore on a shoaling bottom, the wave becomes unstable at some critical depth, and breaks. The smooth undulation of the sea surface offshore of the breaker zone transforms into a highly turbulent mass of "white" water in the breaking action. The breaker continues to advance to the beach as a "foam" line to dissipate the major portion of the wave energy in the turbulent motion of the advancing foam line.

Breakers are classified into three general categories described by the appearance and action of the onset of instability of the wave. Figure SIF-1 shows a representative picture of each breaker type and the surface profile change as the breaker is formed.

(1) Spilling Breakers: The wave becomes unstable at the crest to form white water at the crest. The white water expands slowly down the front face of the breaker. The breaking action is mild.

(2) Plunging Breakers: The wave crest advances faster than the base of the wave to fall on the front face with a violent action. The resulting white water appears almost instantly over the complete front face. The white water is highly agitated.

(3) Surging Breakers: The wave crest tends to advance faster than the base of the wave to suggest the formation of a plunging breaker. However, the wave base then advances faster than the crest, the plunging is arrested, and the breaker surges up the beach face as a wall of water which may, or may not, be white water.

The beach slope and the incident deep water wave steepness, the ratio of the deep water wave height to the deep water wave length, determine the breaker characteristics. The following table serves to illustrate the relative major differences of breakers as a function of beach slope and of incident wave steepness.

Deep Water Wave Height Feet	Wave Period Seconds	Deep Water Wave length Feet	Beach Slope	Breaker Height Feet	Depth at Breaking Feet	Backwash Depth Feet	Backwash Velocity Ft.per sec.
5	8	328	1:50	5.9	6.7	5.6	2.8
5	12	740	1:50	7.3	9.4	7.3	3.0
5	12	740	1:10	9.1	9.4	4.5	11.7

For the same deep water wave height and on the same beach, waves with the longer periods produce higher breakers that break in deeper water. The effect of beach slope is pronounced. The same waves advancing on a 1:10 beach slope as contracted to a 1:50 beach slope produce higher breakers with a smaller depth of water in the trough shoreward of the breaker. The seaward velocity in the trough is also higher on the steeper slope. A further comparison of slope effect is shown on Figure SIF-2. Breakers are more likely to plunge or surge on the steeper slope as contrasted to breakers on the flatter slopes.

The type of breaker is related to the backwash depth and the backwash velocity. High backwash velocities promote the retardation of the base of the advancing wave to establish the plunging tendency. When the backwash depth becomes too small the retardation is not effective and the advancing wave tends to surge.

Other related characteristics of breakers are detailed in Section I-F. These include crest heights, breaker shapes with front face and back face angles, and velocity fields of typical breakers.

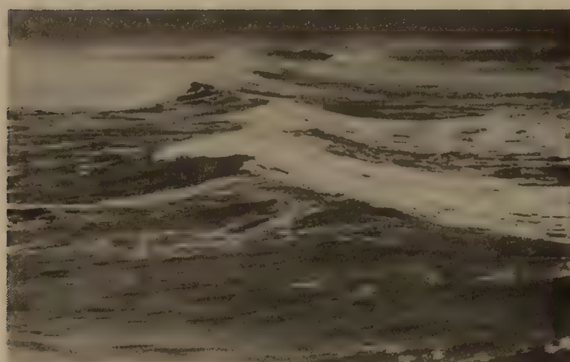
No detailed information is available for breakers on beach slopes which are flatter than 1:50. The effect of slope becomes less important as the slope becomes flatter. The effect of bottom friction becomes increasingly important on waves advancing over flatter depths. The net effects have yet to be reconciled into a working set of criteria.

FIG. SIF-1

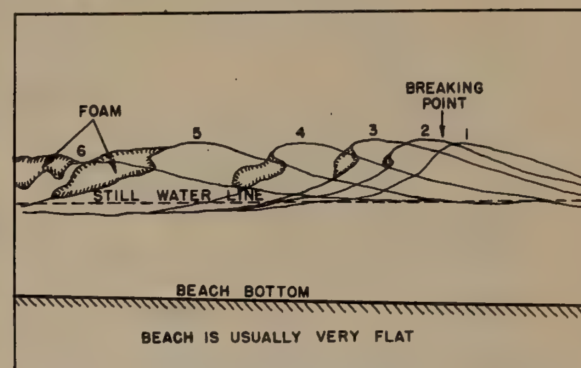
SURF ZONE

(c) Breaker Types

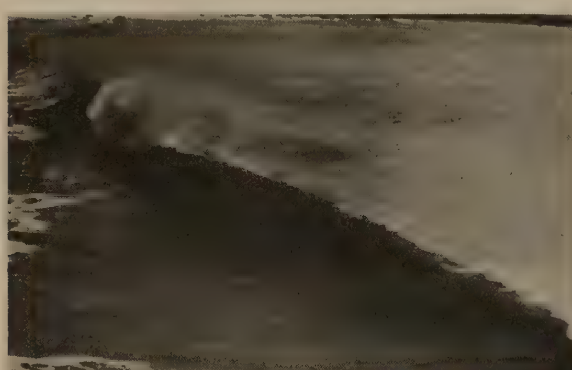
Both photographs and diagrams of the three types of breakers are presented below. The sketches consist of a series of profiles of the wave form as it appears before breaking, during the breaking, and after breaking. The numbers opposite the profile lines indicate the relative times of the occurrences.



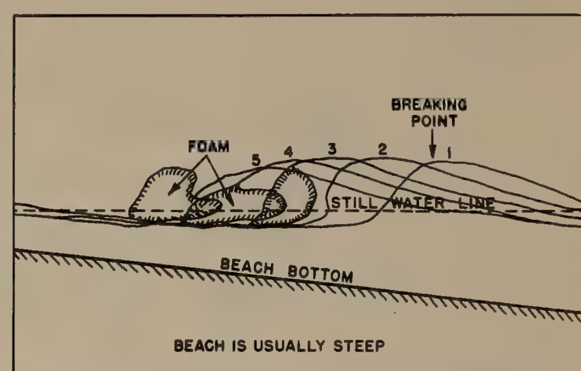
SPILLING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF SPILLING BREAKERS

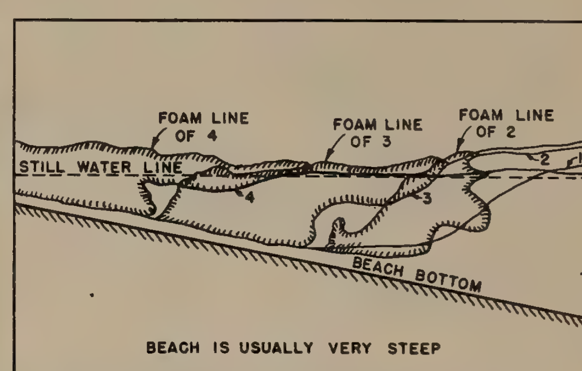


PLUNGING BREAKER

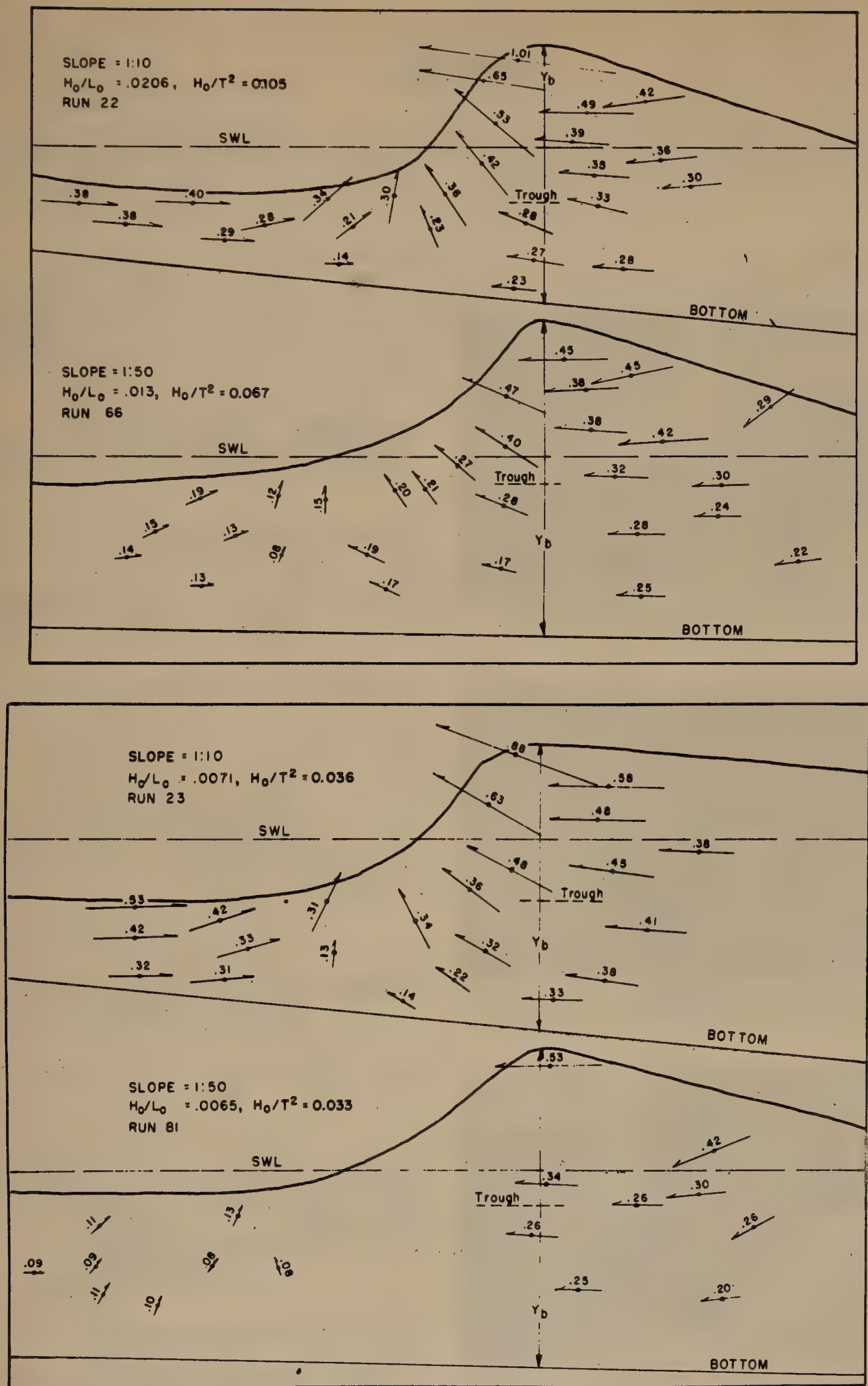


SKETCH SHOWING THE GENERAL CHARACTER OF PLUNGING BREAKERS

SURGING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF SURGING BREAKERS



NOTE: Numbers are = Velocity/$\sqrt{g\, Y_b}$

Equal breaker heights

FIG. SIF-2 - KINEMATICS AT BREAKER POINT

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION I. WAVES, TIDES AND BEACHES

F. BREAKERS AND SURF

by

H. W. Iversen

Table of Contents

F. BREAKERS AND SURF

1. Introduction

The energy of a wave nearing a coastline is in part dissipated in the actions which occur on, or near, the coastline, and in part reflected. Mostly, the dissipation occurs in the breaker and the subsequent movement of the broken wave up to the shoreline. The major portion of the wave energy is absorbed in the breaking process. On steep beaches a noticeable reflection may take place. On flat beaches reflection is not important.

The breaking of a wave is an instability phenomenon in which the crest of the wave travels faster than the main portion of the wave. The smooth surface profile of the wave degenerates at the crest into a mass of unsupported water that becomes highly agitated. Air is trapped in the agitated mass to give the appearance of white water. The wave energy is dissipated as the agitation moves shoreward as a foam line.

The wave velocity in the breaker zone is not as great as that in deeper water. Yet the breaker zone is the most dangerous for craft and other objects. The wave shape changes in the region of breaking from a relatively smooth undulation of the sea surface seaward of the breaker zone to a non-symmetrical profile with a steep front face and a flat back face. The wave appears to tip in the direction of its movement. A floating object cannot move vertically upward rapidly enough to ride up the steep face of the breaking wave. Hence, the wave impacts on objects with considerable force. The resulting motion of the object will depend upon its shape, size, mass, mass distribution, the relative velocity between the object and the breaker, the size of the breaker and the shape of the breaker.

Breaker shapes may be divided into three main classifications; (i) spilling, (ii) plunging, and (iii) surging. See Figure IF-1.

(i) Spilling breakers are characterized by the appearance of "white" water at the crest. As the breaker moves shoreward the "white" water expands down the face of the breaker in a relatively mild fashion.

(ii) Plunging breakers are characterized by the crest overrunning the body of the wave as a sheet of water which projects ahead of the wave face to crash upon the face of the wave. The breaker front is steep and concave. After the impact, the front of the wave becomes a wall of white water.

(iii) Surging breakers appear as steep front faced waves in which the complete front face becomes unstable at the same instant. The action appears to be a race between the breaker crest and the base of the waves. At first the crest moves faster than the base to suggest the formation of a plunging breaker. Then the base surges ahead to remain under the crest. Instead of the crest either spilling or curling over, the entire wall of water surges up the beach face.

2. Breaker Characteristics:

a. Discussion:

The important features of breakers are determined by the incident wave, the wave action previous to a particular wave under consideration, the beach slope, the refraction of the wave, and the bottom topography. Breaker action has not been explained by analytical approaches. Experimental evidence must be relied upon to present the important characteristics of breakers.

The incident wave has been shown to be defined by the wave height and period. The breaker which results from an incident wave, of a series of identical waves, should then be characterized by the incident wave. The beach configuration, upon which the breaker is formed, will introduce a parameter that will influence the resulting breaker. Thus, the breaker height (and other geometric and kinematic features of the breaker) will depend upon the incident wave and bottom slope as the two main variables.

$$H_b/H_o = \varphi \; (H_o/T^2, \; y/x) \qquad \text{(IF-2.1)}$$

The representation of Equation (IF-2.1) is obtained from measured values of the variables. Two approaches are possible, laboratory controlled experiments, and measurements of ocean waves and breakers. Information is available from both sources (References IF-1, 2 and 3). Compilation of all results into single valued functions of the parameters is complicated by the manner in which the results were obtained, and by the limitations of the facilities which were available. Measurements of ocean waves were made from piers which in themselves affect the waves. In addition, the incident deep water wave was of necessity computed from measurements of wave heights seaward of the breaker in depths less than that corresponding to "deep water". As has been noted in the discussion of wave transformations on a sloping bottom (Section I-C of this Manual) the depth at which the wave measurement is made must be greater than a specified minimum in order for the theory to be used successfully to obtain deep water wave heights. The majority of the reported ocean wave measurements do not comply with the requirements.

Laboratory measurements on model waves generated in wave tanks give complete control over the desired range of variables. However, frictional effects due to the wave channel side walls and bottom introduce factors that must be taken into consideration. Confirmed analyses are available to reduce the laboratory results to reliable conclusions.

The breaker is characterized by its shape and kinematics. Since breakers assume a variety of shapes with corresponding kinematics, a simple representation to describe all breakers completely is difficult to make. Certain features of breakers can be obtained to note the essential details of breaker shapes and kinematics with their relation to incident wave and beach slope. The important definable elements of breakers are defined as follows and diagrammed in Figure IF-2.

(i) Breaker height: the vertical distance from the tip of the wave crest at breaking to the trough line. (The trough line is essentially at the same elevation as the trough immediately preceding the breaking crest.)

(ii) Depth at breaking: the vertical distance from the still water line to the beach bottom at the breaker point.

(iii) Crest elevation: the vertical distance from the crest tip to the bottom at the breaker point.

(iv) Backwash depth: the vertical distance from the trough preceding the breaker crest to the beach bottom. The trough position is taken at the intersection of the trough envelope and the surface profile.

(v) Front face angle: the angle between the vertical and a line from the forward part of the crest to the intersection of the front face profile with the still water line.

(vi) Back face angle: the angle between the vertical and a tangent to the back face.

(vii) Forward stagnation position: the location of a region between the breaker crest and the preceding trough at which the water has no horizontal movement.

(viii) Velocity field: the motion of each water particle at the instant a wave breaks. The velocity field includes many different velocities in magnitude and direction. Those which can be specified easily are

(a) Breaker velocity - velocity of the wave crest at breaking

(b) Backwash velocity - the average velocity of the backwash in the trough preceding the breaker

The information which is available on the above-listed breaker characteristics has been obtained from laboratory investigations. The quantitative results are presented to show the nature of breaker formation, the effect of beach slope on breaker formation, and the orders of magnitude of the characteristics as a function of beach slope and incident wave. The available reliable results are meager and do not cover all conditions of the important variables that occur on natural beaches. The results which are presented are for limited ranges of the variables of incident deep water waves and beach slope. Limitations of the laboratory wave channel size limited the scope of the variables. Extrapolations are made beyond the range of the measured results to cover expected natural ocean conditions. However, the results are usable for forecasting breaker conditions from known deep water waves and beach configurations, within the accuracy of establishing the deep water wave conditions.

The various features of breakers are shown in a series of dimensionless relationships in Figures IF-3 to 10. The forecast or measured deep water wave variables are the height and period. The deep water wave is characterized

by the deep water wave steepness, H_o/L_o. The length is related to the period by $L_o = (g/2\pi)T^2$. Since g is constant, the deep water wave steepness is also characterized in terms of the prime variables by H_o/T^2. The measured relationships of Figures IF-3 to 10 are shown as functions of H_o/T^2 for ease in use.

The geometric relationships of breaker crest height, backwash depth, depth at breaking, and forward stagnation position are formed as geometric ratios in terms of both the breaker height, and also the deep water wave height. Measured values are shown only with the ratio with the breaker height. This technique was followed due to the scatter of results in relating the breaker height to the deep water height, Figure IF-3. The best curve has been located for each beach slope on Figure IF-3. Data for the other geometric features, when related to the breaker height, resulted in reasonable trends with a minimum of scatter. The curves of Figure IF-3 were then used with the geometric relationships related to breaker height to eliminate the breaker height and to form the relationship with the deep water wave height. For example,

$$(d_b/H_b)\ (H_b/H_o) = d_b/H_o$$

Results are presented for beaches with slopes of 1:10 to 1:50. The results of the 1:50 beach slope should be used for beaches which are flatter than 1:50 until reliable results become available for the flatter beaches.

An example of two waves of the same deep water height, but of different period is carried through the following discussion to emphasize comparative effects of initial wave steepness and of beach slope on the geometry and kinematics of the resulting breakers. The beach slope is assumed to be plane with contours that are parallel to the waves. The deep water waves for the example have a height of 5 feet and periods of 8 and 12 seconds. Corresponding initial, or deep water wave indeces, are $H_o/T^2 = 0.078$, and 0.035, respectively.

b. Breaker Height, Figure IF-3:

Deep Water Wave Height	Wave Period	Breaker Height, H_b - feet		
H_o - Feet	T - Seconds	1:50 Slope	1:10 Slope	from H.O. 234
5.0	8	5.9	7.8	6.3
5.0	12	7.3	9.1	7.8

The same deep water wave produces higher breakers on the steeper beaches. The example includes breaker heights from the previously accepted forecasting curve (Reference IF-3). The results as now available indicate that the previously used curve, is approximately valid for beach slopes of 1:50. For steeper beaches, the beach slope effect becomes important and must be considered.

c. Depth at Breaking, Figure IF-4:

Deep Water Wave Height H_o - Feet	Wave Period T - seconds	Depth at Breaking, d_B - Feet Slope 1:50	Slope 1:10
5.0	8	6.7	6.7
5.0	12	9.4	9.4

For the same deep water wave height, the breaker occurs at greater depths as the period increases. The depth at breaking is independent of the beach slope. The distance from the shoreline to the breaker is found from the beach contours at breaker depths as found.

d. Crest Height, Figure IF-5:

Deep Water Wave Height H_o - Feet	Wave Period T - seconds	Crest Height, Y_B - Feet Slope 1:50	Slope 1:10
5.0	8	11.5	13.0
5.0	12	15.3	15.7

The elevation of the crest of the breaker above the beach bottom increases as the wave period increases. Beach slope has a small effect which is not consistent in trend but is a function of the wave period.

e. Backwash Depth, Figure IF-6:

Deep Water Wave Height H_o - Feet	Wave Period T - seconds	Backwash Depth, D_{BW} - Feet Slope 1:50	Slope 1:10
5.0	8	5.6	3.5
5.0	12	7.3	4.5

The backwash depth increases with wave period. The effect of beach slope is pronounced. The backwash depth on the steeper beach is considerably less than on the flat beach. The major difference in breaker height results from this backwash depth effect. The breaker crest on flat and steep beaches is approximately the same for the same incident wave. The smaller backwash depth, which is approximately the breaker trough elevation at the point of breaking, is less on the steep beach. Thus, the wave height is greater on the steep beach.

f. Forward Stagnation Position, Figure IF-7:

Deep Water Wave Height H_o - Feet	Wave Period T - seconds	Forward Stagnation Position, X_s - Feet Slope 1:50	Slope 1:10
5	8	10.5	9.2
5	12	13.5	11.0

The forward stagnation position is an indication of the breaker front face steepness. The small distances from the breaker crest to the forward stagnation position on the 1:10 slope as compared to those on the 1:50 slope indicate steeper front faces of the breaker with a consequent greater tendency to plunge on the 1:10 slope.

g. Breaker Face Angles, Figure IF-8:

Deep Water Wave Height H_o - Feet	Wave Period T - seconds	Front Face Angle, α Slope 1:50	Front Face Angle, α Slope 1:10	Back Face Angle, β Slope 1:50	Back Face Angle, β Slope 1:10
5	8	60	42	74	80
5	12	60	42	75	83

The breaker face angles are indications only of the breaker steepness at the time of breaking. The angles show the front face to be steeper than the back face to support the conclusions of the crest moving faster than the base of the wave. The breakers on the 1:10 slope are steeper on the front face than those on the 1:50 slope. The action of the relative movement of the crest and the base of the wave is more pronounced on the steep slope. Thus, breakers on the steep slope show a greater tendency to plunge than do those on the flat slope.

h. Breaker Velocities, Figure IF-9:

Deep Water Wave Height H_o - Feet	Wave Period T - seconds	Breaker Crest Velocity V_c - Ft. per second Slope 1:50	Breaker Crest Velocity V_c - Ft. per second Slope 1:10	Backwash Velocity V_{BW} - Ft. per second Slope 1:50	Backwash Velocity V_{BW} - Ft. per second Slope 1:10
5	8	17.6	18.0	2.8	8.3
5	12	12.6	21.0	3.0	11.7

The most striking effect of beach slope is the rapid increase in backwash velocity as the slope becomes flatter. This contributes to the smaller backwash depth and the net effect of an increase in the retardation of the base of the breaker to promote plunging tendencies.

i. Breaker Type, Figure IF-10:

Figure IF-10 is a block diagram on which the breaker type has been classified as a function of deep water wave and beach slope. Spilling breakers occur on all beaches at the larger wave indeces, or wave steepness. The range of spilling breakers is greater for the shallower slopes. At low values of the initial wave steepness, the breakers are plunging or surging. The line of demarcation between the catalogued types of breakers is not definite. In the example carried through the previous sections, the 8 second wave on the 1:50 slope is a plunging breaker, but not a violent plunging breaker since the index is near the demarcation between the plunging and spilling types. The 12 second wave on the 1:50 slope is definitely plunging. The 8 second wave on the 1:10 slope is definitely plunging. The 12 second wave on the 1:10 slope is surging with some plunging tendencies.

j. Breaker Velocity Fields, Figure IF-11:

The complete velocity field in the water of a breaker cannot be generalized due to the multitude of variables upon which it depends. Figure IF-11 includes velocity fields for two initial wave conditions and for beach slopes of 1:50 and 1:10. Low velocities, parallel to the bottom except in the forward stagnation region, occur near the bottom. Maximum velocities occur in the front face of the breaker near the crest.

The various comparative features of the geometry and kinematics of the breakers of approximately the characteristic of these which have been discussed in the example are shown in Figure IF-11.

Cross-sections through breakers for given initial wave conditions, and given slopes may be prepared from the various geometrical relationships of Figures IF-3 through 11. The shape will be approximate since the curvatures of the front face and back face are not amenable to simple descriptions. Figure IF-11 may be used as a pattern to produce faired surface profiles of the approximate natural character. The face angles may be taken from the crest tip to the still water line. The backwash depth location is approximately two breaker heights shoreward of the crest. The resulting breaker shape is not as wide across the crest as an actual breaker due to the face angles being taken from the crest position.

The geometric and kinematic features of the breakers as described were obtained from laboratory studies of regular periodic series of generated waves each of identical character. In all cases the beaches were plane slopes with incident waves parallel to the beach contours. Wave conditions, and breaker conditions in the ocean and on natural beaches are not regular. The assumption is made that an individual wave described by a height and period exhibits the geometrical transformation as described regardless of the preceding and proceeding wave history. This assumption is open to question in view of the marked effect of the backwash on the wave character. The backwash character will depend upon the history of waves preceding an individual wave. For example, a series of small waves preceding a large wave will produce smaller backwash velocities than those expected from a regular series of the higher wave. Thus, the higher incident wave will tend to break closer to the beach with a smaller breaker height and tend to be more of a spilling breaker than a corresponding wave of a series of identical waves. The converse will be noted for a low incident wave preceded by a group of higher waves.

3. Bars:

All results of Figures IF-3 through 11 were obtained from waves on plane beaches. The slope was constant in the length over which the symmetrical wave transformed into the breaker. Slopes of natural beaches are variable. Some beaches approximate a constant slope, others show bars with depths that increase shoreward of the bar.

Breakers occur at the bar if the depths of water over the bar are equal to or less than the values of Figure IF-4. At water depths over the bar approximately those that result from Figure IF-4, the breaker occurs on, or a distance of four to five times the bar depth, inshore of, the bar. The effects are dependent upon the bar profile and have not been generalized.

After passing the bar, a breaker which occurs at the bar may reform into two or more smaller unbroken waves and proceed shoreward with each new wave breaking on the beach. The complete character of this transformation has not been investigated. Waves which do not break on the bar proceed shoreward to break in the normal manner on the shoaling beach inshore of the bar.

4. Bottom Frictional Effects:

The motion of the water adjacent to the beach bottom produces a dissipation of the wave energy by friction. The passage of the wave and resulting change in pressure on the bottom also tends to set up circulations in the permeable beds of the beach bottom. This results in an additional dissipation of energy. The combined effects tend to reduce the wave height. An analysis of the magnitudes of the reductions in height due to the energy dissipations has been made (References IF-4 and 5). The amount of the reduction depends upon the bottom roughness and upon the permeability of the bottom. For average conditions on sand beaches, the reduction in height is not significant on beaches with slopes of 1:10. For beaches with slopes of 1:300 the reduction of breaker height of approximately 20% is shown for waves of the size carried through in the previous example.

No generalization is possible on the magnitude of the frictional effects due to the varied beach bottom roughnesses and permeabilities that occur on natural beaches. The reported analyses have not been checked by measurement. Since the frictional effects reduce the breaker height, the use of Figure IF-3 will result in breaker heights that will tend to be high. The error is on the safe side in forecasting for expected breaker conditions.

5. References:

IF-1. Hydrographic Office, U.S. Navy - "Breakers and Surf; Principles in Forecasting" - H.O. No. 234, November, 1944.

IF-2. Iversen, H. W., Larocco, M.J. and Crooke, R.C. - "Laboratory Investigations of Breakers; Section B, Breaker Heights and Depths at Breaking; and Section C, Breaker Geometries and Kinematics" - Series 30, Issue 3, Institute of Engineering Research, University of California, Berkeley, California, 1950.

IF-3. Iversen, H. W., Crooke, R.C., Larocco, M.J., and Wiegel, R.L. - "Beach Slope Effect of Breakers and Surf Forecasting" - Series 29, Issue 38, Institute of Engineering Research, University of California, Berkeley, California, 1950.

IF-4. Putnam, J.A. - "Loss of Wave Energy Due to Percolation in Permeable Sea Bottom" - Transactions, American Geophysical Union, Vol. 30, No. 3, June, 1949.

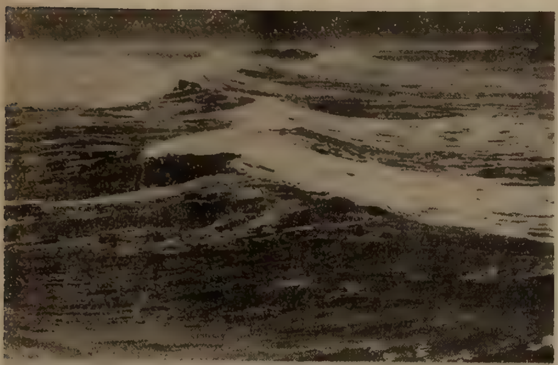
IF-5. Putnam, J. A. and Johnson, J.W. - "The dissipation of Wave Energy by Bottom Friction" - Transactions, American Geophysical Union, Vol. 30, No. 1, February 1949, pp 67-74.

FIG. IF-1

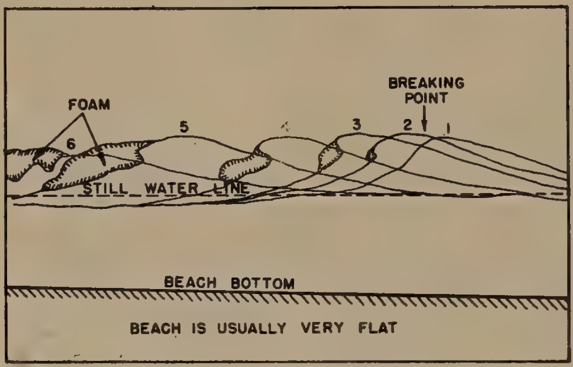
SURF ZONE

(c) Breaker Types

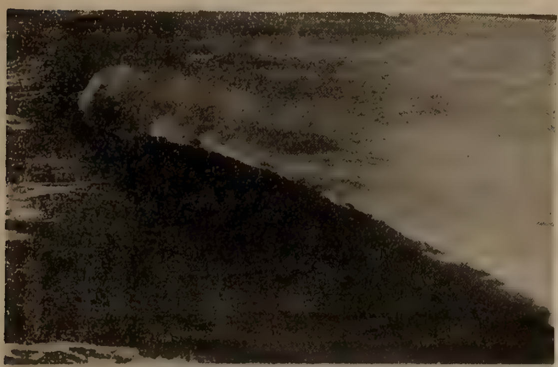
Both photographs and diagrams of the three types of breakers are presented below. The sketches consist of a series of profiles of the wave form as it appears before breaking, during the breaking, and after breaking. The numbers opposite the profile lines indicate the relative times of the occurrences.



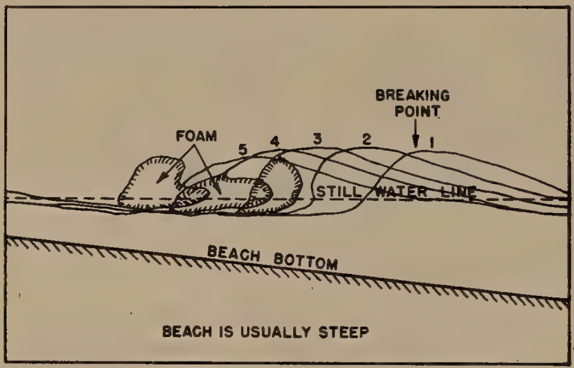
SPILLING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF SPILLING BREAKERS

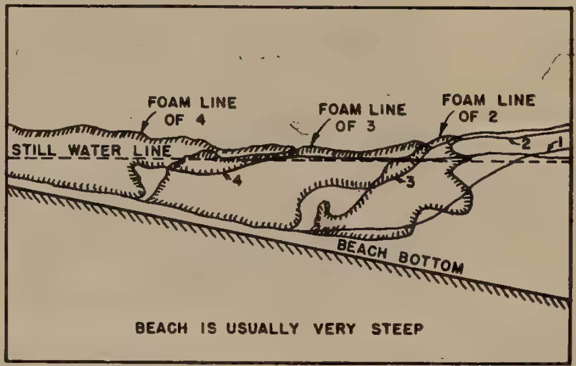


PLUNGING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF PLUNGING BREAKERS

SURGING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF SURGING BREAKERS

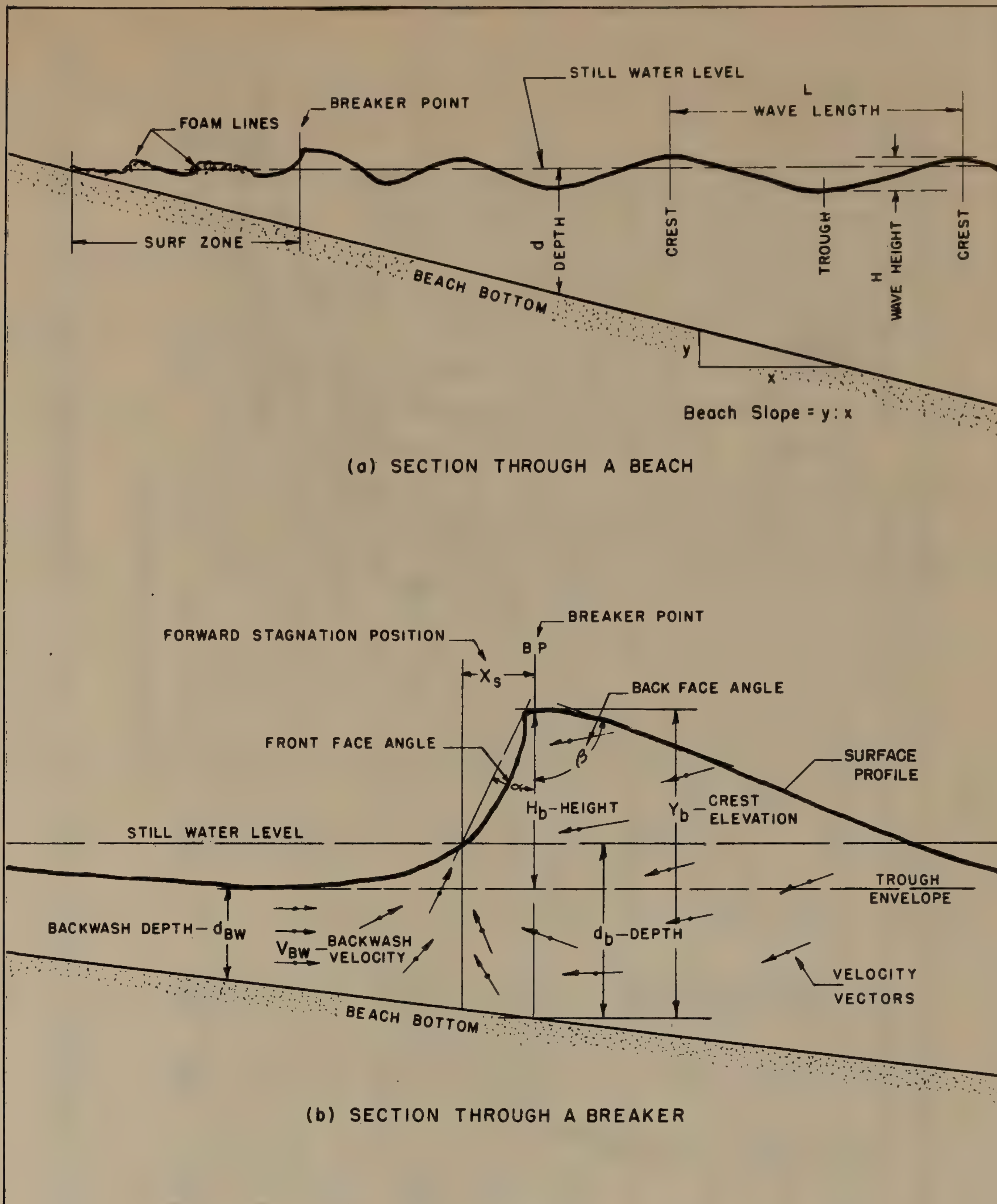


FIG. IF-2 - WAVE AND BREAKER TERMINOLOGY

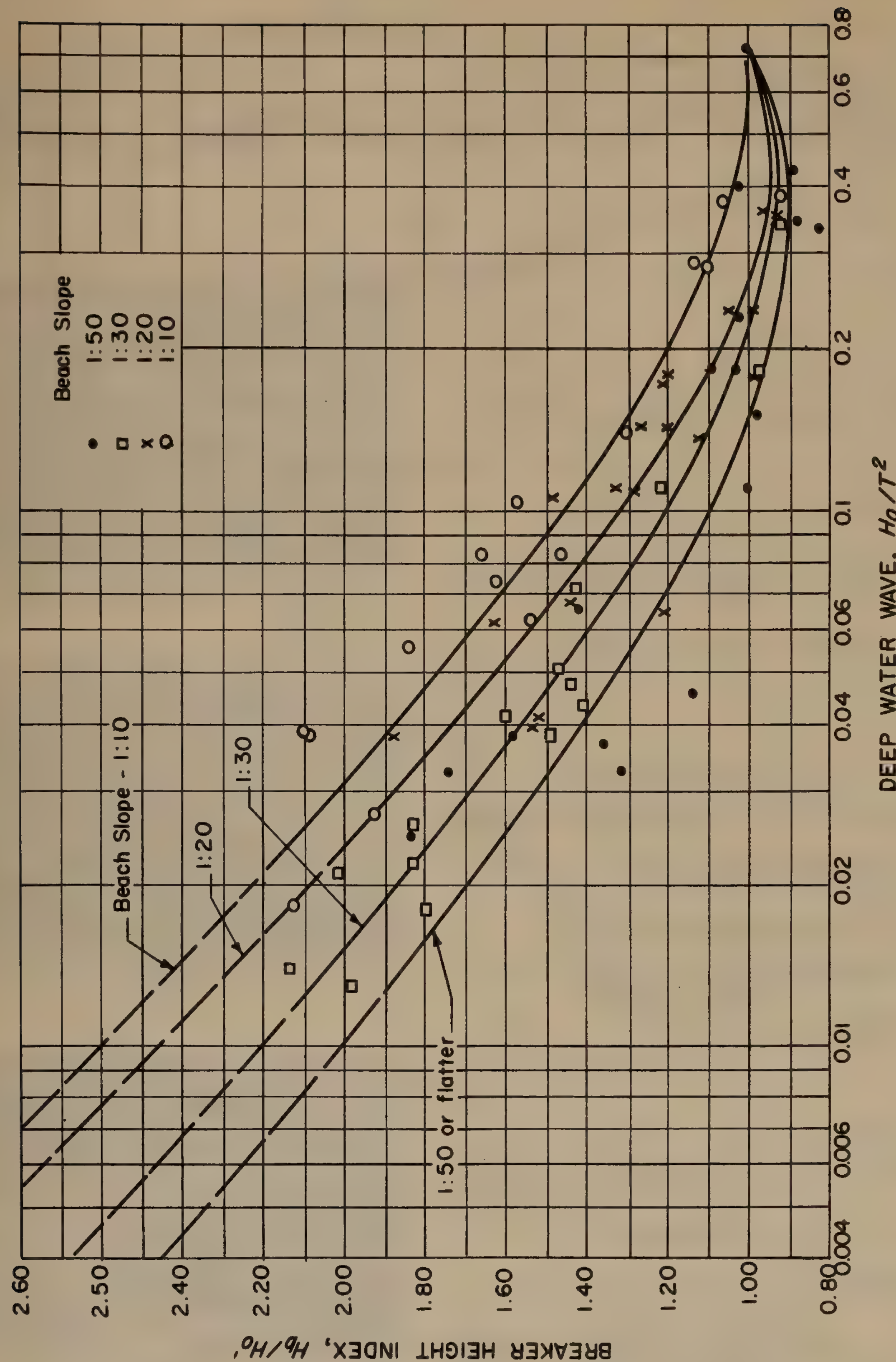


FIG. IF-3 - BREAKER HEIGHT INDEX AS A FUNCTION OF DEEP WATER WAVE AND BEACH SLOPE

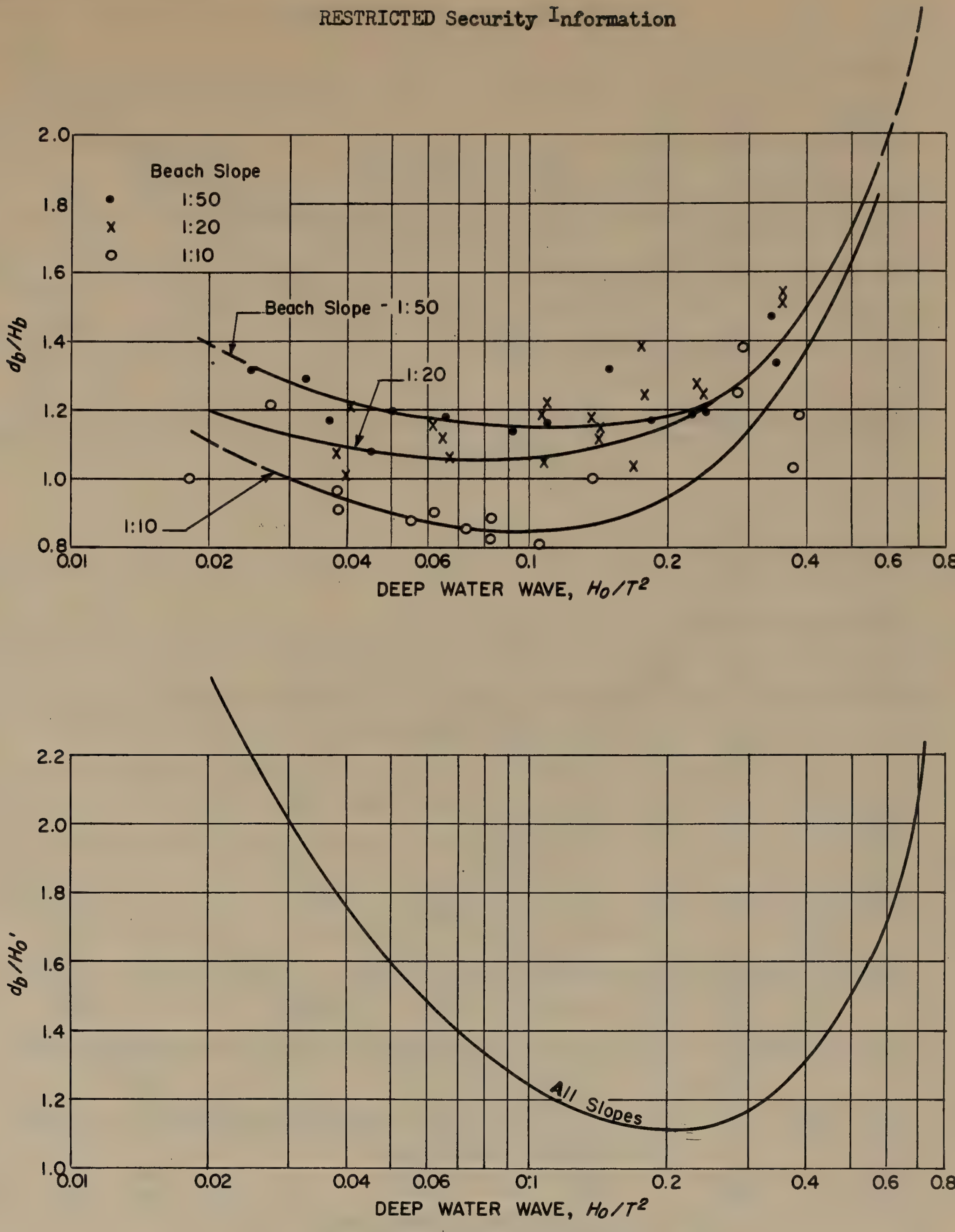


FIG. IF-4 - BREAKER DEPTH INDEX

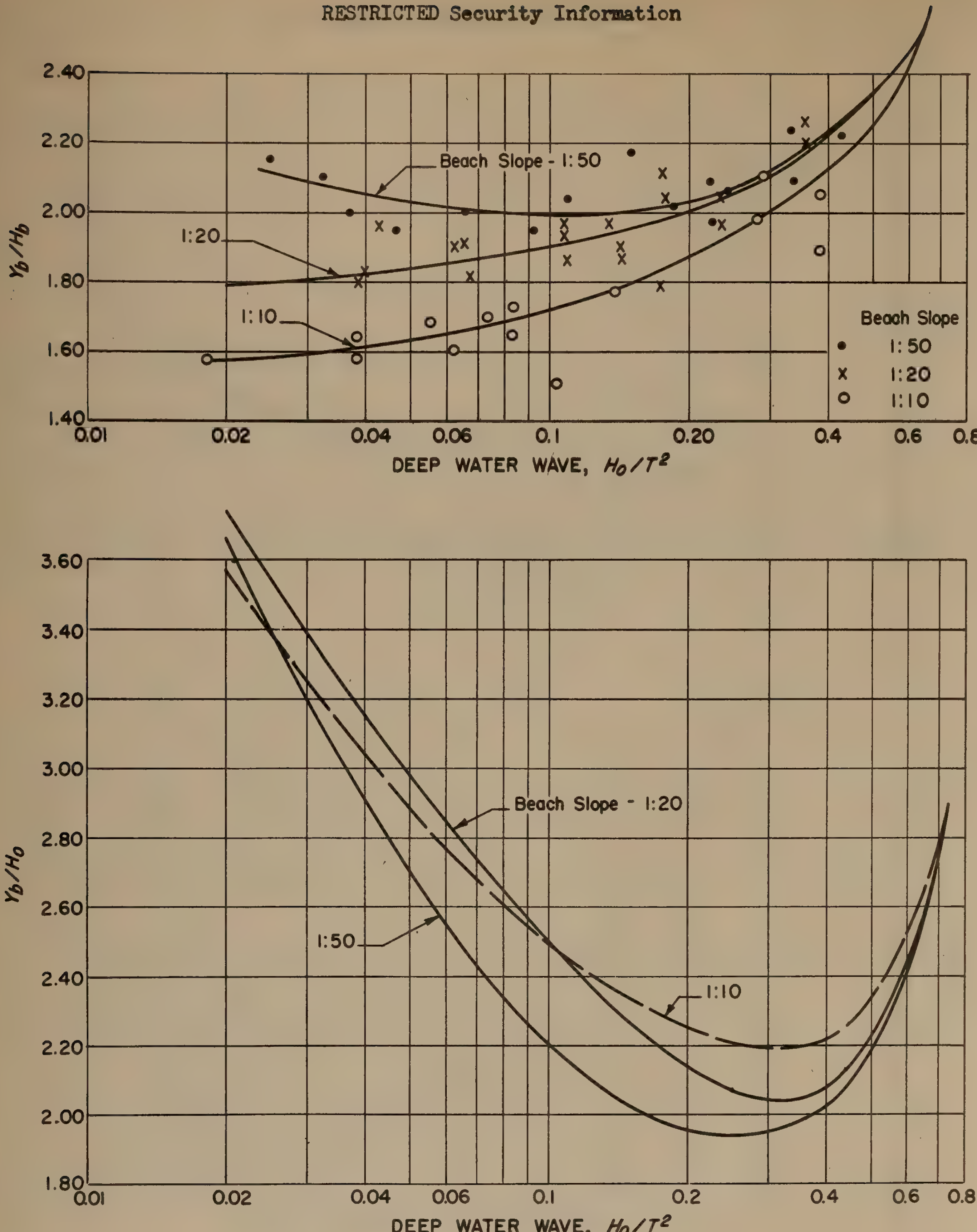
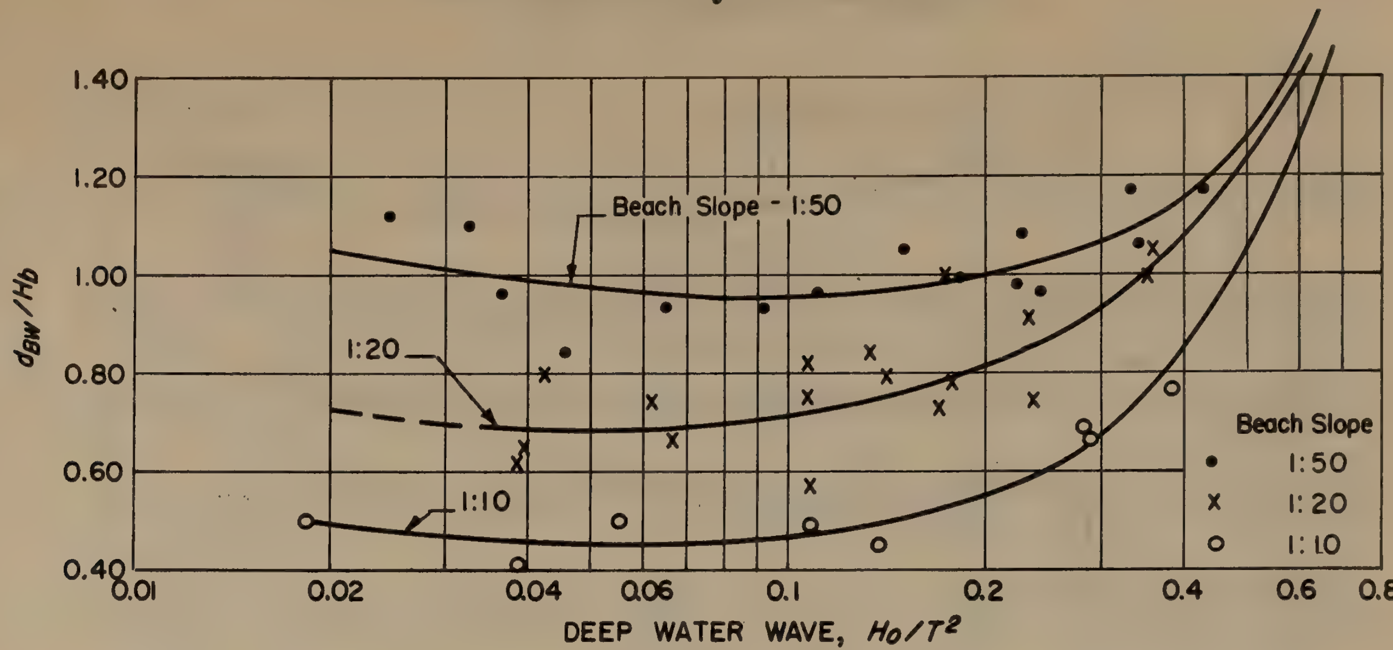


FIG. IF-5 - BREAKER CREST INDEX



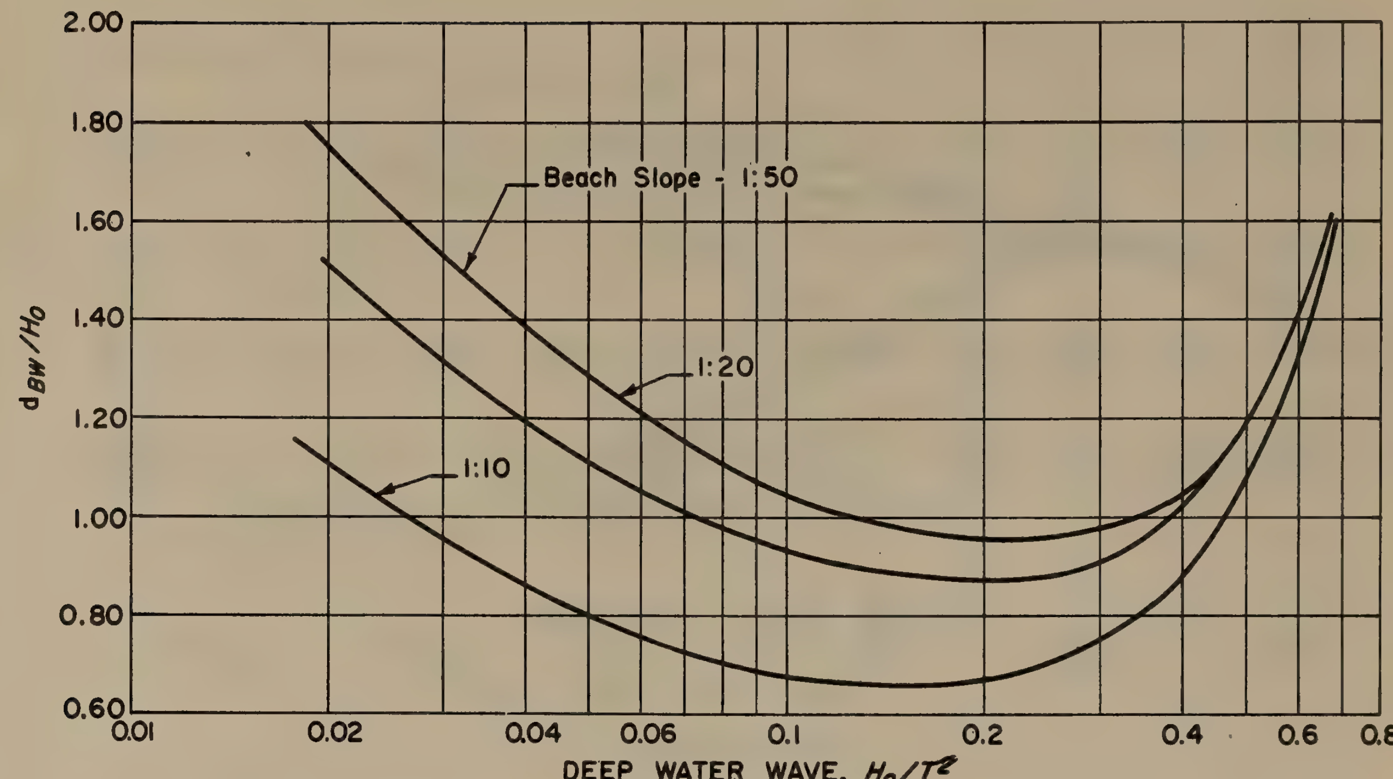


FIG. IF-6 - BACKWASH DEPTH INDEX

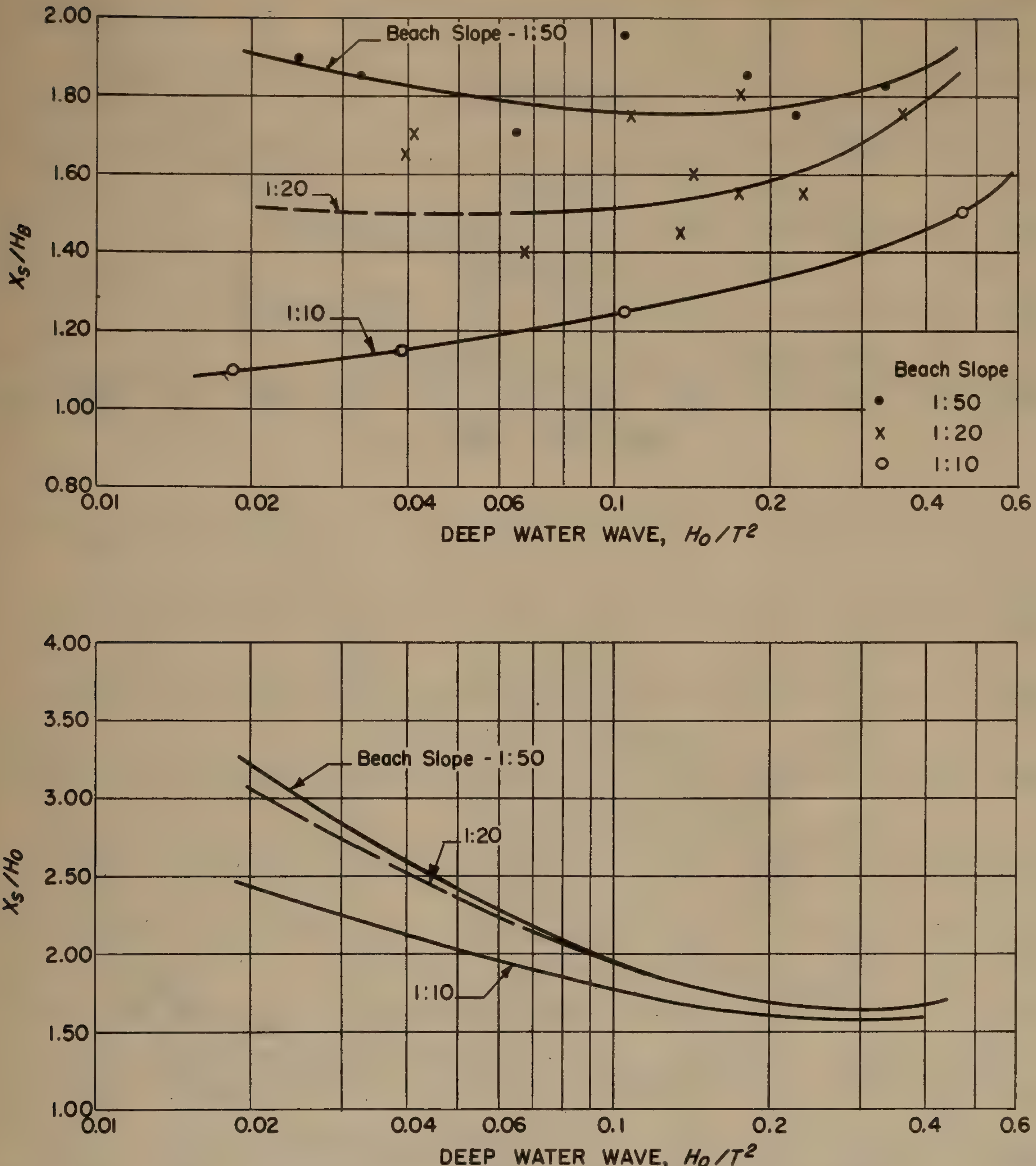
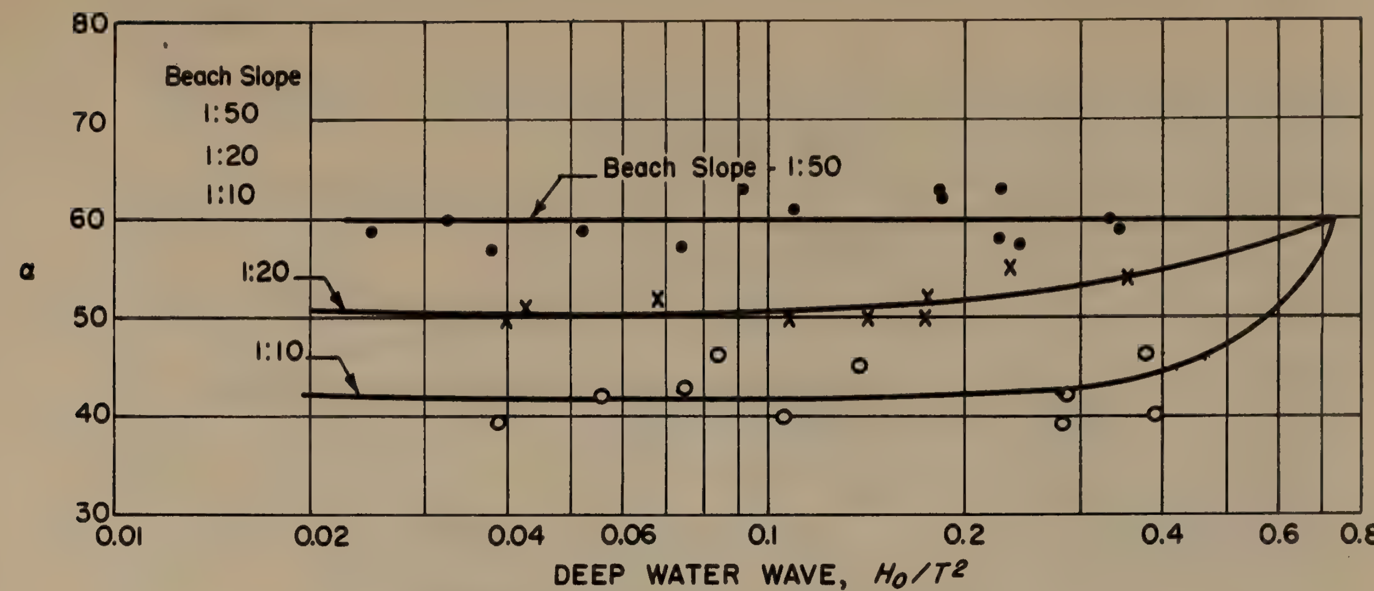
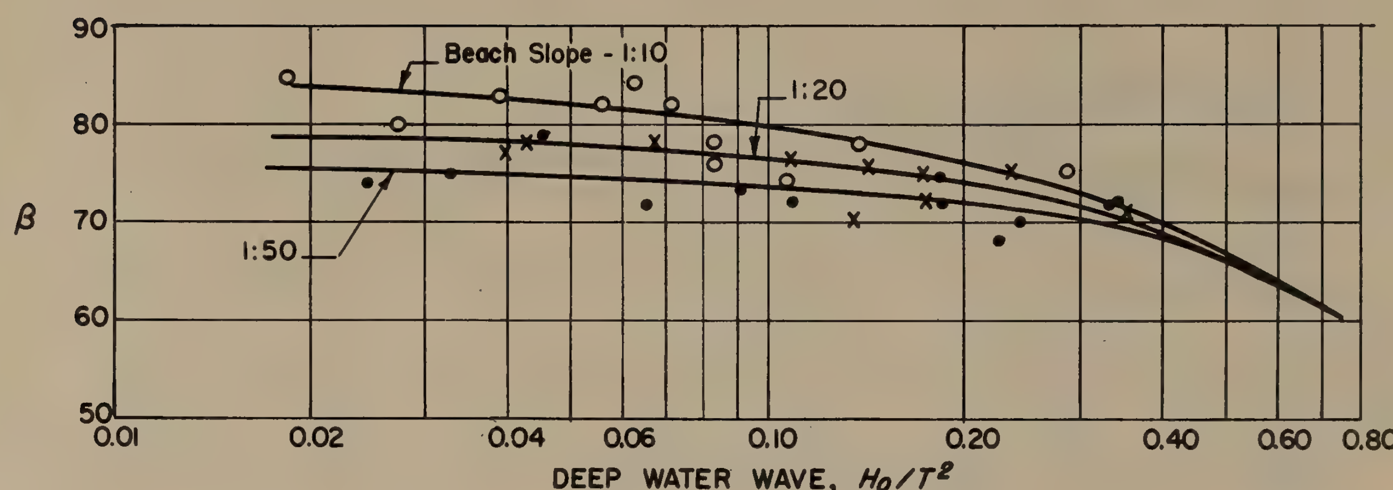


FIG. IF-7 - FORWARD STAGNATION POSITION INDEX

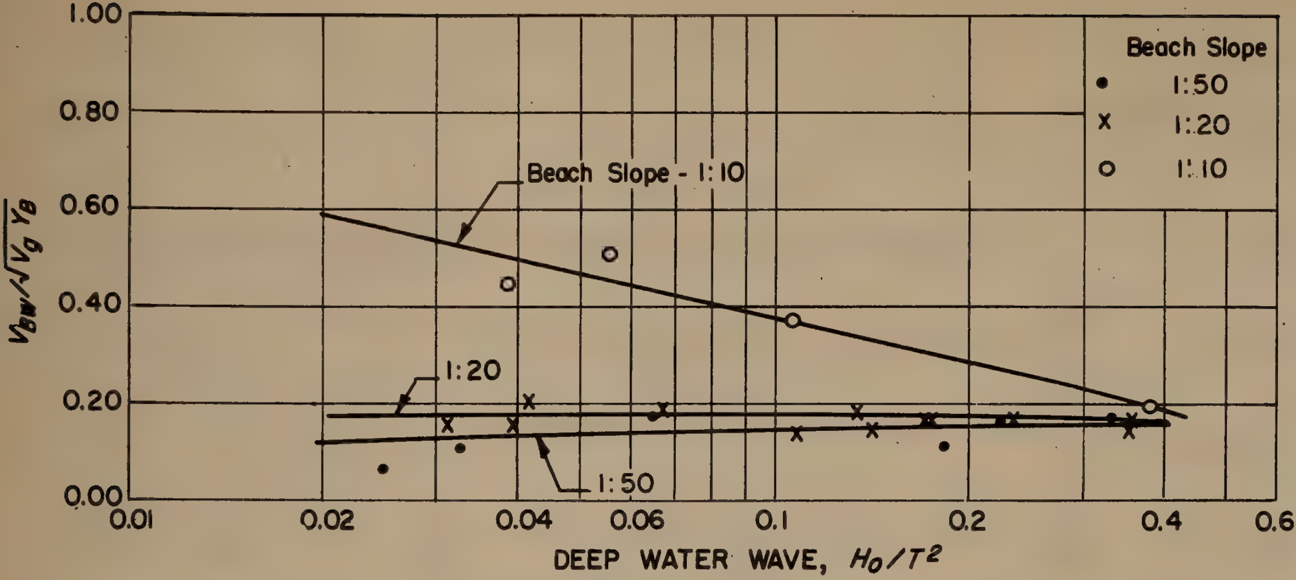


FRONT FACE ANGLE INDEX

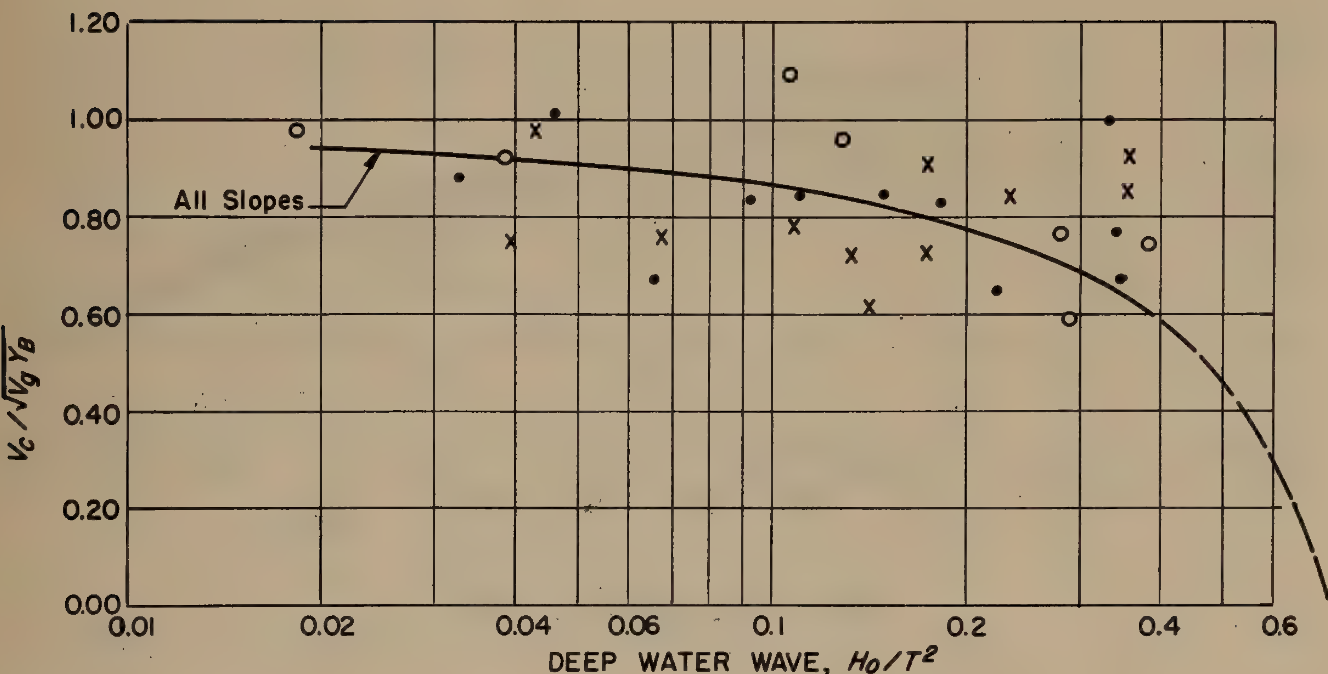


BACK FACE ANGLE INDEX

FIG. IF-8



BACKWASH VELOCITY INDEX



CREST VELOCITY INDEX

FIG. IF-9

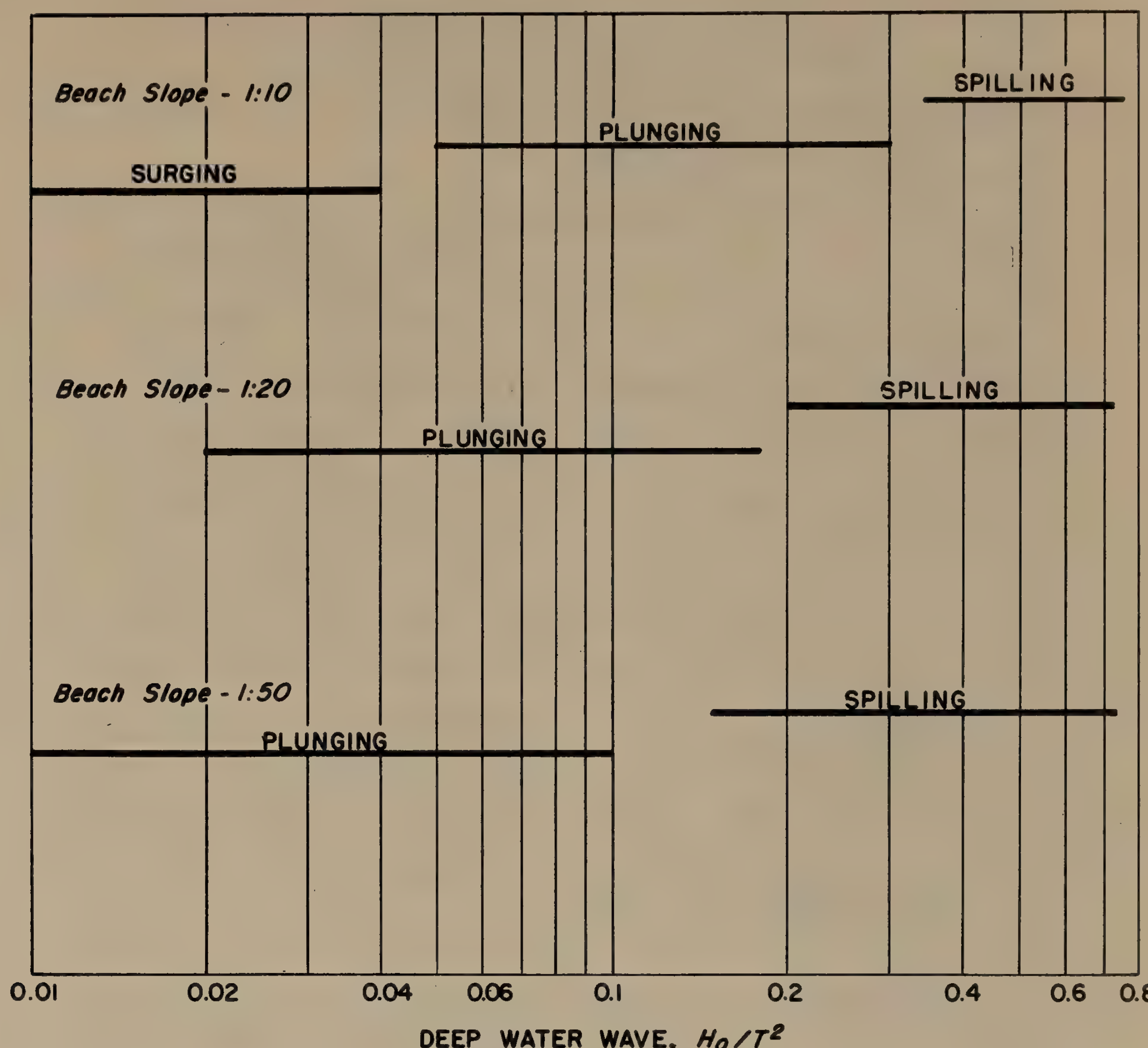
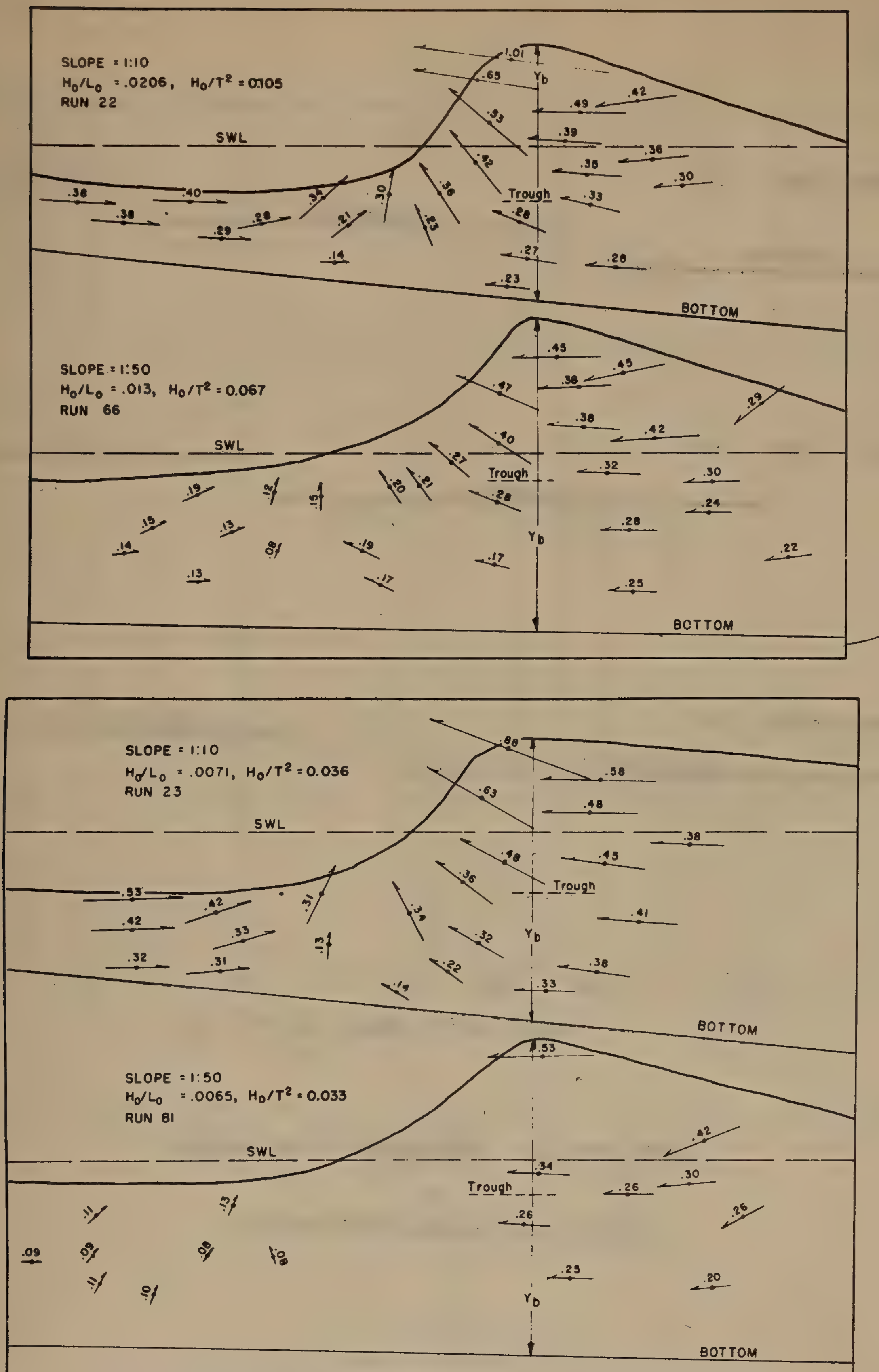


FIG. IF-10 - BREAKER TYPE INDEX



NOTE: Numbers are = Velocity/$\sqrt{g\, Y_b}$ Equal breaker heights

[redacted]-11 - KINEMATICS AT BREAKER POINT

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION I. WAVES, TIDES AND BEACHES

G. CURRENTS IN THE SURF ZONE

BY

H. W. IVERSEN

Table of Contents

G. CURRENTS IN THE SURF ZONE

1. Introduction:

Long crested waves that advance over a plane sloping shoreline with crests parallel to the bottom contours result in velocities of the water that are variable in magnitude, but with directions that are perpendicular to the bottom contours. The velocity is toward the beach in the water under the crests of the waves, breakers, or foam lines, and away from the beach in the water under the troughs.

Natural beaches do not have absolutely plane bottoms, nor do the waves approach the beach parallel to the bottom contours. Thus, currents are generated in, and adjacent to, the surf zone. These may be broadly catalogued as longshore, or littoral currents and offshore, or rip currents (surf rips). In general, the littoral current is generated from the wave advancing at some angle other than the perpendicular to the bottom contours. The littoral current has a direction parallel to the beach face and bottom contours. Rip currents (surf rips) are generated by breakers and foam lines advancing over non-parallel bottom contours. Wave fronts advancing over such contours are refracted to cause convergence or divergence of the energy in the wave. The result is higher waves and breakers in a region of convergence, and lower waves and breakers in a region of divergence. Although most of the wave energy is dissipated in the breaking action and travel of the foam line to the shore, some is reflected and returns seaward. The high concentration of energy tends to fan in the surf zone toward the low energy concentrations. Currents result that have an offshore component. The action, when once generated, retards the incoming wave at the location of the offshore velocity or rip with a consequent sustaining stability tendency.

Irregularities in the beach face in the zone of uprush also tend to promote surf rips. The action is similar to that due to the convergence or divergence of energy. A beach cusp that promotes an uprush beyond the average limit of uprush, tends to produce a delayed backrush with consequent retardation of the next incoming foam line at that location. The advancing foam line on both sides of the retardation tends to spread into the retardation, particularly at the beach face. Thus, the rip is generated and sustained.

Some information is available to predict littoral currents in magnitude and direction. Rip currents are predictible as to possible origination, but magnitudes have not been generalized.

2. Littoral Currents:

When waves advance over a shoaling bottom with the wave crests at some angle other than parallel to the bottom contours, the motion of the water in the wave has a component parallel to the contours (Figure IG-1). A series of waves of essentially the same character will produce the same velocity component in the same water depth. This longshore movement is retarded by bottom friction. An analysis of the resulting motion has been made (Reference IG-1). The expression for the average littoral current velocity is:

$$V = \tfrac{1}{2}a\left[\sqrt{1 + (4\,C_b \sin\alpha / a)} - 1\right] \qquad \text{(IG-2.1)}$$

where: V = Average littoral current - feet per second

$a = 2.61\ m\ H_b \cos\alpha / kT$

C_b = Velocity of breaker crest - feet per second
α = Breaker angle with bottom contours.
m = Beach slope expressed as a decimal fraction
H_b = Breaker height - feet
T = Wave period - seconds
k = Bottom friction coefficient

The reliability of this expression has been checked in the field on ocean beaches and in the laboratory (Figure IG-2). Use of the expression necessitates knowledge of the breaker height, velocity, period, and crest line angle with the bottom contours. In addition the average beach slope in locale of the breaker must be known. In forecasting of the breakers from a deep water wave condition the needed variables are known from the forecast. The one remaining variable, that of the bottom friction coefficient, has been determined for natural sand beaches as approximately equal to 0.008. The friction coefficient depends upon the bottom roughness. Hence, pebble beaches or rock beaches will have k values higher than 0.008. The magnitudes have not been determined.

The order of magnitude of littoral currents on an ocean beach may be seen from Figure IG-2. As a further example consider an 8 foot breaker with a period of 12 seconds and with a crest angle of 30 degrees on a beach with a 1:50 slope. From Section I-F of this manual the breaker velocity is found to be 23 feet per second. Solution of Equation (IG-2.1) results in a littoral current of 5.0 feet per second. The direction of the littoral current will be in the direction of the breaker velocity component parallel to the beach.

Equation (IG-2.1) was developed for plane beaches with regular bottom contours. When the beach contours are irregular, different littoral currents may result at different locations along the surf zone. Equation (IG-2.1) applies only to the region immediately shoreward of the breaker line or breaking point. The foam lines inshore of the breaker continue to refract to become more nearly parallel to the bottom contours. In addition the wave energy is dissipated in the movement of the foam line to the shore. The littoral current decreases from a maximum given by Equation (IG-2.1) for the region immediately shoreward of the breaker, to a negligible amount near the shoreline.

One feature of littoral currents should be stressed. The velocity of the littoral current is considerably less than the breaker velocity. Equation (IG-2.1) permits evaluation of the average littoral current. The instantaneous current is in the order of magnitude of the generating breaker velocity component.

The broaching tendency of craft in the breaker zone has been attributed to the littoral current. The relative magnitudes of the breaker velocity and the backwash velocity in the trough preceding the breaker, have a much greater effect upon broaching than the effect of the littoral current. A craft handled by a competent operator should maintain a course which is perpendicular to the wave or breaker crest. In this relative position, the craft motion is in the direction of the wave motion. The littoral current effect is essentially negated since the craft also has a similar component parallel to the shore.

Broaching is due mainly to the longitudinal craft axis deviating from a perpendicular to the wave or breaker crest line. As the wave or breaker passes a craft which is proceding shoreward, the bow is in the region of backrush and the stern in the forward motion of the wave crest. The

resultant on a non-perpendicular craft is a turning couple produced by the impingement of the opposing velocities on opposite sides and ends of the craft. If the craft control mechanisms cannot compensate for the turning couple the craft will broach.

3. Circulating Currents at the Shoreline:

Near the shore line the longshore currents are usually irregular depending upon the beach contours in the region of the uprush or backrush. The final action of foam line as it rushes up the beach depends upon the shoreline configuration. A cusp will allow a greater uprush distance, and usually a delayed return of the backrush, than the adjacent higher beach faces. The next incoming foam line is retarded at the cusp. Circulations are initiated, towards the sides of the cusp during the uprush in the cusp, and towards the center of the cusp during the backrush in the cusp, (Figure IG-3).

A beached craft in the circulation areas will tend to swing on the grounded bow with the stern following the circulating currents. The next incoming foam line will accent the swing as the forward velocity of the water impinges on the exposed side of the craft. Unless the craft retracts from the beach the craft will continue to swing until it is parallel to the beach and grounded over its total length (Figure IG-4).

The action of the circulation adjacent to the shore line may be erratic. An irregular series of incident waves and breakers will produce circulations that will change from time to time at any particular location on the beach. On flat beaches, with slopes on the order of 1:50 or flatter, the uprush and backrush velocities are usually small in magnitude. Consequently the circulating currents are small. On steep beaches, with slopes greater than 1:50, the circulating currents are serious and should be considered with respect to beached craft.

4. Rip Currents (Surf Rips):

Surf rips are currents that move seaward in a direction essentially perpendicular to the beach face and the crest line of the waves. A typical surf rip is shown in Figure IG-5. The appearance of the rip is marked by a channel of discolored water from the turbulent motion of the water which carries beach sand and intrapped air in suspension. The breakers in a rip are usually lower than those in the undisturbed breaker on either side of the rip. The current of the rip, together with the irregular eddying currents in the rip, tends to dissipate the energy of the oncoming wave and breaker.

A strict definition of rip currents could include any current between the breaker line and the beach face that has an offshore component. However, the term is arbitrarilly restricted to a sustained seaward moving current that is plainly visible as a Figure IG-5.

A rip current is divided into three components, the feeder currents, the main rip current, and the head, (Figure IG-5(a)) (Reference IG-2).

The generation of surf rips results from feeder circulating currents in the foam line area that become stable due to the beach configuration and the incident wave system. Typical situations that are conducive to the formation of rips are shown in Figure IG-6.

The main surf rip that results from the feeder currents is a narrow band of water that moves seaward in a general direction perpendicular to the beach face. Beyond the breaker line the rip current broadens to the head

with reduced velocities wherein the energy of the current is dissipated in internal eddies.

A seaward current, once initiated, tends to become stable until a change in the incident wave train or the bottom contours takes place. The seaward current opposes the incoming wave velocity to retard the wave in the rip area. Thus, the wave on both sides of the rip advances faster than the wave in the rip. At the edges of the rip, particularly in the shallow water of the surf zone, stable feeder currents result from the wave fronts on the sides of the rip to continue an inflow to the rip.

A rip may seem advantageous in regard to craft operation in the surf in that the breaker action in the rip appears to be less violent than that removed from the rip. However, a craft moving shoreward must buck the rip current and is thus in the breaker zone for a longer period of time than one proceeding through the normal breaker zone. In addition, while the direction of the rip current is predominately seaward, the currents in the rip are composed of numerous eddies and whirls that make craft control difficult. A craft proceeding seaward in a rip will experience the same control difficulties.

Large craft or, craft with adequate control, such as the LCMs and LCVP's, can use a rip to advantage. The breaker action is less than in the breaker adjacent to the rip. In addition, the outward current of a rip cuts channels through offshore bars to provide passages through the bars with a greater depth of water than on the normal beach.

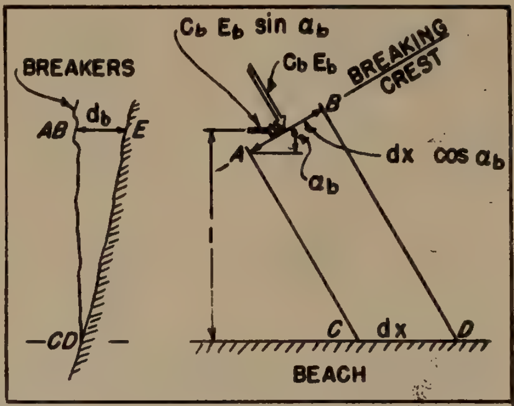
Strong swimmers in the surf can use a rip current to advantage to swim from the beach to beyond the breaker line. Weak swimmers in the surf should avoid rip currents since the current will carry them seaward for distances beyond their ability to return to the beach. A swimmer should not attempt to swim against the current of the rip to try to regain shore. The rip current is normally a narrow band. By swimming parallel to the beach face, a swimmer will, within a few feet, get out of the influence of the rip and will be able to swim shoreward in the normal surf zone.

5. References:

IG-1: Putnam, J. A., Munk, W. H. and Taylor, M. A. - "The Prediction of Longshore Currents" - Transactions American Geophysical Union, Vol. 30, No. 3, June 1949, pp. 337-345.

IG-2: Shepard, F. P., Emery, K. O. and La Fond, E. C. - "Rip Currents, A Process of Geological Importance" - The Journal of Geology, Vol. XLIX, No. 4, May - June, 1941, pp. 337-369.

a. Aerial photograph of swell breaking at an angle to the shoreline, thus causing a longshore current in the direction shown.



b. Schematic diagram for energy and momentum considerations of longshore currents.

FIG. IG-1 - LITTORAL CURRENT

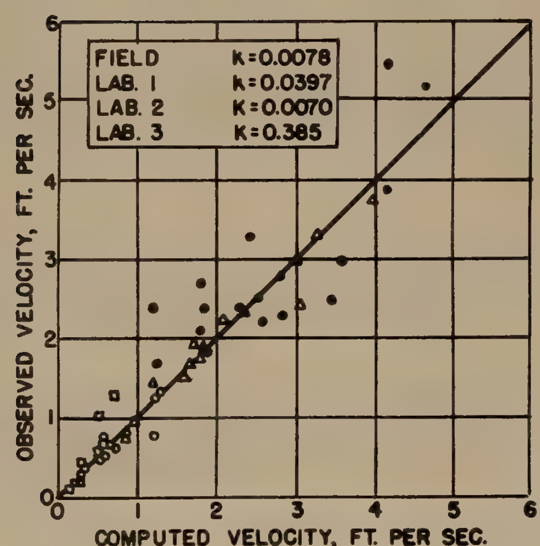


FIG. IG-2 - Comparison between observed and calculated velocities of longshore currents

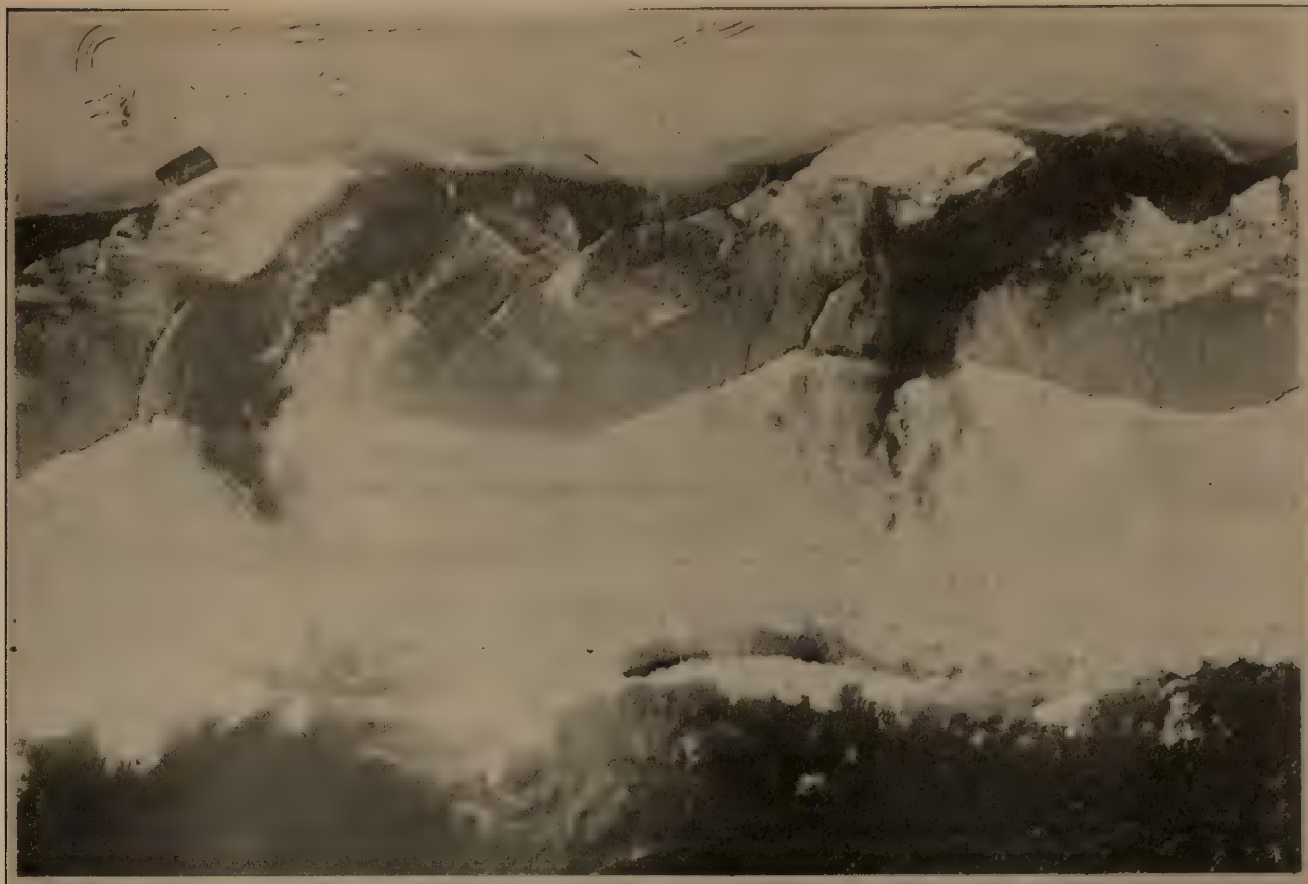


FIG. IG-3 - Vertical aerial photograph of surf rips on a steep beach.

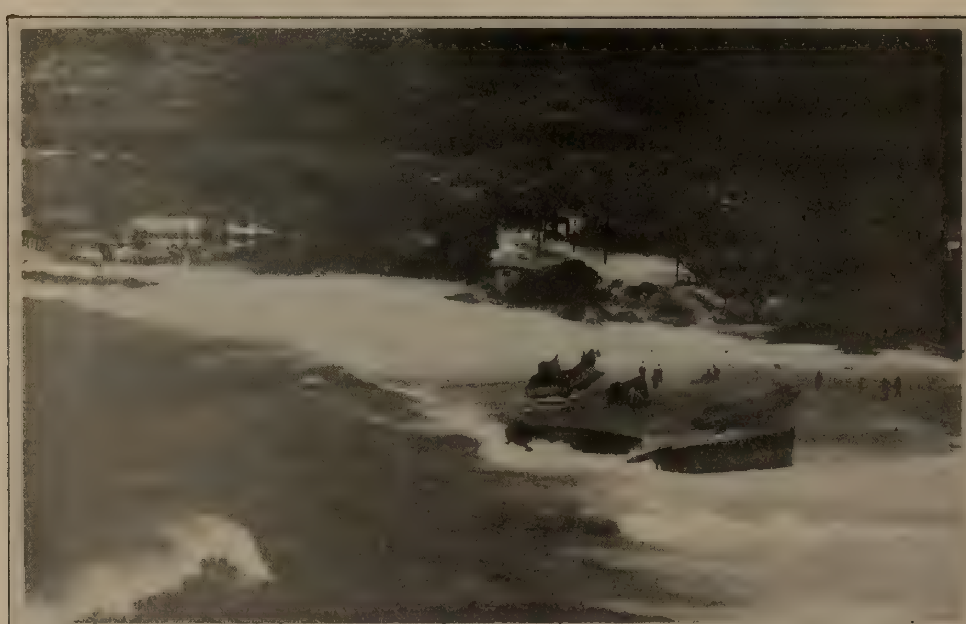
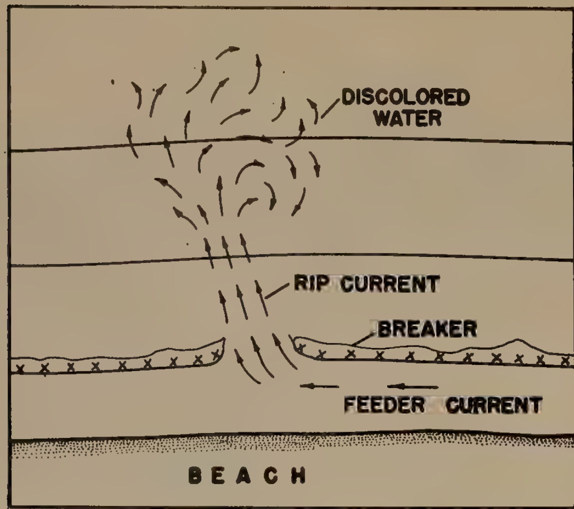
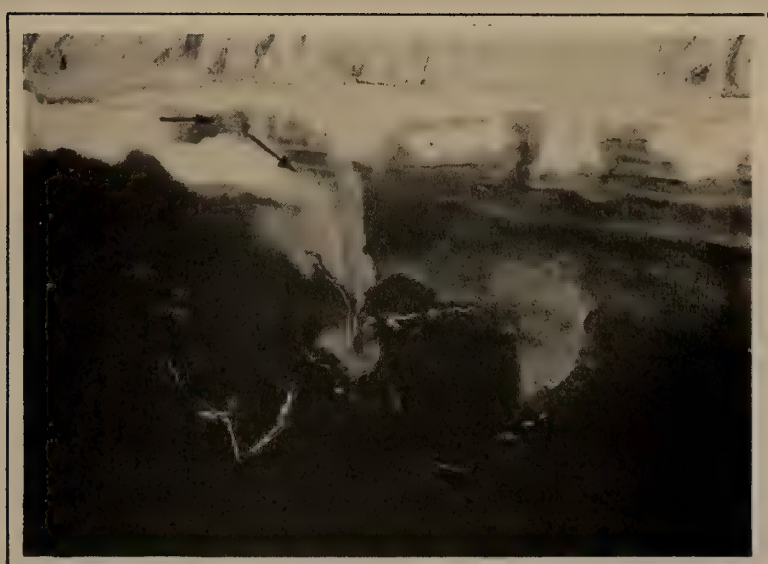


FIG. IG-4 - Broached landing craft.

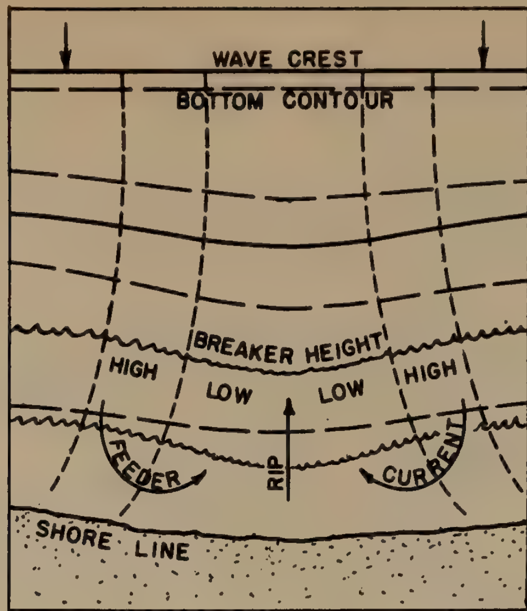


a. Idealized rip current; the feeder current may, and often does, approach from both sides.

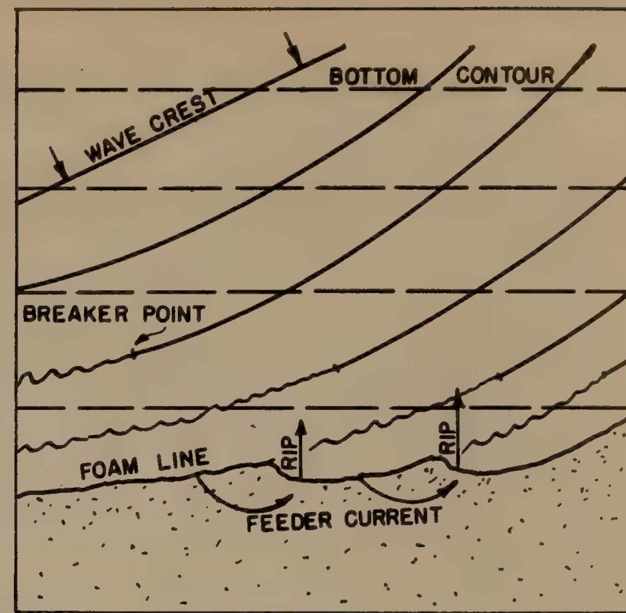


b. Aerial photograph showing rip current

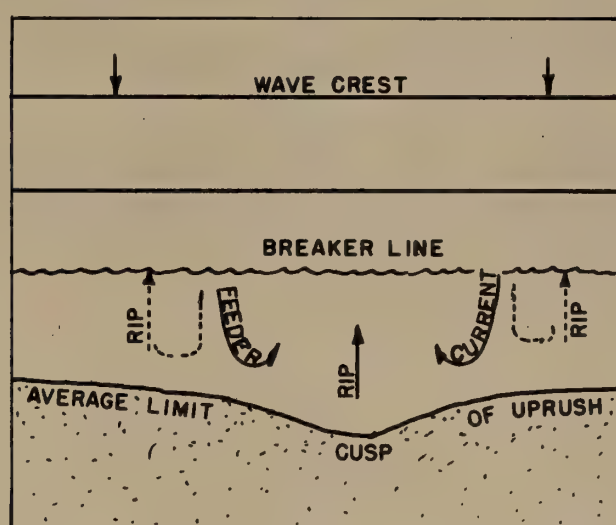
FIG. IG-5



a. Surf rips from depression in bottom in surf zone.



b. Surf rip from wave train approaching the beach at an angle.



c. Surf rips from cusp on beach in uprush zone. Rips may become stable in center of cusps or on sides, depending on wave height and period.

FIG. IG-6 - Wave and beach configurations conducive to the formation of surf rips.

SUMMARY OF SECTION I-H

STATISTICAL PROPERTIES OF OCEAN WAVES

BY
R. R. PUTZ

The well-known variability of successive ocean waves has been found to exhibit certain statistical regularities which can be used for prediction as well as for the simplification of wave system description and analysis.

A knowledge of the average wave height (or period) of a wave system observed over a short interval of time at a given location appears sufficient to determine quite closely the percent of waves whose height (or period) falls within an arbitrary range of values. The basis for this conclusion is suggested by Figures SIH-1a and SIH-1b, which show the frequency distribution of individual wave height for each of two typical wave systems, representing, respectively, high and low mean wave-heights. Both distributions have the same approximate shape, and it will be seen that the waves of the distribution with the higher mean value show a proportionately high degree of dispersion, or scatter, about their mean. The facts are much the same for individual wave periods, which show in most cases, nearly the same amount of statistical regularity.

It appears impossible to predict with any accuracy the height (or period) of a given wave from a knowledge of its period (or height), or indeed, from a knowledge of the heights and periods of all waves occurring immediately before the wave in question. Such a state of affairs does not preclude the possibility of the propagation from one location to another of wave groups whose individual waves maintain, relative to each other, at least, their characteristics. Data are lacking on such transformations of wave systems.

A statistical estimation of the average wave height (or period) for an entire wave system may be made with an accuracy which depends on how many waves of the system are observed. For a given desired accuracy an approximate number of waves can be specified to correspond to any desired probability of attaining this accuracy.

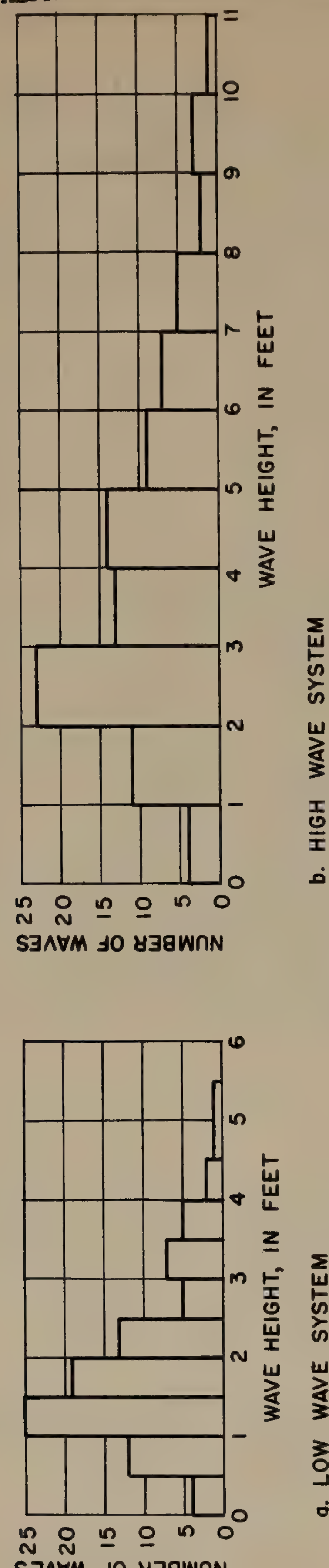


FIG. S-IH-1 - TYPICAL FREQUENCY DISTRIBUTIONS OF WAVE HEIGHTS

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

PART I: WAVES, TIDES, AND BEACHES

H. STATISTICAL PROPERTIES OF WAVE SYSTEMS

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H. STATISTICAL PROPERTIES OF OCEAN WAVES

1. Introduction:

The variability of ocean waves is well known. While wave systems may appear quite regular on casual inspection, they are usually composed of waves that vary both in height and in period. Data about the statistical properties of wave systems have been obtained by observing waves at a fixed point over a short interval of time. The conclusions regarding statistical properties of waves presented here are based primarily on records made by Pacific Ocean underwater pressure recorders located offshore.

These subsurface pressure recorders were placed at locations corresponding most nearly to systems of swell in shoaling water, rather than to systems in the surf zone or in deep water. (Figure IH-1 shows a 5 minute section of a typical pressure record made by waves in shallow water). While these systems do not necessarily approximate other wave systems of interest (e.g. systems of breakers, of swell in deep water, or of wind waves), the indications are that - once sufficient data is obtained for these other systems - it will be possible to analyze them and obtain results analogous to those presented below.

Since a statistical property is defined relative to a variable observed under definite conditions, it is obvious that wave systems have a large number of statistical properties that can be investigated. Research into some of these properties has indicated that there are some definite regularities in wave systems, and that these regularities promise to be of some value for predicting the heights and periods of waves.

Various different problems of prediction may arise, depending upon what is assumed to be known about the wave system. It is impossible, of course, to predict the height of an individual wave occurring in a specified position in the system, because of the large number of physical variables that would have to be identified and measured. However, as methods of utilizing meteorological information for wave forecasting are improved, much can be said about the average value of the wave height. Moreover, when the latter is known, a reasonably adequate prediction can be made about the probability of an arbitrary wave height falling between two given magnitudes.

2. Individual Waves:

a. Wave Heights:

Subsurface pressure record data indicate that the relative frequencies of waves of given heights observed to pass by a fixed point follow a rather definite pattern such as shown in Figure I -2. This graph shows, for example, that 10.6% of the waves have heights between one and three-eighths feet and one and five-eighths feet. The graph also shows, when the frequencies are cumulated, that 81% of the waves are below 3.1 feet in height. The average or arithmetic mean, μ of the wave heights is indicated at 2.20 feet by the dotted line.

For many purposes the cumulative frequency distribution is useful, particularly when the horizontal scale is transformed to render the plotted distribution an approximate straight line. The result of this transformation and the corresponding plot of the distribution function shown in Figure IH-3.

Several observations may be made about the typical distribution represented in these Figures. One is the fact that waves of nearly all heights occur from the highest observed height down to zero height. Another is that more waves

occur with heights below the mean than with heights above the mean, although wave heights above the mean may deviate relatively more from the average than may those below the mean. This asymmetric, or skewed, quality is characteristic of wave-height distributions.

The significant wave height $H_{1/3}$, defined as the arithmetic mean of the heights of the waves which are in the upper one-third of the height distribution, has also been indicated in Figure IH-2 by the dotted line at 3.68 feet. It may be seen that $H_{1/3}$ occurs at approximately the 86% point in the wave-height distribution. Available data (Seiwell, 1949, IH-5; Wiegel, 1949, IH-7) show that the ratio between $H_{1/3}$ and the overall mean wave-height μ, is relatively constant - averaging 1.57.

Present evidence (Putz, 1950, IH-1) indicates that the height distributions of all wave systems are nearly constant in shape, with practically the same degree of skewness for all systems. Furthermore, the relative variability of the distribution remains nearly invariant from one wave system to another. These two facts of regularity permit the use of a rather simple approximate model to represent the statistical distribution of the heights of the waves composing an arbitrary wave system. Figure IH-4 may be used to read off the approximate wave height below which any given percentage of waves will fall when the mean of the system has a given value. Thus, on the curve corresponding to a mean of 5.0 feet, we find that 90% of the waves will be below 8.4 feet in height.

b. Wave Periods:

The distribution of the periods of waves (inferred from subsurface pressure records taken at a given location) show certain regularities. A distribution of wave periods with average characteristics is shown in Figures IH-5 and IH-6, the separate Figures corresponding to the two modes of representation employed in the preceding section. Note that the pressure recorders sense waves of all periods from about 5 seconds up to 21 seconds. Moreover, the wave periods are scattered in approminate symmetry about their mean, (that is, the distribution is not materially skewed toward either end).

Individual wave systems are likely to have wave-period distributions that are slightly skewed toward either the low or the high periods. Present evidence indicates that for wave systems of average long-period waves, the period distribution tends to be skewed toward the shorter periods; while for systems of average short-period waves the period distribution tends to be skewed toward the longer periods.

Available data (Putz, 1951, IH-2) suggest the approximate model of Figure IH-7 to represent the statistical distribution of the periods of the waves composing an arbitrary wave system for which the mean period is known. Reading the curve for a mean period of 13.0 seconds, we see that the indicated percent of waves with period below 16.0 seconds is 81%.

c. Wave Height and Wave Period:

Present evidence (Putz, 1951, IH-3) indicates no material degree of statistical relationship between the height and the period of an individual wave. Thus, the fact that a wave has a long period (or a short period) does not increase our knowledge of its probable height, and inversely, knowledge about its height does not add to our knowledge of its probable period. A plot of wave height versus wave period for the waves of a typical wave system is shown in Figure IH-8. It may be seen that height and period for an individual wave are virtually independent in the statistical sense.

Likewise, when wave systems are compared, no significant statistical relationship was found between mean wave height and mean wave period. Thus a prediction of the mean wave period for a system of waves gives us no additional information about its mean wave height.

3. Successive Waves Within a Wave System:

a. Wave Heights:

While for the average wave system a small degree of statistical relationship has been found between the heights of two successive waves passing a given point, this relationship is not sufficient to predict one wave height from a knowledge of the preceding wave height. (Putz, 1951, IH-4) Higher-than-average waves tend to be followed by higher-than-average waves, and lower-than-average waves tend to be followed by lower-than-average waves. Figure IH-9 shows the plot of the height of a given wave in relation to the height of the following wave. It may be seen that relatively little information about the following wave is obtained, on the average, from a knowledge of the given wave. For waves farther distant (e.g. waves separated by one, two or more waves) the heights rapidly become less related. Even if the heights of all the waves in the immediate vicinity, were known, it still seems impossible to accurately predict the height of a given wave.

Certain wave systems exhibit a somewhat greater degree of relationship between successive wave heights than that shown in Figure IH-9, while others exhibit somewhat less. No relation has been found between the degree of this relationship between successive wave heights and any other wave-system characteristic. At present, therefore, no way is known of predicting those systems in which the degree of relationship is most pronounced (hence approaching a degree useful for purposes of individual wave-height prediction).

b. Wave Periods:

The periods of a pair of consecutive waves passing a given point show (Putz, 1951, IH-4) on the average a small degree of statistical relationship to each other. As with wave heights, this relationship is too small to predict the period of a given wave from those of other waves. A typical plot of the periods of two successive waves is shown in Figure IH-10. As with the heights, the periods of waves separated by other waves show a rapidly decreasing degree of relationship to each other. No way is known of predicting which wave systems will exhibit higher-than-average relationships between the periods of the waves composing it.

c. The Grouping of Waves:

The waves of a wave system have often been classified by observers according to a more or less definite structure in which groups of waves appear. The characteristics defining such groups are related to the heights and periods of the waves within them. Much remains to be said about the properties of such groups of waves, but the extent to which such a group is propagated from one location to another is one of the most important aspects of such wave groups. Some distance offshore, in a moderately irregular system, there are always adjacent successive waves of nearly the same height and period. However, if one considers the wave system a short time later at a location nearer to shore, data is lacking upon which to base sound predictions as to the probability that a high wave, for example, will there be followed by another high wave.

If a group of waves be defined by the existence of a high wave within the set of waves taken as making up the group, then the pattern in which such high waves occur might be used to describe, in part, the occurrence of wave groups. However, the data available indicate that within a wave system the spacings between waves above a given height show little tendency toward regularity and, in particular, do not seem to be predictable from other wave system characteristics.

4. Applications of Statistical Properties of Wave Systems:

a. Estimation of Mean Height Wave from Observed Waves:

Estimation of the mean wave height which may be expected to prevail at a given location for the waves in the immediate future is a matter on which statistical analysis can only arrive at probabilities for given sizes of errors in the estimated value. If the intervening time interval is sufficiently short so that the average wave-system properties may be assumed to remain constant, the expected mean wave height at a later time may be taken as the average of a set of n consecutive wave heights observed at the present time. The maximum relative error (in percent) to be expected with various given probabilities under these conditions may be read from the curves in Figure IH-11, which has been drawn for a typical wave system. For example, we see that if n = 10 consecutive waves are observed to have a mean height of 4.0 feet, 80% of the time our error will be at most 20% in assuming the expected mean value a short time later to be 4.0 feet. If we wish to expect 10% to be our maximum error 80% of the time, we must observe 40 consecutive wave heights.

b. Estimation of Mean Wave Period from Observed Waves:

As with mean wave height, the estimation of mean wave period depends upon a knowledge of the mode of change of the wave system as time passes. If we assume, as before, a short enough time interval to give an effectively steady-state wave system, the curves in Figure IH-12 apply for a typical wave system. We see that in order to expect a 10% maximum relative error with a probability of 80% it is necessary to observe the periods of 10 consecutive waves.

c. Analysis of Wave Records:

Certain features of the regularities found in wave systems show promise for the simplification of the routine day-by-day analysis of wave records taken regularly over long periods of time. For example, the constant form of the statistical distribution of wave heights (mentioned above) indicated that one can use relatively few points of the distribution curve to estimate the entire distribution exactly enough for many purposes. At the same time a theoretical statistical model based upon a constant distribution form serves to explain the persistence of values for certain ratios of wave-height statistics computed by Wiegel and Seiwell. A summary by Snodgrass (1950, IH-6) shows a comparison of various reported ratios of the quantities H_{max} (the highest observed wave height), $H_{1/10}$ (the mean of the highest one-tenth of the wave heights), $H_{1/3}$, and H_{ave} (the mean of all of the observed wave heights). Snodgrass' table, slightly modified, is reproduced below.

TABLE IH-I

Comparison of Wave-Height Ratios
for Various Pressure Recorders
and a Theoretical Frequency Function Model

Basis of Calculations	Computed Ratios				Remarks
	$\frac{H_{1/3}}{H_{ave}}$	$\frac{H_{1/10}}{H_{1/3}}$	$\frac{H_{max}}{H_{1/3}}$	$\frac{H_{max}}{H_{1/10}}$	
Point Arguello, California wave recorder		1.30	1.85	1.42	3 months of data
Point Sur, California wave recorder		1.27	1.85	1.46	14 months of data
Heceta Head, Oregon wave recorder		1.30	1.91	1.47	14 months of data
Cuttyhunk, Massachusetts wave recorder	1.57				10 months of data
Bermuda wave recorder	1.57				4 months of data
Average of wave record values	1.57	1.29	1.87	1.46	
Theoretical statistical frequency function MODEL	1.57	1.29	1.81	1.41	Model based upon 25 wave records (extending over a 3-year period)

5. References:

IH-1: Putz, R. R. (1950), "Wave Height Variability; Prediction of the Distribution Function", Technical Report No. HE-116-318, Institute of Engineering Research, University of California, Berkeley, California, December 1950.

IH-2: Putz, R. R. (1951), "Wave Period Variability; Prediction of the Distribution Function", Technical Report No. HE-116-325, Institute of Engineering Research, University of California, Berkeley, California, June 1951.

IH-3: Putz, R. R. (1951), "Joint Variation of Wave Height and Wave Period for Ocean Swell", Technical Report No. HE-116-328, Institute of Engineering Research, University of California, Berkeley, California, October 1951.

IH-4: Putz, R. R. (1951), "Heights and Periods of Pairs of Successive Waves for Ocean Swell", Technical Report No. HE-116-330, Institute of Engineering Research, University of California, Berkeley, California, October 1951.

IH-5: Seiwell, H. R. (1949), "Results of Sea Surface Roughness Determinations in the Vicinity of Woods Hole, Massachusetts and Bermuda", Papers in Physical Oceanography and Meterology, Vol. X, No. 4 (1949).

IH-6: Snodgrass, F. E. (1950), "Wave Recorders and Wave Data", Paper presented at Institute of Coastal Engineering, Long Beach, California, October 11-13, 1950.

IH-7: Wiegel, R. L. (1949), "An Analysis of Data from Wave Recorders on the Pacific Coast of the United States", Transactions, American Geophysical Union, October 1949.

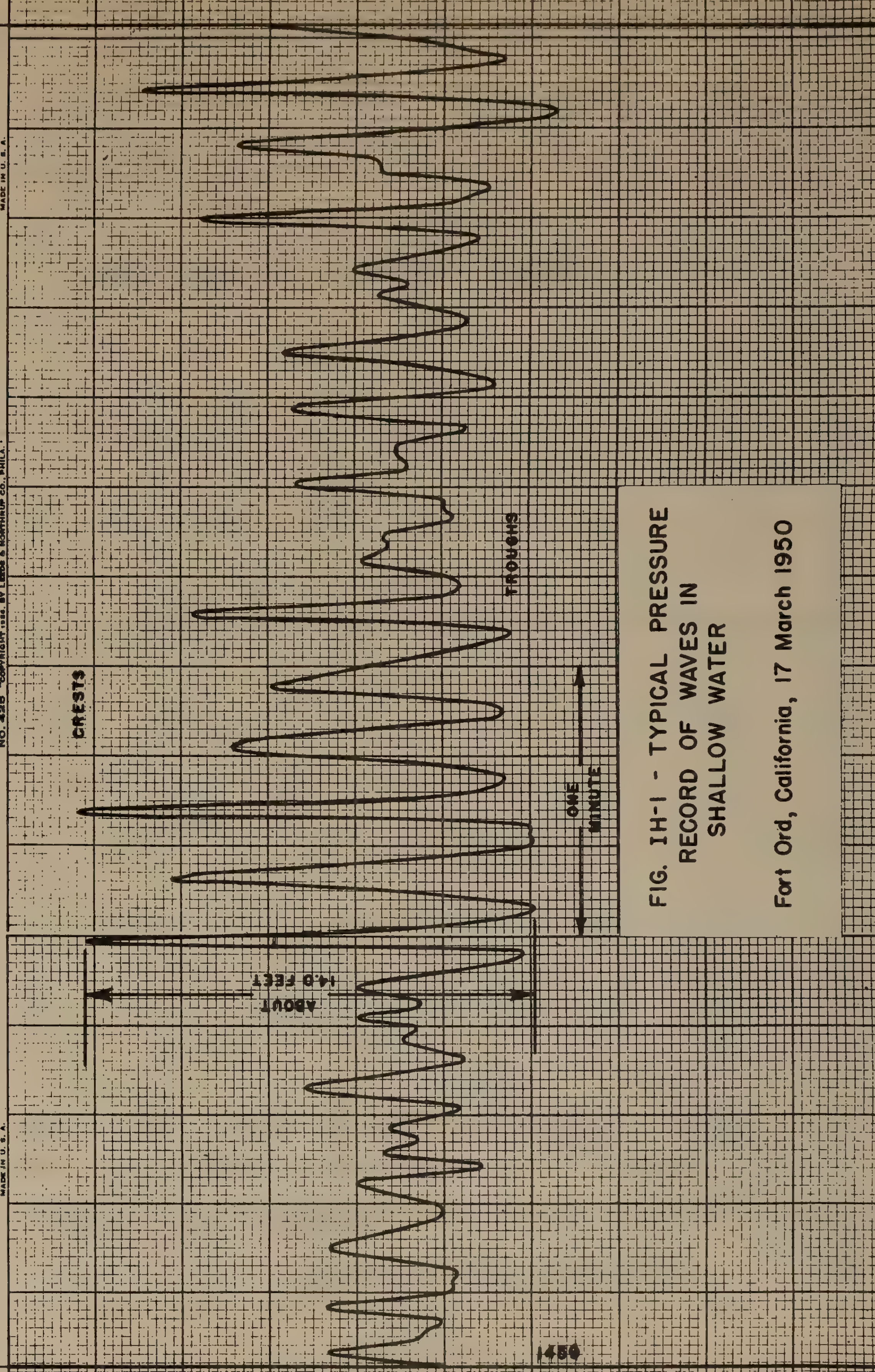

FIG. IH-1 - TYPICAL PRESSURE RECORD OF WAVES IN SHALLOW WATER

Fort Ord, California, 17 March 1950

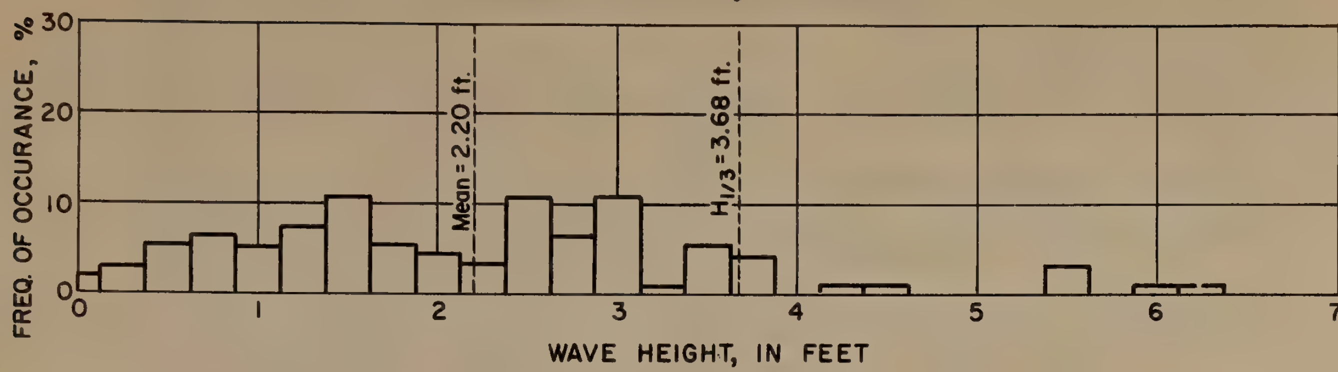

FIG. IH-2 - TYPICAL FREQUENCY DISTRIBUTION OF WAVE HEIGHTS

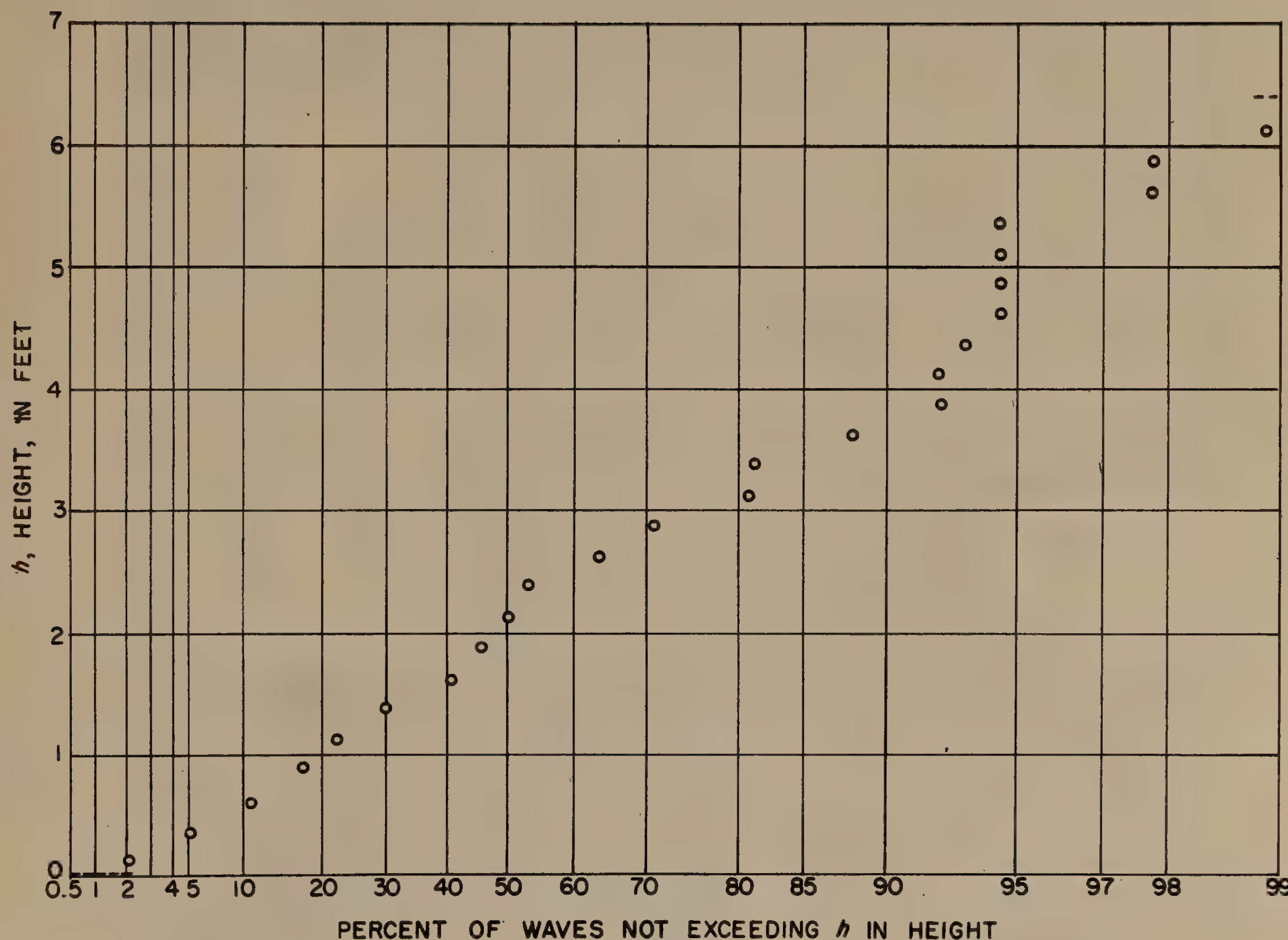

FIG. IH-3 - TYPICAL CUMULATIVE DISTRIBUTION OF WAVE HEIGHTS

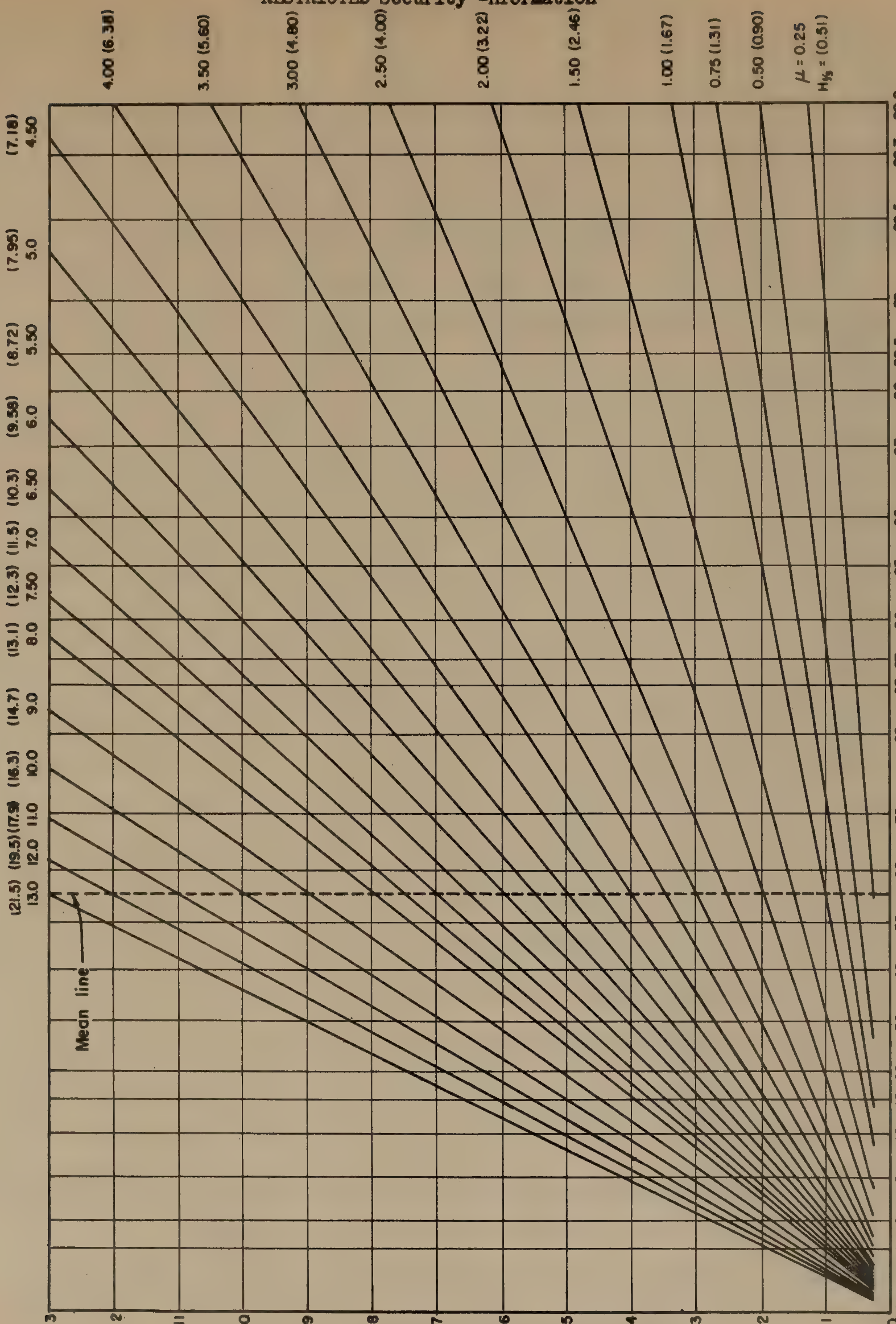

FIG. IH-4 - PREDICTION LINES FOR WAVE HEIGHT DISTRIBUTIONS FOR VARIOUS MEAN WAVE HEIGHTS
(Significant wave height, $H_{1/3}$, shown in parentheses)

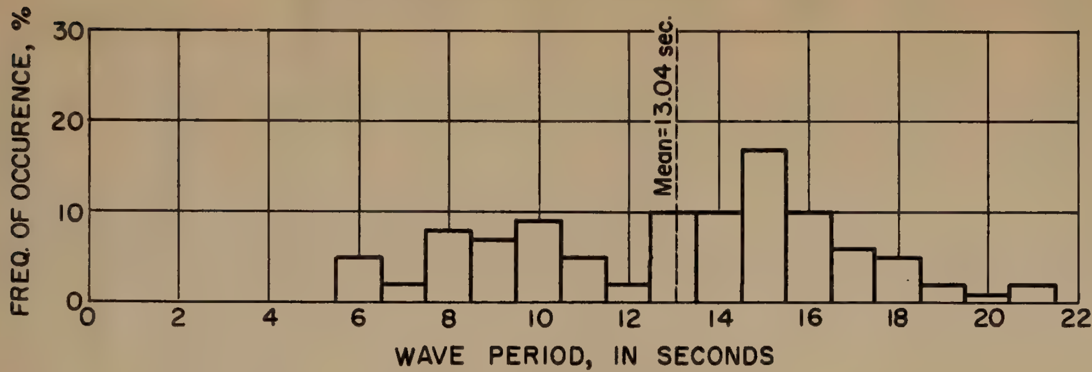

FIG. IH-5 - TYPICAL FREQUENCY DISTRIBUTION OF WAVE PERIODS

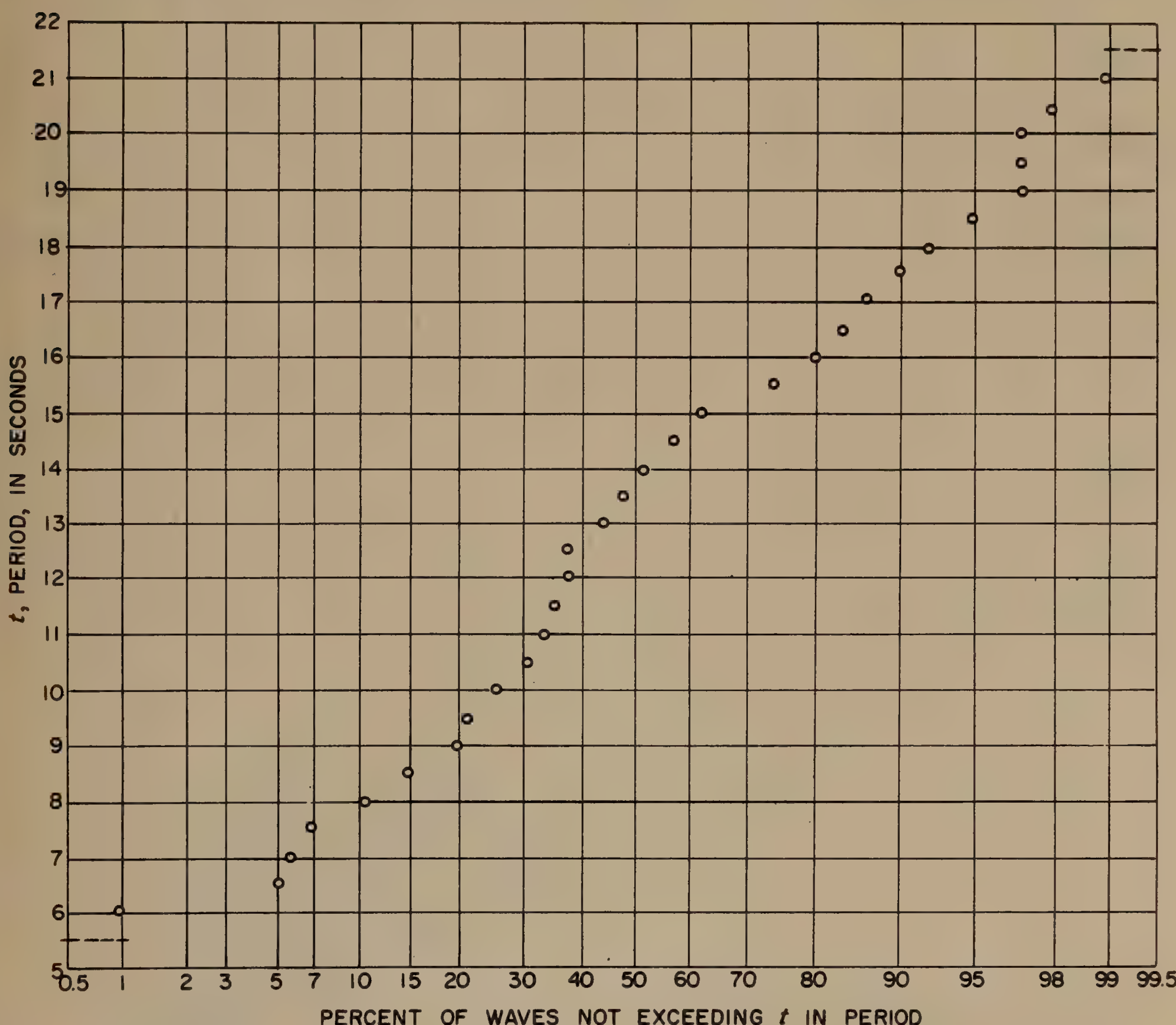

FIG. IH-6 - TYPICAL CUMULATIVE DISTRIBUTION OF WAVE PERIODS

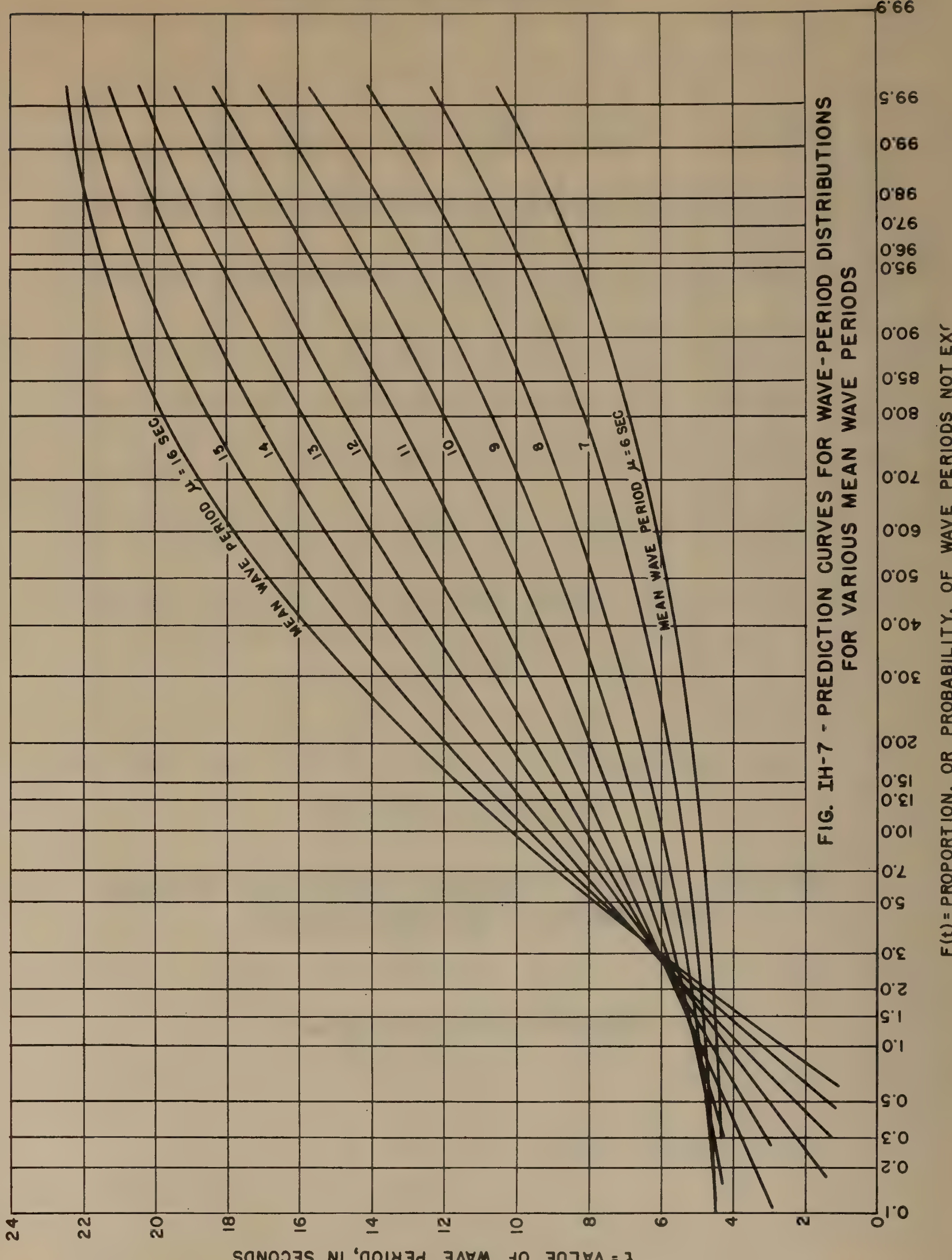

FIG. IH-7 - PREDICTION CURVES FOR WAVE-PERIOD DISTRIBUTIONS FOR VARIOUS MEAN WAVE PERIODS

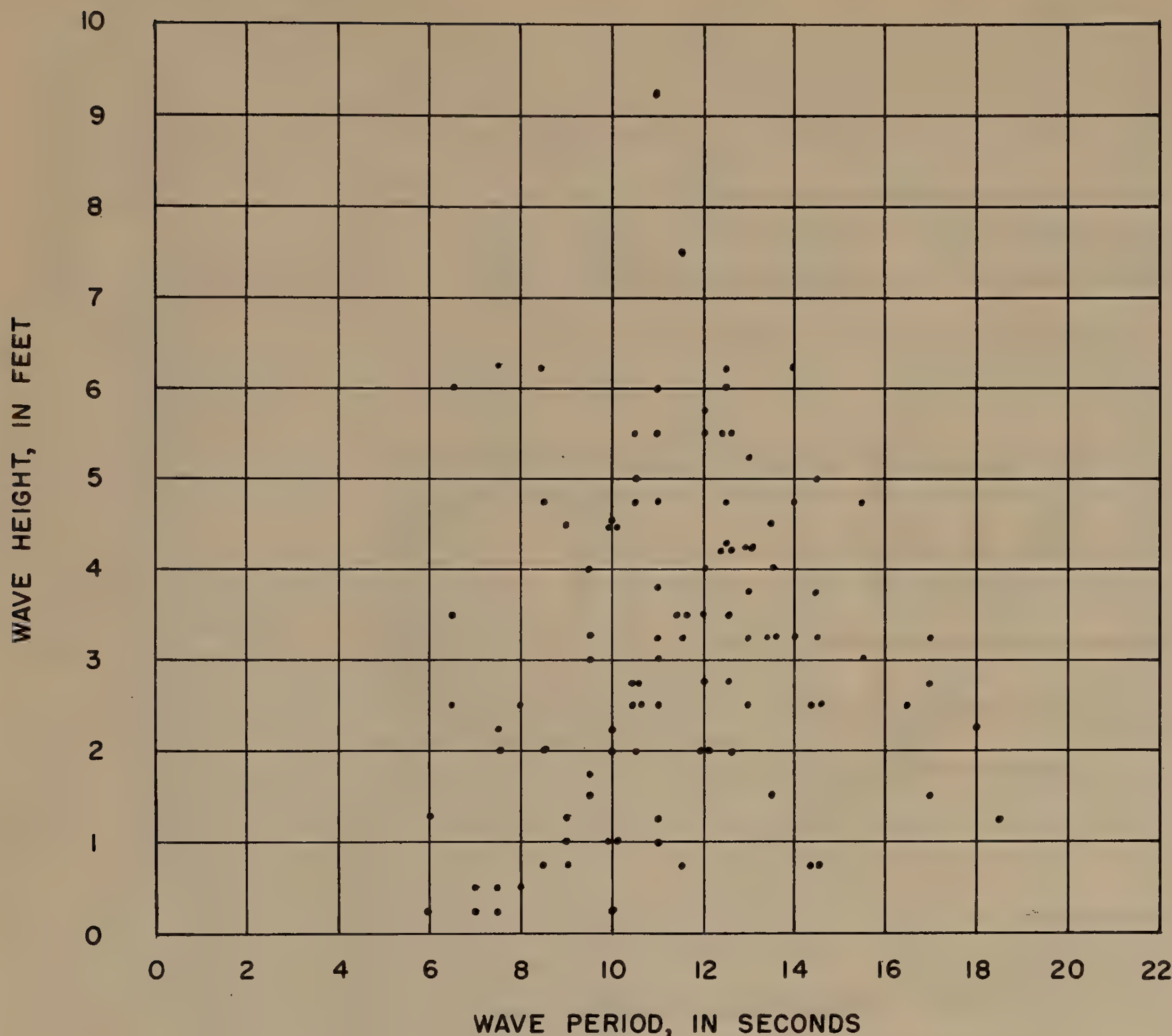

FIG. IH-8 - TYPICAL FREQUENCY DISTRIBUTION OF HEIGHT AND PERIOD OF INDIVIDUAL WAVES

FIG. IH-9 - TYPICAL FREQUENCY DISTRIBUTION OF THE HEIGHTS OF A PAIR OF CONSECUTIVE WAVES

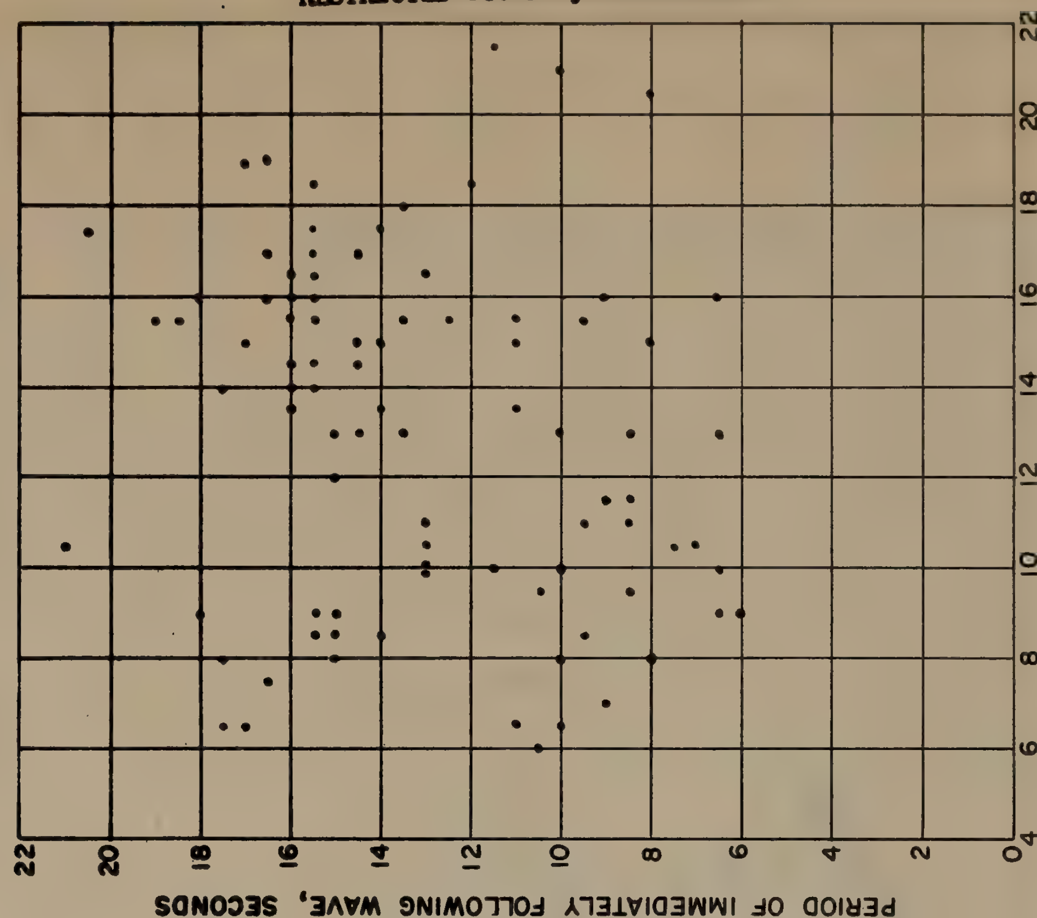

FIG. IH-10 - TYPICAL FREQUENCY DISTRIBUTION OF THE PERIODS OF A PAIR OF CONSECUTIVE WAVES

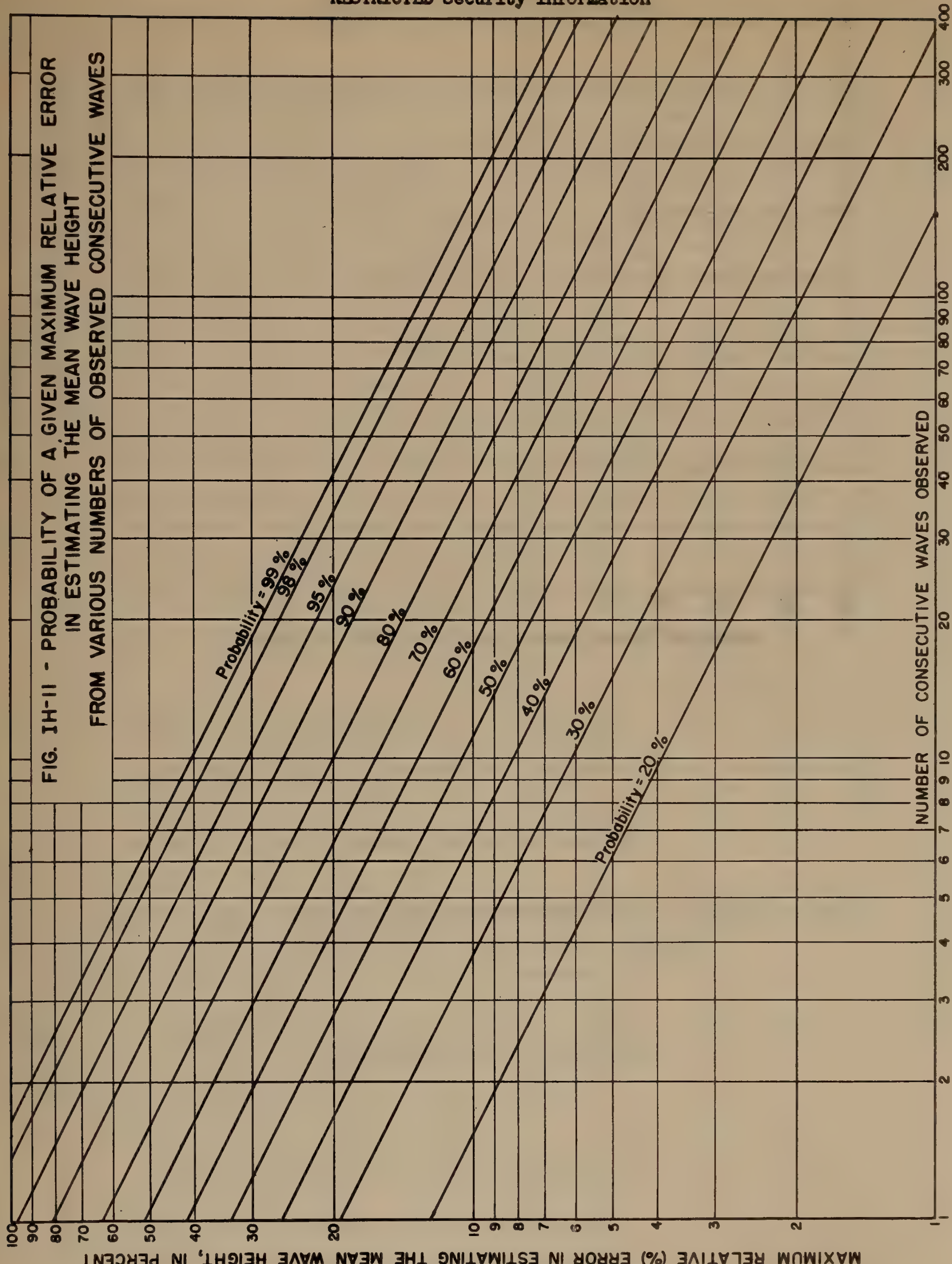

FIG. IH-11 - PROBABILITY OF A GIVEN MAXIMUM RELATIVE ERROR IN ESTIMATING THE MEAN WAVE HEIGHT FROM VARIOUS NUMBERS OF OBSERVED CONSECUTIVE WAVES

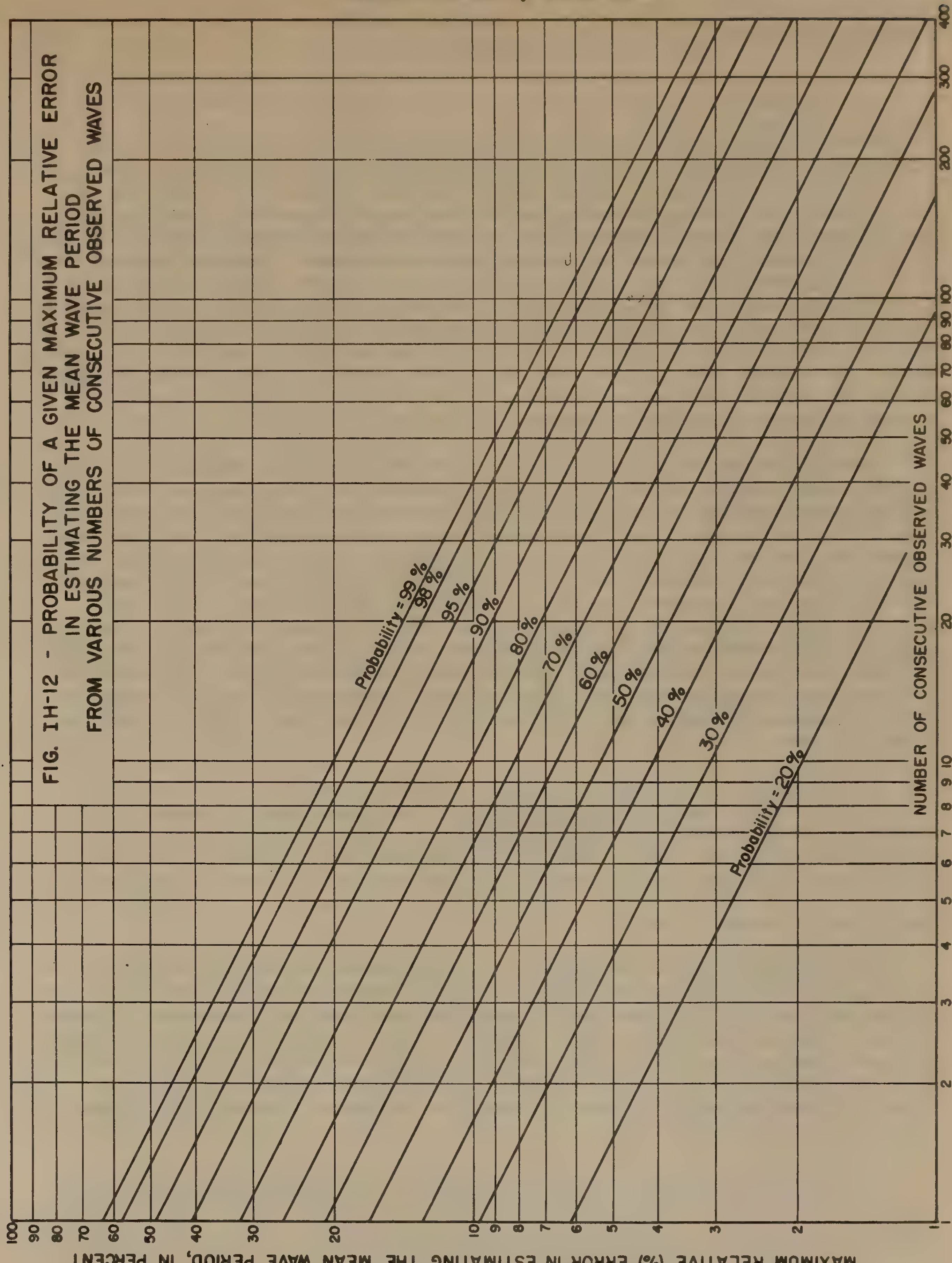

FIG. IH-12 - PROBABILITY OF A GIVEN MAXIMUM RELATIVE ERROR IN ESTIMATING THE MEAN WAVE PERIOD FROM VARIOUS NUMBERS OF CONSECUTIVE OBSERVED WAVES

SUMMARY OF BEACH CHARACTERISTICS

BY
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General Features:

Beaches are of four main types: Rocky, sandy, muddy, and coral. Each of these types affects landing operations in different ways. No single one of these types has the same characteristics throughout the world. Individual beaches also may differ materially in properties within short distances or even in short intervals of time, particularly when subjected to strong wave action. The characteristics of beaches and of the processes that influence them are described in detail in the section which follows. The relationship of beach characteristics to trafficabiltiy are described in another section. For strategic and tactical planning of operations on particular beaches, reference should be made to intelligence reports. A few remarks of the general nature of the four types of beaches, however, should help in a better understanding of the problems of landing operations.

Amphibious operations on beaches consist of three main phases: (i) Landing the water-borne craft on the beach; (ii) Getting the vehicles, equipment and men on and up the beach; and (iii) moving off the beach and onto the adjacent land terrain . These three maneuvers are closely related to the three principle parts of the beach: (i) the offshore, or shallow water area seaward from the shore line at low tide; (ii) the beach proper, or the part of the beach exposed between high and low tides and wet with water from wave action at high tide, commonly more or less steep in slope; and (iii) the backshore, or flat area behind the sloping part of the beach, generally composed of loose sand. The wind blows this sand about, and in many places forms dunes at the rear, which may or may not be covered with vegetation. The different parts of the beach are illustrated in Figure 1-I-I of the detailed report on beaches, which follows.

That part of the offshore extending from the water line to a depth of about 30 feet is called the offshore slope. The place where plunging waves strike the sand is called the plunge line. The level of the plunge line is very close to the mean water level seaward from shore. A small furrow, containing sand or fine gravel, coarser than the sand on the beach proper marks the position of the plunge line. This furrow occasionally is so abrupt as to hamper the movement of landing vehicles.

The offshore slope deepens more or less uniformly away from shore. If the gradient of the bottom is uniform landing operations are relatively simple, unless the slope is very gentle. On many beaches, however, one or more bars are built upon the offshore slope. These bars are the result of wave and longshore current action. They shift their position and change in size, particularly during storms. The channels between bars differ in depth and width on different beaches, and they vary in depth on individual beaches from place to place and from time to time.

Of particular concern to landing operations is the development of moderate or strong waves, striking obliquely upon the beach. These oblique waves set up longshore currents, which may scour the channels behind the bars, sometimes to such an extent as to interfere seriously with landing operations, particularly if the landing craft cannot cross the bar and the land vehicles must pass through unexpectedly deep water between the bar and the shore. Intelligence reports should carry warnings as to whether or not particular beaches are subject to scouring of this sort.

The sloping part of the beach above low tide level and adjacent to the water line is called the foreshore slope. The upper part of the foreshore slope generally is softer than the lower part, and hence, may cause trafficability problems. On many beaches the foreshore slope merges at its upward end into a flat or nearly flat bench, or varying width, and commonly a foot or more below the flat backshore. On some beaches the bench merges imperceptibly into the backshore. These flat parts of the beach, are called berms. The berm adjacent to the foreshore slope is called the summer berm, and the berm that forms the backshore in front of the dunes is called the winter berm.

The berms are formed by overflow of the highest waves at high tide. Such waves rush up the foreshore slope and slop over the crest depositing their load of sand upon the flat berm as the water sinks into the sand. Strong waves at high tide tend to wash away the front part of the berm. The strong storms of winter cut the summer berm back and may remove it entirely, forming a winter berm at a higher level than the summer berm. Extreme storms may even cut back the winter berm that forms the normal backshore of the beach.

As the waves of lesser storms do not rise above the crest of the winter berm, the winter berm in the course of years may build seaward until such a time as it is again cut back by a severe storm. The smaller waves of summer result in the formation of the summer berm. In the course of the summer the summer berm may advance a few hundred feet, only to be cut back again in the following winter. Moreover the width of the summer berm varies from day to day and from full moon to full moon, some times so much as to cause problems in landing operations.

It, therefore, should be borne in mind that sandy beaches are not stable. They usually are more stable where the tidal range is low than where it is high. Landing mats may be buried beneath sand from one tide to another, or sand beneath them may be scoured. Burial of landing mats is more likely to occur when the height of high tide decreases from day to day than when it increases. Intelligence reports should carry information as to the stability of beaches in particular areas.

All types of beaches possess the above-mentioned characteristics though in varying degree, depending upon the extent to which the waves move the rock, sand, mud, or coral particles that form the beach.

Rocky Beaches

Rocky beaches generally are found on coasts off hilly and mountainous areas, though on almost all rocky coast lines, some or all the coves between promontories possess sandy beaches on which it is possible to land. If wave action is strong and the offshore slope is so steep that it will not support masses of cobbles or of sand, bed rock alone is exposed along the shore, with the result that landing operations are hazardous, waves are not reduced in force as they approach the beach, and both the offshore and foreshore slopes are rough. Furthermore, the approach from the beach to adjacent land terrain is likely to be steep and rough.

Rocky coasts generally have beaches of cobbles or sand between some or many of the promontories, and little or no beach around the ends of the promontories. In areas of strong wave action the beaches generally consist of cobbles, unless the offshore slope is gentle, as along the coast of Washington or Oregon, or where a plentiful supply of sand is brought into the streams by the rivers. In such places the beach generally consists of sand. However, if wave action on the beach is strong and rocks are supplied the beach by the rivers or waves, the beach will consist of cobbles. Cobble

beaches have steep slopes, in many places as much as 1 on 4. The water offshore from cobble beaches deepens rapidly, thus permitting craft to land on the beach directly. Wave action rounds the cobbles, thus lessening the friction or wedging action between the cobbles, with the result that traction is likely to be poor when vehicles climb the beach. The approach from a cobble beach to the adjacent terrain may be difficult because of roughness of the land.

Volcanic islands may have rocky beaches composed of cobbles or of sand. They also may have beaches consisting of volcanic ash, such as Iwo Jima, in which case they may be steep, high, and have poor trafficability. Local intelligence reports and airplane photographs should give information on the presence of volcanic ash.

Beaches composed of cobbles are found in some places along gently sloping or low coasts, in areas where glacial deposits occur, or where the rocks that crop out along the coast contain pebbles (conglomerates). The beaches in such areas may be sandy during most of the year, but during the winter, especially in places where the offshore slope is steep and wave action upon the beach is strong, the sand may be removed from the shore, leaving a steep high cobble beach. During the winter such a beach might be difficult to climb and during summer it might still persist as a scarp at the rear of the summer berm.

Sandy Beaches

Most beaches exposed to the ocean consist of sand of varying degree of coarseness. Landing of craft upon sandy beaches depends upon the angle of inclination of the offshore slope. If the angle is gentle, the waves tend to decrease in force as they approach shore, thus favoring landing on the beach. However, gently inclined offshore slopes facilitate the development of bars and channels behind the bars, which may interfere with landing operations. Also in areas where the offshore slope is gentle but irregular, waves are refracted, thus tending to increase in height in some areas (convergence) and spread out in others (divergence). Wave action accordingly differs along such beaches. Rip currents sometimes develop in areas of convergence where waves approach different parts of the beach at different angles.

In landing on sandy beaches where the waves approach the beach at an angle, it is well to bear in mind the possibility of longshore currents flowing in the same direction as the waves approach. If waves are strong and impinge upon the beach at a sharp angle, undesirable channels may be cut between the beach and the bar.

The beach proper varies in height and slope from one beach to another. The angle of slope ranges between 1 to 100 and 1 to 5. As a rule, the coarser the sand, the steeper is the slope. The height of the beach depends upon the height of the tide and the height of the waves. A crude rule gives the elevation of the crest as high tide level plus 1.3 times the wave height. Thus, beaches will be lower in height in places where refraction causes the waves to be lower, such as places in the headlands. Wave action on the beach commonly is greater where the offshore slope steepens rapidly from shore, because waves on such slopes lose little energy as they approach the beach.

The angle of slope of the beach also varies with the grain size, being steep when the grains are coarse and gentle when they are fine. The size of grains depends upon the material supplied the sea by streams and by erosion of the seacliffs. It also varies with the size of the waves. Local intelligence reports should be consulted for information upon steepness of the beach. As a rule, the angle of slope on any particular beach changes very little throughout the year, though immediately after strong storms the angle may be less.

The foreshore slope is gentle near low tide level and increases progressively upward to mid-tide or high-tide level and then is constant almost to the edge of the berm where it rounds off to conform to the flat slope of the berm. The lower part of the foreslope, which is continually wet, is firm and affords better trafficabiltiy than the upper part, which is alternately wet and dry, and hence is softer. The berm, except after rains or extreme high tides or stormy periods, usually is dry and affords poor trafficability.

Approach to land depends upon width of berm and sand dune areas, roughness of boundary between backshore and land areas, vegetation, and wetness of soil. The sandy parts of the approach to land are more trafficable when wet than when dry.

Sandy beaches are more or less unstable. They change in width from year to year and from storm to storm. Some beaches tend continually to progress seaward, with the result that a series of ridges called beach ridges are formed behind the beach. Commonly these ridges are parallel to the beach and form obstacles in the approach to the adjacent land areas, particularly since the soil in the swales between the ridges may be soft. In some areas the ridges approach the shore line at an angle. In such places access to adjacent high land may feasibly be attained by using the rear ridge. In this way vehicles can proceed from the beach along the ridge, which is composed of sand, rather than going up and down across a series of ridges and swales. Such a situation was found at Leyte.

Some beaches are receding. The land areas are eroded during times of severe storms, and cliffs are cut into the adjacent terraine, thus causing difficulty in getting from the beach to adjacent higher land areas. Beaches in such areas are likely to be narrow, though in summer months when the summer berm is growing seaward the upper flat part of the beach may be 100 or more feet in width.

The tidal range affects landing on beaches where the offshore slope is gentle and the foreshore slope is steep. At low tide, landing craft cannot approach the shore closely and may hang up on bars. Landing vehicles then must traverse channels between the bars and the beach. At high tide the water over the gently inclined offshore slope is deep enough to permit craft to land directly upon the beach. At such times waves may cause more trouble then than at low tide.

In many areas where longshore currents are strong, barrier beaches build up in front of bays, harbors, and river mouths. In some places where tidal currents are weak, the bars may completely cross the bay or river, in which case, access may be had to land areas at either end of the bar.

Muddy Beaches

Areas of quiet water, such as inland seas and bays, may have soft muddy beaches. Wave action is not strong enough to wash the mud particles away from the sand particles on the beach, with the result that the sediment on the beach consists of a mixture of mud and sand, which when wet is soft and has low trafficability. In many inland bays, shifting tidal currents may develop bars and troughs. The position of these features changes from time to time in response to the changing currents. The bars contain varying amounts of sand and they vary in firmness. The more sand they contain the more trafficable they are when wet.

The beaches in inland bays generally are flat and gently sloping. If the adjacent terrain is sandy and it supplies sand to the bay, the beaches are sandy. If the water increases in depth seaward at a sufficient rate to allow large waves to reach the shore before breaking, steep beaches may develop. Such a condition is particularly true of areas of high tidal range and where wave fetch is several miles in length. Beaches of this type may be soft and muddy at low tide level and sandy and firm at high tide levels. Rocky beaches are found in some inland bays, where wave action is strong and a supply of rock material exists.

Bays may be bordered by marshes. The sediment in front of the marsh is likely to be soft, and the marsh is apt to be cut up by sloughs with torturous channels. Airplane photographs should give information on this question.

Access from bays and inland seas to the adjacent land varies greatly from one bay to another. Local intelligence reports should be consulted.

Coral Beaches:

In tropical areas off low lying coasts many beaches are composed of aggregates of ground-up coral and shell material. This material commonly is of sand size and is similar in character to ordinary beach sand. In some areas, particularly where wave and current action is strong, the beach proper may consist of barren coral rock, which is more or less rough and steep. In a few places, especially in sheltered areas, where wave action is weak, the material on the beach may consist of limy clay and be soft. Although coral beaches are restricted to tropical or near tropical waters, not all beaches in the tropics consist of coral material. In many places especially where the adjacent land terrain is hilly, the beaches are composed of sandy or volcanic material, just as they are in temperate latitudes. Conditions on coral beaches vary so much from one beach to another that where ever possible local intelligence reports should be consulted.

The principal characteristic of many coral beaches is a low flat area in front of the beach, - the so-called coral reef. The water on this flat area is shallow, generally only a few feet deep. In many places it is exposed at low tide. Though the reef in general is remarkably flat, its surface is likely to be somewhat irregular. The low places commonly are covered with masses of coral sand. Here and there they contain masses of living or dead coral, which rise varying distances above the reef floor and may interfere with navigation. The high places on most reefs are rough and rocky, though on some reefs, they consist of soft limy mud. In either case they are hazards to navigation. Wind can change the level of water on reefs sufficiently to interfere with landing operations. Sustained on-shore winds tend to pile water up, making it deeper; off-shore winds tend to blow it away making it shallower. The water on the seaward side of most reefs is deep. In the South Pacific, sea-going vessels commonly can approach within a few hundred feet of the edge of the reef, as water several hundred feet deep is found within 100 feet of the reef.

The reefs vary in width from a few feet to more than one mile. As a rule the reef is wider on the windward than on the leeward side of islands. The outer edge of the reef commonly is higher than the adjacent flat part of the reef. This outer fringe is irregular in shape, height and width. Ordinarily it is less than 100 feet wide and 10 feet high. It definitely is a hazard in landing operations. Openings or passages across it commonly are a considerable distance apart. They are irregular in shape, depth, and course. Masses of coral commonly protrude above the bottom of the channels or extend outward from the sides. Many of these protruding masses are composed of living coral. This living coral can grow at a rapid rate, in places as much as a foot a year. As growth is a function of food supply, changes in size of coral masses are more likely to be rapid near the outer edge of the reef than near shore.

In many places in the south Pacific, coral atolls are found. Atolls are more or less circular masses of reef material enclosing a body of water. They may be as little as one mile in diameter or as much as 100 miles. The water within the atoll ordinarily ranges in depth from 20 to 200 feet, though it may be several hundred feet deep. The reefs that form the outer edge of the atoll are similar to reefs that border islands. The flat part of the reef may be many hundred feet wide and crossed by only a few channels deep enough for navigation by sea-going vessels. Masses of coral material protruding above the sea floor sufficiently to interfere with navigation may be found at any place within an atoll. In places the ring-like reef that surrounds the atoll rises above sea level to form islands. The passages between many islands are sufficiently shallow to permit landing vehicles to cross them at low tide. The beaches on the inside of the islands generally are similar to the beaches on the seaward side, except that the size of constituent particles may be smaller, because of the generally lesser wave action. Some atolls have islands of volcanic rock rising within them. The beaches on such islands correspond with the type of rock found on the island.

BY W. N. BASCOM

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I. BEACH CHARACTERISTICS

1. Introduction

a. Definition and Scope

"A beach is a deposit of material which is in transit either along shore or off- and on-shore." This definition by Johnson (I-I-37) and Gilbert (I-I-21) seems to best express the concept of a deposit of unconsolidated material which is subject to movement by wind and waves; it will be used as the basis for this discussion. "In transit" is extended to mean subject to movement by the storms of a decade or so rather than movements measured in geologic time or due to abnormalities such as tsunamis.

A definition which is in more general use is worded as follows in a U. S. Naval Photographic Interpretation Center report: "A beach is the zone containing unconsolidated material extending inland from the waterline (shoreline) to the effective limit of normal storm waves." As Johnson has pointed out, the interest is really in the deposit rather than in the zone and historically beach has always meant the material rather than its location. Moreover, a discussion of beaches whose seaward limit is placed at the low waterline would necessarily omit important underwater features such as bars and rip channels.

Figure I-I-1 illustrates beach terminology by means of a "typical" profile; since there is not full agreement between authorities on terminology, the reader may notice small differences between this diagram and others previously published. It is, however, consistent with the thoughts expressed in the text. Such a diagram seems to be more satisfactory than a series of descriptions and no attempt will be made to further define the parts shown.

b. Waves and Surf

Beaches are dependent on wave action for their origin, existence, configuration, and a short review of the important aspects of wave behavior may be worth while. Unquestionably, beaches are somewhat affected by waves of all sizes from ripples to tsunamis; however, it is swell, and particularly the swell from great storms, which produces the greatest and most important effects on a shoreline.

Storm winds at sea generate great waves. Waves radiating outward from a generating area are called swell; as they approach a coast and feel bottom in the shoaling water, they bend to approximately the shape of the underwater contour lines. As the depth gets small these waves peak up, and in water about equal to 1.3 their height, they break. Until breaking takes place, these waves are oscillatory and there is but small forward transport of the water particles; after breaking, if the water becomes continually shallower, the waves continue as foam lines or waves of translation and the water particles move shoreward with the wave form.

On reaching the beach face, this horizontally moving water is diverted upwards along the face in what is known as the <u>uprush</u>; when the uprush slows and thins out to a layer of water about half an inch thick, it is referred to as the <u>swash</u>. At its maximum landward limit on sandy beaches the swash

deposits a thin line of sand grains which are usually of a larger size than the rest of those on the beach. This line is called a swash mark and on receding tides several can usually be seen. Figure I-I-23a shows a swash crossing the berm. When its upward energy is expended some of the water in the uprush sinks through the shore face but the bulk of it returns down the surface of the beach face as the backrush. When the phase relations are right (usually about each third to fifth wave) the backrushing water meets and flows under fairly still horizontal water forming a low curling breaker of "sandy" water several inches high. This is the point of backrush and it is approximately the elevation of the wave troughs in the breaker zone. If one must select a definite point as the water-land boundary at any given time, this is usually the most satisfactory, although it admittedly may vary somewhat in position both horizontally and vertically.

Sometimes backrushing water forms diamond shaped patterns as it flows down the beach face. These are called rill marks and are rarely over six inches in their longest dimension and are of low relief. The sediment carried down the beach in suspension by the backrush usually consists of sand grains much larger than the average - like those in the swash marks. After passing the point of backrush the velocity diminishes abruptly and this coarse material deposits in a low seaward-facing step. Small waves which reach the beach face, break over this step which is called the plunge point. Because of the turbulence caused by the breaking, the water at this point is usually highly agitated and the large sand particles are continually carried up and down the beach face. This is thought to be an important way in which waves sort out large particles and abrade them to uniform size.

Waves transmit large amounts of energy which are hurled at the coast as a result of storms at sea; it is the distribution of this energy by refraction which governs the ultimate shape of a shoreline. Whereas on land the differential erosion of rocks of varying resistance is of the greatest importance, on a shoreline the concentration of wave energy at resistant headlands by the refraction process greatly reduces this effect. This redistribution of energy near shore, indicated in Figure I-I-6, results in the reduction of wave action in embayments; quite naturally then, the detritus churned up at the headlands may be expected to deposit in the bays. The deposition of beach materials in locations of lesser turbulence or where the net forces are small is a principal factor in the creation of all beach forms. Although refraction by the bottom is effective in bending waves to fit the shoreline, the process is not always completed and the waves may strike the beach at an angle. This situation usually sets up a littoral current parallel to the shore in the direction of wave travel. If waves arrive from the same direction throughout the season the current will be a normal condition and there will be a relatively steady drift of sand alongshore with the current. Many of today's shoreline sand problems are the result of man's interfering with the littoral migration of sand along a coast.

Surf is the breaking of waves in shoal water, particularly on beaches; its character depends about equally on the waves and on the bottom configuration. It may be said that waves plus beach equals surf; in any event, the two are inseparable and the beach shapes the waves as much as waves shape the beach. The principal beach changes take place within the breaker zone; however, it should be noticed that most of the wave energy goes into making the water turbulent and only a minor part goes into influencing the beach.

When beach material is supported by (suspended in) turbulent water, very small net forces can move it; this is the key to the understanding of sand movements. It is not necessary to have alongshore currents which themselves exceed the threshold velocity of the material so long as wave currents with no net movement keep the material in suspension.

It is generally appreciated that the high waves can do more damage than small ones to shoreline structures. The effect of waves on beaches are governed by another factor: the rapidity of delivery of the energy to the beach. This is known as the wave steepness or H_o/L_o* and it undoubtedly is the largest single factor in determining the characteristics of a beach at any one time. Because L_o is a function of the period ($5.12T^2$) it can be seen that the value of H/L increases either with height or with the reduction of the period. Therefore short periods as well as high waves contribute to the ability of waves to damage a beach (move the material offshore).

Waves are highly variable. Although it is convenient to think of any wave as being a part of a great train of similar waves all of the same height and period like those in a model tank, such is not the case in nature. The rather remarkable changes in waves from minute to minute may well account for some of the beach changes which are not otherwise readily explainable.

2. Geological Background

It is necessary in describing shorelines and coasts to speak of their origins in terms of major earth movements and widespread coastal and physiographic changes involving great areas over thousands of years. Our most reliable means of discovering the trends which are shaping the world's coasts is to examine the evidences of past changes that are indicated by a study of today's shoreline features; by extension, the shape of future shorelines can be prognosticated. A study of beaches therefore involves some consideration of long term geological trends. A brief account of the influence of the major geological processes, erosion, deposition, and diastrophism will perhaps assist some readers to understand these long term trends a little better.

a. Erosion, Deposition, and Diastrophism

Erosion is the wearing away and removal of rock material at the earth's surface; its principal agent is water, which in the form of rain, streams, ice, and waves, is constantly at work lowering the level of the land masses. Chemicals of the atmosphere, bacteria and large animals, roots, ice, temperature changes, and winds all assist in the erosional process but the effects of water are by far the most important. About 1/3 of the water that falls as rain or snow on the land masses runs off in the form of streams; as it goes it dislodges small particles, dissolves soluble constituents, undermines and abrades the larger rocks, and carries the finer particles in suspension. The ultimate destination of the water and the rock material it moves is the sea. Usually only fine materials reach the sea from larger rivers and the coarse materials which form beach deposits must be supplied from short

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* H_o = Deep water wave height
L_o = Deep water wave length

steep streams which drain near-coast mountains or from erosion of the sea cliffs by the waves. There is no general agreement as to the relative quantities of beach materials supplied by wave action and by supply from streams even in areas which have been the subject of considerable study. Once the detritus reaches the sea it is immediately subject to sorting by wave action. Generally the coarser material appears to remain near shore and the finer material to settle further out in regions of lesser turbulence, particularly on sandy beaches (see Figure I-I-34).

(Many samples of offshore bottom sediment have shown the existence of coarse material in deep water, often in proximity to fine material; however, the origin of these deposits has not yet been satisfactorily explained.) It is plain that erosional and depositional forces work together as great levelers by cutting down the high lands and depositing them in the ocean basins. Three-fourths of the earth's surface is now covered with water which has an average depth of two and one-half miles. The land areas, though, only rise an average of one-half mile above sea level so that if the earth became smooth the sea would cover it everywhere to a depth of two miles. It would seem that with this vast amount of water constantly and effectively attacking the land and moving it to deep water, the land would soon disappear. However, in the 2,000 million years of earth history, this has not happened because of a great counterbalancing process.

Great flexing of the earth's crust called diastrophism occurs from time to time and mountain masses are heaved up. This mountain building usually occurs in the sites of prehistoric seas which received great depths of sediment eroded from pre-existing land areas.

Tremendous lateral forces acting parallel to the earth's surface transpose the sea and land; the sediments are upraised and become elongated mountain ranges. (Note that the principal mountain ranges of the world are parallel and adjacent to the continental margins.) An example given by Longwell, Knopf and Flint (I-I-50) shows how very slight changes in the configuration of earth's crust can produce what we regard as great changes in the ocean basins.

Consider a sphere three feet in diameter. It is dipped into water and then removed; the film of water that adheres to the sphere, would, at that scale, represent a sea one-half mile deep. If the surface of the sphere were warped 1/100 of an inch the change would correspond to the formation of an ocean basin 13,000 feet deep. Comparable up and down warping of the crust must have occurred many times in the earth's history and the land has survived, although much of it shows evidence of having been sea bottom at one time. Since sea level is our datum, it is natural that these changes in elevation of land with respect to sea are particularly noticeable at the shoreline. There is no general agreement as to whether the sea level rose or the land subsided (or vice versa) at any particular time but the relative motion is quite evident. These changes result in raised coastal terraces and beaches, drowned valleys and other features.

Diastrophic movements take place very slowly by man's time standards but they have left abundant evidence as to their direction and magnitude.

We are just now beginning to get a direct record that is long enough to show these movements in the few areas of the world where accurate measurements have been made. Marmer (I-I-51) of the United States Coast and Geodetic Survey (USC&GS) has compared accurate overland level lines with tide gauge records for the last fifty years to show that the Alaskan coast is rising at the rate of about one foot in fifty years.

The U. S. Coast and Geodetic Survey was one of the first organizations in the world to appreciate the necessity for taking great care in the comparison of sea level with the land; serious doubts have been cast on the methods used by foreign surveys before about 1930 and their records are still too short to be of much value. Of much greater immediate importance but possibly of lesser magnitude geologically, are the changes in shorelines due to the erosion and deposition caused by wave action (Keunen (I-I-46) estimates that the yearly contribution of sediments by rivers is 12 cubic kilometers or one hundred times that supplied by marine erosion). Shorelines cannot be regarded as being fixed features and even within the short span of man's observation very substantial changes have taken place. For example, at Lydd, England, the Dungeness Headland has grown at the rate of 25 feet a year for the last two centuries while at Southwold, England, Matthews (I-I-53) estimated the annual rate of erosion at from 15 to 45 feet. The volumes involved in erosion and deposition seem to be about the same on the English coast because the shores which are retreating stand high while those which are building are limited by the height of wave action. A report of the Royal Commission to study erosion indicates that in a 30 year period England, Wales, and Scotland lost 5,507 acres of ground but at the same time gained 40,151 acres. The very short term changes that beaches undergo with individual storms and seasons will be discussed at length later on.

b. Shoreline Features

Geologists are primarily concerned with the erosion and retreat of the coastal rocks and a number of terms have come into general use which are used to describe the seacoast features* which are carved by erosion and built by deposition (Figure I-I-2). When the sea surface comes to rest against a land area at a new level, the attack of waves almost immediately starts to remove the coastal rock just above the water line. As this notch grows and the support of the rock above is removed, caving and landslides occur and a seacliff is formed. Differential erosion of soft rocks at water level and differential wave energy (vertically) sometimes forms sea caves or arches and unusually resistant rocks survive for a time in the shallow inshore water as stacks or outliers. The flat, seaward sloping rock area which has been excavated by the waves is known as a wave-cut bench. These features develop rapidly at first for the waves are able to attack the rock with full force since they have been moving in deep water and have retained most of their energy. The material eroded is of relatively small volume and has but a short way to travel. As the process continues, however, the beach widens and slows the waves, causing them to break and expend their energy before reaching the coast. The photograph of the Washington coast shown in Figure I-I-40 is an example of a denuded wave cut bench with numerous outliers. The block diagram (Figure I-I-2) which illustrates the retreating seacliff might be from a photograph of the California Mendocino coast; actually it is a modification of a drawing by Kuenen who probably intended it to represent the French or Danish coasts.

* These features are not always found; their development depends on the hardness, jointing, structure, and other qualities of the coastal rock.

Concepts of the means by which shorelines are modified by wave action are rapidly changing as geologists are tending to rely on field observations more than on the old theoretical concepts. Generalizations can often be shown to be untrue; however a knowledge of the most probable conditions is frequently helpful. It is to be noted that these processes are generally, but not always, valid.

c. Processes Operating on Submerging Coasts

When the sea rises with respect to the land, the newly-formed shoreline is likely to be most irregular. Rocky ridges of the old terrain become peninsulas, river valleys become elongated embayments and the sea bottom is as rough as the old land surface. Waves begin shaping the shoreline, and refraction causes them to attack the promontories first; the deep water offshore allows the full wave energy to be spent directly against the new coast. Streams bring rock debris from the land mass to fill in the deep quiet water of their now-submerged valleys and the inshore sea bottom is likely to become increasingly smooth. The tendency of marine erosion is to straighten the new shoreline and cliffs, and wave cut benches are formed at exposed headlands while the quiet embayments fill in with river sediment and headland debris. Eventually the combination of wave and stream erosion creates a supply of beach material and bay mouth beaches, spits, and other depositional features may develop. Figure I-I-4 shows a possible progression of events, stage 3 of which is illustrated by the photographs of Nestucca Bay, Oregon (Figure I-I-5). If the cycle of erosion continues undisturbed for long periods of time, the entire shoreline will retreat, both rocky ridges and lagoons will disappear, and the mature coast will be quite straight with low cliffs bordered with a wide sand beach and topography both underwater and ashore will have gentle slopes.

d. Shoreline Processes on Emerging Coasts

Land which has risen from beneath the sea also exhibits certain definite features. The sea bottom is likely to be smooth compared to land surfaces since wave action erodes to a selective depth while stream erosion sculptures bold land forms. When uplifted by diastrophism, wave cut cliffs and benches appear as terraces. Figure I-I-7 illustrates two terraces which were carved by wave action and now are well above the sea; on much of the California coast, terraces can be seen which have heights that vary indicating the magnitude of the uplift. Each rise of the coast requires the waves to start over again; soon a seacliff forms, beach material is created, and eventually a straight shoreline with a continuous sandy beach much like that of the mature shoreline of submergence will develop. Although wave refraction is helpful in keeping the differential erosion of coastal rock to a minimum, in areas where the currents are removing the eroded material from the coast, the soft rocks will retreat as rapidly as the hard ones even though the latter forms headlands and is exposed to greater wave action. This is probably a situation that is eventually rectified when the offshore slopes become moderate and detritus is allowed to remain in the coves and re-entrant areas. The present shoreline of the Little River area (Figure I-I-7) is formed in a metamorphic rock and seems to be more irregular than the older (raised) shorelines. Some of the East coast of the U. S. shows evidence of emergence, but it is a much older area and is quite different from that just described--probably because of the wide and gently sloping

continental shelf. Storm waves, approaching this shore have created distinctive features, the most important of which are the offshore bar and the associated lagoon which are described later. With increasing age, the lagoons fill in and the offshore bar migrates landward until at maturity the coast will be a straight low cliff with a wide sand beach--very much like that of the submergent coast.

3. Classification of Beaches

a. Introduction

The observation of differences and similarities of groups of objects and the clear labeling of each group is a basic part of the scientific method. In order to deal scientifically with any large or complex group of individuals (such as beaches) it is therefore necessary to have a scheme of classification. The classes must be logical and mutually exclusive for the object is to clarify the subject and accent the differences and similarities of the individuals. Once the subdivisions are understood and accepted there will be a meeting of the minds on the characteristics of any particular group. The objective, then, is to clarify the nature of any particular beach and show its relationship to the other beaches of the world using information which might be available from casual sources and from standard hydrographic and weather charts. This classification gives a reasonable basis for the comparison of little known beaches with those which have been studied extensively.

A number of ways of classifying beaches have been suggested at various times, none of which appear to be either comprehensive or practical for general use. They have depended on the location of the beach, its origin, slope, or beach material. Neither is the number of bars that a beach possesses a satisfactory basis although this, at first, any seem to be a rather obvious characteristic. This is because the wave characteristics determine the number of bars at any one time and bar conditions change, and bars are frequently so irregular that it is impossible to know how many there are (Figure I-I-18) without surveys or aerial photos taken under special conditions.

The classification presented here is logical and simple but will define a beach within rather close limits because it is based on the abilities of waves to create and maintain beaches from whatever materials are available at the particular shoreline. Since waves are governed in size by the size of the body of water and are modified by the configuration of the shoreline and the inshore hydrography, it is evident that there are three basic items which control the nature of any beach. These are (i) deep water dimensions, (ii) wave refraction (amount of wave energy reaching the beach under consideration), and (iii) beach material. This classification is therefore based on the exposure of beach material to the prevailing storm waves. It is evident that this is a simplification to keep the variables within practicable limits and it is based on several assumptions: (i) that there are prevailing storm waves, and (ii) that these storms are the principal factor in shaping beaches.

There are regular patterns of winds on the earth which are referred to as the "general circulation of the atmosphere." Within these patterns there is of course much variability with both season and location but recognizable storm tracks exist. Oceanic storms frequently originate within areas which

are limited enough so that for most coasts a "wave climate" can be established for each season. This means that most of the storm waves arriving at any point will be from a certain small sector of the compass (direction is the most important factor in refraction). It is evident that an "unusual" storm may influence the beach at any particular time but it seems probable that the normal beach condition will be produced by the predominant waves. Moreover, it will be noticed that the beach classification may change with the season. This is as it should be, for beaches change character with the season, a matter particularly noticeable in small open bays in which the sand shifts from end to end depending on the wave direction.

Whether the storm waves which exist over a relatively few days of the year are more predominantly the cause of beach shape than the light waves which exist the rest of the year is less certain. There is some impressive evidence to indicate that they are:

(i) Wave energy is proportional to the square of the wave height and so the ability of waves to do work increases rapidly with their size. At Santa Barbara, California, the rate of longshore transport in usual calm seas (1.5 ft. breakers) is about 250 cubic yards per day; a three fold increase in the wave height (4.5 ft.) has run the daily rate to over 1,000 cubic yards which is on the order of the square of the wave height.

(ii) Only the storm waves (often accompanied by storm tides) remove the protective beach to deeper water and then attack the back-beach or seacliff directly. This is demonstrated by the presence of the large beach debris, usually cobbles, evenly piled in a filet at the back of the beach along the base of the cliff. These large rock pieces can only be moved by the largest storm waves; it is obvious that when they are, the forward part of the beach has been removed and that the cliff is being attacked at the same time.

(iii) The beach material is distributed along the inshore bottom by these large waves opposite its eventual location; the small waves which exist between storms can only move it ashore and make minor changes in its location. In other words, although the smaller waves build the above-water beach, they have little chance to shift the material alongshore. In fact, occasional large storms move the sand to such deep water that small waves are unable to return it at all.

(iv) Tombolos are usually aligned with storm wave direction and beaches on islands are likely to have more sand on their lee side.

(v) Long beaches which have varying protection from the swells show marked differences which depend directly upon the amount of wave energy reaching each part (discussed at length later on).

Because beaches often exhibit marked changes within short distances it is advisable to apply this classification by considering the beach as a series of short segments of perhaps 200 yards long. Figure I-I-38 illustrates the necessity for this where protection is variable; on long straight beaches of equal exposure this detail is unnecessary.

b. Outline of Beach Classification

(1) Wave size

(a) Oceans
(b) Large seas
(c) Small seas
(d) Lakes

(2) Types of beach orientation

(a) Convex
(b) Straight
(c) Oblique
(d) Headland-protected
(e) Embayed (open, tight)
(f) Reef-protected
(g) Lee

(3) Size of beach materials

(a) Mud
(b) Fine sand
(c) Coarse sand
(d) Gravel and granules
(e) Pebbles and shingle
(f) Cobbles

c. Wave Size

The size of the body of water principally determines the size of the waves on its coasts because fetch and decay distances are major factors in determining wave height and period. The problems of variability of waves from season to season, day to day, and wave to wave have already been mentioned; here we are concerned only with long range statistics. The larger the water area, the bigger the waves; some useful generalizations can be made on this basis. The type of storm, slope of the basin, depth of water, latitude and direction of storm tracks with respect to the coasts are of secondary importance. The water areas of the world on which waves are capable of forming beaches can be classified by size into four groups: oceans, large seas, small seas, and lakes. This classification group is based on probable wave size and its geographical connotations are inadvertent; it is the intention to keep as far away as possible from location groupings which would describe beaches as "Pacific Types" or "Baltic Types."

d. Type of Orientation (of beach with respect to the prevailing storm seas)

There is much evidence that beach location and configuration is controlled by the size of prevailing storm waves which impinge upon the shoreline. The previous paragraphs have established the probable deep water wave conditions and it now remains to determine simple means of stating how much of the storm energy actually reaches the section of beach that is to be classified. Beaches are protected from waves in a number of ways:

(i) They may be shielded from direct wave attack by some direct barrier such as an offshore island, a promontory, etc.

(ii) They may be protected by refraction which spreads the deep water wave energy thinly along the shore; this may take the form of divergence in an embayment or may result simply because the beach does not face into the prevailing swell.

(iii) Protection will vary somewhat with the depth of the inshore water; this is not taken into account by the classification. The depth of water close inshore might be great which would not allow the waves to refract as might be expected or it might be so shallow that the waves would break prematurely and lose most of their energy before reaching the shore.

These possibilities have been incorporated into seven subdivisions which will individually, or in combination, be applicable to any beach. (Underwater contours are presumed to approximately parallel the shoreline.)

(I) Convex beaches: Beaches which arch outward directly into the swell. Such beaches would have refraction coefficients of greater than one and would be subject to very severe erosion because of the convergence of energy at such a place. Sandy beaches of this configuration are necessarily quite rare but on sandy coasts, rounded points may have cobble beaches. Cape Cod is an example of a convex beach on a huge scale; its size makes it effectively a straight beach, however.

(II) Straight beach: Beach facing directly into the waves. Beaches which are completely exposed are likely to be rough, steep, and irregular. Slight variations in storm direction serve only to make complex hydrography (irregular bars) and no protection is possible. Table Bluff or Pt. Reyes beaches in California typify this situation.

(III) Oblique beach: Straight beach facing obliquely into the prevailing seas gave some protection because of the spread of energy by wave refraction. This "protection" is of doubtful value for, although the effective energy that reaches the shoreline is reduced, littoral currents are set up which may do more damage than the waves. The ocean beaches of Long Island, New York are of this type.

(IV) Headland protected beach: Beach with a headland at the windward end which protects the lee side from oblique waves. Such a barrier stops some of the wave energy directly; the part of the wave that passes the headland is spread by diffraction and refraction. Protection decreases as the distance from the headland increases; where it becomes negligible, the beach may become a member of the previous group. The beaches south of Cape Lookout, Oregon or Pillar Point, California demonstrate this type of protection.

(V) Embayed beach: Beaches in bays which face directly into the swell are protected by the divergence of wave energy in the bay. If the bay does not face directly into the swell either of the previous groups may also be applicable. Bays are regarded as being open (width of mouth greater than depth of embayment) or tight (width of mouth less than depth of embayment). Carmel Beach is an open bay, Caspar Creek, California (see Figure I-I-8a) is a tight bay beach.

(VI) Reef protected beach: Beach protected by offshore barrier. Frequently offshore coral reefs, chains of islands, or shallow bars and reefs offer considerable protection to beaches in their lee. The degree of the protection depends on size, spacings, distance from the beach, depth and other factors. Beaches protected by man-made breakwaters or other structures belong in this group. Madagascar, for example, protects the mainland in its lee from Indian Ocean storms.

(VII) Lee beach: Beach facing directly away from the swell like those on the lee side of islands or headlands. These beaches are almost completely protected from the storm seas of the major water area but are, of course, subject to the waves of the sea that they face directly. The beaches on western Japan are protected from the Pacific but face into the Sea of Japan.

e. Size of Beach Material

In order to form an intelligent opinion of the nature of a beach it is essential that one know the approximate size of the beach forming material. The first two steps in this classification dealt with the energy available to

work the beach material; this step is concerned with the materials that are present. Waves do not make beach material (although they do continually reduce the particle size); they must work with what is available and their primary job is sorting it by size which they do very well. This means that on any one beach, the range of material sizes is likely to be quite small and that when a beach is described as being composed of coarse sand (for example) one can be sure that there will be a very small percentage of finer and coarser material mixed in with it. If a second size of material is present in large qualtities, it will be neatly segregated as shown in Figure I-I-31c. (In a later section material sizes are discussed at length and distribution of sizes along and across a beach are given.) There is a direct relation between size of material, exposure to wave action, and beach face slope (Figure I-I-39) so that if the beach is properly classified the slope and the probable trafficability are known. For the purpose of the classification, beach material sizes have been divided into six groups arbitrarily defined in millimeters which should be easy to identify from even the most vague descriptions. The size limits are not intended to be used rigorously.

TABLE I-I-1

Size Groups

Mud	Less than .05 mm
Fine sand	.05 to .25 mm
Coarse sand	.25 to 1.0 mm
Gravel and granules	1.0 to 4.0 mm
Pebbles and shingle	4.0 to 64. mm
Cobbles	Greater than 64. mm

f. Examples

In order to use this classification it is necessary to know the body of water and the orientation of the beach (relative amount of protection), the approximate size of the beach material, and the season. The following examples (actual observations) show the inferences that can be drawn from short beach descriptions made on the basis of the classification.

(1) "Fine sand beach oblique to the North Sea" (Skallingen, Denmark, winter)

(i) Breakers 9-10 feet high

(ii) Flat beach (beach face slope about 1/50)

(iii) Strong littoral currents

(iv) Probably one or two bars (depending on tide range)

(v) Jeeps and 4 x 4 trucks can probably use the beach

(vi) Beach forms (berms, bars) of low relief

(2) "Coarse sand open embayed beach with some reef protection on the Mozambique Channel" (Courrier Bay, Madagascar--any season)

(i) Well protected beach, breakers rarely over 5 feet

(ii) Steep beach (face probably 1/12 or steeper)

(iii) Doubtful if any bars present

(iv) High relief beach forms

(v) Probably even jeeps would have hard time getting about

(3) "Straight shingle beach on South Atlantic" (near Cape Tres Puntos, Argentina--summer)

(i) Exposed beach but summer (November to March) surf rarely over 5 feet high--breaking on beach face.

(ii) Very steep beach face slopes 1:4

(iii) No vehicles can move anywhere on beach

(iv) Large and regular berms, well developed

(v) Probably no bars are present in the summer (maybe deep bar not breaking from previous year)

(4) "Fine sand beach slightly oblique to the North Pacific" (Clatsop Spit, Oregon--winter)

(i) Exposed beach with surf from the larger storms breaking 16 feet high

(ii) Very flat beach face 1:70

(iii) Probably two or three bars present

(iv) Usable by most types of vehicles (especially after erosion)

(v) Low relief beach forms

(vi) Strong littoral currents

(5) "Sand beach slightly oblique to Pacific Ocean storm centers (fine sand with many shell fragments) protected by offshore islands and by a promontory" (Coronado, California--winter)

(i) Fairly well protected, breakers rarely over 8 feet high

(ii) Intermediate slope 1:25

(iii) Probably not usable by vehicles without four wheel drive or by heavily loaded vehicles

(iv) Distinct beach forms

(v) Strong littoral currents

It cannot be expected that this classification will completely define a beach because there are other factors which are sometimes of importance, such as tide range, ice in water, amount of sand and local wind; naturally these will be added to any description when they are known.

4. Beach Environments

The classification of beaches was designed to describe beaches as independent features made of materials which can respond to varying wave action. It is important to realize that they can be considered alone and that, except as the terrain may influence the configuration of the shoreline, beaches do not depend on the kind or shape of coastal rock. Even the beach material may be brought in from elsewhere and may bear no relation to the rock of the nearby land. However, the beach setting is of primary importance in the use of the beach and in order to describe a beach as a land-water bridge, which it properly is, the character of the nearby sea and land areas must be considered. The sea environment is fairly well described by the size of the body of water and the bottom contours which influence the refraction. This section deals with the land environment, particularly with the beach boundary. Since a beach is "material in transit," and it is not always easy to decide where the transit ends; it is realized that the boundary may be vague at times, particularly where dunes are present. The phrase "back-beach" is used to mean the transition zone between the beach and the coast, and often has no definite shoreward boundary.

Beaches may end shoreward against (i) the main landmass (plain or cliff) or (ii) against water (lagoon, bay or marsh), and the discussion will be divided along those lines. The picture of the lagoon beach (Figure I-I-3) illustrates four different back-beach conditions in a short distance although it is evident that the continuous beach is everywhere the same. At the extreme right is a walled pocket beach ending against a cliff; next is a bay-mouth beach ending against a lagoon; then a sea cliff; and at the left the back beach is a flat river bottom area.

In outline form, back-beaches are divided thus:

I. Beaches ending landward against the main coast

A. Land which is the same level as the beach or slopes gently upwards

1. Flood plains and river mouths
2. Streams at bayhead beaches
3. Shore intersecting undulating terrain at a low point
4. Dunes which make a smooth transition
5. Low islands

B. Beaches which end against cliffs of any height

1. Beaches which have exits along the beach
2. Pocket beaches

II. Beaches which end landward against water

A. Barrier beaches
B. Bay-mouth bars
C. Spits and hooks
D. Tombolos

(It is evident that for purposes of amphibious warfare with present equipment, only beaches in section (A) of the first group are of importance.)

a. Beaches Ending Landward Against the Main Coast

A logical division of back beach types can be based on the height of the land shoreward of the beach. We will consider two groups:

(1) Land above the sea level which is on about the same level as the beach or slopes gently upwards from the beach.

(2) Cliffs of any height.

The principal means the land has of being graded to sea level is by stream action--either erosion or deposition. For this reason most beaches in the group will be found where river valleys meet the ocean. Where a valley reaches the sea between cliffs, the back beach will be of the first type opposite the valley mouth, the second on either side.

(1) Land Which Is the Same Level as the Beach or Slopes Gently Upwards: There are several types of the back beaches which are nearly level with the beach.

(a) Flood Plains and River Mouths: The most important, already briefly mentioned, is that of the river flood plain which intersects the open coast. Valley bottoms may be as narrow as a few yards or maybe a score of miles wide; often they will offer the only reasonable access to the interior. Since the river supplies the waves with a large amount of debris, the beach across the valley mouth is likely to be more straight and regular than similar areas not so supplied.

(b) Streams at Bay-head Beaches: A special case of the low back beach at a stream mouth is found at some bay-head beaches; most bays exist because the land there is lower than the surrounding terrain and a stream is likely to be present; if so, the back beach usually has gentle slopes. Bay-head beaches are generally formed by debris which is contributed by the stream but some may be brought from the headlands by the process shown in Figure I-I-4. From many such beaches there may be gentle inland gradients.

(c) Shore Intersections Undulating Terrace at a Low Point: By pure chance some back beach areas will be low simply because waves have cut into undulating terrain. Such a low place will drain some of the land

behind it but will not necessarily support a stream which can appreciably change the configuration. The back beach may meet the beach proper at its own level for some little distance as at Grenville Bay (Figure I-I-15).

(d) Dunes Which Make a Smooth Transition: Dunes of windblown beach sand may create a rather smooth transition between the beach and the terrain behind, and occasionally dunes have covered serious obstacles which existed previously.

(e) Low Islands: Low islands, such as Pacific atolls, which have never existed high enough above the water to form a cliff may also be placed in this group.

(2) Beaches Which End Against Cliffs: Of the beaches which end landward against cliffs there are two simple divisions.

(a) Beaches with Exits: Beaches from which some exit can be found by moving along the beach, i.e., to a beach in the other type. (The back beach at Oceanside, California is a cliff with exits at the Santa Marguerite River and Aliso Canyon.)

(b) Pocket Beaches: Beaches from which no exits exist. Beach ends laterally and longitudinally against cliffs. Bay-head beaches which do not have a stream floodplain or other low land to their rear are also pocket beaches. Some beach areas may be pocket beaches at high tide but have side exits past rocky points at low tide. Figures I-I-5 and I-I-8b illustrate these conditions.

b. Beaches Which End Landward Against Water

This apparently ambiguous title describes a relatively common situation since beaches frequently separate coastal marshes, lagoons, and bays from the sea. Wave and current action builds natural sand embankments which are called barrier beaches, spits, hooks, bay-mouth bars and tombolos. Local conditions and large quantities of sand sometimes make unusual combinations of these features which are given special names; the laws of sand movement are, of course, the same and the basic forms can be recognized through these complexities.

(1) Barrier Beach: A barrier beach (sometimes called an offshore bar) is a narrow strip of sand raised above the water surface by wave action and lying parallel to, but some distance from, the main shore. Between this distinctive sandy island and the coast there is usually a shallow tidal lagoon or marsh area. Barrier beaches or barrier islands vary greatly in dimensions and features; they may almost touch the coast or they may be as much as 30 miles offshore. Widths range from a hundred yards to a mile or more and the sand may rise from 15 to 50 feet above sea level. The spacing of the tidal inlets is an important feature since it determines the lengths of the bar sections; it varies from an average of one every five miles on the south New Jersey coast to the 100 miles of unbroken bar at the mouth of the Rio Grande (Padre Island). (See Figure I-I-9.) Large cities such as Atlantic City, New Jersey, Galveston, Texas, and Miami Beach, Florida, are built on offshore bars which gives some indication of their permanence and stability.

A barrier beach is a permanent feature physiographically but is rather temporary geologically. Underwater (longshore) bars can be formed by a single storm but an offshore bar probably takes centuries; for this reason their origin must be considered in more general terms and "storms" may mean many hundreds of severe storms. Where the sand supply is large and the offshore slopes gentle, large storm waves break a considerable distance out from the shore. In the theory advanced by Beaumont in 1845 (I-I-11) and still generally accepted, these waves cut into the sea bottom and, while part of the debris is carried out to deeper water, another part is thrown upon the landward edge of the submarine cut to form a submarine bar roughly parallel to the shore. Eventually wave action builds the bar above the surface of the water; this new strip of land separates a lagoon from the ocean. The bar widens as wind moves the sand shoreward into the lagoon and new sand is thrown up by the waves. Eventually large dune areas may develop (as at Kitty Hawk, North Carolina) and the lagoon fills enough to become marsh land. The actual position of the shoreline may shift back and forth considerably in a century and a propensity of real estate brokers for accepting its maximum seaward limit as the permanent line has led to many serious "erosion" problems.

Barrier beaches make up a considerable portion of the shorelines of Holland, Brazil, Poland and the East and Gulf coasts of the United States; they do not, however, comprise a very large percentage of the world's coastlines. Where they do exist, the beach features found thereon are like those found on mainland beaches.

(2) Bay-mouth Bars: These are beaches which extend from headland to headland across bay entrances creating a quiet lagoon or bay which may or may not be open to the ocean. The U. S. Pacific Coast has many excellent examples of bars of this kind. The lagoon beaches of northern California typify the closed kind (although in some seasons the lagoon water cuts a channel to the ocean which lasts for a few days) (Figure I-I-3). On the Washington coast, Willapa Bay and Gray's Harbor are tidal bays separated from the ocean by bay-mouth bars (Figure I-I-10). These bars are formed by a combination of two processes:

(i) The embayed area shoals gradually as the headlands erode and retreat, until the bay mouth bottom is shallow enough to be seriously affected by waves and an underwater (longshore) bar forms; as shoaling continues, the bar is built higher and higher until it finally projects above the surface. (This process is much like that described under offshore bars.)

(ii) At the same time, coastal currents transport material alongshore. They remove debris from the eroding points and distribute it along the newly formed bar. The bar grows as the additional sand is added and becomes a relatively permanent feature; if the bay is large enough or has an appreciable source of fresh water, a tidal bay develops and the tidal currents keep a channel open through the bar. Eventually the bar and headland become a straight beach broken only by the river entrance (Figures I-I-4, I-I-5 and I-I-10). Discontinuous bars of this nature are called spits although their origin is somewhat different from the spits which are about to be described.

(3) Spits and Hooks: Shore debris, carried out into open water by current action, frequently deposits in the form of a long narrow embankment. As long as the free end of the embankment is surrounded by water, this ridge of sand is called a spit. Spits require a supply of sand and a dominant current for their growth; as the sand is transported alongshore, any slackening of the current such as may be caused by rapid deepening of the water allows the sand to deposit. Spits usually build fast at the beginning where the water is likely to be shallow, and slower as the increasingly deeper water requires more sand.

Since abrupt deepening of the water caused the current slackening which allows spit building, it may be expected that they will generally form at irregularities in the coast which alter the course of coastal currents. This is true; spits often form on the down-current side of a deflection. The current which supplies the sand, controls the direction in the beginning; as the spit progresses, however, other forces may act to change the direction. This results in recurving. Spits which are sharply curved at the end are called hooks, and those which show alternating changes of direction are called recurved spits.

The growth of Sandy Hook (Figure I-I-11), a spit at the entrance of New York Harbor (an extension of the New Jersey offshore bar) indicates the nature of spit growth. Cape Cod, another famous spit, is undergoing similar changes.

Spits will sometimes be limited in extent by strong transverse currents which remove new sand as rapidly as it is deposited. This is particularly true of the spits which create tidal bays (bay-mouth bars); the tidal currents are dominant and the channel shifts at the expense of either boundary spit (Figure I-I-10). This shift of channel between sand spits creates a need for structures which will maintain a fixed channel suitable for navigation; for this reason jetties (rock structures which confine the tidal flow) are usually built on spits. Since the longshore currents which originally built the spit are still operative, jetties frequently act as groins and stop the flow of sand along the coast. This of course results in beach widening on the up-current side and retreat on the lee side.

The size of the bay enclosed by a spit governs the tidal currents and the width of the entrance because the volume of water which passes through the entrance on each change of tide is controlled by the size of the tidal prism and in turn controls the size (vertical section) of the entrance. This means that there is a minimum size of tidal entrance (a maximum size of spit) for any confined body of tidal water. This would apply most frequently to the bay-mouth bar type of spit although spits like that which creates Presque Isle Bay on Lake Erie do occasionally form the complete enclosure.

(4) Tombolos: A tombolo is a bar which ties an island to another island or to the mainland; it is literally a "connecting bar." These bars are quiet water features which form (typically) on the lee side of near shore islands where wave action is a minimum. Waves striking the island are partly reflected; the remainder of the wave refracts around the perimeter, the energy being spread thinner and thinner as the wave front is extended. On the lee side, the fronts from the same wave arrive from opposite sides and

meet head on; at this point the net transporting power is zero and here the sand begins to deposit. The first effect is the formation of a small beach in the lee of the island and the widening of the beach at the point of greatest protection of the mainland; these beaches grow until they become connected with an underwater ridge of sand which eventually rises above water and then widens along the mainland side. The sides of the tombolo are smooth curved beaches which parallel to the wave fronts that formed them; they exhibit all the features of other curved beaches protected from the prevailing swell. Tombolos illustrate the basic rule of sand movement; sand will leave turbulent water and deposit in calm protected water such as an island's lee.

The city of Santa Monica, California has a would-be tombolo in its harbor resulting from an artificial island (offshore breakwater) that was built to protect small craft from wave action (Figure I-I-13a). The tombolo effect can be observed on any beach which has a rock or wreck projecting above the sand. It appears as a pile of sand on the shoreward side of the obstruction.

Streams entering the ocean are opposed by the action of the surf which will frequently dam them with a temporary berm and cause the stream to seek a new outlet. In some areas where there are rocks or islands offshore, the stream will almost invariably enter the ocean in the lee of the rock where it can maintain a channel with the least effort. This is because the berm, through which the stream must pass, is lower where the waves are lower. At La Push, Washington (Figure I-I-13b) a tombolo has formed in the lee of James Island with the sand brought by the Quillayute River which enters the ocean there. The river entrance is well protected and the shelter became so popular with small fishing craft that jetties were built and the river was dredged.

Ships which run aground and are broached (nearly parallel to the beach) are endangered by the formation of a tombolo amidships and scouring under the bow and stern. This results in severe flexing of the ship on each wave so that the bottom plates over the tombolo fail in compression and the ship's "back is broken." When the Kenkoku Maru grounded broadside a tombolo formed as just described but the ship held together. In the first pull by salvage crews, the ship moved 200 feet along the beach; within 20 hours the tombolo had completely reformed opposite the new location in spite of the fact that the sea was fairly calm.

5. Beach Features

a. Longshore Bars

Longshore or beach bars are ridges in the fragmented material of which the inshore bottom is composed. They are the result of storm conditions which create steep waves and consequent seaward-flowing bottom currents that remove material from the beach face and carry it seaward. When these currents encounter the normal landward moving currents which transport sand ashore, the net current is zero and the sand deposits to form a bar. Between bars and between the innermost bar and the beach are depressions which are called troughs which are really a part of the bar since they originate from the same currents; these troughs tend to become channels for rip and feeder currents and as such may be severely modified. The reader is referred to Figures I-I-28 to -30 which are profiles of various beaches and show bars and

troughs as they exist on various beaches. Longshore bars are termed "balls" by the British and the troughs, "lows," terms which are sometimes encountered in the literature. The number of bars on a beach depends on the tide range, the size of the waves and the bottom slope; they may be found in materials ranging in size from fine sand to cobbles.

Consider a beach with an even sloping bottom (barless beach profile) which is subject to light wave action; sand is moved slowly ashore by the waves because of differential pressures on the sand grains and higher wave current velocities in the direction of wave travel. A storm arises and the waves become much higher; bottom velocities increase and more bottom is affected because the waves feel bottom sooner. Sand continues to move ashore from deep water. When these waves break and become translatory the water moves forward with the wave form and piles up on the beach face; the returning backrush carries sand with it and becomes a seaward-flowing bottom current which transports sand eroded from the beach face. This seaward-moving sand meets the shoreward-moving sand under the line of breakers (where the transformation from oscillatory to translatory waves takes place); at this point the opposing currents neutralize each other and the sand deposits to form a bar. Once the process starts the effect is progressive. The rising mound of sand (the bar) causes more waves to break since it is shallower; it also concentrates the breakers of slightly varying height (which formerly broke in their respective depths) in one place. Sand moving toward the bar from either direction is now stopped by the bar as well as by the reduction in current velocity. Waves are sorted by the bar; only those less than some critical height pass across without breaking--when these reach some appropriate depth, they also break and an inner bar may be formed. When bar formation has progressed sufficiently, the bar projects well above the surrounding sand and the trough to its landward side is frequently wide and deep. This results in a further adjustment since broken waves are now progressing in deeper water rather than gradually shoaling water. The translatory waves created by breaking are thus able to reform into oscillatory waves again and although a great deal of energy has been lost in the original breaking, the reformed wave is as large as many of the smaller ones which passed over the bar without breaking; these waves break again on the inner bar (Figure I-I-17). For this reason, practically all waves break on the inner bar but the breakers there are much smaller than those on the outer bar; for those waves which break twice, the difference in height between the two breakers is a measure of the amount of energy expended in the outer breaking. (Note that the height of a breaker is proportional to the square of the energy.)

A bar is the result of a storm condition since the steep* waves required for seaward transport of sand from a beach exist only during storms. J. W. Johnson (I-I-39) has related wave steepness to beach profile in model sand studies and reached the conclusion that if the steepness is greater than .03 a bar will always form, if less than .025 a bar will never form. These figures have not been shown to be entirely valid for ocean beaches but the general relationship is probably the correct one. Large waves and small

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* Wave steepness H_o/L_o or $H_o/5.12T^2$ is discussed in more detail in a later section.

periods are seen to increase the steepness values and thus increase the likelihood of bars forming. The smaller waves are not able to remove deep bars since they do not affect the bottom sufficiently; however, as already mentioned, they do move sand ashore from shallow water and thus will tend to plane off bar tops and fill in troughs. This is abetted by the absence of scouring rip currents in light surf. The modification of the inner bar and the filling of the trough between it and the beach face often results in a nearly flat surface which is called a low-tide terrace. As might be expected, these low-tide terraces are generally found on partly protected beaches or on exposed beaches which have experienced a long period of small waves. This makes them a summertime feature in most localities; a sudden storm will cause rips to develop in these terraces perpendicular to the beach face which have no appreciable feeder currents (Figure I-I-19).

Thus, a large storm may create a deep bar which will remain largely unchanged through months of calmer weather (until another group of large waves exists to change it) although at the same time there may be considerable change in shallower bars and the beach inshore. This means that the depths of the several bars are not necessarily related except immediately after a large storm and that valid comparisons of bars and waves require continued observation of the beach before, during and after changes. Little is known about the movement of ocean sand below Mean Low Water because direct measurements are difficult to obtain. Experiments by the Beach Erosion Board with artificial bars at Long Branch, New Jersey and Santa Barbara in the past decade gave negative information. In the hope of nourishing these sand-starved beaches, underwater sand ridges were placed in 38 and 18 feet of water respectively, the idea being that the wave action would bring the sand onto the beach. According to Hjulstom's formula (I-I-25) for threshold velocity, the sand should have been moved by the waves that existed; however, the sand did not move appreciably and the mounds are still there. It should be noted, however, that no bars are known to exist even on the most exposed beaches on the U. S. Pacific Coast which have crests more than 20 feet below mean lower low water. Figures obtained from averaging tabulated data from 29 surveys of exposed U. S. Pacific Coast beaches with two or more bars have turned up some interesting facts:

Bar depths

Depth of inner bar	-	1.0 ft.	
			6.5 ft. difference
Depth of second bar	-	7.5 ft.	
			5.5 ft. difference
Depth of third bar	-	13.0 ft.	

Beach slopes (measured between MLLW and -30 ft. or 3000 ft. offshore)

Less than 1/75	three or more bars
1/75 to 1/50	two bars
Greater than 1/50	one bar

Note: All exposed beaches had at least one bar.
No individual beach varied greatly.

The phrase "exposed beach" was discussed previously and the importance of exposure has been emphasized; however, the specific application of this to bars is of interest. Like other beach features, bars have lower relief and are composed of smaller particles as protection is reached. Where protection is at a maximum, no bars exist because waves never get steep enough to form them. Figure I-I-15 which shows the bars at Grenville Bay illustrates this well. At the left of the picture where protection is greatest, the waves break on the beach face; as the beach becomes less protected, bars form and at right two well developed bars can be seen. It is emphasized that the waves in the picture are not necessarily the ones which formed the bars. Attempts to correlate bar spacing with wave length or other characteristics have been unsuccessful to date but there is a relationship between bar depth and wave height as already indicated. This cannot yet be stated quantitatively because of the large number of variables involved. Kueligan (I-I-41) compared bar depths with trough depths and decided that the depth of bars to the depth of troughs was a fairly constant relation; i.e., the depth of the bar divided by the depth of the trough (both measured from still water level) is approximately 1.69. Shepard (I-I-62) measured additional profiles and found that the ratio was generally less (1.16 to 1.63). The author is skeptical about the value of such data, having observed extreme variability in short distances and, in one instance, a trough twenty feet deeper than a bar whose crest was at about minus three feet.

Since bars appear on tideless seas, obviously no tides are required for their formation; however, the previous statistics show that the difference in elevation between pairs of bars is nearly the same; this happens to be about the same as the tide range at those beaches. Water level is continuously changing because of the tide's surface but for about an hour before and after high and low tides, the water level is fairly constant. This means that there is considerable time for a bar to develop at a particular elevation during the high and low periods; in the transition period, the level changes too fast for the sand to follow. Because of this, the waves are able to form bars which are in use at both high and low tide; the outer bar at low tide will be the inner bar at high tide (Figure I-I-17). At intermediate stages of the tide, unusually large waves will break three or more times, on each of the bars and the beach face. At lowest tide, the inner bar may be exposed, or nearly so, and occasionally a ridge of sand appears on the seaward side of this bar (Figure I-I-29); this profile may be a vestigal remnant of a berm which started to form at extreme low tide. The tide range probably influences the number of bars that can exist since a large range would bring the water level into contact with widely separated bottom areas. Steep beaches exposed to the same wave action as flat ones have fewer bars because even at lower tides the rapidly deepening bottom is below the limit of wave action except on the flanks of the existing bars. The remarks about the depth of water in which waves influence the bottom must also be modified to take into account the usual rise in actual water level during a storm above that predicted. Although the average depth of the third bar is given as 13.0 feet below MLLW, the waves that cause that bar may only exist during a storm tide and the effective depth of that bar may be several feet more. Moreover, the slope of the bottom influences the depth of breaking and a bar obviously offers a steeper slope than the barless profile. These comments are based on information obtained by making detailed soundings of bars before, during and after heavy surf conditions on a variety of beach types.

A great deal of valuable information about bars can be taken from aerial photos and from observation of the surf from a high vantage point. Generally the two opposites of surf conditions are best for making these observations. Very calm weather may allow the suspended material to settle and the water to clear so that one can actually see the bars on the bottom (Figure I-I-18a). This is a rather rare condition on most beaches; a more likely possibility is high surf. The large waves break on the bars which show up as lines of white water; rip channels and troughs are usually dark. The description of bars which suggests that they are regular ridges parallel to the shore is a considerable oversimplification; although this occurs frequently, it is by no means the rule. Bars are created and modified by waves and currents which are highly variable in direction, velocity, height, and period; there may even be concurrent storm from two directions. As a result, there are many irregular mounds of sand formed which are complex variations of bars (Figure I-I-18b). These bars shift about, migrate landward or seaward, and often intersect the shore at an angle. Occasionally bars appear to be a series of mounds at regular intervals which are called "knolls"; at other times bars assume forms something like huge underwater cusps.

Bars have been observed on beaches ranging in size, and subject to the wave action of, model tanks, lakes, seas and oceans. Many types and sizes have been described; Hagen (I-I-24) who first suggested their origin in 1863 saw five on a single beach; Gilbert (I-I-21) mentions bars on the shores of Lake Michigan* that could be traced for "hundreds of miles" (he probably did not mean that these bars were continuous, but even so, it is quite remarkable). Substantially unbroken bars 20 miles long have been seen on the Washington Coast (Isaacs, I-I-32). It is hard to say just what the length of a bar is, since bars are most easily judged by the breakers on them; breakers are highly variable in height along the crests and a large number of waves would have to be observed in order to say whether a low point was in the bar or in the waves.

The word bar also has a more specific usage for a similar bottom feature of about the same origin as the longshore bars in the surf zone. River or tidal-entrance currents transport sand seaward until they encounter landward-moving wave currents or until they are dissipated through widening of the channel; in either event, they drop their sand. The result is the <u>harbor bar</u>--much like those in the surf but usually further offshore and of limited extent. Like longshore bars, harbor bars are subject to shifting as the currents change and so must be frequently charted and buoyed accordingly. Only the largest waves break on this bar (usually at low tide); a breaking bar is therefore much feared by navigators and often stops all ship movement. Channels are maintained in the bar by dredging and Bar Pilots Association exists for the principal purpose of navigating ships across this narrow strip of dangerous water. At the Columbia River, the pilot boat (I-I-49) has observed breakers 40 feet high (spilling) on the bar a number of times, and at Coos Bay, Oregon in 1950 a destroyer was damaged and two men killed by a bar breaker.

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* The level of Lake Michigan varies as much as seven feet because of rainfall, evaporation, seiching and wind tides; these bars therefore are not necessarily related to the water level as observed.

Great banks of sand exist off the U. S. New England Coast near Cape Cod and Martha's Vineyard which shift so rapidly that surveys cannot be kept up to date; these were formerly a great hazard to larger shipping but now are fairly well marked. The sand assumes the various forms described in this book and spits of all descriptions, giant ripples and longshore bars appear in complex profusion because of the abundance of sand.

b. Rip Channels

Surf rips (or rip currents) are narrow currents of water that flow about in the surf zone scouring channels as they go. Their ultimate destination is the open sea but they are influenced by existing sand forms and their routes are often very devious. Rip channels are cut in bars by currents which are caused by the same steep waves that form the bar; for this reason they exist only on exposed beaches. Like other streams of water, rip currents follow the line of least resistance; they flow in existing channels whenever possible, altering and modifying them as necessary. Because practically all bars are a modification of some previous bar which had its own system of channels, probably no definite time of origin can be assigned to a rip channel; it starts its cycle half-built and, when the storm waves calm, leaves a channel which remains virtually intact until another storm.

The three major parts of a rip current have been assigned names by Shepard et al. (I-I-64), i.e., feeder, neck and head. The feeders are currents which flow parallel to the beach inside the breakers and gather the water which flows out through the neck. The neck is the part of greatest interest for there the current is fast and the channel narrow and deep where the breaker zone is traversed. Once beyond the breakers the current widens and the water spreads and diffuses, sometimes in eddies, as shown in the photos.

Storm waves, after having broken on a beach, become masses of water in the form of foam lines which move landward above the still water level. Since this action results in a bottom return of water which scours sand from the beach face, a bar is formed as described in the previous section. As the bar grows in height it becomes a sort of dam whose hydraulic damping effect, assisted by newly oncoming waves, supports the water between the bar and the beach face at a higher level than that of the average sea level outside the bar. This higher water inside (estimated to be as much as two feet higher in a heavy surf) escapes by means of breaches in the dam (old rip channels in the bar). Once started, these currents gain momentum and become systematic; the troughs between the bars become feeders or tributaries and water is brought to the rip channel from a considerable area, something like a watershed. The current through the bar is now powerful enough to reshape the channel to its needs and it can modify the route with surprising rapidity.

Rip channels are readily observed from the air or from nearby high points, but from the sea or the beach they may not be discernable. They usually can be recognized by the following characteristics:

(i) The water appears dark since it is deeper than its surroundings.

(ii) Waves are generally lower over the channel than on either side and there is much less likelihood of breakers.

(iii) The movement of driftwood or the water itself may frequently be seen (since the current flowing seaward opposes the oncoming waves, the surface may appear choppy).

(iv) Outside the breakers there are large muddy areas of suspended material carried out by the currents; these occasionally show vortex lines.

(v) "Feeder" currents flow alongshore and may be more noticeable than the main current.

Rip current velocities up to about four knots have been measured in high surf and it is quite possible that much higher velocities existed in inaccessible areas of the same surf zone. Generally currents are likely to vary from one-half to one and one-half knots changing with variability of the waves that create them. Rip currents occasionally exist in protected areas or on barless beaches but are most likely to be found on exposed beaches where there is considerable irregularity in the wave action; although surf rips are irregular and sometimes hard to see, they may be the only possible route through a heavy surf for small craft, since they create a breakerless path of deep water--the rip channel. At low tide in any surf, the rip channel may be the only path to the beach through the bar which may be either out of water or have too little water on it to pass a boat. For these reasons, it may be well for persons working in the surf to locate the rip channels before attempting a passage.

A popular misconception of the nature of surf has led to a general belief in a mysterious current called undertow that "sucks the swimmer out along the bottom." As has been pointed out, the bottom currents are of such small magnitude as to transport sand only under favorable conditions. Swimmers are much more likely to be affected by "rip currents" and they can save themselves by swimming across and out of the stream, which is frequently rather narrow, rather than heading straight for shore.

Some confusion has arisen as to whether all narrow currents in the surf should be called rips. While this usage is acceptable, it is the author's feeling that up-currents should particularly apply to currents generated by a difference in head, as described, and that currents caused by waves striking the beach at an angle should be known as littoral currents regardless of their dimensions.

c. Berms

A berm is a nearly horizontal formation of beach material brought ashore by the waves. Berms may be said to be the opposite of bars since they are the depositional sand form and bars are the erosional sand form; as described before, the wave steepness appears to control which of the two will form. There is no general agreement as to the criteria for determining whether any beach has a berm, for on some beaches berms are difficult to recognize. If it is necessary to have a flat surface or a well-defined crest, many beaches will never have a recognizable berm. On the other hand, if the accretion of sand by the beach is to be the standard, obviously every beach must have a berm part of the time in order to exist. The berm is the beach, or at least is its essential sand replenishment. There appears to be no basic difference between the sand deposits that waves leave on very flat

beaches (which are said by some to have no berms) and the sand deposits on intermediate slope beaches which have well-defined berms by any standard. On beach areas which have varying protection from wave action, this is readily observed. On the most exposed portion, the beach may have a definite berm with a rather ill-defined crest (coarse grained beaches rarely have clean-cut berms); on the part of the beach where the sand is intermediate in size and exposure is moderate, there will be a sharp crest and a flat berm top; as the most protected area is approached, this berm fades into a fairly flat beach and is no longer discernable. It will be noted that each of these areas falls within the definition and has a "nearly horizontal formation of beach material brought ashore by the waves"--the point is that on the beach which is always "nearly horizontal" the berm is difficult to see.

Since berms are formed by wave action, height of the crest of the berm is a function of the height of the forming waves above the sea level at the time of formation. Experiments in a wave channel by Bagnold (I-I-2) indicate that the height of the berm is 1.3 x H_o of the waves that formed it. Although this research has not been extended to include ocean beaches as yet, it is the author's opinion that such a relationship will eventually be shown in which some factor (possibly 1.3) multiplied by the refraction coefficient will give the height of the crest of the berm at any point. For this reason, the berm formed by any one set of waves is lower in protected areas than on exposed beaches. This effect has been observed at a number of places including Monterey Bay, California where, on occasion, the crest of the berm is found to be 16 feet above MLLW at Fort Ord, decreasing to 10 feet at Del Monte Creek and disappearing near the harbor.

Berms are depositional features placed above water by wave uprushes. The action of the wave is something like this: Great turbulence exists in the surf zone, particularly at the line of breakers, which churns up the bottom sand and maintains it in suspension. After a wave breaks, the water rushes forward up the beach carrying the suspended sand, losing velocity as it goes because it is opposed by both gravity and friction, and at some point stops completely. When the velocity drops below the threshold of transport, the suspended material deposits. Since beaches are permeable, some of the water sinks down through the sand leaving a lesser amount to return along the surface as backrush. The water that does return on the surface must start from zero velocity; consequently a large part of the suspended sand which was carried up the beach remains near the termination of the uprush. The backrush reaches a high enough velocity to remove some sand from the lower part of the beach but the balance is in favor of deposition. The saturation of the beach face largely controls the relationship between the volumes of uprush and backrush water and it is readily seen that the short period waves keep the beach wetted best because the intervals between saturations are smaller. As already noted, the short period waves increase the H/L values which are the criteria for erosion or deposition. It may be reasoned then that a most important erosional characteristic of waves is their ability to keep the beach wetted face saturated; by the same token, berms build more readily on "dry" beaches since there is less water returning along the surface to transport the sand seaward again.

As this process continues, the sand builds seaward; since the height of the waves above sea level controls the height of the berm crest, the tide

has a considerable influence on berm height. If ocean waves were all of the same height, a rapidly growing berm surface might show undulations in response to the tides. However, berms grow evenly because of the variation in the heights of waves and although the seaward growth depends largely upon the average waves, the upward growth depends only on the largest waves. Where the berm crest is well developed, the uprushes of these largest waves pass completely over the crest and deposit the bulk of their sand load on the landward side. Since this can only occur on the highest tides, the crest grows higher as it builds seaward and the shoreward side of the crest sloped inland. This has two effects on the growth of the berm.

(i) The water rushing shoreward down the inside of the berm levels off irregularities by depositing new sand and scouring off high points with the result that the berm may be quite flat (Figure I-I-23a).

(ii) This water eventually saturates the beach completely and then stands in large puddles; eventually, when enough water gathers it is able to breach the crest of the berm in some weak point and flow back out to sea. Future uprushes crossing the crest deepen this cut and the result is a drainage channel, something like a "rip channel" in a bar but forming on the upper beach (Figure I-I-23b).

This vertical growth of the berm caused by large waves creates an interesting paradox. Storm waves which erode a beach also build a berm during the erosion. The uprushes of the large waves carry sand up and deposit it on top of the berm, adding to its top, and when the storm condition subsides, a narrow but higher berm remains. Since large storms frequently occur in the winter when the beach is narrow, these higher berms are called winter berms and are found near the back part of the beach. They will survive until progressive erosion or a larger storm removes them. A prograding beach may have a series of abondoned berms at its landward side.

Since a berm is a depositional feature, its width is dependent on the sand supply and the length of time that waves exist which are capable of moving sand shoreward. Because of the effects just described the most recent addition will frequently cover and obliterate traces of the previous berm limits. The example of the growth of the berm at Carmel Beach, California (Figure I-I-22) illustrates this point. This particular berm disappears completely in the winter time and builds out as much as 300 feet by the end of summer. The growth suffers set-backs with each storm and technically each additional deposit might be termed a "new berm"; however, the total flat surface area is generally referred to as "the berm."

Erosion frequently results in the formation of steep scarps on the seaward side of the berm which may be as much as five feet high (Figure I-I-4). This is particularly true if the erosion takes place during the neap tides and the effect is one of undercutting. Berms with ill-defined crests may not even be recognized until this scarp forms. Since the berm "is the beach," it is evident that berms can be formed in any sort of material that may compose the beach. The impressive cobble berms of Baja California, Mexico (Figure I-I-31c) are as much as three or four tiers high, 30 feet above MLLW, and extend unbroken for at least ten miles. At Fort Bragg, California the city dumps its waste into the ocean and a very regular berm of old tin cans exists nearby.

Frequently berms have a rather soft surface when compared to recently eroded beach areas of about the same slope; this can be used at times to decide whether the beach is eroding or building. At Clatsop Plains, Oregon, a very flat beach where clear cut berms never exist, it was noticed that on one day a DUKW traveled 20 mph slower along the beach than it had the same place the previous day; although no beach changes were evident, a survey showed that about a foot of new sand had come ashore during the night.

The Growth and Retreat of a Beach (Figure I-I-22) illustrates many of the items mentioned in this section and is worthy of a little additional explanation. Remember that there is a vertical exaggeration of 1:10.

(i) The berm built gradually seaward at the rate of about forty feet a month during most of the summer when the light waves existed; in August and September the rate increased. No further surveys were made until December and February; by then the beach was found to have retreated almost to the cliff.

(ii) The winter berm crests, formed as the beach retreated before high storm seas, were several feet higher than the summer berm crests.

(iii) The growth of the berm from 20 August 1946 to 4 September 1946 was upward and outward partly because the height of high tide increased during that period (until the 26th) and partly because the waves were slightly higher.

(iv) Each successive addition to the berm was higher so that the surface sloped landward to a drainage channel.

d. Beach Ridges

Shorelines which are advancing seaward frequently leave behind a series of low ridges indicating former positions of the shoreline. These lines of growth are called dune ridges, beach ridges or storm beaches in this country and fulls in England. Beach ridges appear as low, elongated ridges of sand or shingle parallel to the shore on the upper beach frequently with marsh or soft ground between them (caused by a rising water table and wind-blown fine sand) (Figure I-I-24). Although they are found on practically every prograding (building) shoreline they are a fairly rare shoreline feature. Some examples of shorelines which are building and exhibit well developed beach ridges are Cape Carneveral, Florida; Darss and Swinemunde, Germany; Sandhammaren, Sweden; and Dungeness, England.

Since the existence of a series of ridges is evidence of beach growth, it is evident that the ridges require that sand be supplied to the ridge area faster than it can be removed by the waves; this implies considerable erosion elsewhere and longshore sand movement into the area. Beach ridges are in the author's opinion berms built by great storms and then abandoned at the back of the beach as it builds seaward. Since it takes a considerable amount of sand to extend the beach, it is evident that every great storm cannot form a lasting ridge. After a period of considerable deposition, though, a storm can form a new ridge which is subject to erosion and adjustment by future storms--as shown by the fact that ridges occasionally merge. Thus a ridge might be the work of several storms or of one. The heights of

the ridges are related to the heights of the waves (plus the tide) which formed them; the crests of the highest ridges having been formed by the highest waves although many of the ridges are low enough to have been formed by something less than a "great" storm. The survival of the ridge depends more on the length of the relatively quiet period after its formation than on the exceptional magnitude of the storm which built it, the basic question being whether the longshore currents can put more sand on the beach than can be eroded by the next large storm. Ridges are often marked by lines of similar vegetation which make them easier to see from the air than on the ground.

To quote Douglas Johnson, "A ridge beach plain is a very imperfect record of a complex history." However, ridge counts seem to be such a good way of obtaining data on rates of beach growth that many investigators have worked on the problem, particularly at Dungeness, England where the records go back hundreds of years. There the average rate of ridge formation is about one each 20-40 years. At Rockaway Spit (New York Harbor entrance) five ridges were formed in 23 years and the rate of growth varied from one ridge in 8 years to one ridge in 2 years. Studies by Solger (I-I-67) on the Swinemunde tombolo indicate that ridges form there at the rate of about one every 35 years, and he attempted to correlate this with Bruckner's climatic cycle (now thought to be invalid). Although there is variation in these figures, each may be quite correct for the specific locality; they point out that there is no worldwide growth rate. The question of whether a coast is rising or subsiding, a common geologic problem, may be resolved sometimes by the relation between the heights of a succession of these ridges.

e. Cusps

Cusps are a series of regular depressions, concave to seaward, which occur in the upper part of the tidal zone of beaches. The nomenclature, shape variation and characteristic wave action is illustrated in Figure I-I-25. The distance from apex to apex along the beach is called the intercusp spacing or length; the distance from the apex to the back of the bay is the width; and the vertical distance from the apex to the lowest point on line with the next apex is the depth. Cusps may vary greatly in both size and shape. Johnson (I-I-37) made cusps three to nine inches long in the laboratory and described them as frequently being eight to ten inches long at Lake George. The author (I-I-3 and -10) has observed cusps varying from six to one hundred feet long at Carmel beach at various times and has mapped cusps ninety feet long at Fort Ord and one hundred fifty feet long at Seacliff (see Figure I-I-23b). On one occasion at Table Bluff, California nine regular cusplike indentations in the beach were measured with a DUKW milometer and found to have an average length of 1,180 feet. Cusps can apparently form in any beach material; cusps in very coarse sand and pebbles have been seen at Cambria and Pebble Beach, California and Isaacs (I-I-38) reports the existence of cusps in large cobbles on Mexican beaches which have steep sides and depths of as much as six feet. The unusually well developed cusps of San Simeon Bay are in fine sand. There the lengths and widths are about equal; however, extremes of variation have been observed elsewhere as shown in Figure I-I-26a. Like other beach features, cusps have more striking relief, and are less regular, on coarse exposed beaches than on the apex points compared with that in the bay; investigators are not yet agreed whether this is a cause or a result of cusp building.

Although cusps were among the first beach features to be described (by Palmer, I-I-58, in 1934) and dozens of investigators have considered the problem of their origin, there is still no general agreement as to their meaning or how they are caused. Several factors that seem to contribute to cusp formation have been noted by various investigators.

(i) Waves must strike the beach squarely and be unconfused by local wind waves or other disturbances. This accounts for the prevalence of cusps in protected bays where the refraction has filtered out all waves but the swell which reaches the shore undisturbed and breaks in even plunging breakers.

(ii) The width of the intercusp spaces varies with the height of the waves. According to D. W. Johnson (I-I-37), a light breeze on a pond made cusps about a foot long; waves 2½ feet high make cusps 30-60 feet long and storm waves build cusps 100 feet or more apart. Deliberate attempts to change this spacing by trenching and arbitrary excavations were useless; the waves reformed the cusps at appropriate intervals.

(iii) A number of researchers have suggested that an original irregularity is needed to start cusps forming: Kuenen (I-I-47) suggests slight depressions; Jefferson (I-I-36), seaweed.

(iv) There is apparently no significance to the location of any part of the cusps. Experiments have shown that if a series of cusps is obliterated and the wave conditions continue unchanged, new cusps of the same size and shape will form with no regard to the location of the previous cusps.

(v) Cusps are generally better developed in places where the tide range is small or during the neaps. This seems to be because the waves have a longer time to shape the beach.

(vi) The direction of the wind, the slope of the beach, and the stage of the tide appear to exert little if any influence on cusp formation. Branner (I-I-12), however, theorized that cusps are the result of interference of two sets of waves of translation on the beach.

(vii) Although none of the proffered theories seemsto adequately explain the origin of cusps, it is apparently easy to see how wave action maintains them. (Numbers in the following discussion refer to the diagrams at the bottom of Figure I-I-25.)

The cusp cycle starts with a straight foam line rushing up the beach:

(i) It is split into segments by the apex points and deflected along the sides of the bay until its force is spent or it meets the water from the opposite side;

(ii) once stopped, it is influenced solely by gravity and returns by the steepest path

(iii) which leads it to the channel in the center of the bay.

(iv) The rather considerable velocity of the water down this channel moves bottom material and a submarine delta is formed at the seaward end of the channel. The next wave meets this jet of water and is largely stopped by a combination of this concentrated backrush and the steep slope of the apex face,

(v) but a part of the water goes through a similar, but smaller, motion which ends in a relatively even backrush as shown in I-I-6.

This two-wave cycle does not repeat every time because variability in wave height and period causes the waves to get out of phase--which is probably the reason that the most regular cusps are formed by the most regular and even waves. It should be pointed out that the high backrush velocities which are associated with the removal of sand from the beach exist only in the channel, which is consequently flat, whereas the highest uprush velocities and least saturation are found at the apex, which is steep and depositional.

f. Ripple Marks

Sand subject to the action of moving water frequently forms parallel ridges and troughs called ripple marks. There are two distinct types, each due to a particular sort of water motion (Figure I-I-27b).

(1) Current Ripples: These are asymmetrical and are formed by water flowing in a single direction; they have a long, gentle slope on the side from which the current comes and a steep, short slope on the lee side. According to Owens (I-I-57) and Cornish (I-I-14), they migrate slowly with the current when water velocities range from .85 to 2.2 feet per second and against the current when velocities are from 2.2 to 2.5 feet per second. Above this upper velocity the ripple marks are swept away completely. Kindle (I-I-42) has concluded that the length of the ripples (crest to crest) varies with the velocity of the current, the volume of material in suspension, and possibly the water depth. Generally ripples are likely to be of the order of a few inches to a foot from crest to crest and less than an inch in depth, but "giant" ripples have been described from several localities which may be as much as 50 feet long and 4 feet deep (Figure I-I-27c). At Carmel Beach in 1946 ripples in shallow water five feet long and over a foot in depth were sufficient to temporarily "stick" a DUKW being used to survey the beach.

(2) Oscillation Ripples: These are created by the bottom currents under oscillatory waves which move in either direction with equal force. Consequently symmetrical ripple marks are built, the shape of which is thought to be related to the length of the waves, the depth of the water and the coarseness of the sand. These are relatively minor features and are rarely seen above water.

The currents in the surf zone are many and complex, and either type of ripple may be formed with any orientation. There is as yet no sure way of determining the position of the shoreline from the direction of the ripple marks although geologists have repeatedly tried to do this, using the fossil ripple marks which are commonly seen in sedimentary rocks.

6. Beach Profiles

A beach profile is a line representing the intersection of a vertical plane perpendicular to the water line with the surface of the beach. It may be extended as far as its use requires or as conditions allow; however, experience has shown that the zone from -30 feet (below MLLW) to the start of the back beach is likely to include all the significant features. Although little is known about sand movements in water deeper than 20 feet, it is evident that changes do take place because submarine cables, crab traps and other objects on the bottom are alternately covered and bared. These changes are of small vertical magnitude compared to those in shallow water since relatively few waves influence the deeper bottom, but they may be very important in explaining sand migrations. The physical problems of locating an exact spot and getting depths below a fluctuating water surface, so that successive soundings can be compared, are difficult; a more accurate way is to measure an object buried in the sand. Variations in average water level as shown by tide gauges indicate that water level changes within very short periods of time are of the same order of magnitude as the probable sand changes. At protected Carmel Beach sand changes deeper than -22 feet are relatively minor; at exposed Table Bluff, 40 feet seems to be the limit. It is to be emphasized that the absence of elevation changes in the sand surface is not a criteria as to whether the sand is moving at or past the point. It has been already pointed out that underwater beach features may be highly irregular and therefore any one profile may not give enough information about the actual conditions there; it is usually desirable to take a series of profiles at equal arbitrary distances in accordance with good sampling procedure. If the intention is to actually make a detailed underwater chart, a rather large number of profiles may be required; only a few are necessary to indicate the slope of the bottom or the character of the bars. Surveys made for the purpose of studying beach erosion or accretion generally require that the elevations of underwater points be known with the greatest possible accuracy, since small errors quickly multiply out into many thousands of yards.

Accurate hydrographic profiling is largely a matter of care and technique; the details of how a lead line is used, how the position of the sounding is located, how the true elevation of the sea surface is established and how the land and sea profiles are matched are all important in determining the final accuracy. The use of a recording echo sounder has been tried and is useful in quiet water; however, in any surf with breakers over about four feet high, these instruments create more problems than they solve. An experienced survey crew can be expected to make a bottom profile with an accuracy of about 15% of the breaker height since the problem of how to use the sea surface as a datum is largely related to the size of the waves. Other errors are introduced because it is necessary to regard the sea surface as a level plane; actually it may surge (rise and fall in the surf zone with periods of several minutes); may slope upward towards the beach; or may not have the level, with respect to land, that is predicted in the tide tables. As a whole, the profile of a beach will change very little but the berms, bars, and channels migrate along it quite rapidly (sometimes changing appreciably before the survey work is even plotted); therefore, the use to which the profiles are to be put most be considered carefully.

Shepard (I-I-62) has been able to establish a continuing program of profile surveys at short intervals by the simple means of sounding from piers that project out into the water; this gives accurate results even in heavy weather, but the piers influence the waves and the sand, do not extend to very deep water and rarely exist where a profile is needed. Hydrographic surveys of inshore waters have been made with swimmers and with rubber boats (using range markers and a distance line) on many Pacific islands; with lobster boats on the New Jersey coast; and with all sorts of small craft willing to venture into shoal water in other parts of the world (most of which obtained their sounding position from sextant bearings of shore points in the manner of a ship). These craft all have the disadvantage of having to work on days when the sea is fairly calm and the breakers are very small or nonexistent. The need for information about the storm profiles of the northern U. S. Pacific Coast beaches lead Isaacs and Bascom (I-I-34) to develop techniques for operating in all but the most violent surf using amphibian vehicles (DUKWS) which operated off the beach. Surveys were repeatedly made on beaches with two or more bars through breakers as high as fourteen feet. The DUKW's position was triangulated by transit as it moved slowly in on a marked range; soundings were estimated from probable still water level and transmitted by radio to the beach. Although the estimation of probable still water level (1/3 of the wave height above the trough) is an apparently inaccurate process (especially if the leadsman is about to be hit by a large breaker) profiles which were repeated as a check were found to be surprisingly similar. Variations of a foot were not uncommon, but the magnitude of the underwater features and the necessary scale of plotting made these seem insignificant.

Beaches are not readily classified on the basis of their underwater slope, because of the difficulties encountered in determining where and how to make measurements on an undulating and irregular surface. The simplest possible classification is used here as a means of convenient presentation in which beaches are divided into steep, intermediate, and flat groups using the following arbitrary criteria:

Steep beaches -- over 30 feet deep 1,000 feet seaward of MLLW

Flat beaches -- less than 15 feet deep 1,000 feet seaward of MLLW

Intermediate -- between the above two

The reader is discouraged from trying to use such a classification rigorously. Note particularly that the steep beach profiles shown are plotted on a scale which reduces their vertical exaggeration by half.

Unfortunately, it is difficult to obtain good profiles of foreign beaches so only a few are presented here which were obtained from declassified military surveys of World War II. These beaches are, of course, much like some of our own and it is reasonable to believe that a thorough knowledge of U. S. beaches and processes will allow one to form accurate mental pictures of foreign beaches from limited information. The profiles of the U. S. beaches were made by Bascom (I-I-34) using DUKW's and the method described. It is believed that these profiles represent very nearly the complete range of beach slopes that exist throughout the world on major bodies of water.

In a following section it will be shown that, at the reference point, the beach face slope is easily measured. There is undoubtedly a relationship between the underwater slope and the beach face slope, but it has not been definitely established; the flat beaches of this grouping would generally have face slopes of less than 1:60 and the steep beaches, slopes greater than 1:15.

7. Beach Materials

a. Introduction

Shorelines and beaches are composed of materials of a wide range of sizes, shapes and compositions. Comprehensive treatment does not seem to be required here and these paragraphs will deal with the subject in general terms.

Beaches are composed of fragmental rock debris and of coral, foraminifera, and shell fragments which may be formed in place by the encroachment of the sea upon the land or may be carried to the sea by streams. It is often difficult to determine the origin of any particular sand since most beaches have materials derived by both processes and a classification on this basis is therefore impracticable.

The work of the waves in reducing materials to smaller sizes is of greatest importance. Waves may attack the seacliff directly; they may agitate and abrade the materials brought by streams thus creating smaller particles; they may attack the underwater rock, subjecting it to high currents and pressures; and they continually eliminate the smaller size materials that might act to reduce the inter-grain impact. All of this is carried out in the presence of highly corrosive salt spray and is aided by the other factors that cause erosion and decomposition inland (temperature changes, etc.). The nature of the rock being attacked is very important. Hard, homogeneous rocks (granite, basalt, quartzite) which have no natural planes of weakness and are likely to form rounded cobbles which abrade each other into small sand.

Bedded (hard) sedimentary rocks, schists, and slates break naturally into tabular shapes which eventually wear to flattened pebbles (shingles) that have a hydrofoil upper surface and are particularly susceptible to transport. Even small particles from these rocks may retain the flattened appearance until the individual grains are freed. This stage is actually the return of a rock to its original constituents (in some cases the rocks may be old beach deposits) and is probably common in places where sedimentary beds make up the seacliffs. A similar situation occurs when unconsolidated material is eroded by waves; the very fine components go into suspension and the larger materials stay on the beach and are "reworked." Figure I-I-31a of the Mexican beaches illustrates the erosion of an alluvial plain. Underwater rock is attacked in several ways besides the actual pressures caused by waves. For example, many Pacific beaches are composed of coral sand eroded from below the water surface; large storms break off coral heads and roll them around on the reef--grinding sand as they go. Other beaches have cobbles on them which are brought ashore by kelp; the kelp grips the rock firmly and in storms it is not unusual for the rock to give way and be floated ashore by the kelp.

Streams have a somewhat more prosaic job in transporting material to the sea from its inland origin; bedrock decomposed under physical and chemical weathering and the resulting pieces migrate seaward in brooks, streams, and rivers. As the pieces move, they are abraded, rounded and segregated; the smaller pieces move faster than the large ones, and they may be quite small and well rounded when the river finally spews them into the sea. Only exceptional floods are likely to bring any of the larger particles (cobbles) to where the waves can reach them.

b. Size

Beach materials may include all sizes from microscopic particles to large boulders; a description of these sizes is given in the accompanying table. Two fortunate circumstances simplify the problem greatly.

(i) Most beaches are composed of materials which fall within a rather limited size range and it is not necessary to give detailed consideration to the extreme sizes. (Boulders, silt and clay can be neglected as beach materials although they may be found in exceptional circumstances.)

(ii) At any beach the size range is usually small as a result of the excellent sorting by the waves. On some beaches like those of western Mexico (Figure I-I-31c) there is good sorting on two levels; these beaches have well sorted fine sand (.20 mm median diameter) below MLLW and cobbles (100 mm median diameter) which compose the berms and the part of the beach above water.

Practically all the beaches of the United States are sand beaches; this has led to the general impression in this country that sand is synonymous with beaches; originally, however, the word beach actually meant shingles or pebbles when it was created in England. Quite probably there are Alaskans who think that most beaches are cobbles and Koreans who think they are usually muddy. The size of the beach materials, as well as the degree of rounding, may be a clue as to the distance that the material has traveled to reach the beach. Since transport connotes abrasion and rough treatment, and since the conditions for moving large pieces rarely exist, it can be generalized that the large rough pieces are likely to be nearer their point of origin than smaller smooth ones.

Krumbein (I-I-45) has pointed out that the emphasis on statistical summation of mechanical analysis data has the disadvantage that there is no general agreement among investigators as to which statistical parameters need to be used or are most satisfactory. Generally it has been found that geometric and logarithmic measures are more useful in interpreting sediments than arithmetic ones. He has listed seven measures that are in common use; median diameter in millimeters (mm); sorting coefficient; modal diameter, geometric mean diameter; logarithmic standard deviation; quartile skewness and the logarithmic skewness. Beach sands seem to be best described by the first two and the others will not be considered here. The median diameter, in millimeters, is the average grain size at the middlemost diameter of the distribution. The sorting coefficient

$$\sqrt{\frac{Q_{25}}{Q_{75}}}$$

is the measure of the geometric spread of the central half of the distribution. These two figures are most easily arrived at by plotting the distribution of percentages of sand sizes, connecting them with a smooth curve, and taking the figures directly from the curve. Figure I-I-35 shows typical distribution curves from reference points of greatly varying sandy beach types. There are two systems in use for obtaining the size distribution of a sand in a sample. The "standard" method is in general use in testing laboratories throughout the world and is best described in the A.S.T.M. handbook (I-I-1). Briefly, it consists of drying and weighing the sample and putting it on a set of vibrating screens for a fixed length of time. The cumulative per cent retained on each screen is calculated and used to plot a distribution curve.

In recent years experiments aimed at making the analysis of beach sand more simple and accurate have been conducted by Emery (I-I-17) and Mason (I-I-52). Their methods use long vertical tubes and allow a small sample of sand to settle through water much as it would in its natural state; at intervals the pressure or the amount of sediment at the bottom is read and a distribution is plotted. The method seems to be fast and accurate and is still under test by the Scripps Institution of Oceanography and the Beach Erosion Board; it has the advantage of putting platy minerals like mica in their proper classification and the disadvantage of having small countercurrents in the tube which seem to confuse the results.

Typical beach sands of three varying sizes are shown in Figure I-I-33.

TABLE I-I-2

Size in Millimeters and Corresponding
Names of Beach Materials

British (Tyrell)		U. S. Bureau of Soils		
Boulder	>256 mm			
Cobble	64 - 256	Gravel	2.0 - 1.0	Probable Range of Beach Materials
Pebble	4 - 64	Coarse sand	1.0 - 0.5	
Granule	2 - 4	Medium sand	0.5 - .25	
Sand	2 - 0.062	Fine sand	.25 - .10	
		Very fine sand	.10 - .05	
Silt	.062 - .0037	Silt	.05 - .005	
Clay	Below .0037	Clay	Below .005	

Both tables are given because the British table includes the larger sizes that are found on many beaches. There is no essential difference since the naming is rather arbitrary.

Screen Sizes in General Use in Size Analysis

1/2 inch	48 mesh	12.70 mm	0.295 mm
3/8 inch	65 mesh	9.53 mm	.208 mm
4 mesh	80 mesh	4.699 mm	.175 mm
8 mesh	100 mesh	2.362 mm	.147 mm
14 mesh	150 mesh	1.168 mm	.104 mm
28 mesh	200 mesh	.589 mm	.074 mm

c. Shape

The shapes of particles change as they are subjected to rough treatment by moving water for long periods of time; their tendency is to get smoother and more spherical. For purposes of comparison these two characteristics are called roundness and sphericity and they are most easily determined for any specimen by comparison with the charts of Figure I-I-32; for a group of fragments, the figures are averaged.

Roundness is the ratio of the average radius of curvature of the several corners or edges of a particle to the radius of curvature of the maximum inscribed sphere. In describing roundness the following words or numbers are used:

Angular	0. - .15
Subangular	.15 - .25
Subrounded	.25 - .40
Rounded	.40 - .60
Well rounded	.60 - 1.00

Sphericity is the ratio of the surface area of a sphere with the same volume as the fragment in question to the actual surface area of the fragment (written s/S). For a perfect sphere the ratio is 1.00; for all other solids, the ratio is less than one.

The shape of the particles which compose a beach may be of considerable importance to the investigator in several ways. Particle shape makes a considerable difference in the trafficability of a beach; generally highly rounded, spherical grains are difficult to traverse since they shear easily. Coral sands are likely to be highly rounded because they are soft and easily polished. Grain shape may be a clue to the distance that a sand has traveled from its source and to the length of time that it has been "worked" by the waves.

Grain shape is a help in identifying the minerals (through their cleavage) and it may give an indication of the nature of the parent rock. Mineral character governs particle shape in the small sizes as rock character does the large sizes. Minerals with good cleavage in one or more directions are likely to be platy or tabular (mica or feldspar) and those that have no cleavage--and homogeneous rocks--are likely to be well rounded (quartz or coral). Shape also influences the settling velocity of the particles and thus determines the susceptibility to transport by waves and currents--the roundest particles settle fastest.

d. Composition

There are a large number of minerals and rocks that are found occasionally on beaches (in fact, probably all are at some time or other), but a rather small group is thought to comprise about 90% of the beach material of the world. The accompanying list summarizes these in what is estimated to be their relative importance. Sand grains which are original rock pieces (lithic fragments) are of course present to an extent commensurate with the volume and proximity of the supply to the beach. The waves work with whatever material is supplied them; the composition characteristics of minerals which determine their suitability as beach materials are:

(1) Hardness: Soft minerals are easily abraded and are not usually found in with a large percentage of hard minerals although they exist by themselves very nicely. Coral sand, for example, with a hardness of only 2.5 would probably be quickly ground up if mixed with volcanic sand of hardness of 5 or 6.

(2) Cleavage: Some minerals have the property of breaking along perfect planes parallel to some possible crystal surface; this is called cleavage and is obviously of great importance in determining how easily a mineral will break, and into what shape. The decomposition of a granite, for example, (which is composed of nearly equal amounts of quartz and feldspar) will give a sand which is predominantly quartz even though the two minerals have about the same hardness because the feldspar has good cleavage in two directions (and breaks easily), the quartz none.

(3) Specific Gravity: Variations in the specific gravity of beach sand particles results in the heavier sand working its way to the bottom (a principle long used in the mining business to sort ores). In winter, or after a storm in which the upper sand has been removed, the heavier minerals which often compose a layer on top of the underlying rock are seen. Beaches in metamorphic rock areas are likely to be underlain with garnet, ilmenite or other heavy minerals to the extent that the beach is a dark red color; in graphite areas, monazite or magnetite may be found.

(4) Chemical Composition: Occasionally beaches have been of interest for their mineral value. Nome, Alaska had beach gold deposits; near the Coquille River, Oregon chromite and zircon sands were mined during World War II; and there has been interest elsewhere in monazite and corundum. Sand for use in glass and concrete is of considerable commercial importance in a number of places at both present day and fossil beaches.

(5) Color: Beaches range in color from bright white sands to nearly black; any generalizations about size, slope or composition based upon color alone are probably valueless.

TABLE I-I-3

Common Beach Sand Minerals*

Name	Formula	Hardness	Gravity
Quartz	SiO_2	7.0	2.6
Orthoclase	$KAlSi_3O_8$	6.0 - 6.5	2.4 - 2.6
Plagioclase	$NaAlSi_3O_8Ca{+}Al_2Si_2O_8$	5.0 - 6.5	2.5 - 2.7
Magnetite	Fe_3O_4	5.5 - 6.5	5.2
Biotite	$K(Mg,Fe)_3Si_3AlO_{10}(OH)_2$	2.5 - 3.0	2.7 - 3.1
Hornblende (Amphibole)	$Ca_2Na_{0-1}(Mg,Fe)_4(Al,Fe)(Si,Al)_8O_{22}(OH)_2$	5.0 - 6.0	2.3 - 3.4
Augite (Pyroxene)	$Ca(Mg,Fe,Al)(Si,Al)_2C_6$	5.0 - 6.0	3.3
Garnet	$(Ca,Mn,Mg)_3Al_2(SiO_4)_3$	6.5 - 7.5	3.1 - 4.3
Olivine	$(Mg,Fe)_2SiO_4$	6.5 - 7.0	3.3
Ilmenite	$FeTiO_3$	5.5 - 6.0	4.5 - 5.0
Rutile	TiO_2	6.0 - 6.5	4.2
Muscovite	$H_2KAl_3(SiO_4)_3$	2.0 - 2.5	2.7 - 3.0
Epidote	$Ca_2(Al,Fe)_3Si_3O_{12}(OH)$	6.0 - 7.0	3.2 - 3.5
Hypersthene	$(Fe,Mg)SiO_3$	5.0 - 6.0	3.5
Tourmaline	$NaMg_3B_3Al_3(Al_3Si_2O_{27})(OH)_4$	7.0 - 7.5	3.0 - 3.2
Sphene	$CaOTiO_2SiO_2$	5.0 - 5.5	3.5
Zircon	ZrO_2SiO_2	7.5 - 8.0	4.7
Kyanite	Al_2SiO_5	5.0 - 7.0	3.6
Actinolite-Tremolite	$Ca_2Mg_5Si_8O_{22}(OH)_2$	5.0 - 6.0	2.9 - 3.4
Monazite	$(Ce,La,Di,Th)\ PO_4$	5.0 - 5.5	5.0 - 5.5
Cassiterite	SnO_2	6.0 - 7.0	6.8 - 7.1
Hematite	Fe_2O_3	1.0 - 4.0	5.0
Glauconite	$FeKSi_2O_6H_2O$		2.2 - 2.8
Glaucophane	$Na_2Mg_3Al_2Si_8O_{22}(OH,F)_2$	6.0 - 6.5	3.1

* From Dana's "Textbook of Mineralogy" (I-I-16)

8. The Relation of Sand Size to Beach Face Slope

There is a definite relationship between the size of the sand, the slope of the beach face, and the intensity of the wave action. Although this has been recognized for some time, only recently has a method of measurement been reduced to simple terms so that the conditions are readily comparable (Reference I-I-8). The whole matter revolves about where and how to take the sand sample and make the slope measurement, and the relation of wave conditions to the beach slope (sand sizes vary greatly along a profile as do slopes, and waves steepen or flatten a beach according to their characteristics).

a. Reference Point

In order to standardize beach measuring procedures, a "reference point" is used; sand samples taken there and slope measurements made at that point can be used in making comparisons of beaches across time and distance. The reference zone is the part of the beach face subject to wave action at the mid-tide stage. Any point in this zone is presumed to have approximately the same characteristics (even slope and same sand size). The dimensions are deliberately somewhat flexible since the technique would be impracticable if it were necessary to locate some exact elevation. Mid-tide refers to a level halfway between the previous high tide and the succeeding low; since the backrush is only slightly below and the uprush may be considerably above average sea level, the reference zone will be largely above the mid-tide level. It is not difficult to locate since the width ranges from twenty feet on a steep beach to as much as a hundred on a very flat beach. The use of the reference point has certain advantages:

(i) A lone investigator with simple equipment can take measurements rapidly and easily.

(ii) The variables being sampled are those of the present time (existing waves placed the sand sampled on the slope measured).

(iii) Inter-tidal beach face slope is a major criterion for the potential use of the beach.

(iv) Other beach characteristics can be determined indirectly from this information.

Reference point measurements are most conveniently made and samples taken at a low tide stage. Slopes are measured perpendicular to the water line using simple instruments (a clinometer or Brunton compass). Sand samples are, of course, taken at the same time and place. It is good practice to scrape away the upper cm. of sand and then take the sample from the top 10 cm. A 1,000 cc sample is ample and no special sampling or quartering methods are necessary; quart cardboard cartons are satisfactory as sample containers. Slope, location, and other data can be written directly on the carton, thus insuring a correct combination of data.

b. Size Distribution Across a Beach

A glance at Figure I-I-34 will serve as an ample explanation of why a definite point of sand sampling is necessary if beaches are to be compared on the basis of sand size.

It is quite evident that there is a wide size distribution along a profile and it makes a great deal of difference where a sample is taken. Figure I-I-34 shows the relationship of percentage variations from the median diameter of the reference sample with a stylized profile which represents all beaches. A number of beaches which were systematically sampled and surveyed are plotted and they show unmistakable trends which are apparently characteristic of sorting on ocean beaches. Several important points should be noted:

(i) The largest sands have the widest range of variation along the profile and the finest sands show the least variation.

(ii) The coarsest sand is always found at the plunge point, just seaward of the backrush, which observation indicates is the point of maximum turbulence. The other group of large particles is that on top of the berm; apparently, these are churned up at the plunge point and swept by maximum uprushes across the top of the berm, from where they cannot return (those on the seaward face are carried back to the plunge point by the backrush).

(iii) The sand becomes increasingly finer to the seaward; above water the finest sand is in the dunes.

c. Size and Slope Variations

The range of possible sizes of beach materials is a large one as has been already pointed out; however, if we consider only the beach face and the particles subject to systematic motion by the waves the field is narrowed somewhat. Probably the largest cobbles (over 150 mm) are moved so rarely that they need not be considered; sand less than .15 mm rarely exists even in the most protected locations. These figures refer to median diameter; small amounts of larger and smaller particles are present in each sample, but do not appear to be significant. (As has already been pointed out, sorting by waves is generally very good.) Figure I-I-35 shows the range of sand sizes likely to be encountered; there is little data on pebble and cobble beach slopes and sizes available but there is no reason to believe that it would be basically different. Slopes of sandy beach faces range from 1:4 to 1:100 as shown on Figure I-I-36; cobble beaches like those of Figure I-I-31a and -31c occasionally have slopes as steep as 1:1½. It is to be particularly noted that these slopes are measured at reference point and do not include slopes formed by undercutting (above the reference zone) which may result in a vertical scarp.

d. Slope Changes at a Point

The slope of a beach face will change, even though the sand size is unchanged, if wave conditions change. The rule is simple: Beaches flatten as they erode and steepen as they build. This means that beach face slope can be related to the H/L values of the waves. As explained in detail later, these H/L values appear to control the movement of sand from berm to bar and vice versa. When the values are low, the sand moves ashore and the beach face steepens; when values are high, the sand moves off the beach to the bar and the face flattens. Figure I-I-37 shows short term changes on two beaches which were under close observation during periods of erosion and subsequent deposition. At Carmel, storm seas cut the beach back, flattening it in the reference zone and causing it to steepen near the crest of the berm. This

erosion lasted for about one day; by the following day, wave conditions had changed to one favorable to deposition and the beach had steepened and widened. A week after the storm the beach was wider than ever and considerably steeper. Twenty profiles made of this station during the summer of 1946 showed that there were no greater variations in beach slope than those caused by this storm.

The Santa Barbara example in Figure I-I-37 shows the cumulative effect of two storms on a beach. The upper, original, profile is typical of the summer (depositional) conditions; two severe storms in quick succession cut the beach back to the much gentler slopes.

These bits of information indicate that a relation exists between the slope of a beach (for any given sand size) and the waves impinging upon it. Moreover, it appears that limits can be put on the amount of change that can be expected for any beach.

The appearance of a beach after a storm emphasizes the need for a definite point at which to make measurements. For example, if a berm was formed during spring tides and waves causing erosion occurred during the lower neap tides, undercutting and a steep scarp may result (Figure I-I-41). The beach will then appear to be steeper although it has actually flattened in the reference zone.

e. Size and Slope Variation With Protection

As mentioned previously, there is a very definite relationship between beach characteristics and their exposure to the prevailing storm seas. The diminution in size of berms, bars and channels with protection has been discussed. Two other items are now seen to be responsive to protection: (i) the sand size gets smaller as protection increases and (ii) the beach flattens out.

Laterally, the coarsest material is concentrated at the point of maximum turbulence; this seems to be true longitudinally also. The coarsest sand stays on the most exposed beaches and the fines migrate to the quiet waters. Figure I-I-38 of Half Moon Bay, California shows how slopes and sizes vary with protection. The prevailing swell is from the northwest and considerable protection is afforded to the harbor area (between profiles one and two) by Pillar Point. South along the beach the protection gradually disappears until at profile four, the beach is exposed to nearly the full effect of the prevailing seas. The beach, which is continuous and unbroken, has reacted to fit itself to this environment. At profile one where wave action is at a minimum, the beach is fine grained (.17 mm) and flat (1:41). As the exposure increases, the beach responds characteristically and at profile four the sand is coarsest (.65 mm) and the face is steep (1:8).

This alongshore sorting is typical of the response of sand to wave action and is partially the basis for the classification of beaches on the basis of exposure. The classification could be used to describe the beaches at the four profiles as follows: Profile 1--group VII (the beach facing directly away from the swell); Profiles 2 and 3--groups IV or V (protection by headland and by refraction); Profile 4--group II (straight beach facing the seas obliquely). It was previously said that storm seas caused the beach

to flatten; here it is shown that greater exposure to prevailing or average storm seas results in a steeper beach. This may appear to be a contradiction at first, but it is really the best evidence that the sand size controls the beach slope. The size of the sand at any reference point is a sort of integrated value that represents long term average conditions; it rarely changes appreciably. The slope, on the other hand, changes frequently in response to wave conditions of short duration--conditions that are only a small component of the long term average. Equilibrium therefore exists when the wave conditions of the moment are exactly equal to the long term average. Because of this difference in time required for adjustment, the sand size exhibits no short term response; for this size there is a corresponding slope value for the average wave conditions and that slope is the one referred to when it is said that exposure to prevailing storm seas results in a steep beach. Any one storm, however, gets direct response from the slope (but great lag from the size) and so it can be said that storm seas cause the beach to flatten. The profile of changes at Carmel (Figure I-I-37) already discussed shows this; probably one of the intermediate slopes could be said to be the equilibrium condition.

f. Size-Slope Relationships

The elements of the problem of relating size to slope have each been dealt with individually; the final step is clear. If these relationships are all true, it should be possible to combine the three major parameters into a single statement which shows the relationship for any beach at any time. Figure I-I-39 is an attempt to do this with rather incomplete data and its curves will doubtless need to be modified as additional information becomes available. Because beach slopes change considerably, it is evident that each point should be a horizontal line indicating the range of slopes experienced by any beach--then, from a knowledge of the percentage of time that each slope existed, the average slope could be worked out. This would require exceedingly detailed study of each beach; in lieu of this, the author has rather arbitrarily divided beaches into three classes based on their estimated average exposure and the relative conditions that existed during the time of sampling. The upper line represents average slopes of completely exposed beaches--ones which might have refraction coefficients of around 1.0; the lowest line represents the almost completely protected beaches; and the middle line is an intermediate stage mid-way between the two. These lines are guesses and they represent an ideal, rarely achieved condition. Since a beach is seldom in equilibrium, the slope point can be expected to deviate one way and then the other from the average as indicated by the four horizontal lines. For example, the beaches at Point Reyes, Gold Bluffs, Surf, Raft River, Manzanita and Oysterville are all exposed in about the same degree to Pacific storms and their equilibrium slopes should fall close to the upper line. It will be noticed that they fall on each side of the line with considerable spread; this is because the conditions were not those of equilibrium at the time of sampling. The four Half Moon Bay points, details of which are shown in Figure I-I-38 demonstrate how the slope-size relationship changes with the exposure on a beach of varying protection.

9. Beach Changes

a. Summary of the Causes of Beach Changes

Geological processes along seacoasts have been described and the mechanics by which waves shape shorelines, erode cliffs and grind material were stated briefly. These are long-term processes and were discovered by matching old charts or pictures with recent ones to determine trends and by considering the possible causes. Beaches, however, exhibit very important changes from day to day and from season to season which are readily recognized even by untrained people. It is not unusual to go to a beach on successive days and find that overnight a new berm has formed, a scarp has been cut in the beach face, or a large rip channel has developed. In the sections on bars and berms this was mentioned, but a detailed explanation and summary may be worthwhile.

Beaches respond readily to small changes in the characteristics of the waves striking the beach at any point. These characteristics, length (L), height (H) and period (T) are measured in deep water (deeper than one-half the wave length) and are combined into a single figure called "wave steepness." (The actual slope of the wave front is of no importance in this usage.) Wave steepness or the ratio of height to length is written:

$$\frac{H_o}{L_o} \quad \text{or} \quad \frac{H_o}{5.12T^2}$$

The height used is the height of the waves at the beach corrected for refraction back to the deep water condition; the length is calculated from the period. It will be seen that the steepness increases either with an increase in wave height at the beach or with a decrease in period. Investigators generally agree that steep waves erode the beach face and build longshore bars and that less steep waves move the sand ashore, usually building the beach face at the expense of the bar. Unfortunately, there has not been enough work done on the establishment of limiting conditions so that it can definitely be said which wave conditions cause erosion of the beach face and which cause deposition. However, experimental work in wave channels by J. W. Johnson (I-I-39) and others indicates that waves with H/L values of greater than 0.03 cause beach face erosion and the formation of a "storm profile" (a bar) while waves with H/L values of less than 0.025 create an "ordinary" profile (no bar) and they add material to the beach face. The corresponding response of the beach slope in the "reference zone" has already been described. The question of whether models of beach changes are directly comparable with actual shoreline changes has never been decided and objections to model studies have been raised on two important counts.

(i) Sand grains small enough to give a true scale have entirely different properties; as a result the smallest sand used in models is about equivalent to cobbles on a real beach.

(ii) Waves in nature are highly variable in both period and height and none of the present types of wave generators produce wave groups or variable waves (model wave records are not at all like actual ocean wave records). In an attempt to define "beach equilibrium," the Beach Erosion Board (I-I-52) has kept wave generators in their channels operating

continuously at fixed conditions for many days at a time without ever achieving a "steady state" of the sand. For these reasons many investigators prefer to rely on their own observations of the movement of material on ocean beaches and, as a result, there are differences of opinion as to the mechanisms that cause beach changes.

The following is a series of statements which it is hoped will clarify the relationship between wave changes and beach changes.

(i) Short term beach changes largely consist of the transfer of sand from the berm (beach face) to the bar or vice versa. At the point of MLLW the changes are usually quite small.

(ii) Although waves of various periods have greatly varying velocities in deep water, in shallow inshore water (where the sand transfer takes place) the velocities are about equal. This is especially true of broken waves inside the bar (translatory waves or foam lines).

(iii) Observations indicate that for any one wave the shoreward velocity of the water particles is slightly greater at the time of passage of the wave front than at any other time in the cycle. (On the bottom the vertical component of the orbital motion is zero and the particles follow a reciprocating path.)

(iv) Since the higher velocities are most likely to exceed the threshold of movement of the sand or rock particles and since all waves have substantially the same velocities in the area of interest, sand probably tends to move shoreward with all waves. This establishes one major shoreward force that exists during periods of all but the very smallest waves.

(v) This force is always counterbalanced by the effect of gravity which tends to make the particles move downhill (away from the shore). In addition, any shoreward movement must overcome inertia and friction against other sand grains.

(vi) It is perfectly plain that under wave action the sand always moves shoreward except when waves with large H/L values exist.

(vii) Wave energy in the surf zone is largely expended in turbulence rather than in moving sand directly. It is the unbalance of forces in the turbulent area that results in net currents (and sand movement, since submerged objects tend to do what the water is doing). The actual energy expended on moving sand may be minuscule. This is evidenced by the fact that waves move only ashore and the sand may move in either direction. This raises the question: What is the cause of the reversal of sand movement direction when the H/L values get large?

(viii) The following answers to that question are theories since means of measuring small changes in the surf zone to get direct proof have not yet been devised.

(a) High waves and short periods pile water up at or near the beach face; the high waves raise the surface above mean sea level, the short periods (high frequency) maintain it there by replenishing the supply

rapidly. Water from broken waves is hurled shoreward as foam lines on the surface. This creates an inshore head which must naturally flow downhill towards the sea; because more water is arriving on the surface, this inshore water returns along the bottom, moving sand as it goes. This generally accepted theory was first stated in the most general terms by D. W. Johnson and V. Cornish, who do not agree on the relative importance of the factors involved.

(b) The high waves end in long uprushes which occur in rapid succession because of the short period. This keeps the beach face saturated with water so that the uprush water must return along the surface. This means that a greater volume of water rushes down the beach face, removing material as it goes, than if the beach face were not saturated and a large part of the uprush disappeared into the sand. (This also may account for the increase in beach steepness with permeability.) This suggestion is from a discussion with M. P. O'Brien; it is not known whether it has been published.

(c) Wave heights and periods are most variable and it is difficult to assign definite values to a train of waves outside the laboratory. For this reason the technique of referring to the "average of the upper third" as the "significant height" has developed. This variability may be important in causing seaward forces which move sand offshore. Generally large waves exhibit a larger variation in height and a lesser variation in period than do smaller waves. This is significant because wave energy is proportional to the square of the height and the actual difference in energy between maximum and minimum waves of a train of large waves is much greater than for a train of small waves. This variation between maximums and minimums suggests that the waves may support a head of water for a short while and then release it abruptly. The sudden release of this inshore mass of water may cause sand motion far in excess of that that would result from a continuing head. If this is true, sand may move seaward in pulses corresponding to the times between maximum arrivals. If the release occurred when the waves were at a minimum--as they often are (the large waves seem to "capture" the small ones and are thus followed by extreme quiescence), there would be no landward forces to oppose the movement. The smaller waves, of course, return some sand to the beach. (From an unpublished paper by Bascom.)

(ix) In the light of the above explanation of wave variability and sand movement, beach equilibrium can be regarded in a new light. Beach equilibrium (not yet achieved in a wave channel) may actually mean balance between the maximum and minimum waves of a train in such a manner that the maximums move exactly as much sand off the beach face as the average and minimum waves put on the beach in between maximum waves.

b. Littoral Processes and Problems

The means by which waves move sand off and on beaches and cause certain local beach changes have been considered. This previous discussion treated with "closed systems" of sand movements in which there was no flow of sand in or out of the area under discussion. Such is not always the case, however. When waves strike the beach face at an angle as shown in Figure I-I-42b, they set up an alongshore littoral current which flows close to the beach in the longshore component of wave direction. Even though this current may be of very low velocity it is able to transport sand along the

beach because it need not move the bottom sand but only that already in suspension--churned up by breaking waves. In the discussion of relative H/L values it was pointed out that the shorter period waves are more likely to cause sand to move off the beach face. They are also more likely to set up littoral currents since they refract less and strike the beach at a greater angle than long period waves from the same direction. High waves affect the bottom in deeper water and keep sand in suspension over a wider area than do small ones. It is evident, then, that rates of sand flow along a coast are related to the direction, height and period of the waves and these factors have been correlated with the flow to some extent. When storm waves strike a beach at an angle, the sand inside the breakers is put into suspension by the turbulent water and swept along by the longshore current. It is as though the upper few millimeters of sand throughout the surf were raised everywhere and carried along with a motion like that of sand in a stream bed. Professor Munch-Petersen (I-I-58) has likened this littoral process to a conveyor belt the width of the surf zone whose belt speed is the current velocity. As indicated by the beach classification, many shores have a constant longshore current because they are oriented at an angle to the prevailing waves. This means that there is a fairly continuous flow of sand along these coasts and, of course, erosion and deposition problems. Because man attaches great value to seacoast property, he resents changes made in the configuration of the coast by wave action. Moreover he likes to build structures that influence the ocean and then expects the ocean not to react. Shoreline "problems," then, are largely caused by man's reluctance to accept the continuation of the natural geological processes which formed the shore. These processes go on with surprising speed and the retreat of a coastline can probably not be delayed for long by the works of man. Coastal retreat or erosion means that there is a supply of beach material being formed and that there are currents to carry it away. These prevailing littoral currents that move the sand along coasts are readily upset by shoreline structures such as jetties, groins and breakwaters which often act to interrupt the flow. Usually the moving sediments will be stopped on the upstream side of a structure or in the zone of quiet water that it creates. The waves which set up the littoral currents also act on the downstream side of the obstruction and a new current is formed which continues down the coast; because the structure has stopped all or part of the sand from passing, the downstream beach is deprived of its natural resupply and rapidly erodes. The typical problem is likely to be a dual one: first, the sand moving up to the structure may require changes and additions or may spoil its usefulness; second, the downstream beaches need to be replenished. Various schemes for by-passing the beach material have been tried, their function being to duplicate the littoral movement mechanically. Finally, the sand arrives at its ultimate destination: a prograding sand structure, usually a spit. Here it is deposited with what is sometimes astonishing rapidity for a geological process. Rockaway Spit, at the entrance to New York Harbor, is receiving sand which moves southwest along the Long Island coast. This spit grew at the rate of over 200 feet per year for long period (one mile in twenty-three years). Such changes are less likely to be damaging than erosion but the new land is often nearly valueless; problems such as the moving of lighthouses to keep up with the sand exist even on sparsely populated coasts.

The littoral flow of material is seen to link the place of erosion with the site of deposition; in order to reach any useful conclusions about it, the rate of flow must be known. No satisfactory instruments now exist

for measuring sand flow or currents in the surf zone although numerous sand traps and other devices have been tried. Neither is repeated profiling of any value (the method used to determine offshore-onshore movements) since a great quantity of sand may pass a profile without altering it in the slightest. The most satisfactory rate data to date has been obtained by accurate measurement of the size of the terminal deposit at closely spaced intervals. To some extent these can be correlated with the wave characteristics which determine rates of flow.

Santa Barbara, California is subject to a nearly continuous littoral current and corresponding sand flow; since it has problems representative of the southern California shoreline and similar to those of the New Jersey and Danish coasts, a description of the situation there may be illuminating. Throughout most of the year, waves strike the nearby coast at a considerable angle; these maintain a current which transports sand from beaches to the west. At Santa Barbara harbor a breakwater was built to shelter small craft from these westerly waves. As shown in Figure I-I-43, the sand moving along the coast in the turbulent shallow water of the "conveyor belt" is suddenly dropped into the thirty foot deep, quiet water of the harbor where it builds a spit. The beach just east of the harbor is subject to the same wave action and contributes its sand to the new current; without resupply it naturally erodes rapidly. The U. S. Engineer's solution to this deposition-erosion problem is to dredge the spit from the harbor and place its sand on the eroded beach at two year intervals. There is much evidence to show that <u>all</u> the littoral drift is deposited in the harbor in a relatively small area. Measurements by Bascom (I-I-7) of the changing size of the spit have given some definite figures on rates of flow of sand for short increments of time. On days when the breakers were less than two feet the sand moved at the rate of around 250 cubic yards per day; on days of storm with breakers over four feet high the rate was over 1,000 cubic yards per day. On a longer term basis, the yearly rates seem to follow long period cycles (perhaps eleven years) with daily averages varying from 250 cubic yards to 600 cubic yards.

A littoral problem on the Oregon coast is illustrated by Figure I-I-42a in which southward moving sand was stopped by the groin-like single jetty at Tillamook Bay entrance. When the sand filled and passed the end of the original jetty, the solution was to extend the jetty (which is still accumulating sand). No attempt was made to nourish the beach on the lee side and it rapidly retreated; valuable properties including a hotel and summer resort were undermined and lost to the sea many years ago and the retreat is still going on.

At Santa Monica, California, which is subject to about the same littoral transport as Santa Barbara, a slightly different structure was tried in the hope that the littoral movement would not be interrupted. An offshore breakwater was built (Figure I-I-13a) the intention of which was to shelter small craft but allow the sand to freely pass behind at the same time. This was not successful because the current was not strong enough to carry the sand the length of the breakwater without the assistance of the wave action and the beach in the lee built out rapidly as shown.

On the sandy west Danish coast a number of artificial harbors have been built. These mostly consist of pairs of curving breakwaters built out from

the beach to shelter a small area of water. There is considerable littoral drift and these structures accrete sand on their up-current side while the downstream coast retreats (Munch-Petersen, I-I-55). The Danes have also tried the offshore type of structure and have built a causeway a considerable distance out from the main coast to a ring-shaped breakwater. This technique seems to be fairly satisfactory because of the relative dimensions but the tombolo effect has been observed.

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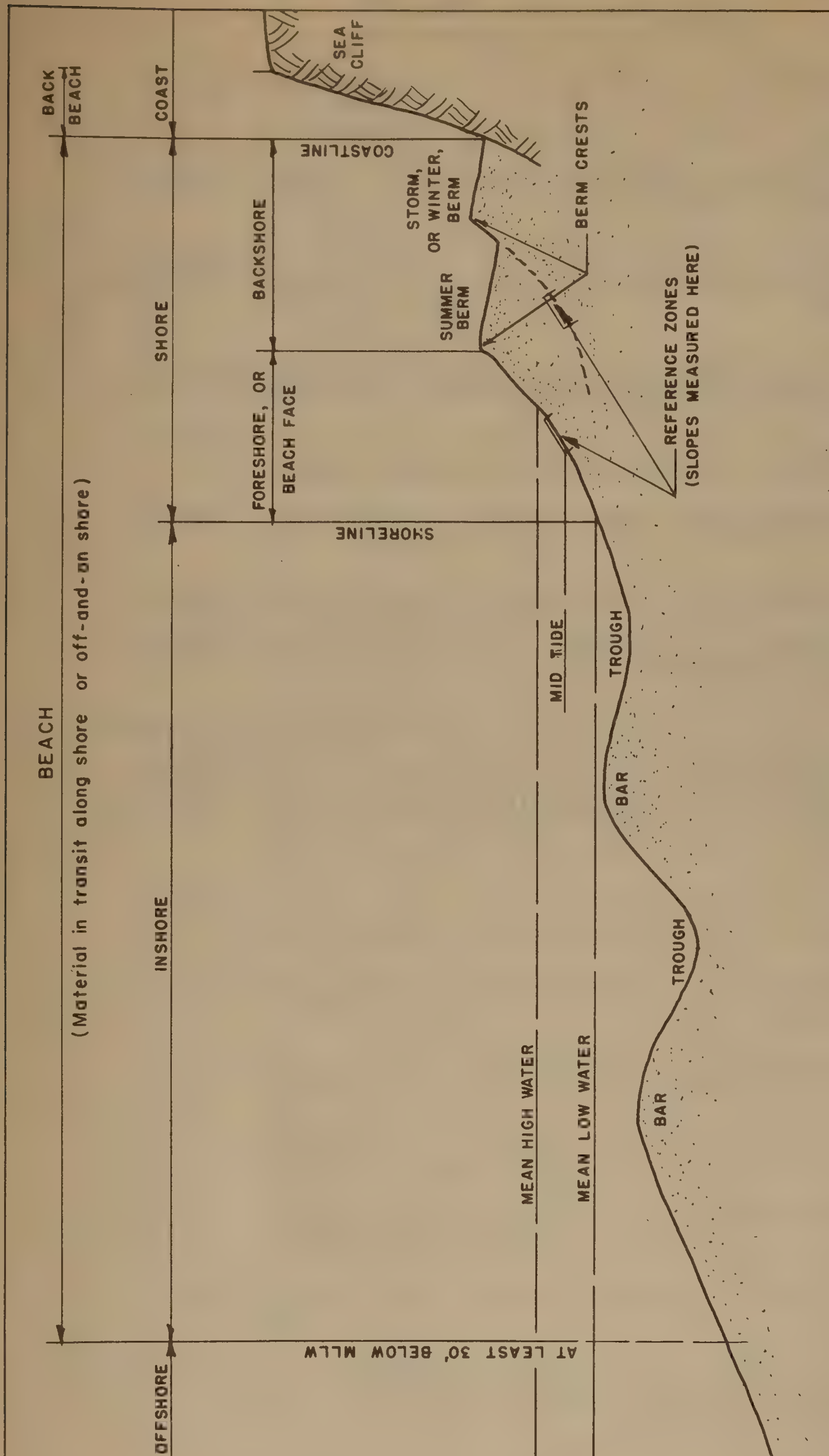
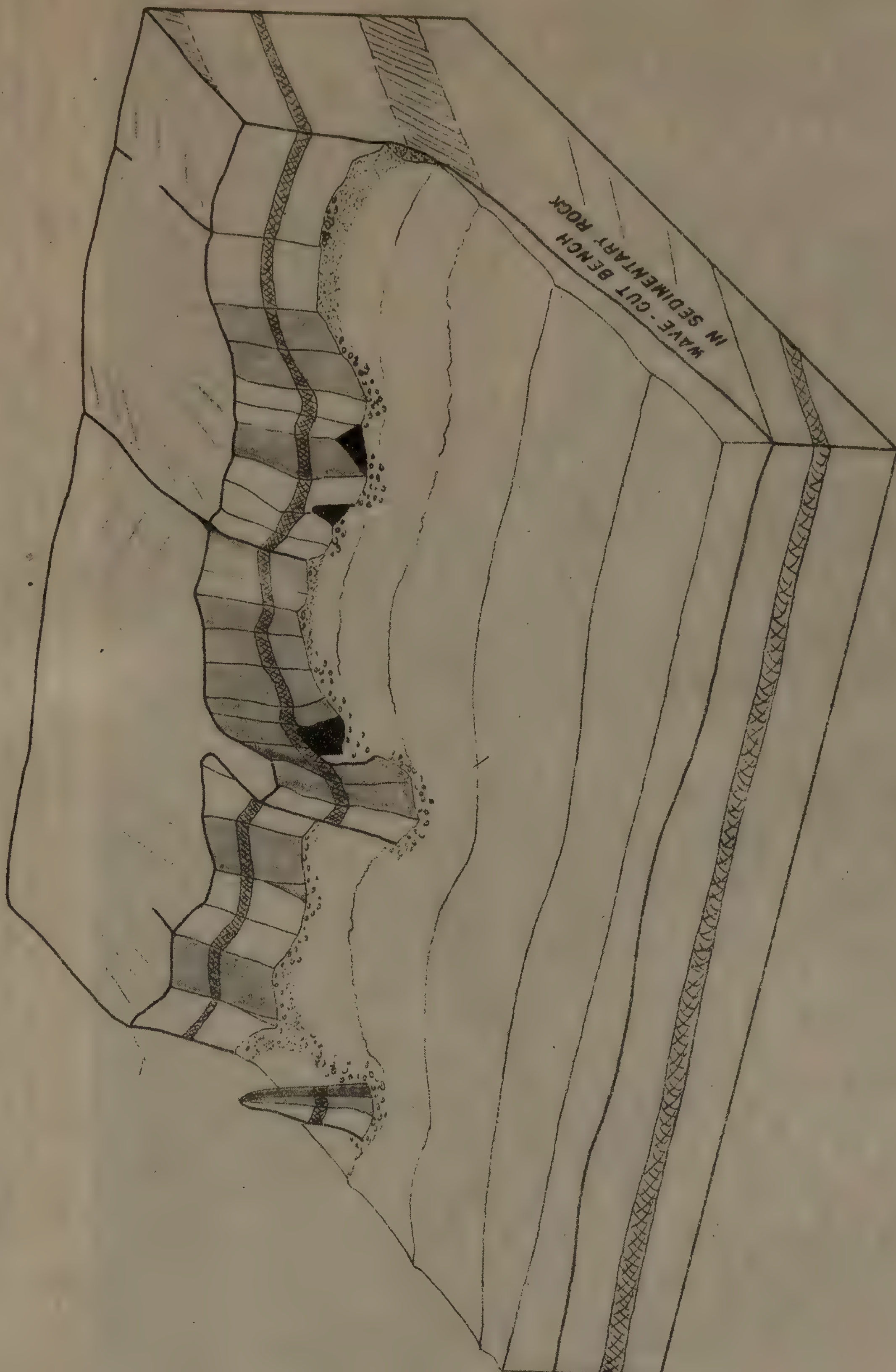


BEACH PROFILE ILLUSTRATING TERMINOLOGY

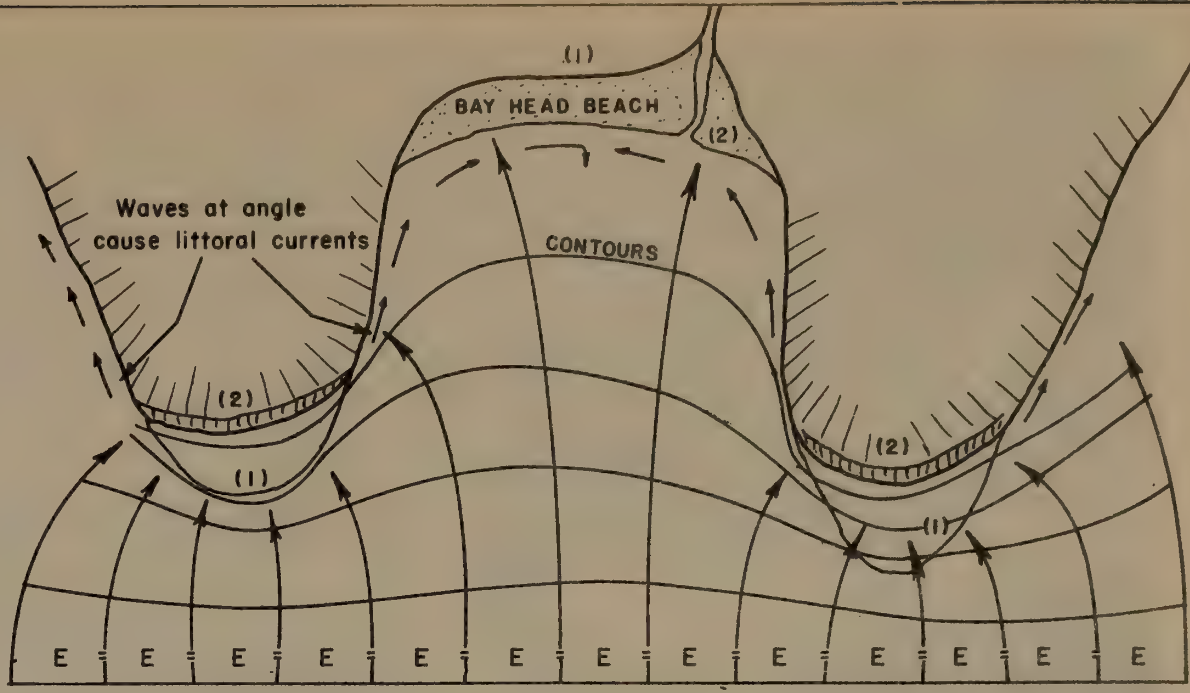
FIGURE I-1-1



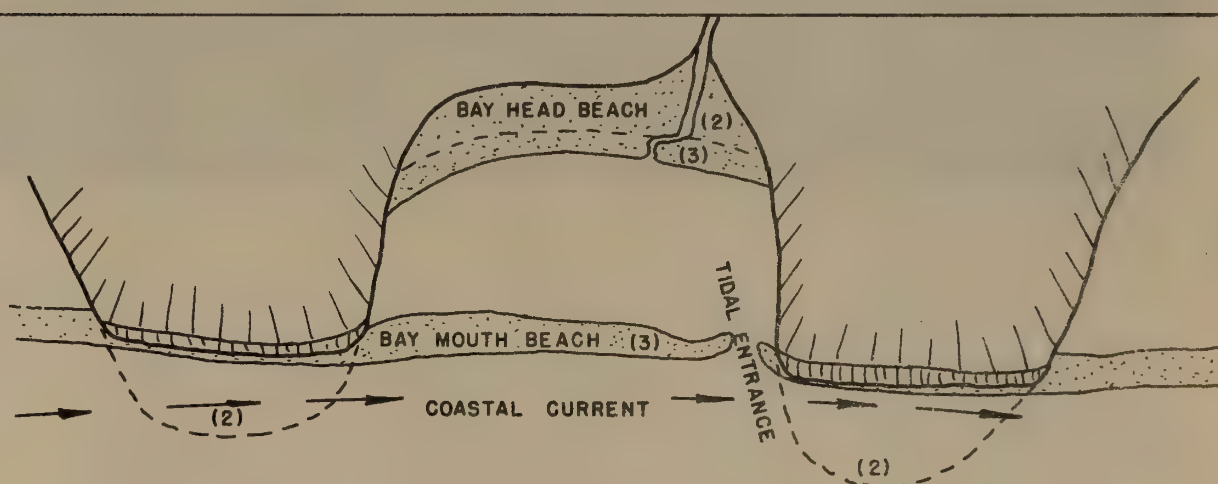
RETREATING SEACLIFF SHOWING CHARACTERISTIC SHORELINE FORMS:
STACK, ARCH, SEA CAVE, NARROW SAND BEACH, WAVE-CUT BENCH

FIGURE I-1-2

FIG. I-1-3 - Bay-mouth bars at the lagoon beaches in northern California. This formerly irregular coastline was cut back to the cliff-line and at the same time received a large supply of sand. The lagoon to the left of the truncated headland was filled with silt from Orick Creek and is now farmland; the lagoon at the right had no comparable supply and is filling much more slowly.



Original rough coastline (1) is subjected to waves which, because of refraction, concentrate their attack on the headlands, as shown by the lines dividing zones of equal wave energy. Littoral currents set up by waves breaking at an angle sweep the eroded material into the bay which shoals; a beach forms at its head. Coastal currents are broken into ineffective eddies by the irregular coast.



Waves continue attack and headlands retreat; bay becomes shallow and underwater bars form. Storms build the bars above water and the now predominant coastal current reinforces them with material from the headlands. A spit (in this case a bay mouth beach) forms and the whole coastline becomes straight with a continuous sandy beach (3).

FIGURE I-1-4

HYD-5364

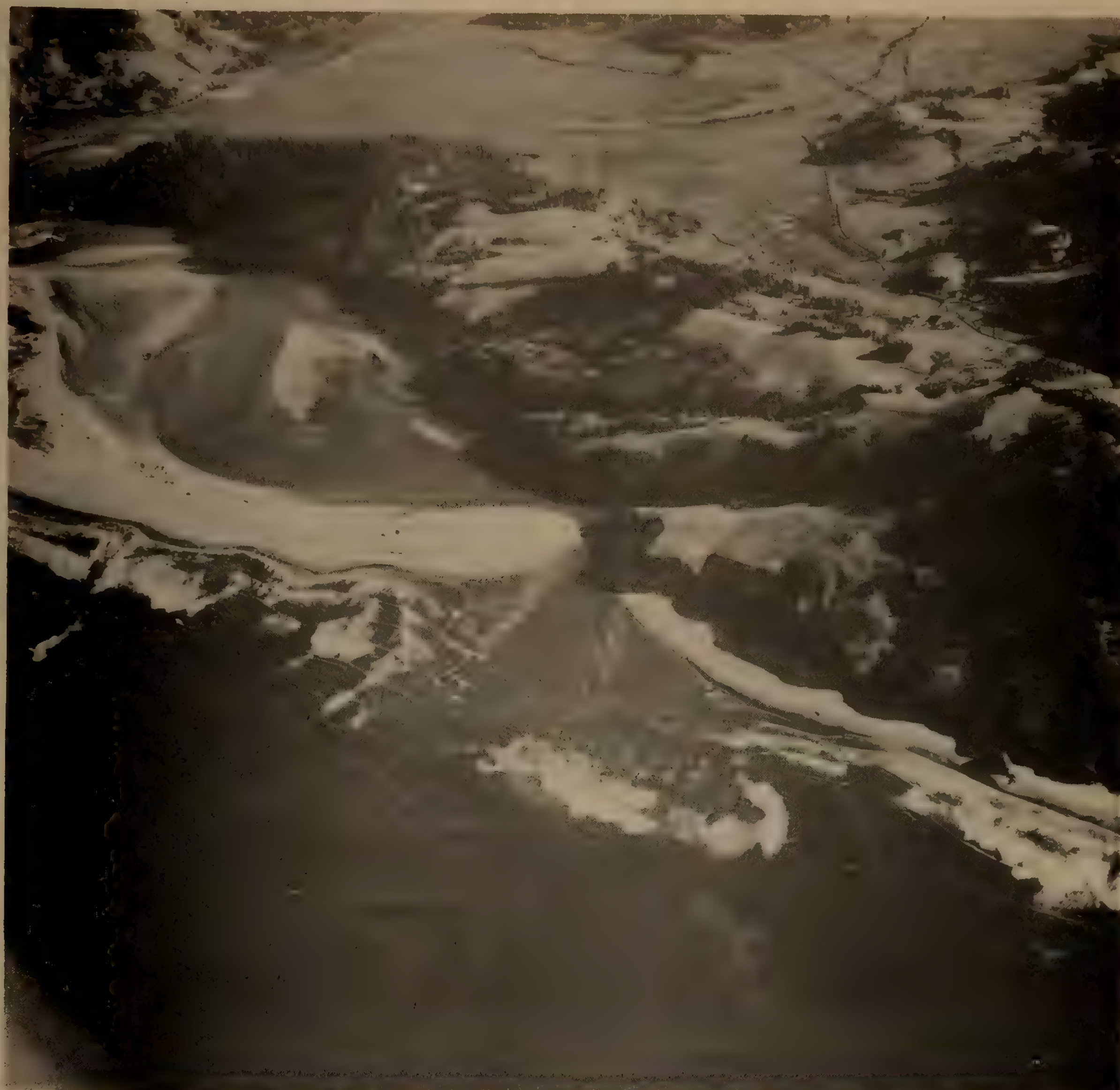
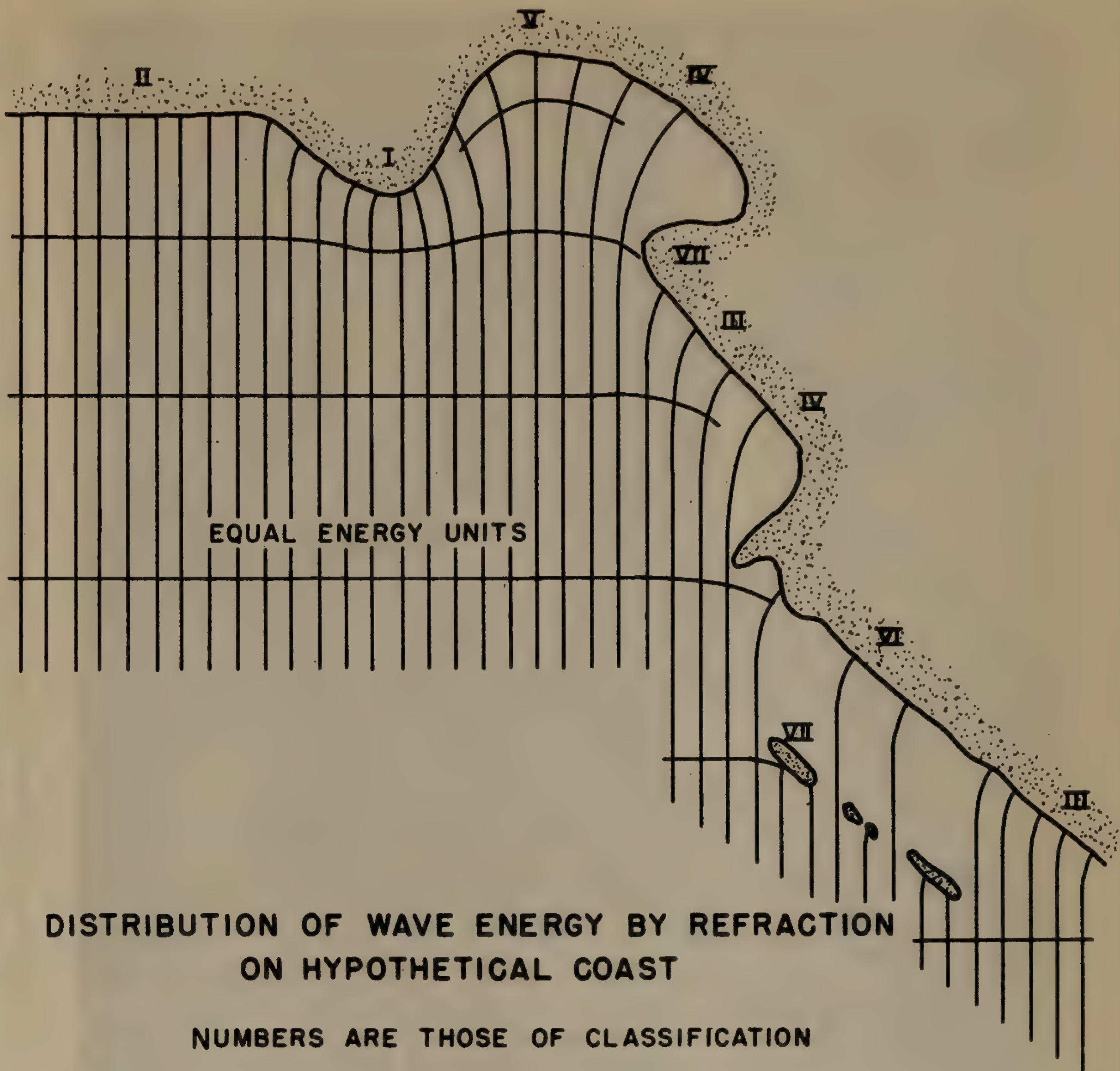


FIG. I-1-5 - The entrance to the Nestucca River on the Oregon coast illustrates the condition sketched in Figure I-1-4. An excess of fine debris has filled in the bay and built the spit; seacliff at the right is protected by a sandy beach and the coast is fairly straight.



DISTRIBUTION OF WAVE ENERGY BY REFRACTION ON HYPOTHETICAL COAST

NUMBERS ARE THOSE OF CLASSIFICATION

Because most storm swell originates in definite areas along storm tracks there is a prevailing wave direction for each season which allows the beach protection to be described as shown above. Since wave energy is responsible for the character of a beach, the relative amount of energy reaching the shore is of paramount importance and beaches can be classified as to the amount they receive.

FIGURE I-1-6

HYD-5389-116

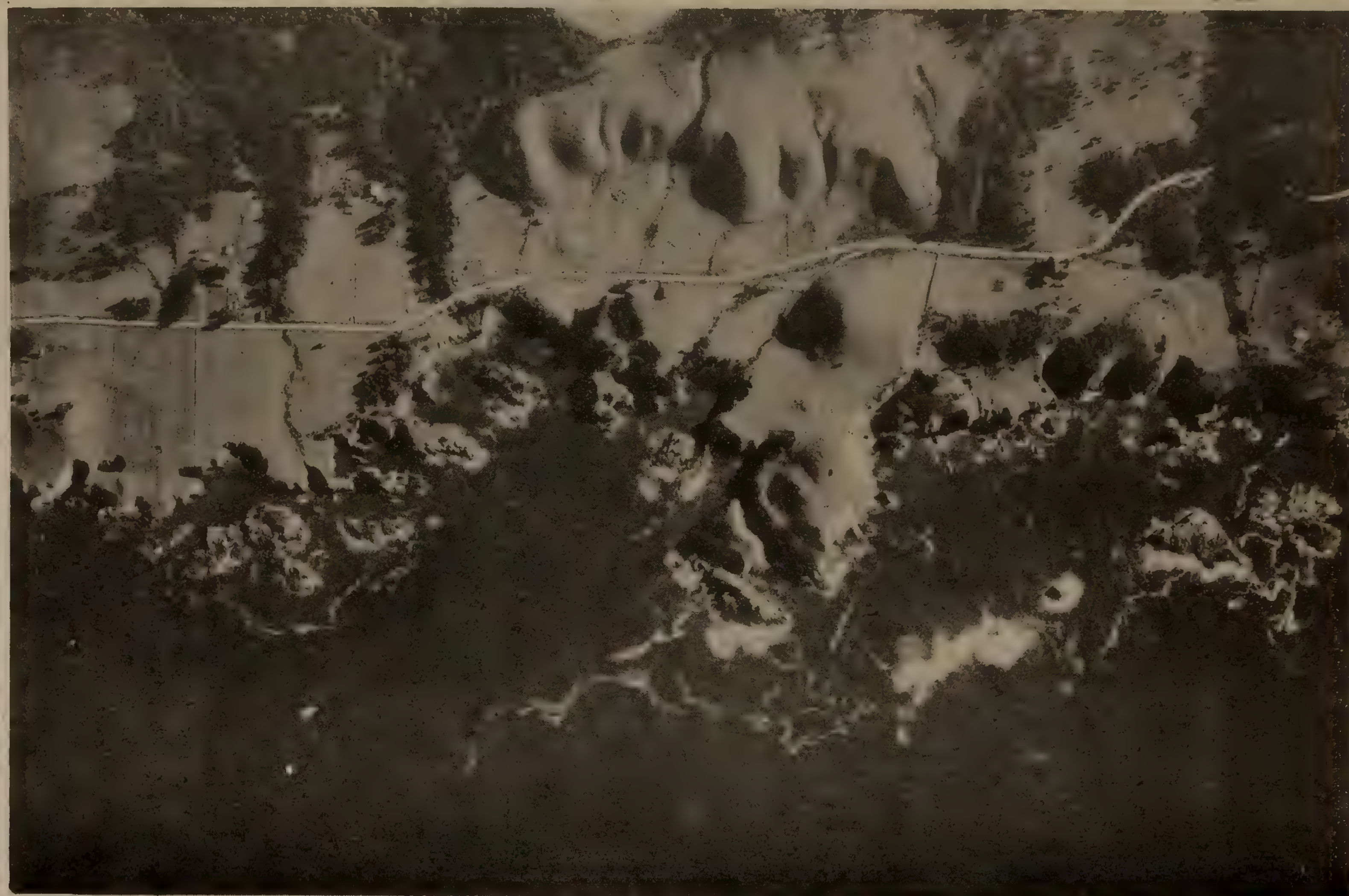


FIG. I-1-7 - Two terraces near Little River, California, are fine examples of wave-cut benches now raised above the sea. A new and very irregular seacliff is now being carved by the waves, and stacks, caves and arches are all present. Note the cave open to the surface at the extreme left and the irregularities in the old bottom which show as hills on top of the point.

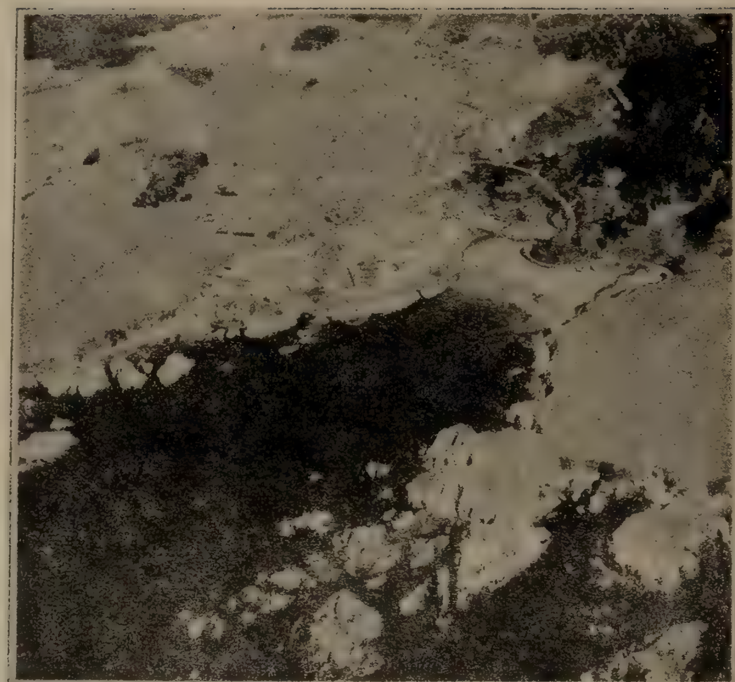
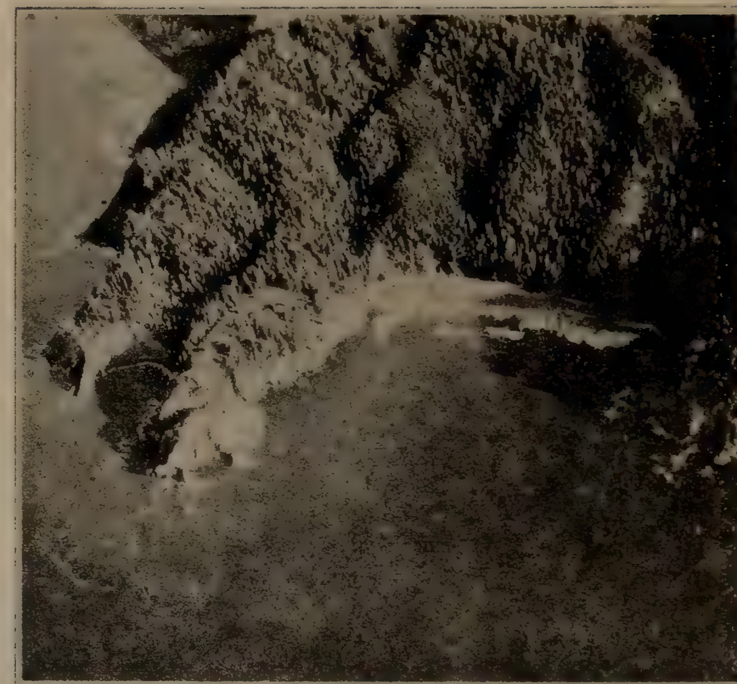


FIG. I-1-8a - Caspar Creek, California, beach is typical of bay-head beaches on a rough coast. Beach material is supplied both by the creek and by the eroding headlands at each side. Beaches like this make ideal landing places because the surf is always low.



b- Cape Falcon and Short Sand beach on the Oregon coast exemplify the erosion of headlands and the formation of a beach from the material. Steep cliffs are formed by the waves which attack the headland from all sides; note that the slope behind the beach is relatively gentle.

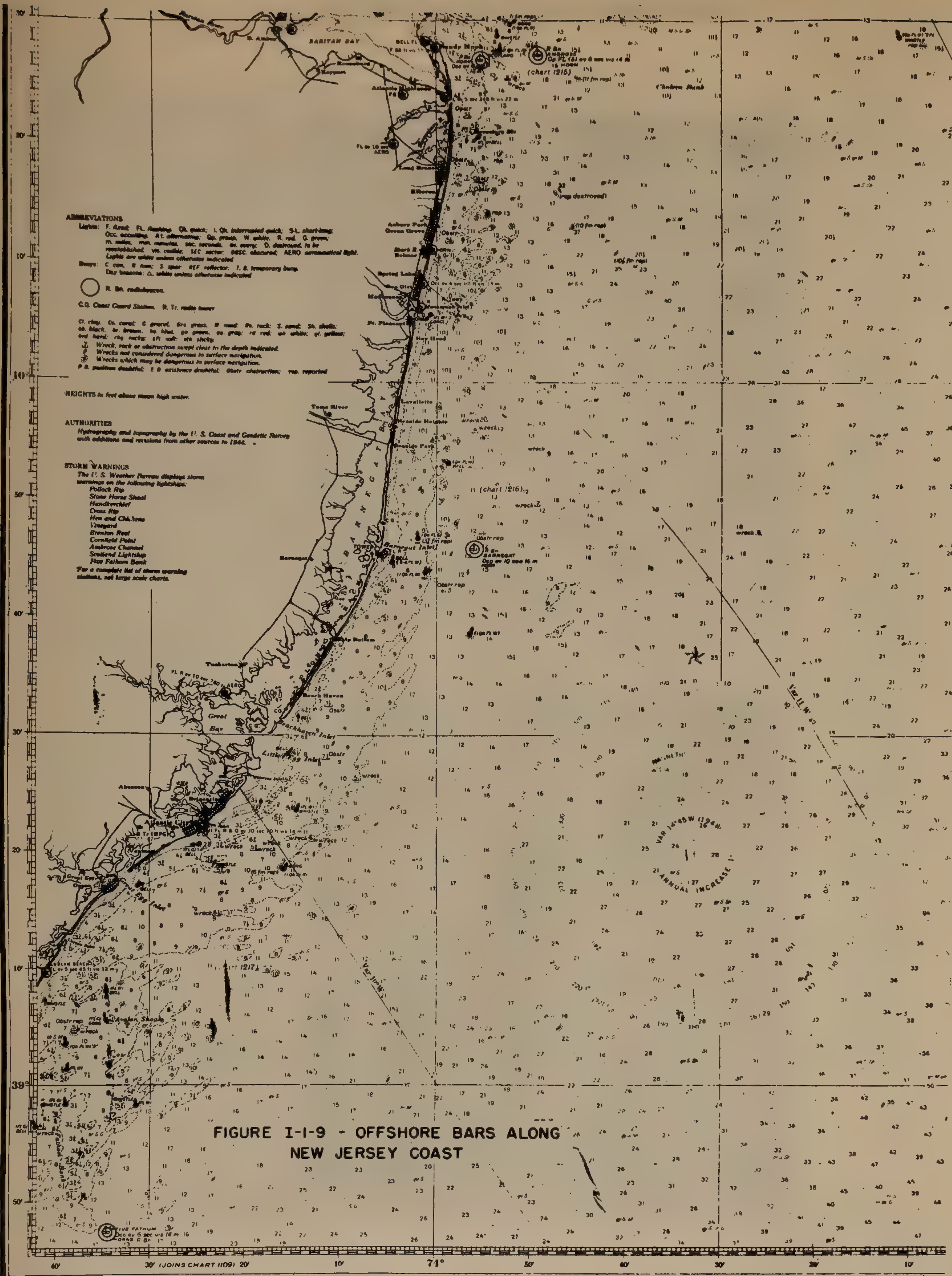


FIGURE I-1-9 - OFFSHORE BARS ALONG NEW JERSEY COAST

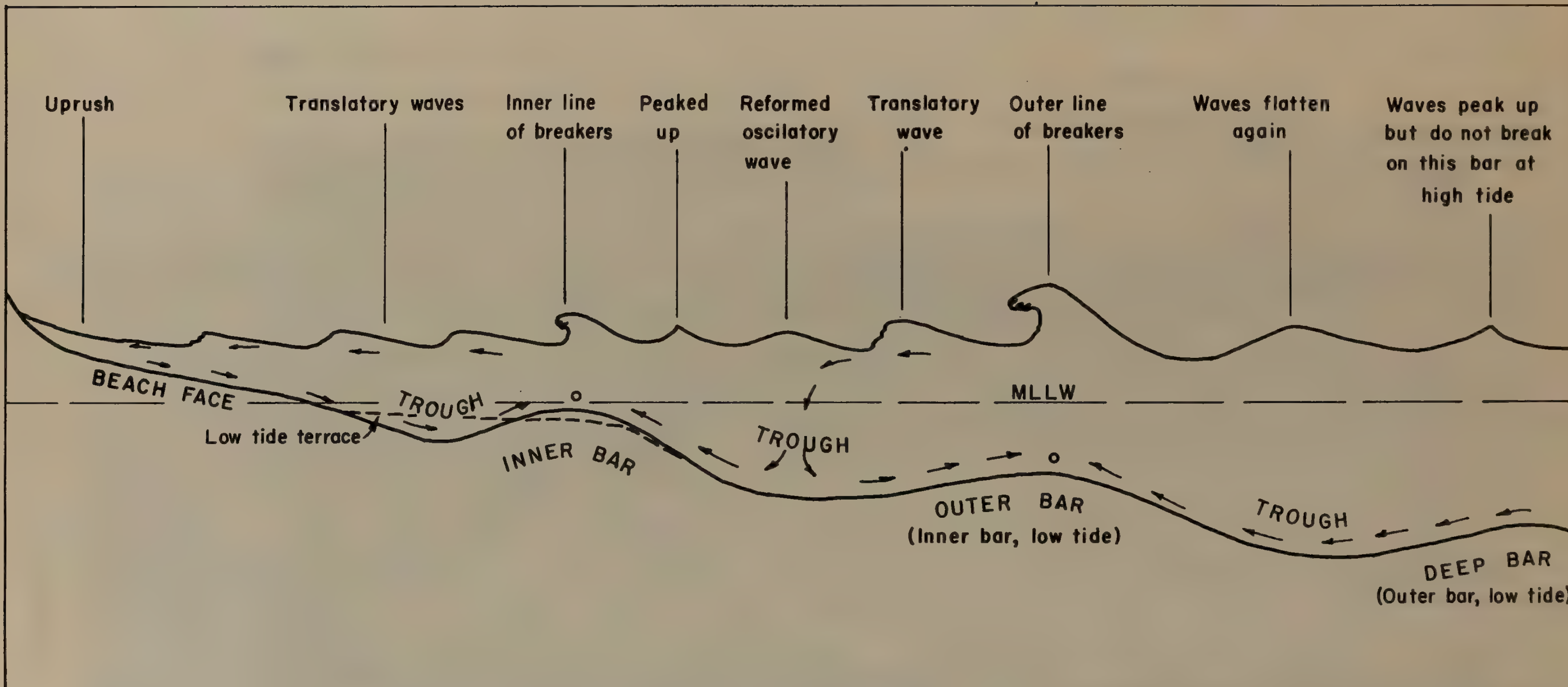
FIG. I-1-10 - Bay-mouth bars on the Washington coast separate large and shallow tidal bays from the north Pacific. Pt. Brown is the prograding beach area illustrated in Figure I-1-24a.

Fig. I-I-11 - Development of Sandy Hooks spit. As the original shore between Seabright and Long Branch was cut back by wave attack, the zone of spit formation north of Navesink Highlands advanced toward the northeast. The fulcrum point, dividing the zone of retrograding shoreline from that of prograding shoreline, shifted progressively from F^1 to F^4. West of the letters "oo" in "Hook" is a small southward-pointing spit built of waves from the northwest out of material eroded from the recurved points of the main spit. (After J. W. Johnson) 124

FIG. I-1-12 - Spits in the Straits of Juan de Fuca

FIG. I-1-13a - The Santa Monica, California, breakwater has caused a tombola to start forming. Sand, migrating along the coast from the left, deposits in the quiescent zone behind the breakwater. Beaches beyond the structure are deprived of their natural supply of sand and retreat.

b - Tombola on the Washington coast. James Island protects coast from the prevailing westerly seas. The Quillayute River, which also supplies much of the sand, takes advantage of this easy outlet to the sea and is now maintained in a permanent position there by jetties - not visible at this high tide. A smaller tombola has formed on the flank of the large one.



WAVES AND NET BOTTOM CURRENTS OVER UNDERWATER BARS AT HIGH TIDE

DIAGRAMED ON PROFILE OF BEACH
AT MANZANITA, OREGON

FIGURE I-1-14

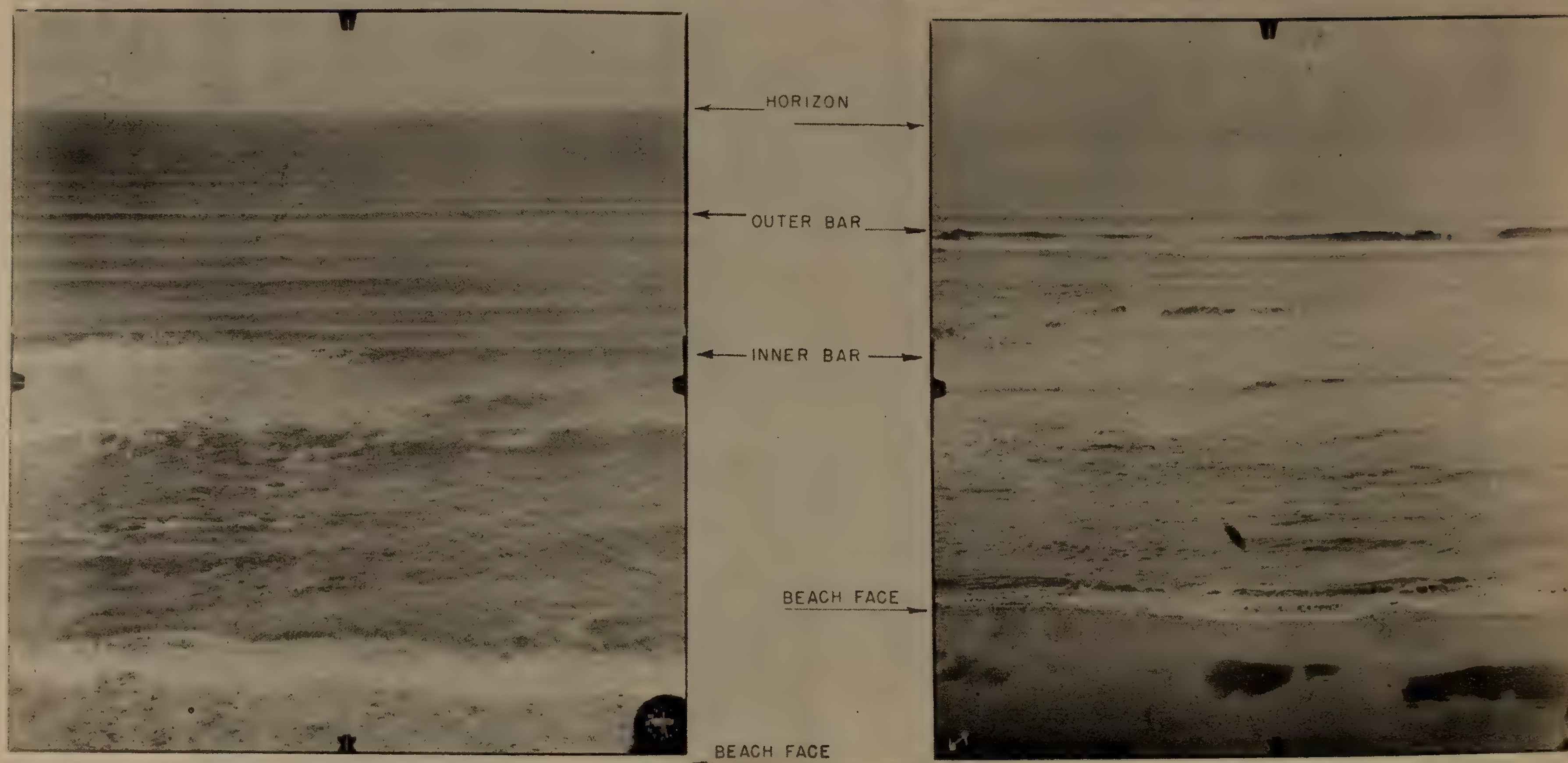
HYD-5361

FIG. I-1-15 - GRENVILLE BAY, WASHINGTON, OCTOBER 27, 1948. POINT GRENVILLE to the north (left) protects the beach at the left from the prevailing northwest swell. There the waves break directly on the beach face; at the right, where there is no protection, there are two lines of breakers (indicating bars) besides those on the beach.

FIG. I-1-16a - Bar at Carmel Beach is awash at low tide. Since there is no return of water seaward across the bar under these conditions, all of the white water seen must flow through the feeder trough in the foreground and thence out to sea by a rip channel which is off the picture to the left.

b - An extreme low tide has exposed this classic example of a longshore bar and rip channel on the flat beach at Cape Mears, Oregon. The steeper beach face can be seen just to the left of the flooded trough.

TABLE BLUFF, CALIFORNIA



HIGH TIDE — 7.1
1-16-46 0925

At high tide longcrested regular swells approach the beach. They peak-up on the outer bar, but do not break until they reach the inner bar. Waves break a second time on the beach face.

LOW TIDE — -0.5
1-16-46 1602

At low tide, even though the waves are somewhat lower, they now break on the outer bar and again on the inner bar. This expends most of the energy before the beach face is reached.

FIGURE I-1-17

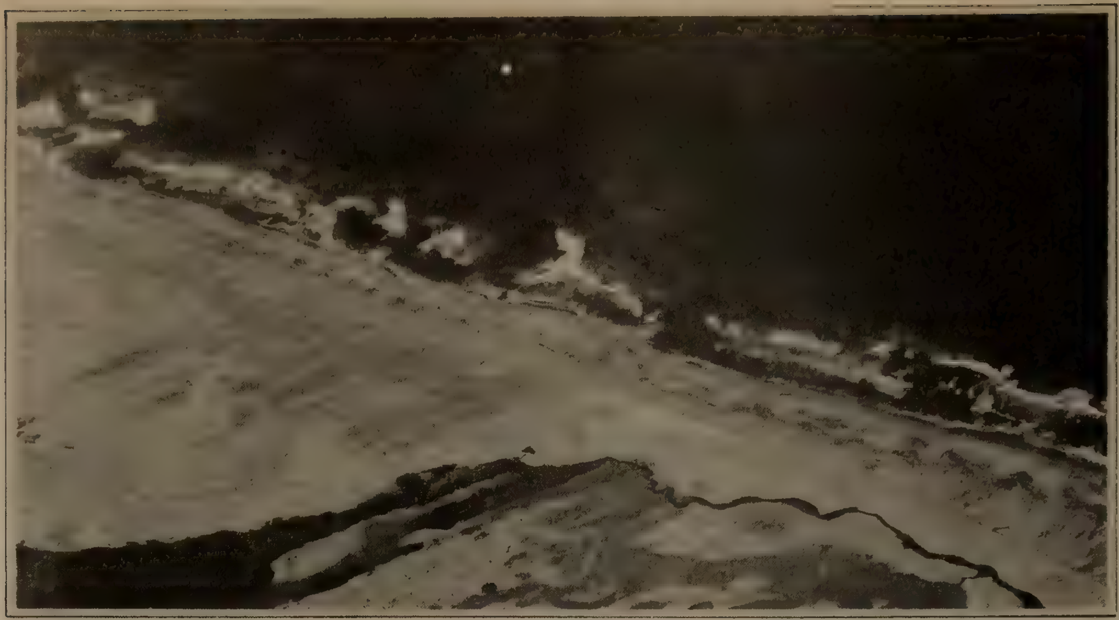


FIG. I-1-18a - Occasionally, when the sea is calm, the suspended material in the surf settles out and the bottom is plainly visible. The shapes of irregular bars and rip channels can be seen in this photo of a beach near Fort Bragg, California. Notice the cusps at the right and the creek that forms a moat at the dune line.

b - Bars are not always even ridges of sand that parallel the shore, as shown in these of complex bars on widely separated beaches (Salinas River, California, left, and Netarts Bay, Oregon, right). Photographs like these which "look along the coast" are often helpful in illustrating the relationships between coastal features.

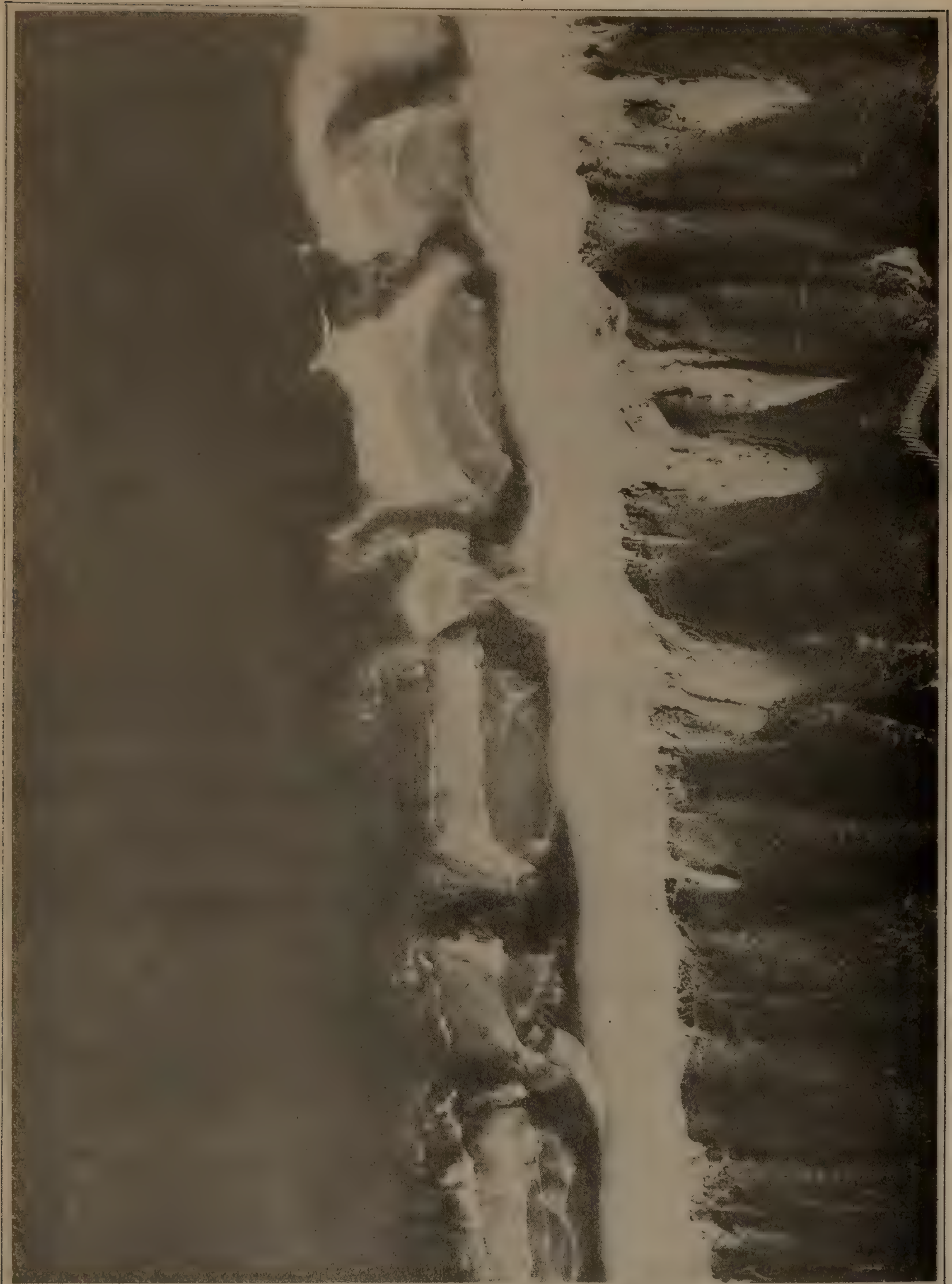


FIG. I-1-19 - Rip channels in a low tide terrace at Fort Ord, California. Deep channels such as these allow access to beach face by small craft even at the low tide stage shown.

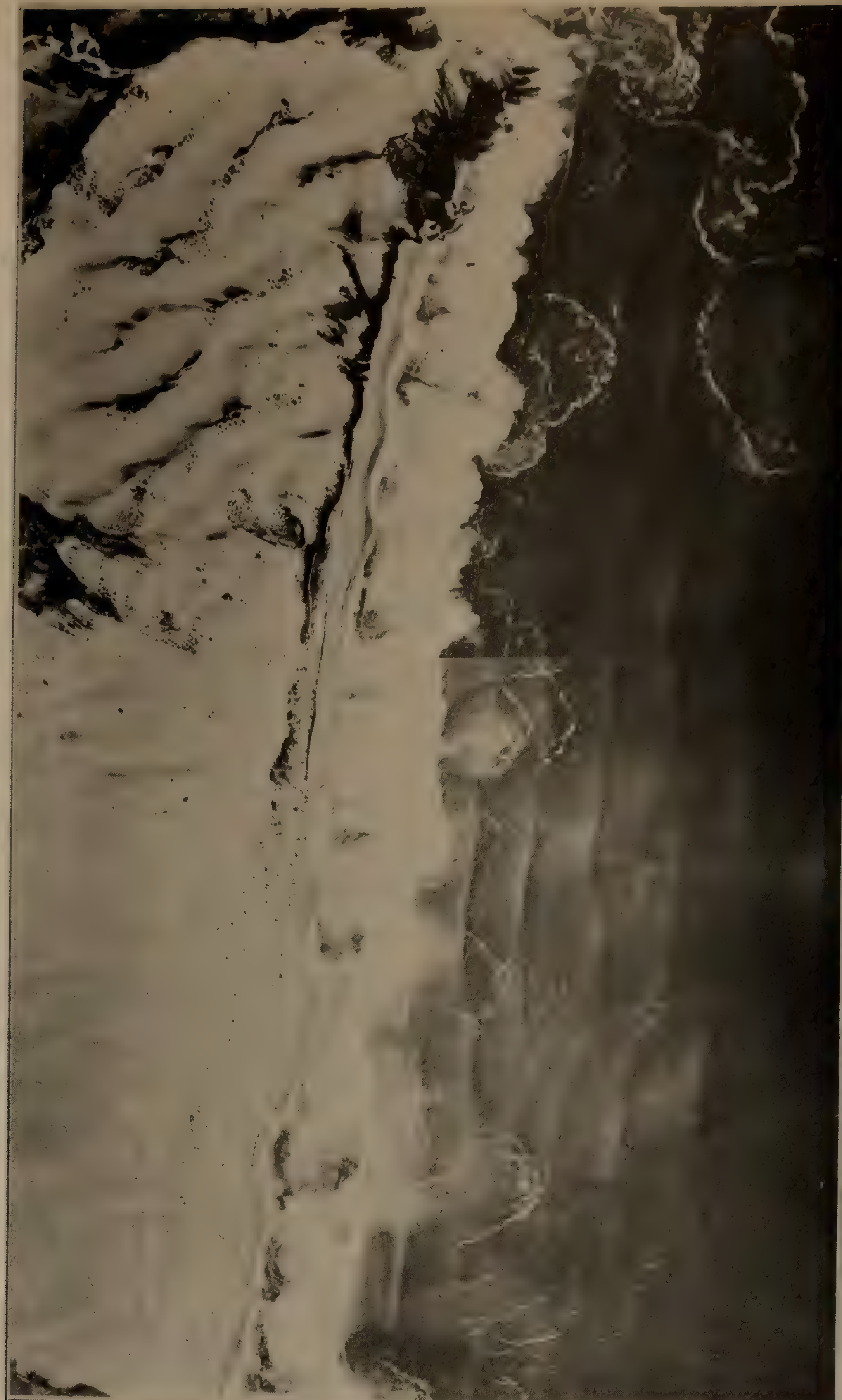


FIGURE I-1-20 - Rip currents at Mussel Point, California, (Camp Cooke) are readily distinguishable from the air by the dark water in the breaker zone and the foam-marked vortices off the beach. The seacliff increases in height to the south and is surmounted by a large dune area. Migrating sand falls into creek and is returned to the sea again.

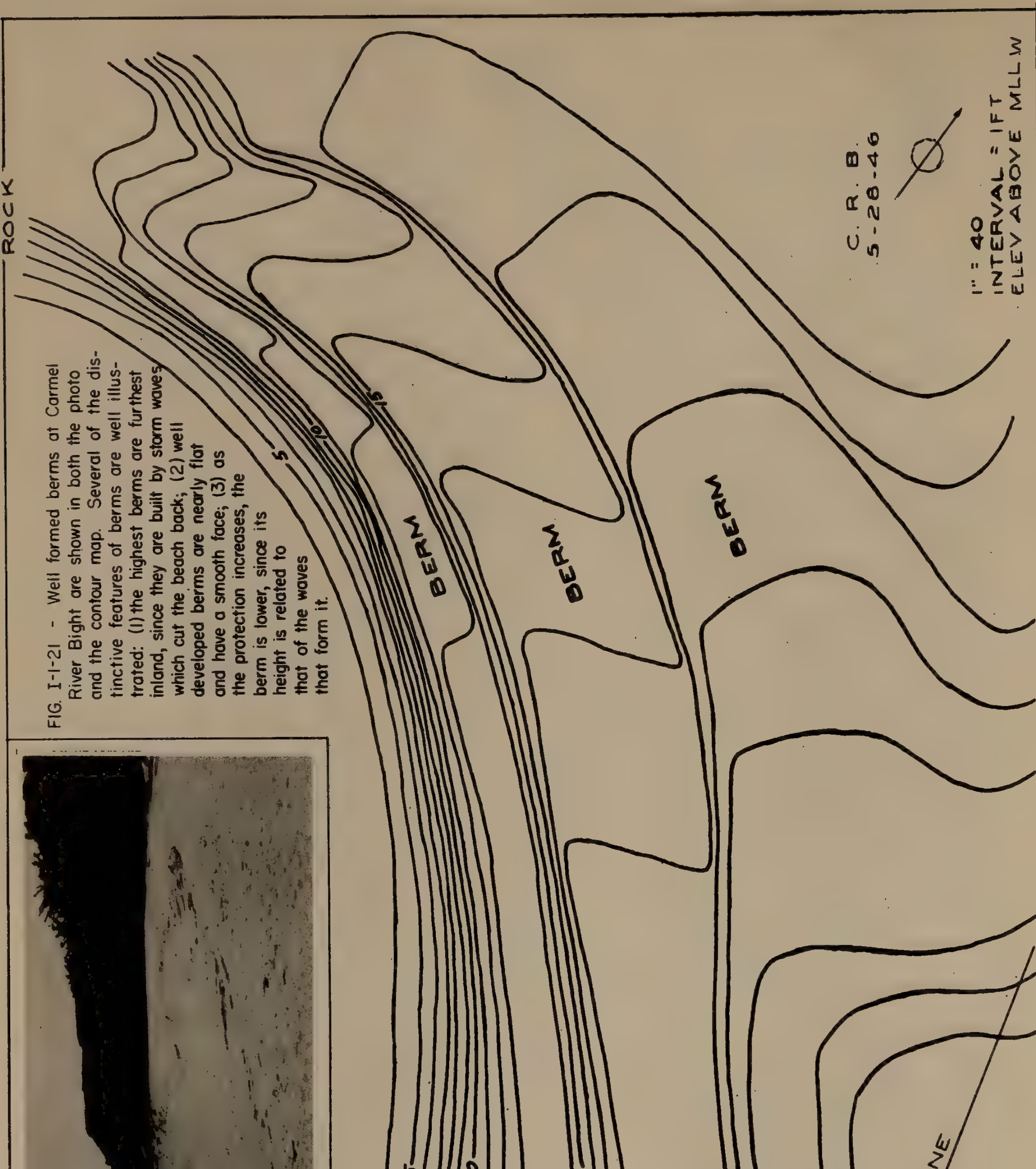
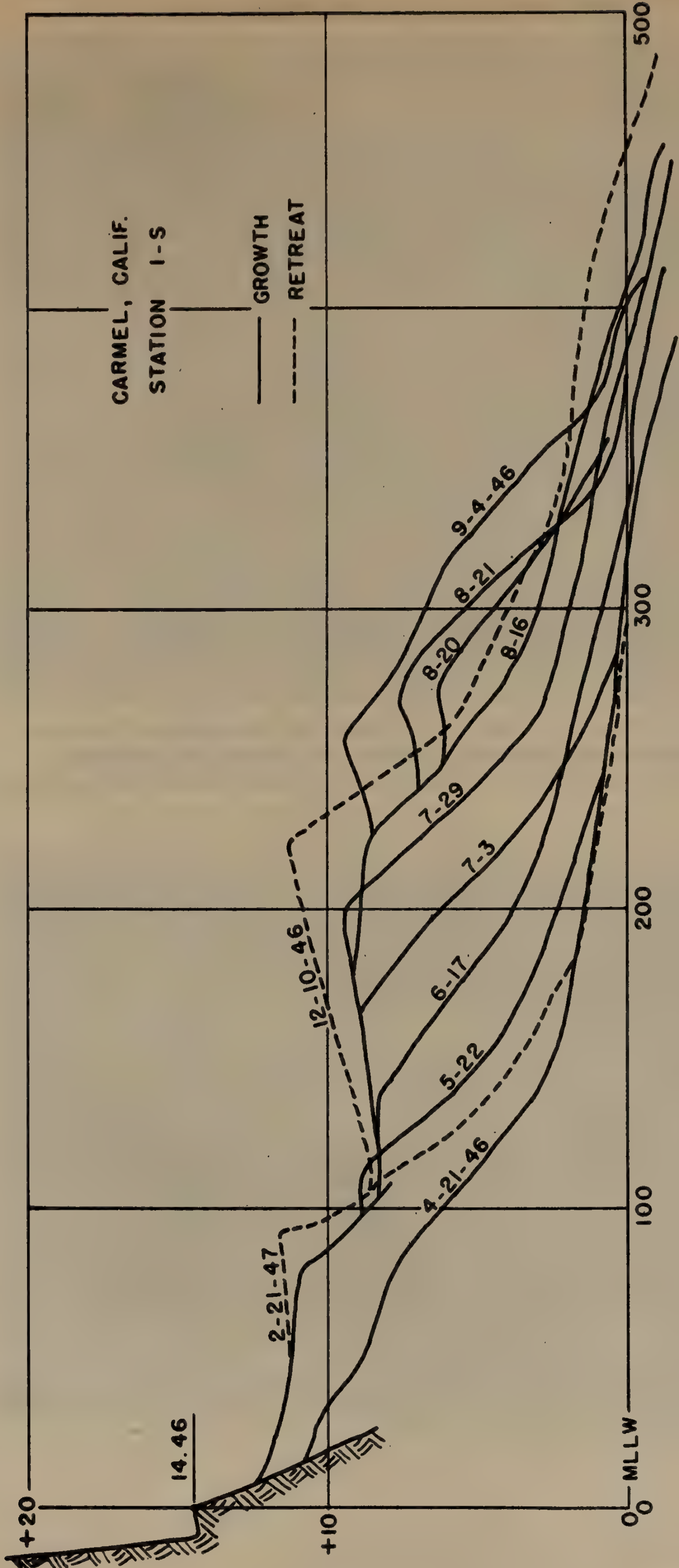


FIG. I-1-21 - Well formed berms at Carmel River Bight are shown in both the photo and the contour map. Several of the distinctive features of berms are well illustrated: (1) the highest berms are furthest inland, since they are built by storm waves which cut the beach back; (2) well developed berms are nearly flat and have a smooth face; (3) as the protection increases, the berm is lower, since its height is related to that of the waves that form it.



GROWTH AND RETREAT OF A BEACH

FIGURE I-1-22

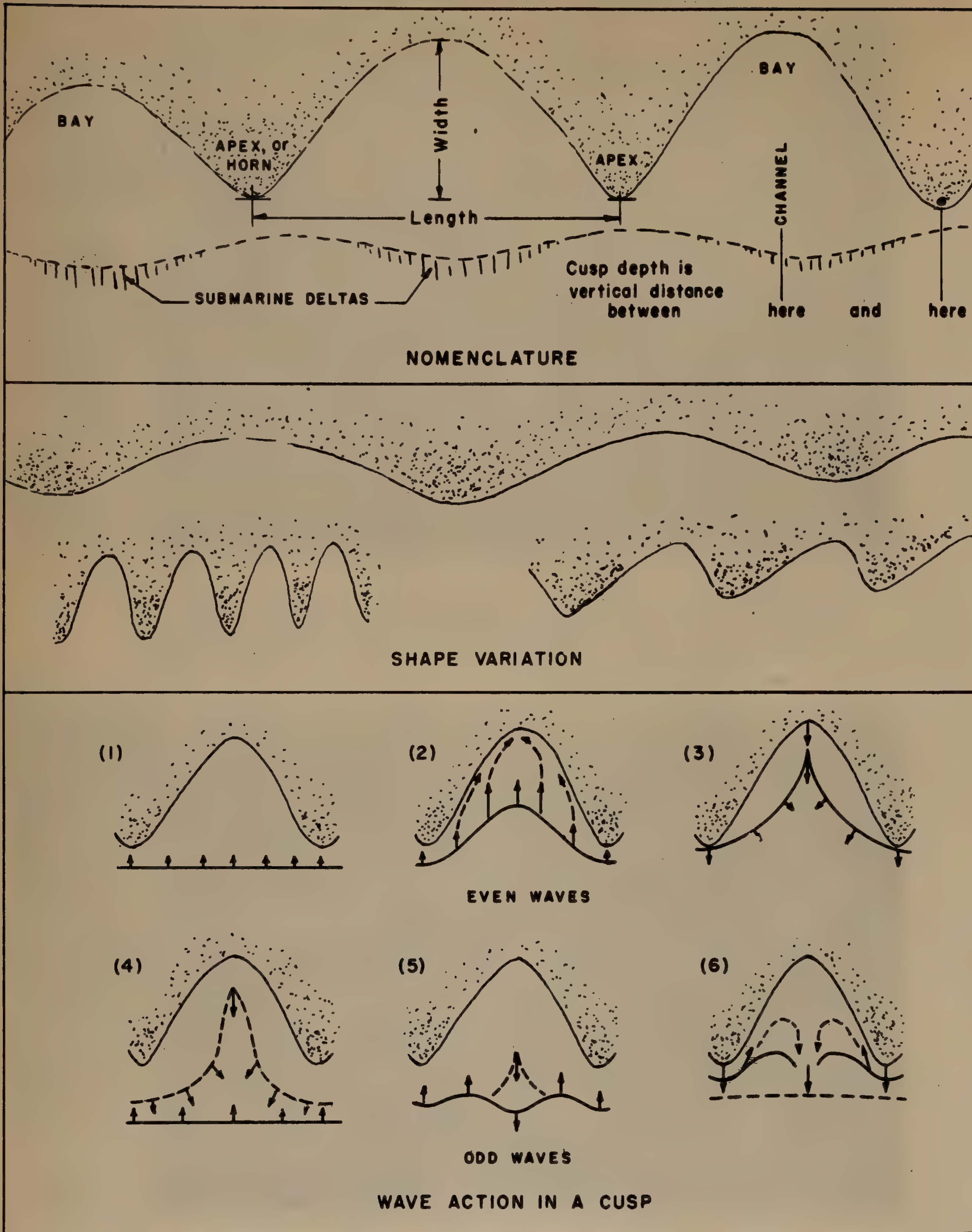
HYD-5360

FIG. I-1-23a - An uprush crossing the crest of the berm. In this manner the berm grows vertically and traces of previous berm crests are eliminated. Excess water returns to sea through the run-off channel at the right.

b - A new berm (under DUKW) has widened the beach at Seacliff, California. The old "winter" shoreline with its large cusps is now abandoned at the back of the beach. Run-off channel is just over of the right hand tree.

FIG. I-1-24a - Beach ridges at Gray's Harbor, Washington, are marked by lines of varying vegetation. Beach is widening more rapidly in the foreground and the ridges converge in the distance. Note the remarkable similarity between this beach and the one below, which is on the opposite coast of the U.S.

b - Beach ridges at Parramore Beach, Viriginia. Each ridge marks an old beach line - probably a berm formed by a large storm. If the building of a berm is followed by a long period of deposition, the ridge is likely to remain.



CUSPS

FIGURE I-1-25

HYD-5362

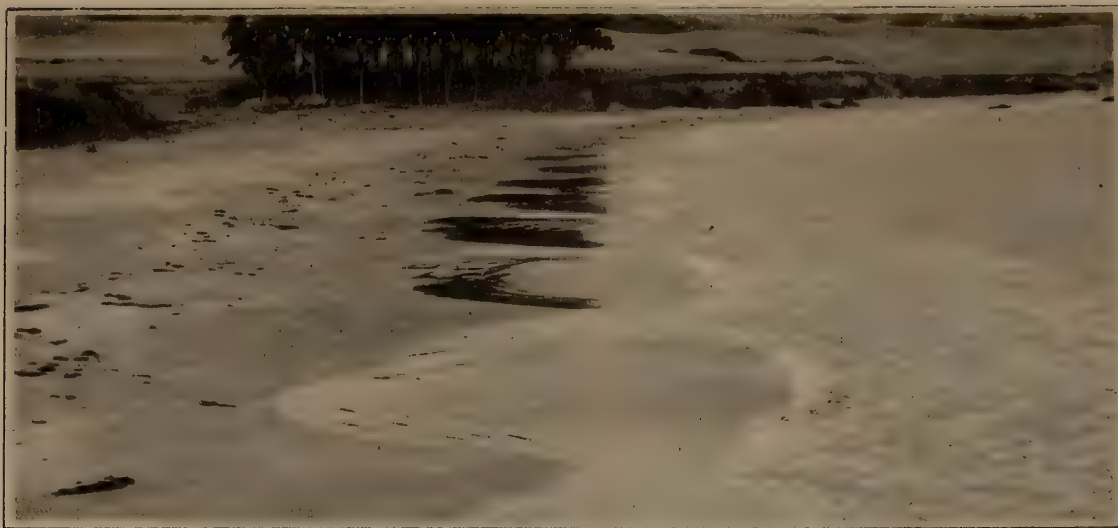
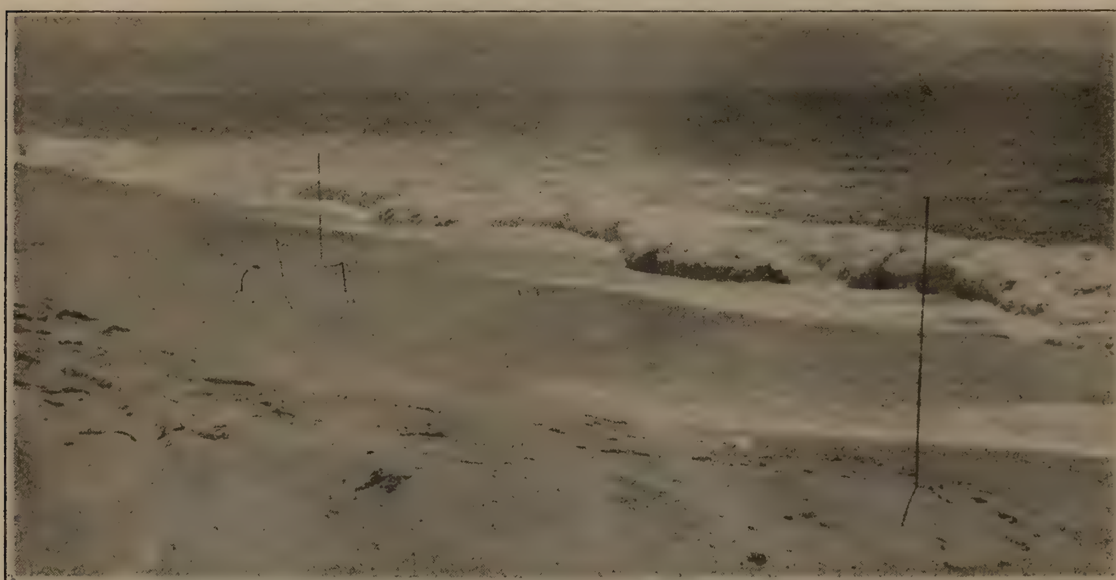
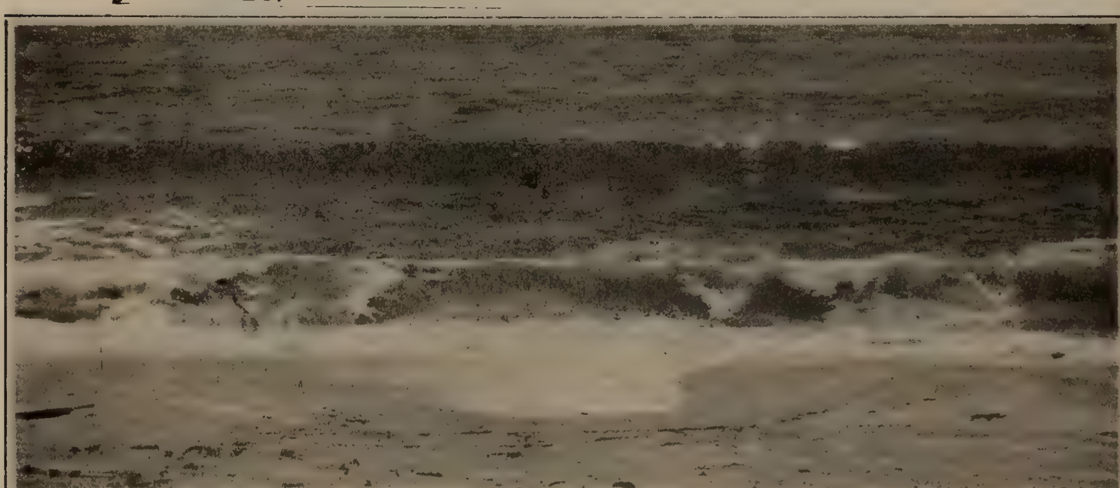


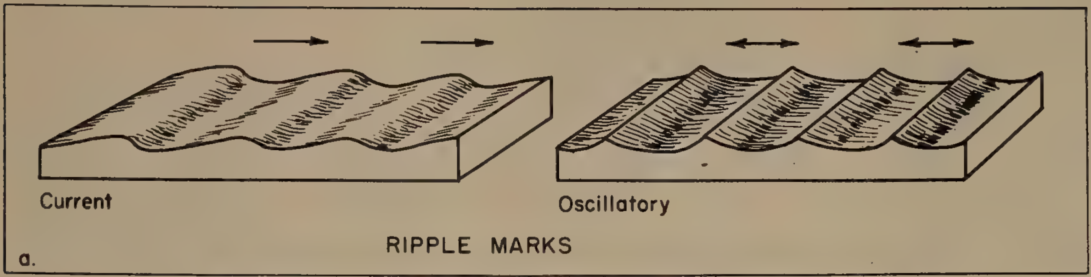
FIG. I-1-26a - Cusps in protected San Simeon Bay, California



b - Cusps on the exposed beach at Fort Ord, California, are bold in contrast to the gentle ones shown above. The wave stage is comparable to 6 in the diagram (Figure I-1-25).



c - The same cusp as above. Situation is now comparable to stage 4.



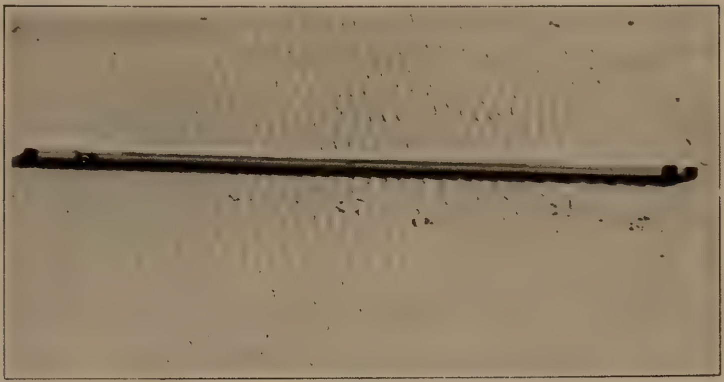


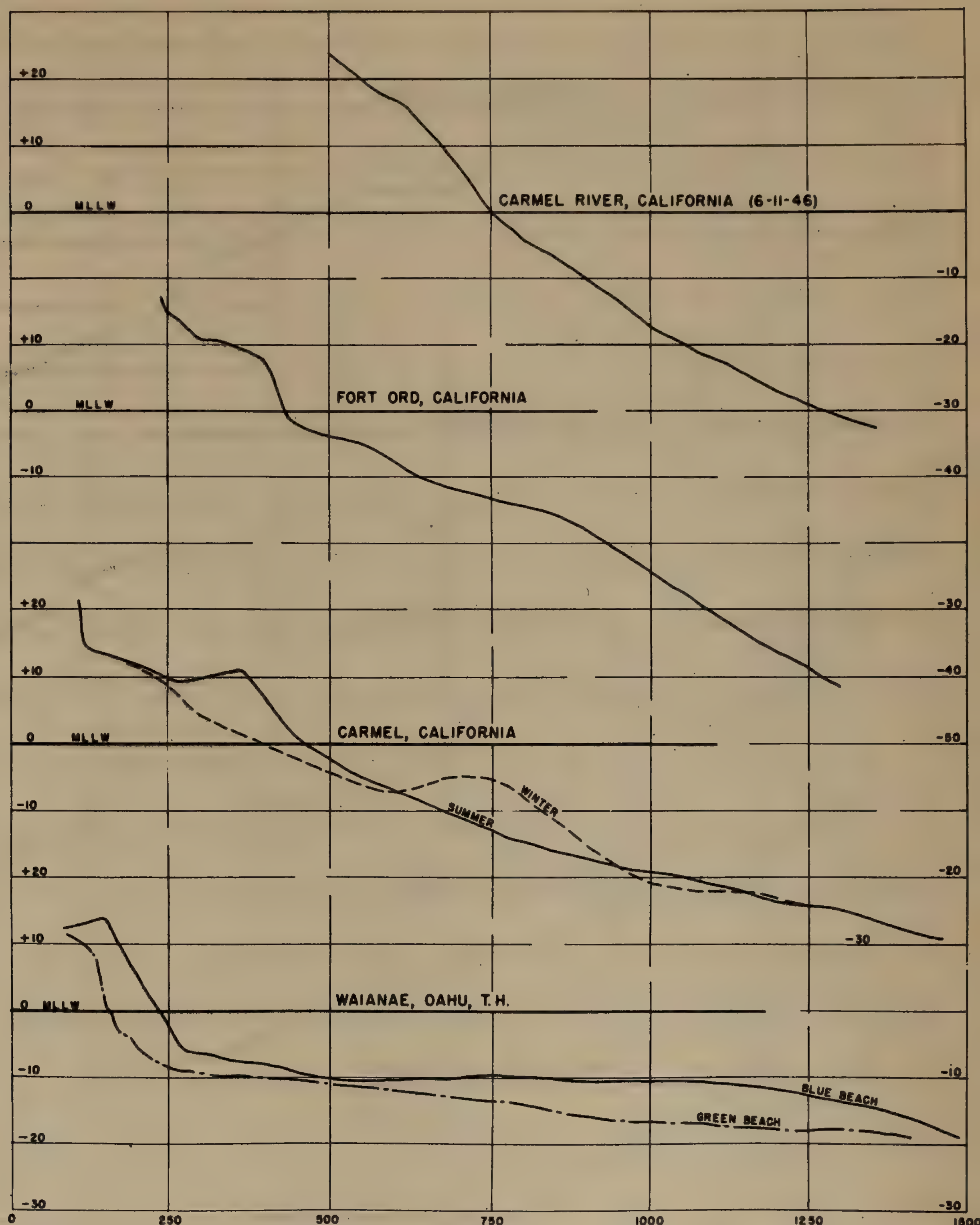
FIG. I-1-27b - Small current ripples at Copalis, Washington, are about 0.15 feet from crest to crest. Current flowed from right to left.

FIG. I-1-27c - "Giant" ripples in Netarts Bay, Oregon, as seen from the air. Those at the lower left are probably 50 feet from crest to crest and several feet deep.

FIG. I-1-28

PROFILES OF STEEP BEACHES

HORIZONTAL:VERTICAL = 1:2.5

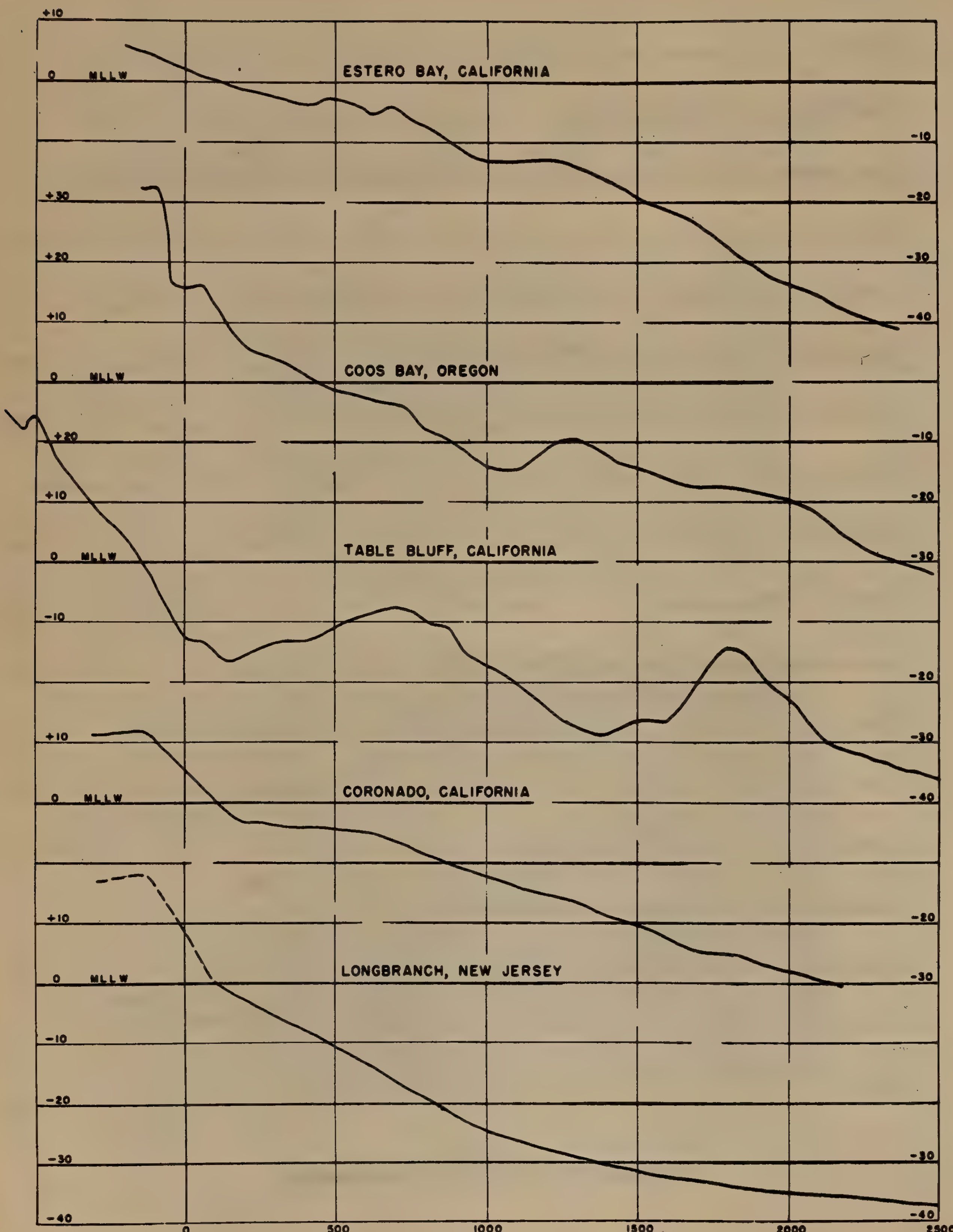


HYD-5393-116

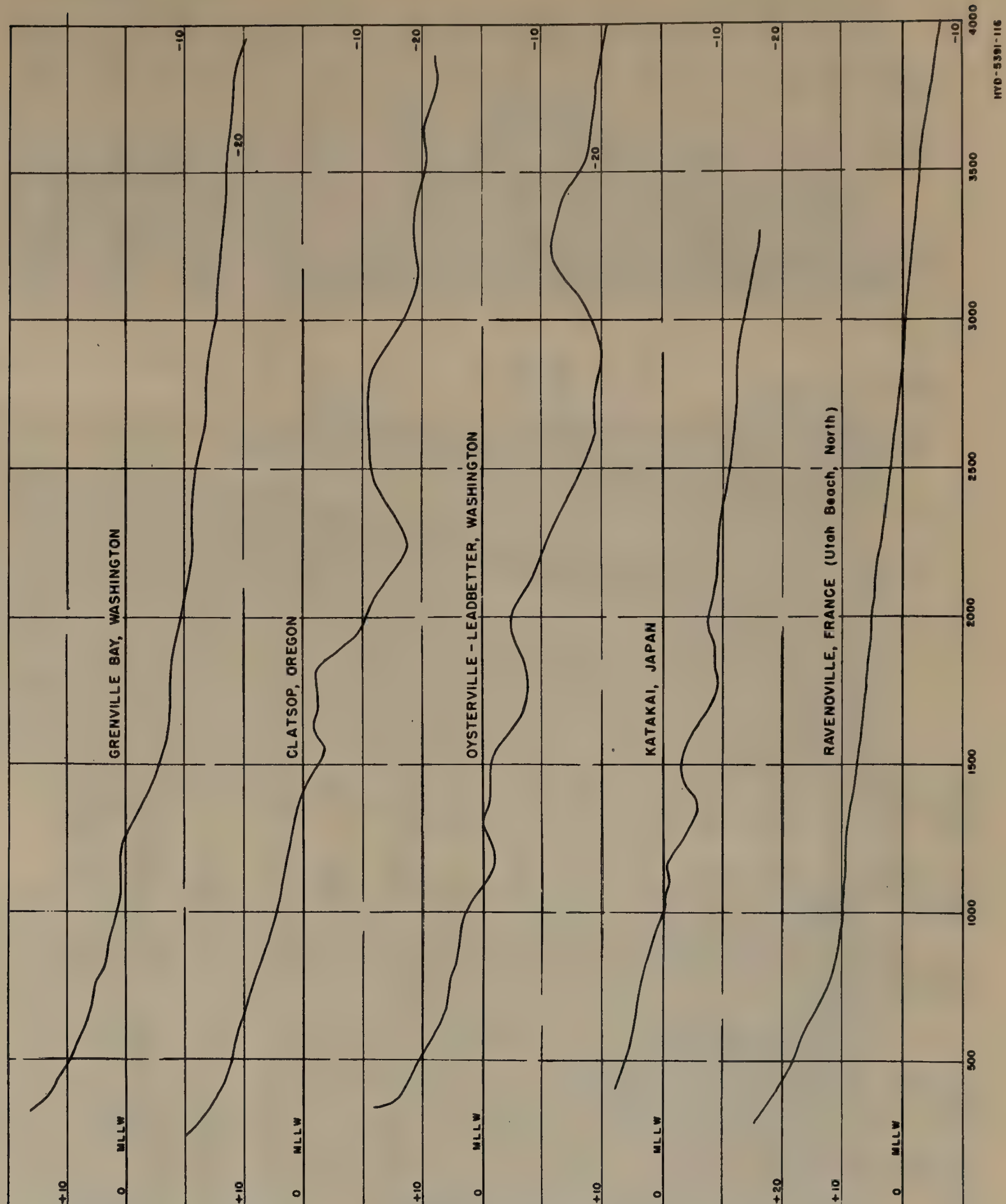
FIG. I-1-29

PROFILES OF INTERMEDIATE SLOPE BEACHES

HORIZONTAL : VERTICAL = 1:5



NYD-8382-116



F. [illegible]-1-30
PROFILES OF FLAT BEACHES
HORIZONTAL: VERTICAL = 1:5

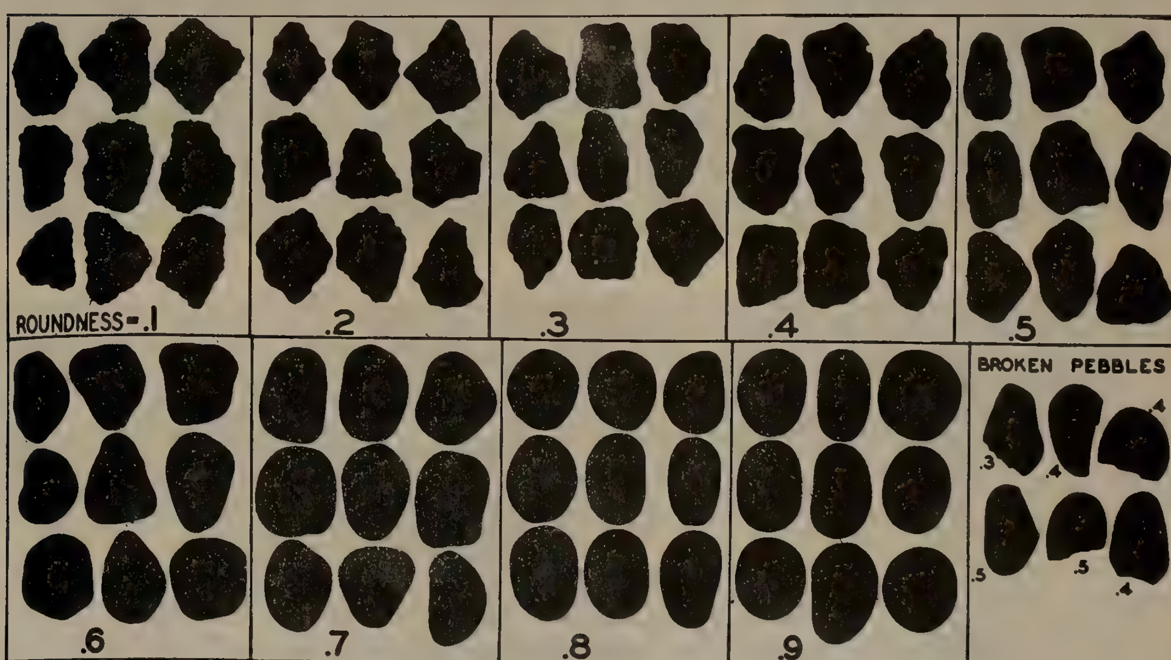
FIG. I-1-31a - Double cobble berms near El Rosario, Mexico. The soft alluvium cliff to the right appears to be the source of the cobbles on the beach after the fine material is transported to sea.

FIG. I-1-31b - Cobble bar exposed at low tide on the beach at Kincheloe, Oregon. All of the major beach features form on cobble beaches as well as on those of sand.

FIG. I-1-31c - Cobble berms, sometimes called ramparts, near Santo Domingo, Mexico, form impressive barriers, such as this 25-foot dam at the stream mouth. Two lower berms may be distinguished; at about MLLW fine sand intersects the cobbles. The upper berm may be of many years standing since it supports plant life; only another great storm will change it.

PARTICLE IMAGES FOR VISUAL SPHERICITY

FIGURE I-1-32a



PEBBLE IMAGES FOR VISUAL ROUNDNESS

FIGURE I-1-32b

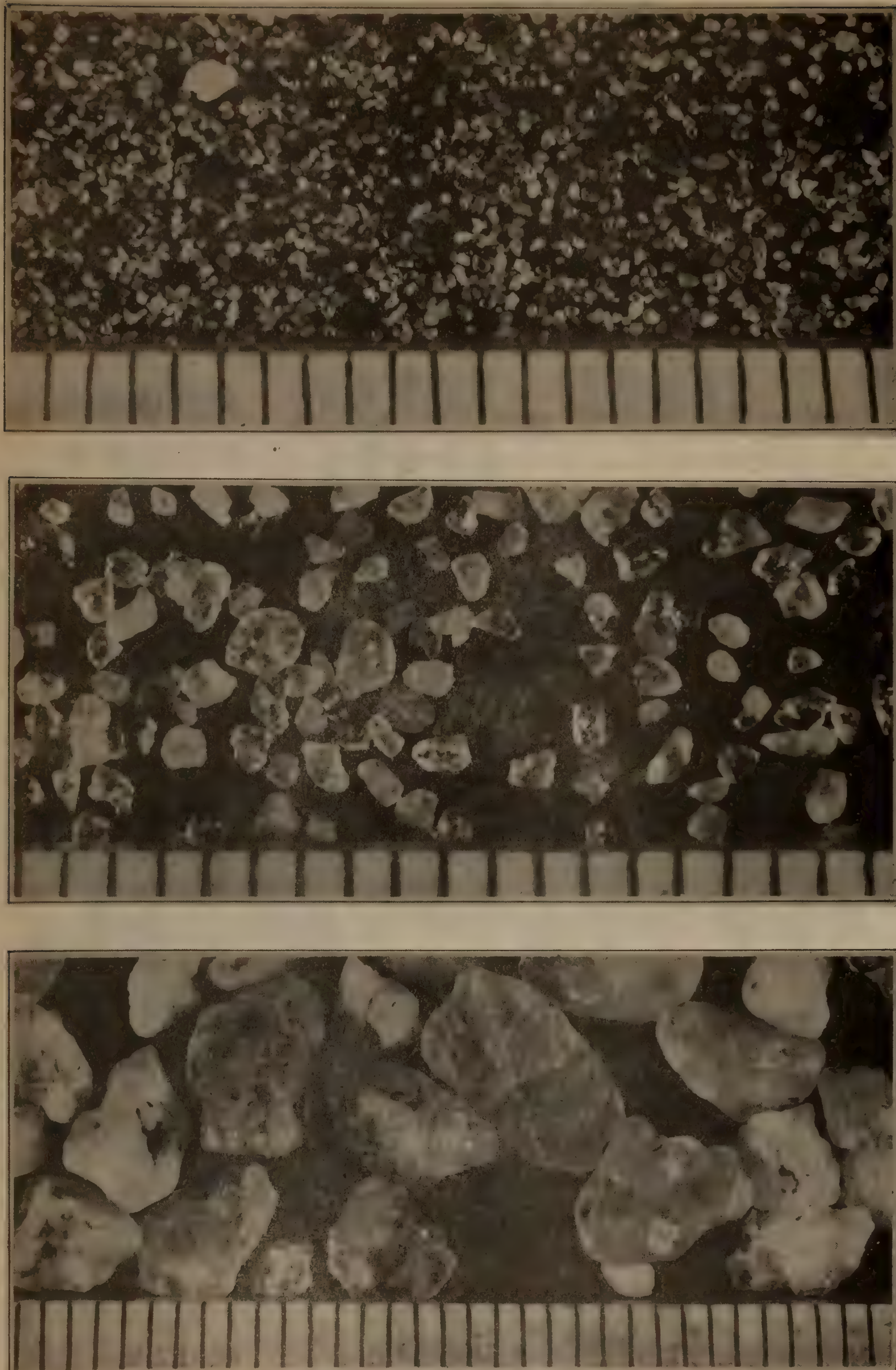


FIG. I-1-33 - Photomicrographs of beach sands showing range of sizes frequently encountered. Scale divisions are 1/64", or 0.397 cm.

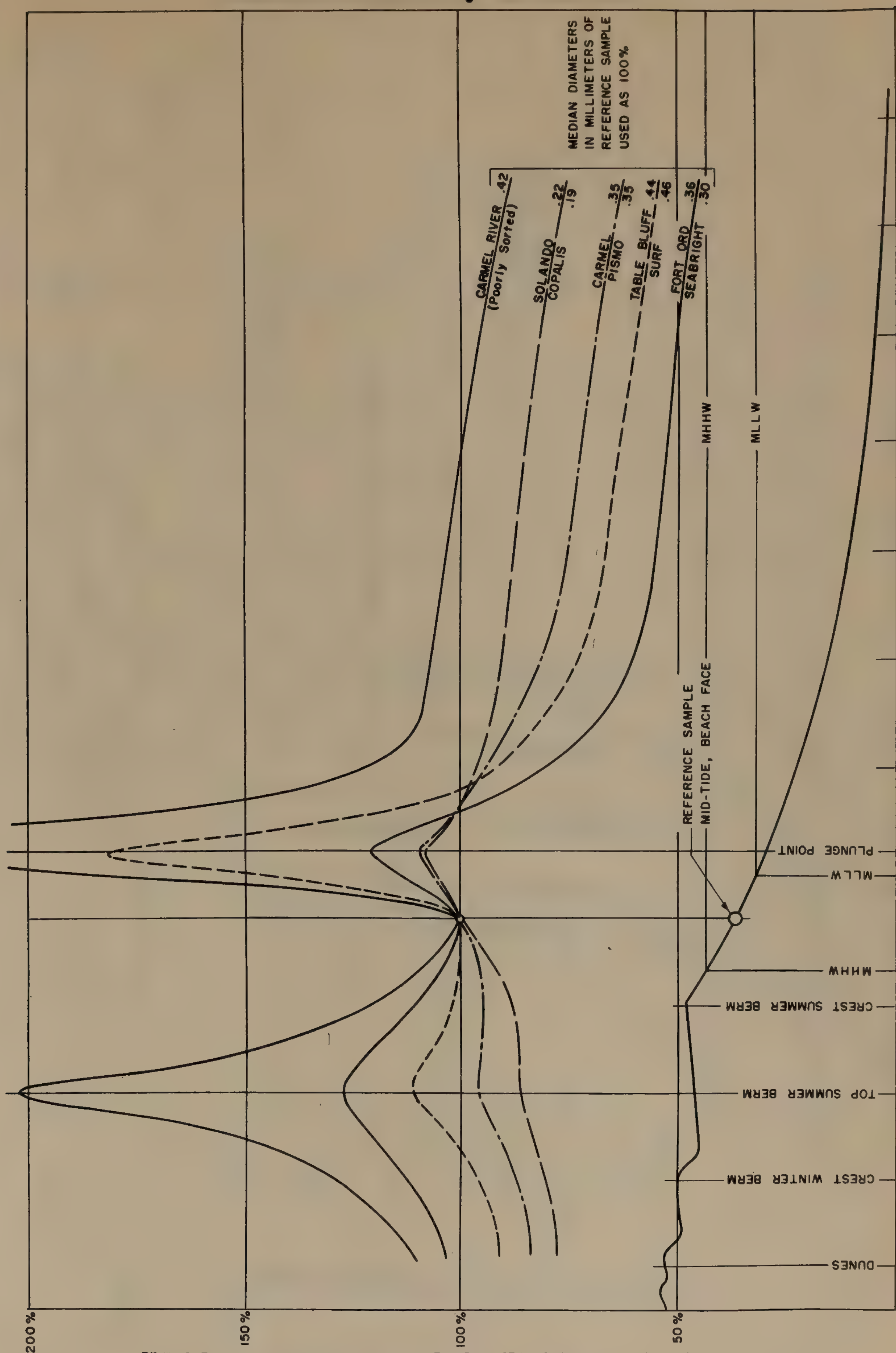


FIGURE I-1-34 - SIZE DISTRIBUTION ACROSS A BEACH

HYD-116-5296

HYD-5252-116

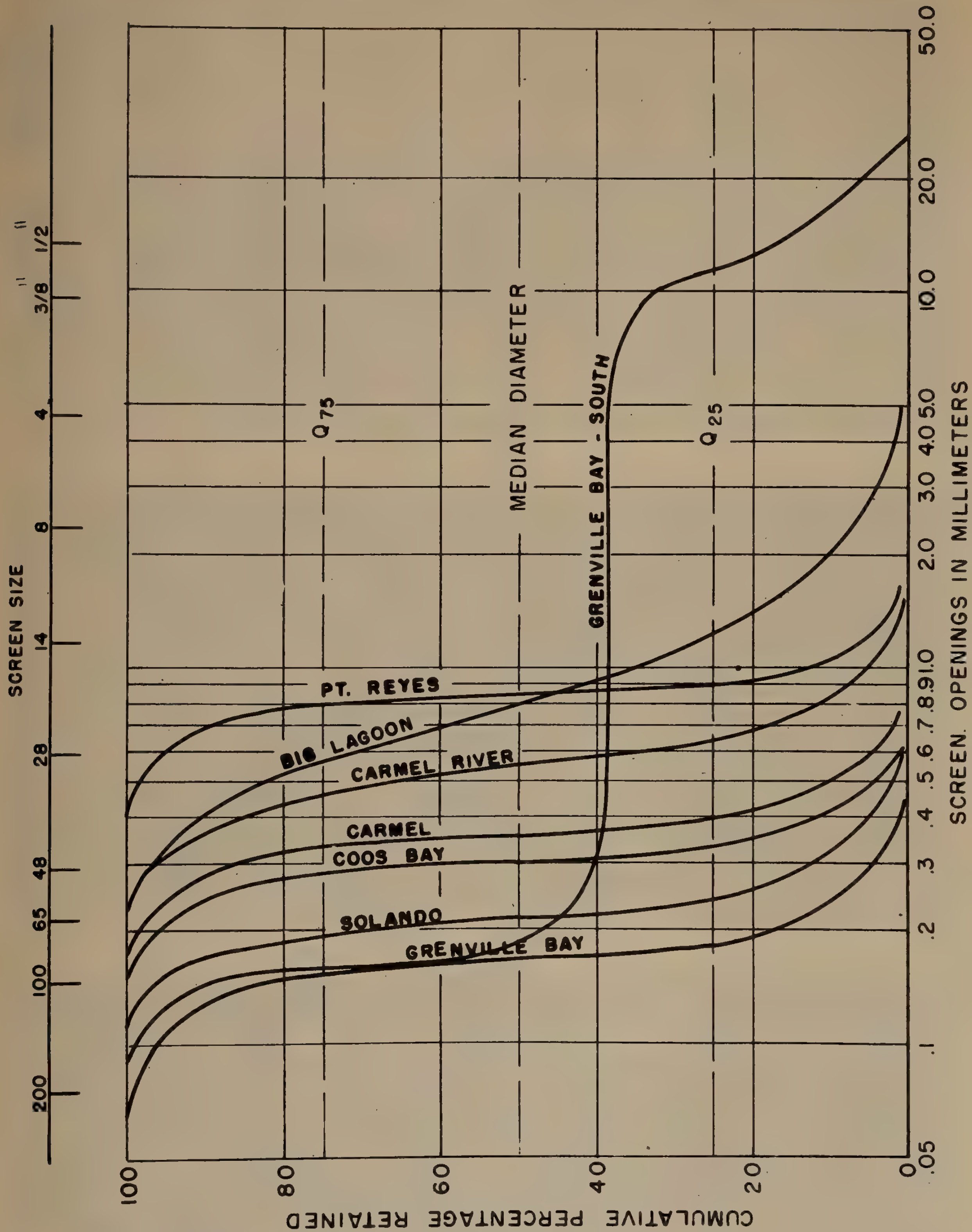


FIGURE I-1-35 - SIZE VARIATION OF BEACH SANDS

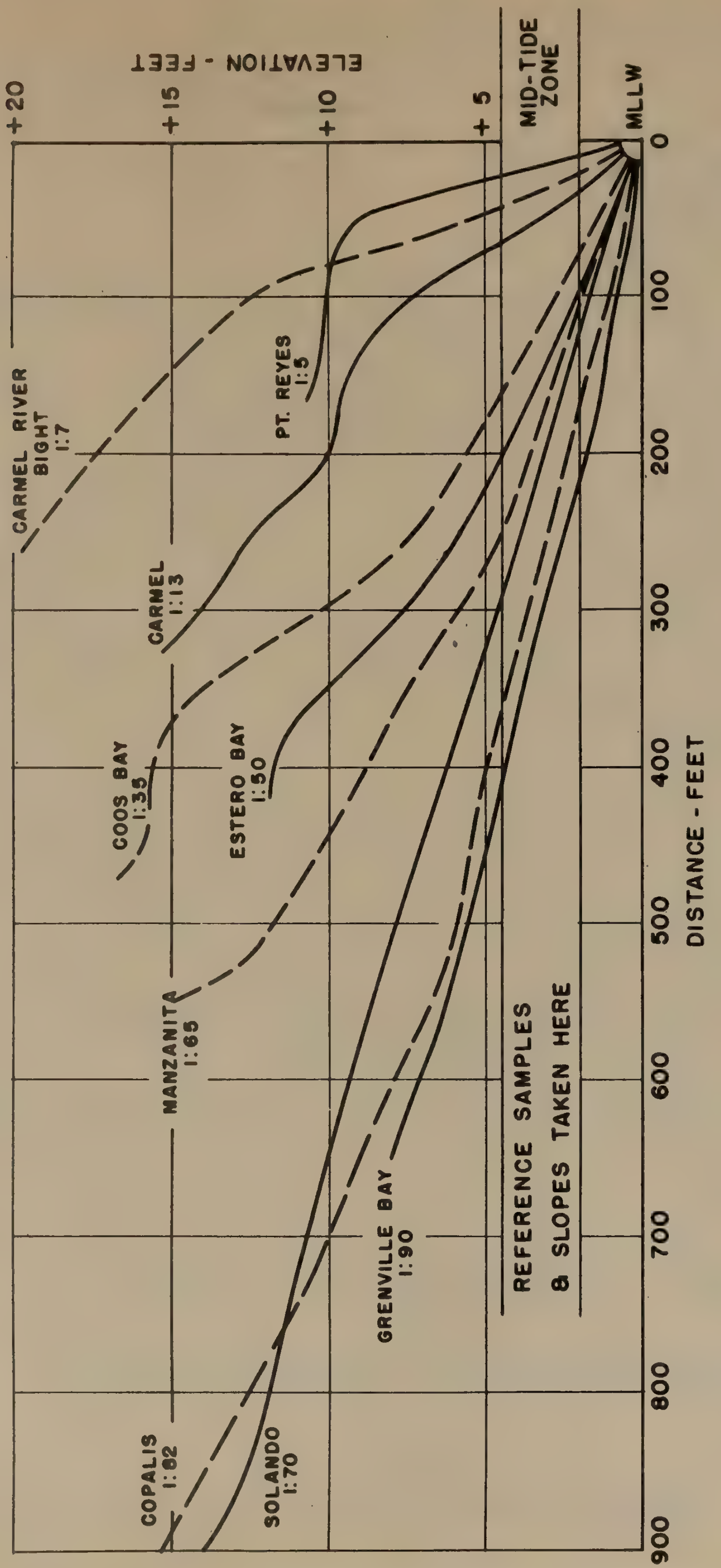


FIGURE I-1-36 - VARIATION IN SANDY BEACH-FACE SLOPE

HYD-5250-116

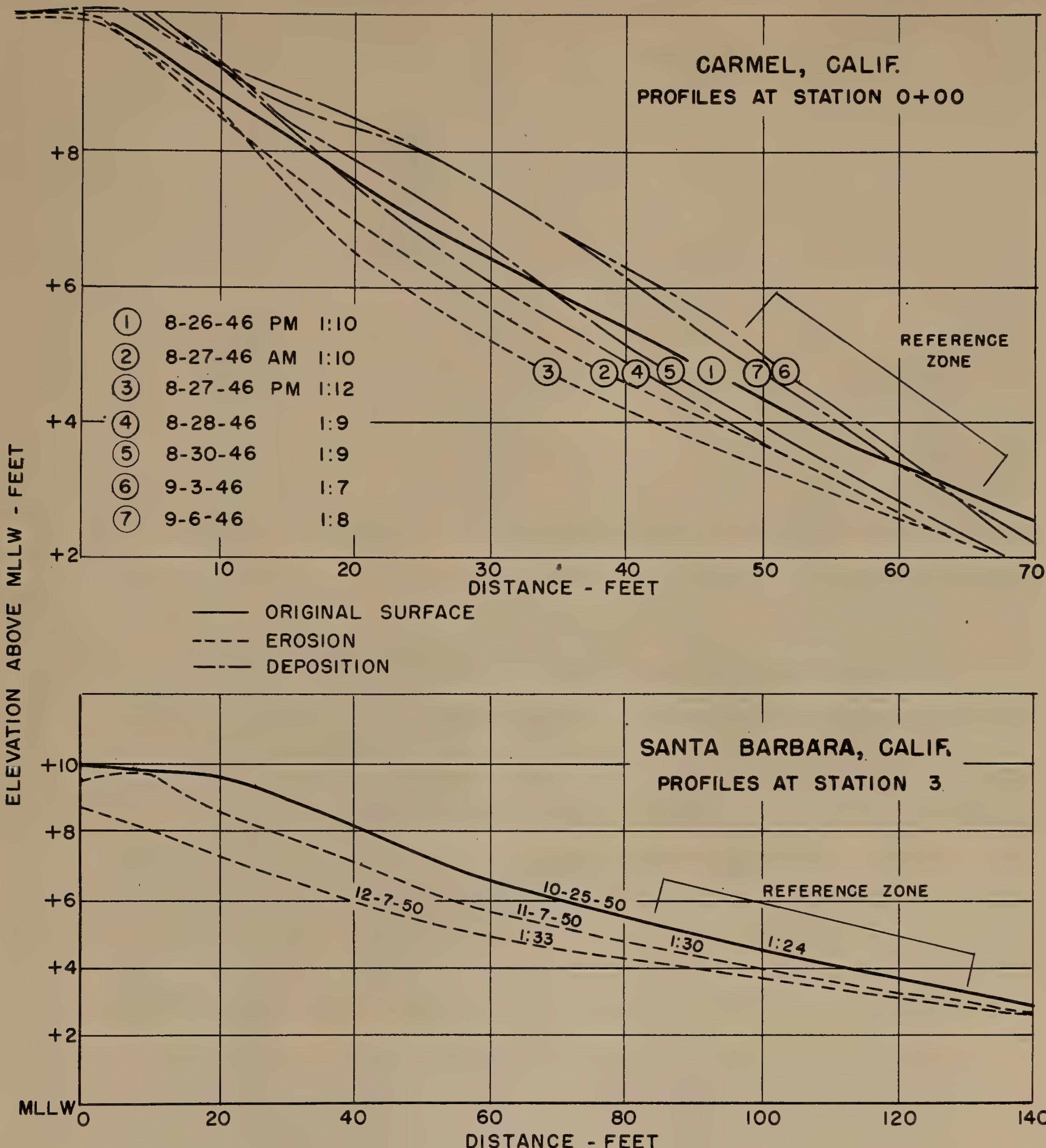


FIGURE I-1-37 - SLOPE CHANGES WITH EROSION & DEPOSITION

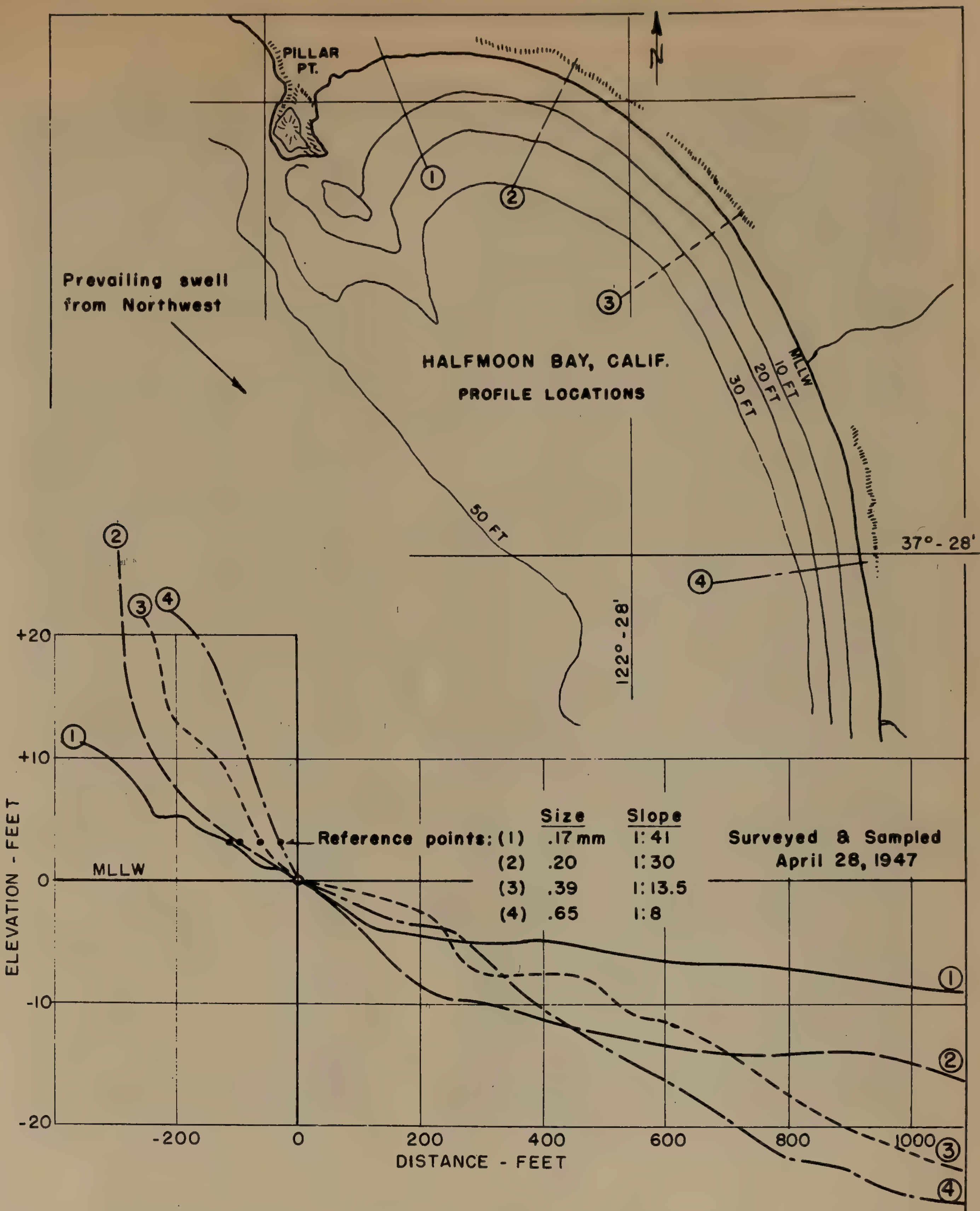


FIGURE I-1-38 - SIZE AND SLOPE VARIATION WITH PROTECTION

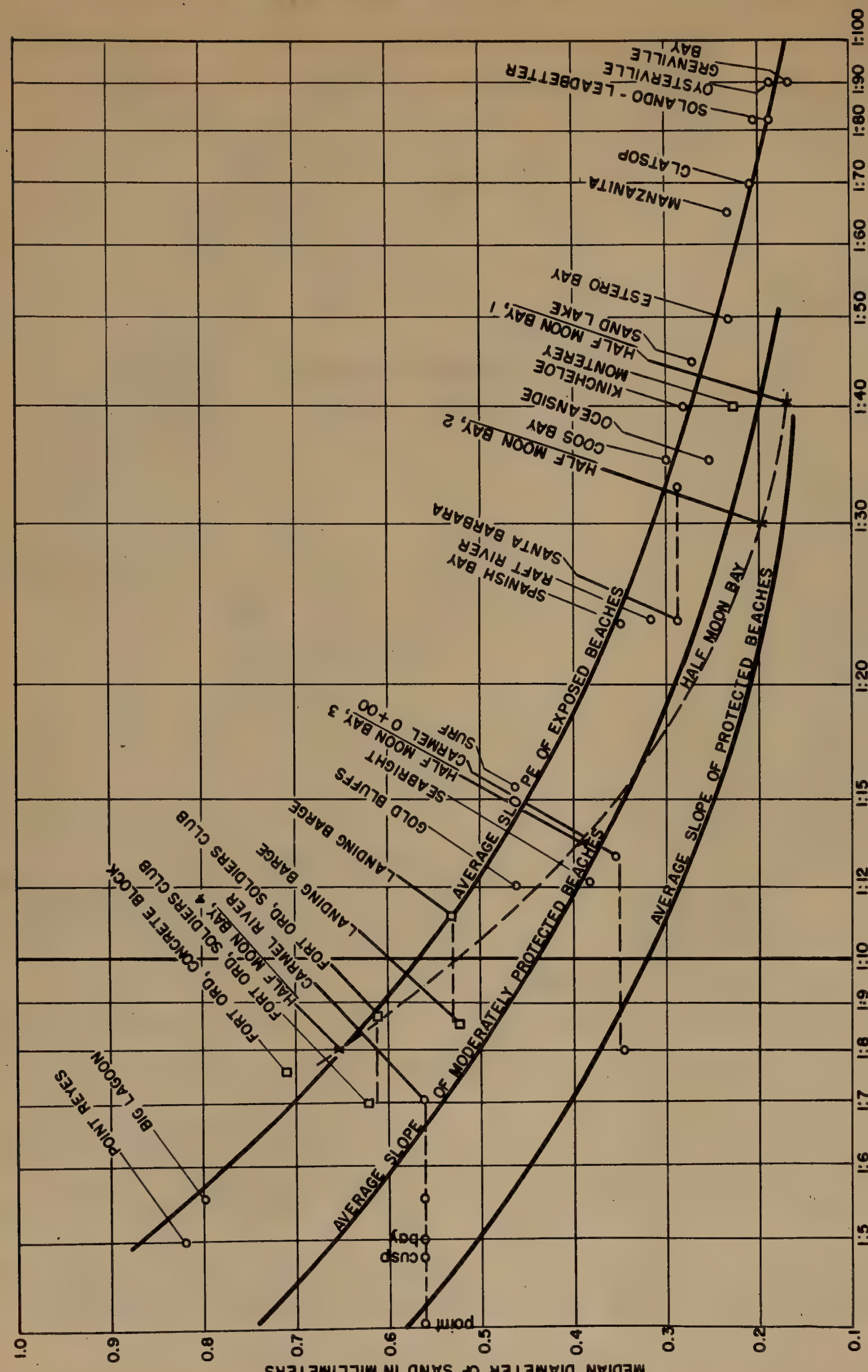


FIGURE I-1-39 - SIZE-SLOPE RELATIONSHIP AT THE REFERENCE POINT

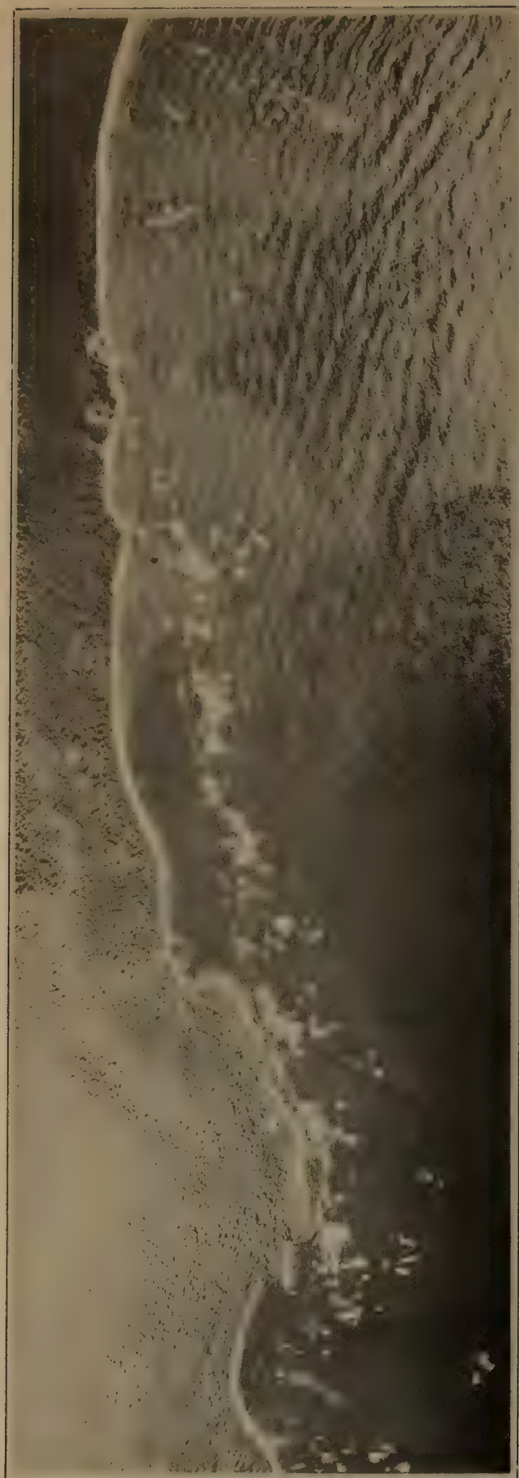


FIG. I-1-40a - High tide, high altitude. The sandy beach appears to have some possibilities as a landing beach. Although there is some indication of rocks, the line of breakers may be on a sand bar.

FIG. I-1-40b - Low tide, larger scale photo. The same area now appears to be a completely denuded wave-cut bench with only a narrow sand strip near high water.

FIG. I-1-41a - Berm at Sand Lake, Oregon, has been eroded by storm waves which have left a vertical scarp about 5 feet high.

FIG. I-1-41b - Same location as above, two weeks later. Calm seas have replaced the eroded sand and the scarp is now only about a foot high.

FIG. I-1-42a - Tillamook Bay, Oregon, entrance. Dashed line indicates approximate shoreline before the construction of the jetty. Southward moving sand has been halted by the jetty, which acts as a groin; the beach to the south is sand-starved and is retreating rapidly (see text).

b - Swell approaching the beach at an angle is never completely refracted and imparts an alongshore component to the water inside the breakers. This sets up the littoral currents which are so important in the movement of sand along a coast. In the example shown, near Oceanside, California, the current is moving from top to bottom.

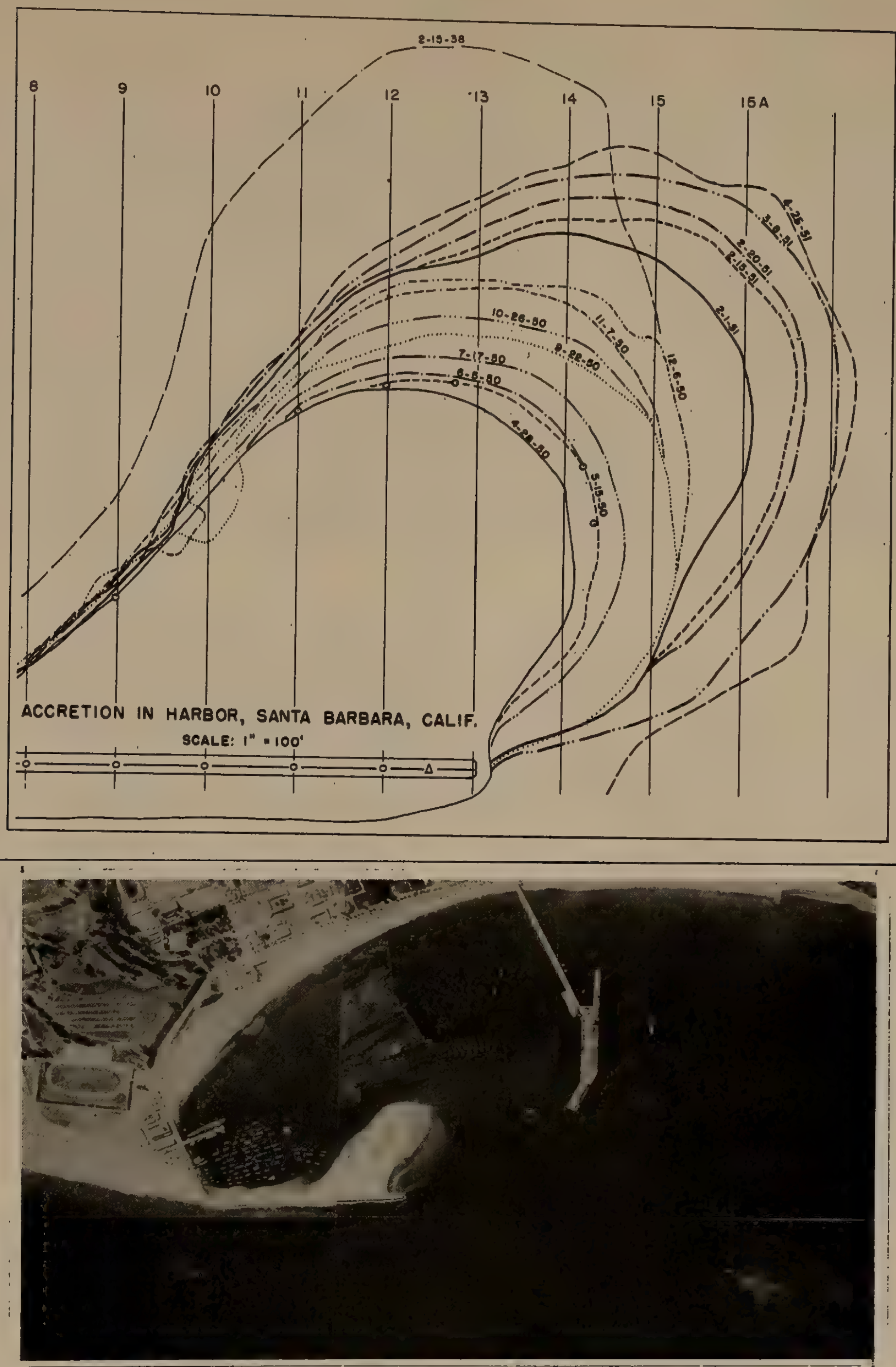


FIG. I-1-43 Santa Barbara, California, has a serious sand and beach problem created by the construction of a breakwater. Sand which moves eastward along the coast (left to right) under the influence of the prevailing westerly waves is unable to pass the breakwater and deposits in its quiet lee. Besides blocking the harbor, this starves the beaches at the right of the picture, which retreat rapidly. The upper diagram shows successive stages in the growth of the sand spit. The sewage, lower right, shows that currents exist even on this calm day.

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION I. WAVES, TIDES AND BEACHES

J. NATURAL BEACH OBSTRUCTIONS
BY
W. N. BASCOM

Table of Contents

J. NATURAL BEACH OBSTRUCTIONS

The word obstruction suggests interference with movement and raises the question of what is going to move and when. Obviously men, wheeled vehicles, boats and tractors are each retarded to a different degree by varying types and sizes of obstacles. The reader can best judge for himself the degree of hindrance constituted by each obstacle mentioned. It must be pointed out that almost every one of these so-called obstructions may have its advantages as well as its disadvantages. For example, at Iwo Jima the steep berm faces which stopped all wheeled vehicles also furnished the only possible shelter from enemy fire. Only natural beach obstacles will be considered here; most of them are unusual and are presented for the sake of completeness.

1. Driftwood:

Most free floating objects eventually find their way to a beach, where, after much tossing about in the surf, they are stranded at the high water mark. While in the surf, even small objects can be a great hazard to craft and swimmers because of their high velocities when hurled by a breaker; on the beach, larger drift wood may be a considerable obstacle to vehicles. Most beaches of the world are relatively free of driftwood, either because the land nearby does not support large vegetation or because the natives gather it up for use as fuel, but on some beaches it is a serious obstruction. The large drift, if undisturbed, marks the line of highest water and it may be surprizingly far inland; it serves as a warning (not always heeded) not to build to its seaward. Drift frequently becomes buried in the beach by wind-blown sand or by the action of the waves in flinging the beach material on top of it and even when buried it may still be a hazard to vehicles or to tractors which have been known to throw a track on much lesser obstacles. The erosion of the beach may expose a bristling ridge of drift (Figure IJ-1a) or it may release buried logs to the surf where they thrash about. During a storm in 1948 a large log was observed to dig up a buried armored submarine cable and snap it like a string with a single lunge -- a cable with a breaking strength of 18,000 pounds. The possible importance of driftwood as a barrier to military operations is indicated by the size and quantity of the material found on our own Washington coast where single logs as much as 8 feet in diameter and 150 feet long have been seen. (Figure IJ-1c)

2. Stumps and Trees:

Mangrove trees are about as serious a beach obstruction as can be encountered. Long looping roots and branches meet indistinguishably at the waters edge and the effect is of a perforated and springy (but exceedingly tough) barrier. The "alligator" was invented by Roebling for the express purpose of crossing swampy mangrove covered land and beaches and it did so quite well, as does the present form of LVT. No other vehicle can operate in them. Rooted stumps are sometimes seen on beaches; these might be a considerable hazard to either craft or vehicles but like mangroves, are quite rare.

3. Ice:

It may be presumed that on many far-northern beaches it is not unusual to have large blocks of floe ice cast upon the beach which would offer a considerable obstacle to any sort of movement although this is not within the writer's experience. There is evidence that such ice, driven by gales, has made extensive and steep sandy ridges on the shores of Prince Edward Island and near the Columbia River mouth. These ridges are high, long and steep and would themselves be a considerable impediment to movement on the back beach.

Another likely possibility is that very cold offshore winds would cause extensive freezing of the beach surface and materially alter the beach characteristics - probably for the worse.

4. Sand Forms:

Almost any of the beach forms or conditions mentioned previously would, under some circumstances, constitute an obstruction. A steep beach slope, a series of berms, a berm face eroded into a steep scarp, beach ridges, or dunes each create problems. Most of the problems will probably be those of trafficability caused by steep slopes and loose material. The reader is referred to sections I-I and V of this Manual for more complete information.

5. Beach Material:

Generally fine damp sand (.17 to .25 mm) is best for vehicle operation. Movement problems increase as materials get finer and coarser until at the two extremes, flat mud and steep cobbles, no present types of vehicles will move and men find progress very difficult. Recent experiments on the Laborador coast indicate that not even tractor type machines can move up steep cobble slopes such as may be encountered on a beach face. (Figure IJ-2) Muds have caused perplexing mobility problems to every army; however, they are not usually found on beaches. At Inchon, Korea the area between tides was muddy with the result that all vehicle unloading took place at higher tides against a narrow sand strip. A condition more likely to be encountered is that found in-between sand dunes at many beaches. Wind-blown fine sand fills in the hollows and the water table rises to within a few inches of the surface; often there are grasses and plants growing there. This ground may feel perfectly solid to a scout and, indeed, may support several vehicle sorties before giving away. When the surface collapses, the area becomes virtually bottomless "quicksand".

6. Reefs, Bedrock:

Offshore rocks in beach approaches are called reefs if they are not exposed at low water, rocks if they are; either may be dangerous to craft or amphibious vehicles. Because of difference in the hardness, strike, or protection, the bed rock which underlie all beaches has a rough irregular surface. Sand, created by the rock's destruction, covers most of the rock and makes up the beach; often though, there is insufficient sand to cover the highest, most resistant rocks and these project above the sand. Since the sand is responsive to waves which are seasonal, the amount of sand available to cover the bedrock varies, and rocks or objects exposed at a beach one time are covered at another. (Figure IJ-3) Thus many beaches exist only after periods of calm weather and reach their maximum in late summer. Some shores have only a narrow sandy beach above mean sea level and are completely composed of ragged barren rock underwater (wave cut bench). When examined at high tide, they are difficult to distinguish from beaches which have ample sand. It will often take ground reconnaissance or large scale aerial photography at low tide to determine this point. Generally though, this condition is more likely to exist in front of rocky cliffs. (Many French beaches are like this.) Underwater rocks (reefs) may often be located by the changes they produce in the surface wave pattern. Reflections which appear as radiating ripples, peaked up waves with direction altered, or actual breakers in heavy weather, are readily seen from the air. Coral reefs frequently border tropical shores at some distance from the beach and often appear very similiar to the wave cut benches just described. (Figure IJ-4)

Beaches often end alongshore against rocky points; sometimes it is possible to go around these points at low tide to the next beach if the sand is continuous. This is a frequently encountered situation which once more points to the need for the correlation of tidal data with beach reconnaissance. As the tide recedes, it will be found that rocks projecting through the sand have mounds of sand in their lee and channels around the rock formed by runoff water. These result in soft areas near the rock in which vehicles may become stuck.

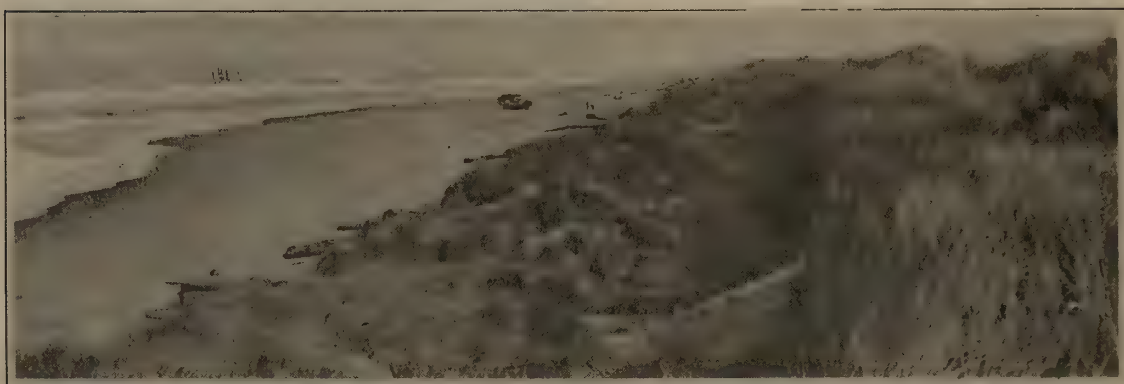
7. Streams:

Streams flowing across a beach frequently create an obstacle to alongshore movement in several ways. Larger streams may simply be too deep to ford and therefore terminate the usable beach; at low tides however it may be possible to cross them at the seaward extremity where the water is likely to flow in a wide and shallow channel. All streams are best crossed as far to seaward as practicable because generally the sand is harder, the banks lower and there is a smaller probability of mud. Even small trickles readily cut a channel a foot deep as they cross a beach. (Enough to stop a heavily loaded vehicle.) Wave action frequently dams up a stream mouth, in which case the stream will flow along behind the beach until it finds an easier place to cross the beach (usually a low point in the berm); this may make a very effective moat along the back edge of the beach (O'Conner Creek in Washington flows along the back of the beach for over a mile).

8. Kelp

Large areas of seaweed, or kelp, may be an obstacle to some amphibious vehicles although the rough bottomed DUKW is able to go through beds of it that are not too dense. Wide beds at low tide would certainly slow all present types of craft and vehicles. (Kelp beds 2000 feet wide exist at Elwood, California) Kelp grows only on rock and it is of some value in marking reefs and showing bottom conditions. Since some plants grow to be 100 feet long, it is of use in determining depth. Large storms tear the kelp loose from its moorings; the kelp may part or a piece of the rock may come loose. In this manner, rocks (some of which are 8" in diameter) are transported to the beach. The kelp is buoyant and floats the rock ashore where it eventually comes to rest at a relatively protected zone. Piles of kelp four feet deep, 20 yards wide, and 100 yards long have been left on Carmel Beach by a large storm.

FIG. IJ-1a - Back beach at Coos Bay, Oregon, consists of an erosional scarp about 10 feet high studded with buried logs. This, or a similar barrier, extended almost unbroken for at least 3 miles.



b - A view of the top and back of the barrier shown above (note pole in both pictures). Evidently ancient berms have been modified by wind erosion resulting in these steep dune ridges.

c - Large logs on the beach at Quilliyute River, Washington. The butt of the log in the center is about 10 feet in diameter. At least half of the Washington beaches have a log barrier.

FIGURE IJ-2 - Cobble beach face

1/29/47 - Landing barge is almost completely exposed

2/12/47 (14 days later) - Only superstructure shows after period of beach building

2/24/47 (12 additional days) - New storm has caused beach to retreat again

FIG. IJ-3 - Short term beach changes are most easily seen when the sand level can be compared with some fixed object such as a rock or a wreck. This sequence shows changes that took place at Monterey Bay, California, in less than a month; this does not represent the maximum possible change here.

FIGURE IJ-4 - Coral reef

SUMMARY OF SECTION I-K

WIND TIDES

BY
DAVID K. TODD

Introduction

Wind blowing over a water surface generates oscillatory waves and at the same time induces a surface current in the general direction of the wind movement. These phenomena result from tangential stresses at the water surface between the wind and water and from differences in wind pressures on the windward and leeward sides of waves. In an ocean the surface current would produce a piling up of the water at the leeward side and a lowering of the water level at the windward side with a return flow of equal volume along the bottom. The deviations from the still-water level caused by wind-driven currents have been defined as wind tides. For practical purposes the more important of these are those wind tides which raise the water level. In limited bodies of water such as lakes, rivers, bays, and seas, wind tides have been measured and found to vary from a few inches for light winds to several feet in extreme cases involving hurricane winds. In general, wind tides produced in shallow water will be higher than those in deep water because of the greater resistance encountered by the return flow currents.

Changes in water levels can also be caused by seiches and by differences in atmospheric pressure. There should be no confusion, however, between these effects and wind tides, since the continued action of wind on the water surface is not involved. Only wind tides are caused by direct wind action, so that as long as the wind continues to blow over a water surface, there will be a transport of water by the surface currents.

In connection with shore installations or landing operations, wind tides are capable of vast destruction. This is particularly true in regions experiencing storms with high wind velocities, such as hurricanes or typhoons. About the only beneficial purpose that wind tides can serve is that of maintenance of harbor channels. This action results when wind tides develop in harbors or inlets and are followed by a change in wind direction or intensity. A large mass of water is then released which flushes or scours the entrance way and thus helps to maintain the channel depth. This contribution, however, is not particularly effective since ordinary small wind tides caused by coastal storms would not be sufficiently strong to have much scouring action, and large wind tides occur so irregularly to be of any value as a reliable aid in channel maintenance.

Relation of Wind to Surface Currents

The surface drag of wind on an ocean surface induces a current in the upper layers of the ocean. Because of the deflecting force of the earth's rotation, the current direction differs from that of the wind direction. In the open ocean it has been found that the surface current will be directed 45° to the right of the wind velocity in the Northern Hemisphere and 45° to the left in the Southern Hemisphere. With increasing depth, the angle becomes larger and the current velocity decreases. Below a depth of from 50 meters to 150 meters, depending upon the wind velocity and latitude, the wind-driven surface current becomes negligible. Because of this turning of the current direction with depth, it has been found that the resultant transport of water by the entire surface current is directed 90° to the right of the wind direction.

This phenomenon has important applications in the upwelling of cold water along the coast of California and certain other coastal regions.

In the case of a completely enclosed sea, it has been found that the slope of the water surface due to the wind tidal section very nearly coincides with the wind direction, regardless of the sea depth. Thus, although the currents may deviate considerably from the wind direction, the earth's rotation has only a small influence on the direction of the gradient in an enclosed sea.

Wind Tide Studies

Based on several theoretical studies, the slope of the water surface due to the wind action has been found to be directly proportional to the stress of the wind on the water surface and inversely proportional to the water depth. Therefore, since the wind stress is a function of the wind velocity, the strongest wind in the shallowest water would create the greatest slope and consequently the highest wind tide.

Numerous observations of surface currents resulting from winds have shown that the ratio of the surface current speed to the wind speed is about .02 to .03, or in other words, the surface current equals 2-3% of the wind speed.

In recent years laboratory studies have been made of wind tides using long, covered glass channels. A shallow depth of water is placed in the channel and a fan is connected at one end. The air moving through the channel over the water acts the same as wind blowing over the ocean surface. The change of the water surface due to the air movement can be measured accurately at various points along the channel, so that curves can be drawn of the water surface profile. By varying the air speed the different effects on the water surface elevations can be determined. In general, these laboratory studies have verified previous theoretical studies of wind tides and also actual observations of wind tides in lakes and seas.

Wind Tide Observations

Wind tide observations have been made at various localities over the world. These observations include the measurement of water levels at or near the shore and measurement of wind velocities over the water surface area. Such observations have been made in the following water areas: Lake Erie; Lake Geneva, Switzerland; Lake Hornborga, Sweden; Lake Okeechobee, Florida; North Sea; Baltic Sea; and Gulf of Bothnia. Figures SIK-1, 2, and 3 illustrate the results of some of these observations for three of the localities. Figure SIK-1 shows a map of Lake Erie, a profile of the bottom, and profiles of the water surface under the action of 10 meter per second winds from the east and west. The wind tidal action is clearly shown by these two water surface profiles, indicating the lowering of the water surface at the windward end of the lake and the raising of the water surface at the leeward end in each case. Figure SIK-2 shows a similar effect on Lake Okeechobee, Florida, during a hurricane in September 1928. The bottom and water surface profiles are both illustrated in this north-south cross-section. The various dots on the figure are high water marks left on trees, ridges, and levees at the southern end of the lake. The computed wind-modified water surface curve, which closely follows the high water marks, was determined from computations based on estimates of the wind during the hurricane. At the northern end of the lake the curve indicates a wide area of exposed bottom. This is verified by reports of people walking more than half a mile out on the lake bottom and picking up fish. This curve thus indicates how accurately it is now possible to forecast wind tides for a given location for a given wind velocity. Figure SIK-3 is a

slightly different illustration of a wind tide on the Gulf of Bothnia in the Baltic Sea. The solid lines are contours of the water surface, measured in centimeters, and the dashed lines are isobars, measured in millibars, showing the distribution of atmospheric pressure at a given time during a storm. Since surface winds blow in a counter-clockwise direction and slightly to the left of the direction of the isobars, it can be seen that the water surface contours are perpendicular to the surface wind with the higher elevations on the leeward side. Thus the water surface slope is parallel to the wind direction. The wind tide values below the normal water surface on the windward side are considerably greater than the wind tide rises on the leeward side because of the general orientation of the storm. The surface currents have transported large masses of water into the open Baltic Sea to the southwest of the area shown in Figure SIK-3.

Lake Okeechobee Wind Tide Project

The Corps of Engineers, Department of the Army, has initiated an extensive investigational project at Lake Okeechobee, Florida, for the measurement and study of wind tides. Present plans call for the design and construction of many miles of levees around the lake for flood control and protection purposes, so that this project will govern design features representing many millions of dollars. Lake Okeechobee is a shallow, saucer-shaped lake with a bottom elevation of zero mean sea level. It is 25 miles wide and 30 miles long, but the maximum depth is only 12 to 15 feet. The land surrounding the lake is relatively flat and levees have been built in the past around the north and south shores to an elevation of 32.5 feet. Instrument stations for measuring wind tides have been established at several points along the shore and in the lake itself. Figure SIK-4 shows an outline map of the lake and the location of the shore and lake instrument stations. Each shore station includes an anemograph, barograph, and water-stage recorder, for recording wind velocity, atmospheric pressure, and water level, respectively. The lake stations, built on steel-girder pylons, also include wave-height recorders. All instruments are battery operated and are visited weekly for servicing and collecting records.

Lake Okeechobee is an ideal body of water for detailed wind tide investigations. It is large enough and unprotected by surrounding terrain to be fully exposed to surface winds. Its geographic location places it in the center of the hurricane belt, so that it is frequently exposed to high winds. Its extreme shallowness makes it subject to wind tides of greater magnitude than would be experienced by lakes of comparable size but of greater depth. During the period 1947-1949 the lake experienced three major hurricanes, one in each of the three years. With the instrumentation on the lake during this period, detailed measurements were made of wind tides and meteorological conditions. The records of these measurements constitute probably the most precise and complete data ever available on the subject. To illustrate the results of some of these measurements, a typical collection of data is shown in Figure SIK-5. This figure represents conditions at 0100EST, August 27, 1949, during a hurricane. The dashed arrows indicate the wind direction over the lake, and the short dashed lines are lines of constant wind speed over a 10-minute period. The solid lines are water surface contours. It can be seen that a portion of the lake in the west has been exposed by the wind action. Reports at this hour also indicate a large overflow from the lake occurred in the flat area along the northwest side. A maximum difference in elevation of 11.5 feet in a distance of only 20 miles existed between the western lake station and the shore station at the northern end of the lake.

For the design of levees and shore protection installations, wave heights must be computed for maximum winds and added to wind tidal elevations. For example, using standard wave forecasting procedures, it has been estimated that a hurricane on Lake Okeechobee could develop waves 8 feet high. For coastal areas bordering an open ocean, this value would be even larger.

Conclusions

Both theoretical and observational studies of wind tides today are fairly well in agreement. Although some discrepancies still exist, further studies based on Lake Okeechobee data and model studies should tend to eliminate many of these. The end result of this research, of course, is to be able to estimate wind tides at any given location for design purposes, and also to be able to forecast wind tides for military and civilian needs.

Wind tides are capable of some very damaging effects along open coastal areas. This is particularly true in regions where the ocean bottom slopes gradually away from the coast. The most disastrous wind tide known occurred at Galveston, Texas, in connection with the hurricane of September 8, 1900. This city, located on a long, low sand island paralleling the Texas coast, was exposed to the full force of the wind tide. It was inundated to a depth of from 15 to 20 feet by the high water and some 6000 persons lost their lives in a city of 38,000 population. A similar occurrence happened during the 1938 hurricane that struck the Connecticut coast, although fortunately the loss of life was considerably less. Recently, during a cyclonic disturbance along the Atlantic Coast, wind tides flooded many low-lying areas in and around New York. Figure SIK-6 illustrates graphically the effect of wind tides on air bases located in coastal zones. This New York storm occurred at the end of November and was not a hurricane, hence the damage there could have occurred at many other coastal cities just as well.

These wind tidal effects have been mentioned to stress their importance in the design and location of shore installations and in the protection of existing works. For amphibious landing operations, adequate meteorological data and personnel for the detection of major storms which might cause excessive wind tides become essential. The probability of occurrence of a destructive wind tide at any given location, other than in the common hurricane belts, is usually small, but this does not justify ignoring it. The damage resulting from large wind tides is considerably larger than the small investment required for providing warnings and protection against wind tides.

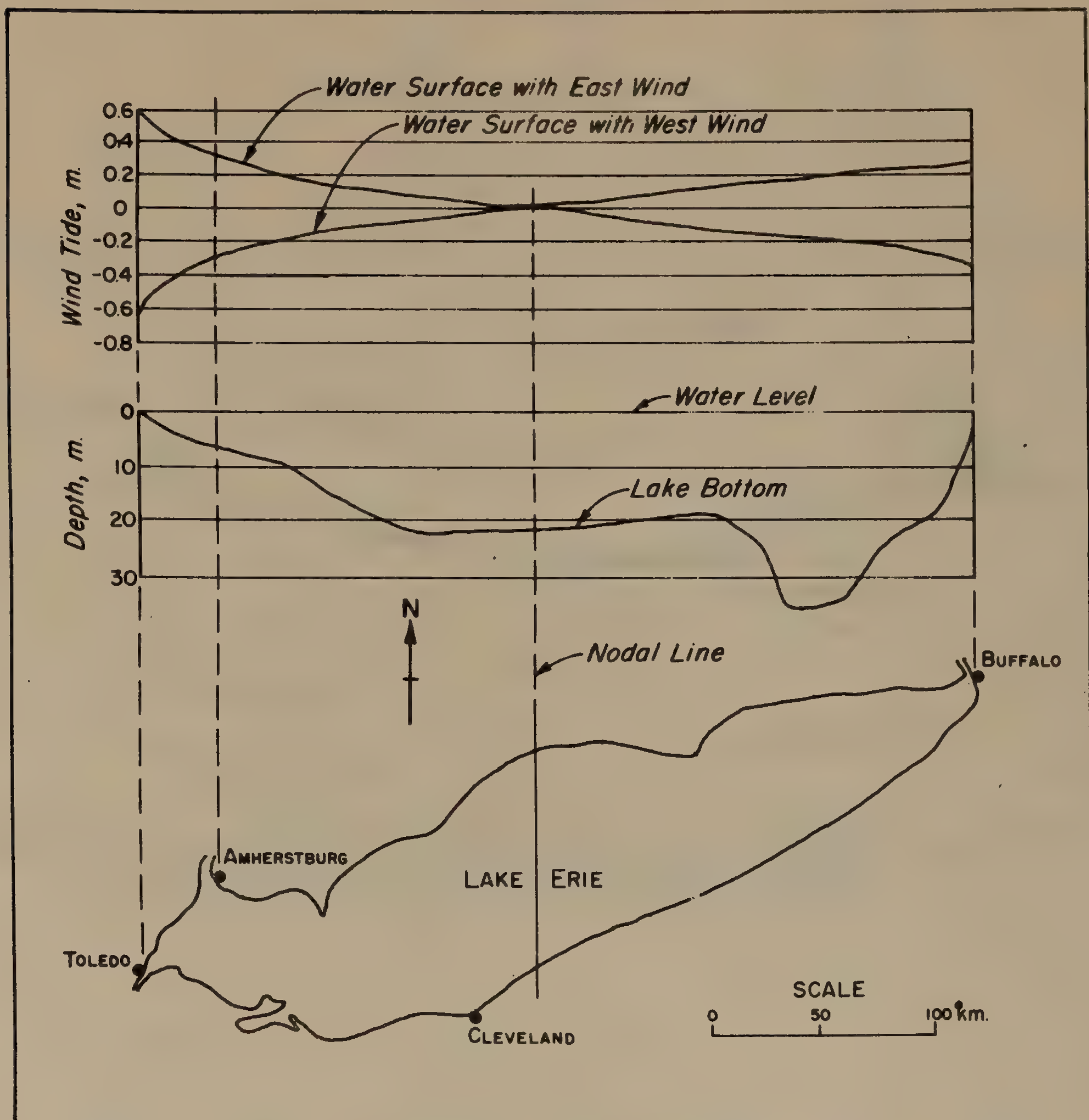


FIG. S.I.K-1 - Map of Lake Erie, showing bottom profile and wind tides with east and west winds (after Hellstrom)

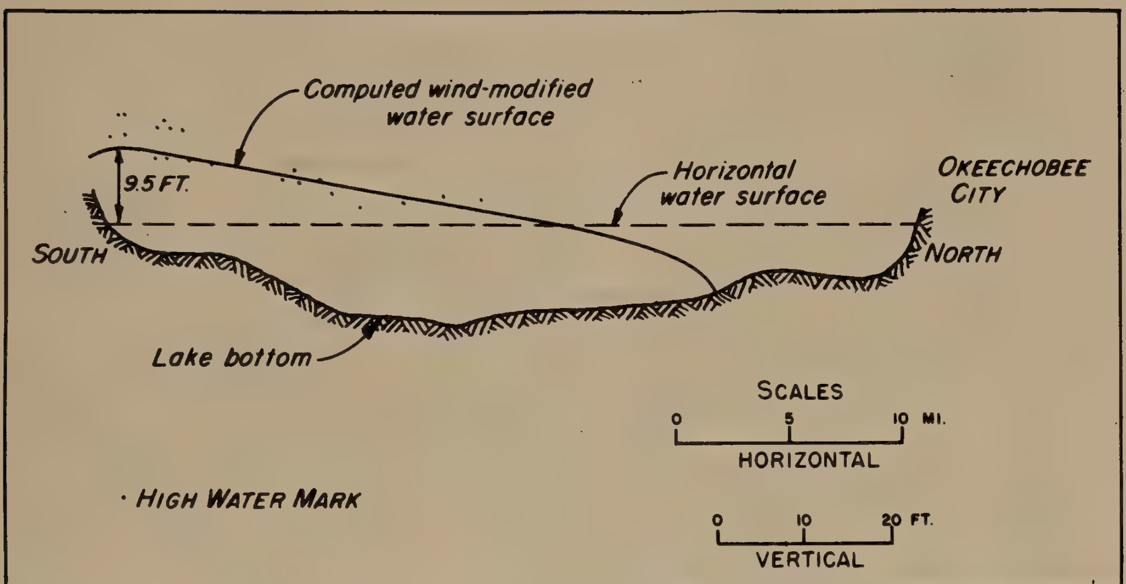


FIG. SIK-2 - Profile of Lake Okeechobee from north to south during first half of September, 1928, hurricane (after Hellstrom)

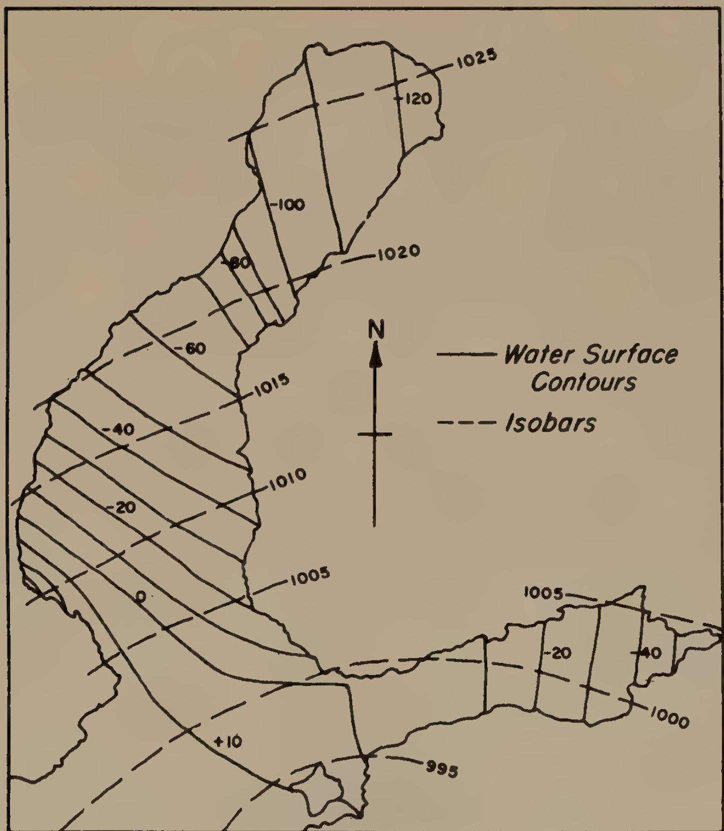


FIG. SIK-3 - Map of Gulf of Bothnia showing isobars, mb, and water surface contours, cm, at 1400h, 4 October 1936 (after Palmen)

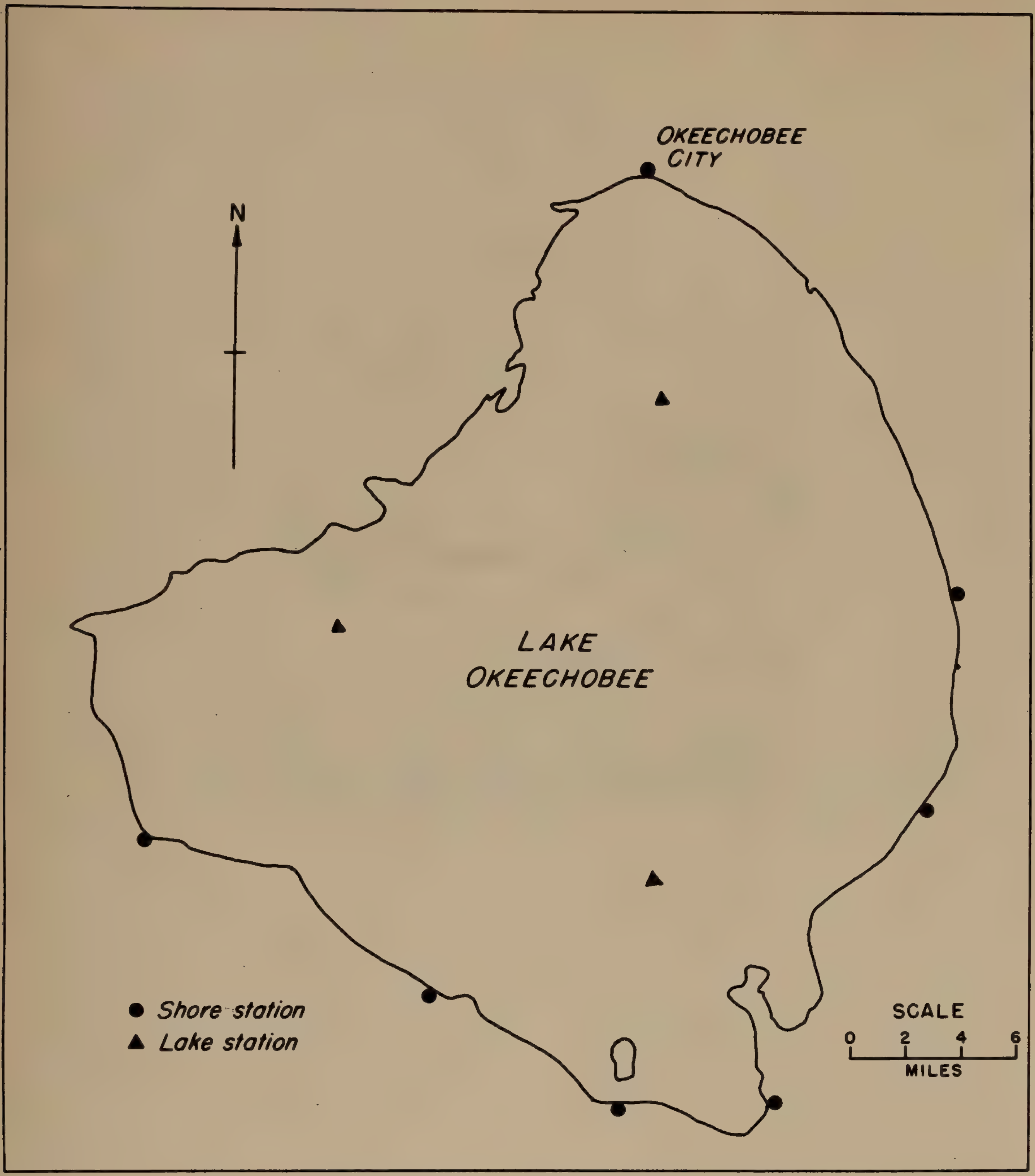


FIG. SIK-4 - Map of Lake Okeechobee, showing locations of shore and lake instrument stations for measuring wind tides.

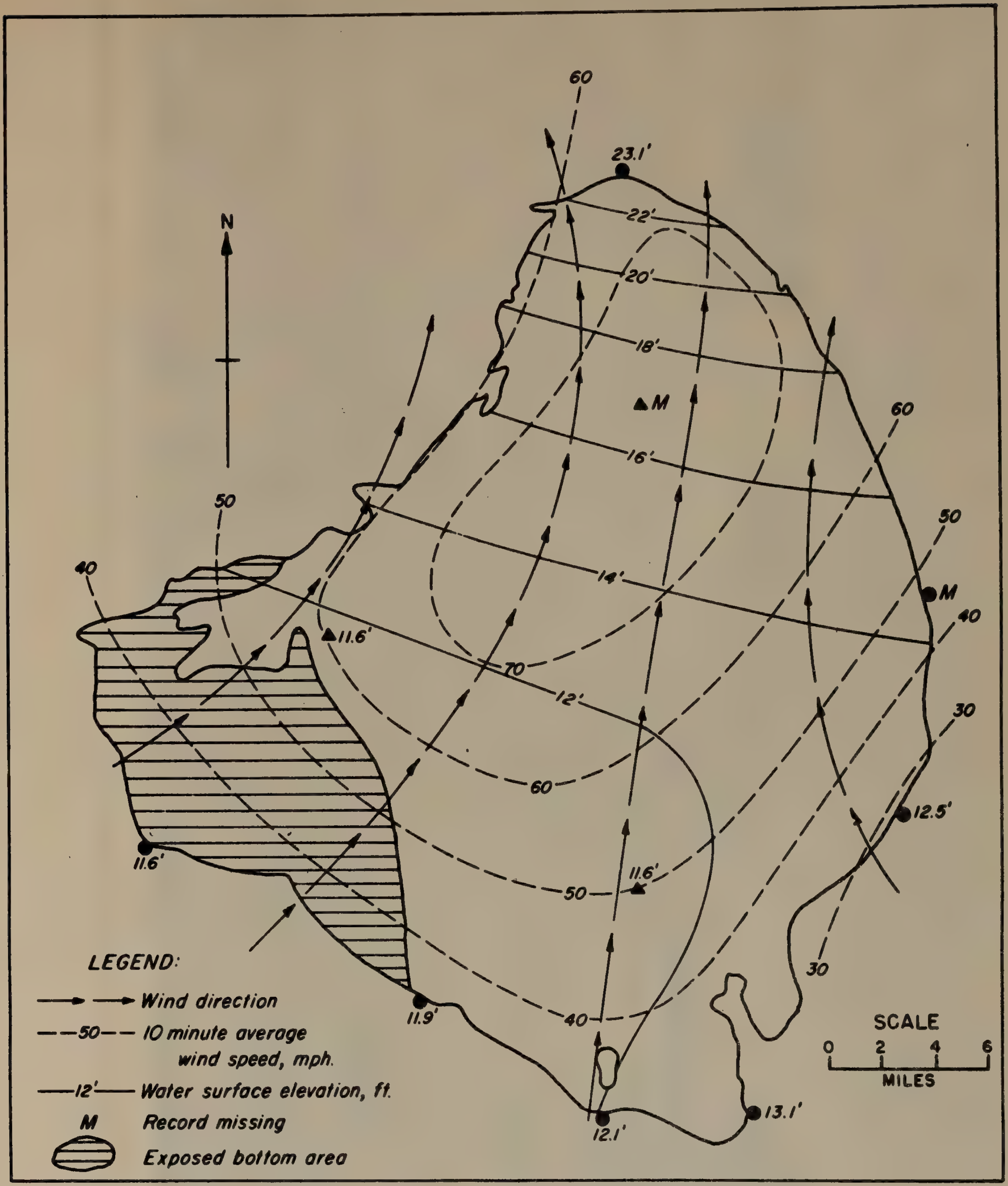


FIG. SIK-5 - Wind and water surface conditions at Lake Okeechobee at 0100EST, 27 August 1949, during a hurricane.

FIGURE SIK-6 - Wind tides on LaGuardia Airport, New York, under the influence of a 90 mph wind (Courtesy Time, Incorporated)

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION I: WAVES, TIDES, BEACHES

K. WIND TIDES

by

D. K. TODD

Table of Contents

LIST OF SYMBOLS

A rather well defined set of symbols has been used in the literature on wind tides. Hence, the following symbols are defined for this section of the Manual, as distinguished from the list of standard symbols for the rest of the Manual.

a	a constant
A	a constant
B	a constant
C_1	a constant
C_2	a constant
C_3	a constant
d	a constant
D	depth of frictional influence
E	energy required for steady motion of surface current
E'	actual energy of surface current
f	Coriolis parameter, $2\omega \sin\varphi$
F	shear stress of water
F_x, F_y, F_z	external forces parallel to coordinate axes
h	depth of drift current
H	height of the gradient wind
k	wind stress coefficient
k_r	a constant
ℓ	mixing length
L	length of basin
m	coefficient of water surface roughness
n	a constant
n_x, n_y, n_z	direction cosines
p	water pressure
R	bed reaction to weight of water
s	wind tide height
S_x, S_y	mass transport of water in the coordinate directions
t	time
T	time required for actual energy to reach energy of steady motion
u, v, w	velocity components along the coordinate axes
V	water velocity
V_{as}	wind velocity at the water surface
V_b	mean velocity at channel bottom
V_g	gradient wind velocity
V_m	average channel wind velocity
V_o	formula characteristic velocity
V_{sw}	surface water velocity
W	weight of water volume
x, y, z	coordinate axes
z_o	water depth
z_{os}	still water depth
z_1	ordinate to bottom of basin
α	angle between surface and gradient wind directions
β	angle of slope of basin bottom
γ	specific gravity of water

γ_1 specific gravity of water at bottom

γ_0 specific gravity of water at surface

γ_m mean specific gravity of water

δ a constant

ϵ average height of surface roughness elements

θ a constant

μ coefficient of viscosity

ν coefficient of eddy viscosity

ρ_a air density

ρ_w water density

τ_B frictional force between water and bed

τ_s tangential stress of wind on water surface

τ_{sx}, τ_{sy} components of τ_s along coordinate axes

φ latitude

ψ a constant

ω angular velocity of earth's rotation

K. WIND TIDES

1. Introduction:

The free surface of an undisturbed body of water will form a level surface. This assumes the only forces acting are uniform atmospheric pressure, a constant gravity field, and the rotation of the earth. Several factors may cause a water surface to depart from this level position, including tides, seiches, changes in atmospheric pressure, rain, and winds. The last named factor is the one of interest in this section. It has long been known that wind blowing over a water surface is capable of generating oscillatory waves and at the same time of inducing a surface current in the general direction of the wind movement. These phenomena result from tangential stresses at the water surface between the wind and water and from differences in wind pressures on the windward and leeward sides of waves. A uniform wind force of constant direction acting over a homogeneous water volume of infinite extent and finite depth on a non-rotating earth would eventually reach a state of equilibrium in which the wind force at the water surface would be balanced by the frictional drag produced at the bottom by the currents moving in the same direction as the wind. However, in the more realistic case of finite extent and depth, the surface current would produce a piling up of water at the leeward side causing a return flow along the bottom. This return flow would result from the hydraulic gradient existing between the leeward and windward sides and, in a steady state condition, would volumetrically equal the surface current transport. The deviations from a still-water level surface elevation caused by the transport of surface water and the consequent greater head developed to overcome the resistance to return currents in the lower layers have been designated wind tides. Strictly speaking, then, wind tides are deviations of water level above or below the normal or still-water level; however, for practical purposes the more important are those wind tides which raise the water level. In limited bodies of water such as lakes, rivers, bays, and seas, wind tides have been measured and found to vary from a few inches for light winds to several feet in extreme cases involving hurricane winds. In general, wind tides produced in shallow water will be higher than those in deep water because of the greater resistance encountered by the return flow currents.

The distinction between variations in water levels caused by seiches and by wind tides should be emphasized. Seiches are standing waves having free oscillations of a period depending upon the horizontal and vertical dimensions of a closed or partially enclosed body of water (Sverdrup et al, 1942, IK-47). These long waves may be caused by variations of wind forces acting over a water surface, by differences in atmospheric pressure, and possibly by impact forces due to rain. Having once been generated by an external force, seiches may continue for a considerable period of time, being acted upon only by gravity and frictional forces. Seiches become most pronounced in deep bodies of water with uniform bottom contours, where frictional forces of the oscillatory motion are minimum. Amplitudes of seiches have been measured from a few inches to several feet and have been found to occur in practically all lakes (Hellstrom, 1941, IK-22). Periods vary from little more than one minute to several hours. Two early papers containing descriptions of seiches and numerous observational data are those of Harris (1907, pp. 467-482, IK-19) and Honda et al (1908, IK-25). Defant (1929, IK-11) developed a method for computing

periods, vertical displacements, and nodal line positions of seiches in lakes and bays of any shape. Patton and Marmer (1932, IK-38) reported seiches occurring even along the open coast of New Jersey, apparently due to the bay-like shape of the Atlantic Coast in this general region. While seiches may be induced by winds, it should be realized that they are a rhythmic, oscillatory motion, whereas wind tides result from a direct piling up of water along a shoreline ahead of a wind and continue as long as the wind force exists. Both phenomena may occur together, causing the water level to be either higher or lower than that produced by the wind tide alone, depending upon the orientation and period of the seiche.

To avoid possible confusion a distinction should be made also between water level variations due to atmospheric pressure and wind tides. Normal atmospheric sea level pressure is approximately 29.92 inches of Mercury, while over any given expanse of water this pressure will deviate from this value as a result of the cyclonic and anticyclonic systems existing in the atmosphere. If we assume a static condition of pressure distribution over a given area, that portion having a greater pressure than the mean of the area will cause the water surface to be depressed, while that portion having a lower pressure than the mean will be elevated. The total pressure above any given submarine level will be constant and will be composed of the water and atmospheric components above that level. Under ordinary meteorological conditions, atmospheric pressure varies by only a small fraction of an inch Hg, so that the variation of water level is usually a small amount. Even with extreme pressure gradients, as are sometimes found in hurricanes, the gradient may equal only one inch Hg in 100 miles. This would correspond to a water surface difference in elevation of only 13.5 inches. Actual storm conditions would tend to reduce even this figure because of storm movements and frictional effects. Hayford (1922, IK-20) has studied this effect on the Great Lakes and Ogura (1925, IK-32) on the western part of the North Pacific Ocean. The problem of a lag between pressure changes and sea level changes was treated mathematically by Proudman and Doodson (1925, IK-41), and recently Unoki (1950, IK-51) showed that variation of sea level lags behind that of atmospheric pressure on account of resistance action on sea water. Hellstrom (1941, pp. 18-21, IK-22) presented a straightforward analysis of the static case. In relation to wind tides the water variation due to atmospheric pressure is generally of a smaller order of magnitude. Where situations are discussed hereinafter involving pressure differences over a body of water, corrections have first been applied to eliminate this effect before computing wind tidal heights.

In connection with shore installations or landing operations, wind tides are capable of vast destruction. This is particularly true in regions experiencing storms with high wind velocities, such as hurricanes or typhoons. Specific examples will be included later. About the only beneficial purpose that wind tides can serve is that of maintenance of harbor channels. Brown (1928, IK-3) mentions this point when he says, "Into bays or lagoons connected with the sea by inlets the waters driven by the wind penetrate and accumulate. They then escape, when the wind has changed in intensity or direction. This produces in the inlets a powerful flushing effect or scour, which is very beneficial in the maintenance of depth". It is dubious, however, whether small wind tides, occurring frequently with ordinary coastal storms (Todd and Wiegel, 1951, IK-50), are effective in this action. The effectiveness of large wind tidal action on the other hand is overshadowed by its relative infrequency of occurrence.

2. Theoretical Studies of Wind Tides:

Any analysis of the subject of wind tides involves several different variables. These include wind velocity, water depth, energy transfer from wind to water, and stratification of the water volume. In conjunction with these factors, assumptions must be made regarding the flow conditions and viscosities at and near the air-water boundary. Although several investigators have studied this subject or problems closely akin to it, such as the relation between winds and surface currents, only Hellstrom (1941, IK-22) appears to have given his attention to all of the phases involved. Frequent reference will be made to Hellstrom's work; nevertheless, certain investigators with a more limited scope have gone more into detail on portions of the subject and therefore their contributions are important. In this section an attempt will be made to summarize the approaches and results of the leading theoretical studies relating to wind tides.

a. Relation of Winds to Surface Currents

The surface drag exerted by wind on an ocean surface induces a surface current in the upper layers of the ocean. Because of the deflecting force of the earth's rotation, however, the current direction differs from that of the wind direction. Ekman (1905, IK-14; 1906, IK-15; 1928, IK-16) was the first to analyze this problem, and his contributions to the knowledge of surface drift currents are still the basis of present-day studies.

If u and v are the horizontal velocity components of the water current parallel to the x- and y-axes, taken with the y-axis 90° left of the x-axis, and the z-axis is taken positive downward from the water surface; then the tangential stress of the wind on the water surface may be defined as

$$\tau_s = -\nu \left(\frac{dv}{dz}\right)_{z=o}, \quad \nu\left(\frac{du}{dz}\right)_{z=o} = 0 \qquad \text{(IK-2.1)}$$

where ν is the "virtual coefficient of viscosity", as defined by Ekman. This coefficient corresponds to the presently known coefficient of eddy viscosity. Taking the simplest case of wind and earth rotational forces only, and starting from the equations of motion for an incompressible fluid, Ekman found

$$\left.\begin{aligned} u &= V_{sw}\, e^{-az} \cos(45^\circ - az) \\ v &= V_{sw}\, e^{-az} \sin(45^\circ - az) \\ V_{sw} &= \frac{\tau_s}{\sqrt{2\nu\rho_w\, \omega \sin\varphi}} \end{aligned}\right\} \qquad \text{(IK-2.2)}$$

where V_{sw} is the absolute velocity of the water at the surface, ρ_w is the water density, ω is the angular velocity of the earth, and φ is the latitude. The quantity, a, is defined as

$$a = +\sqrt{\frac{\rho_w\, \omega \sin\varphi}{\nu}} \qquad \text{(IK-2.3)}$$

If the wind velocity and hence the tangential stress of the wind, τ_s, is directed along the y-axis as is indicated by equations (IK-2.1), then it follows from equations (IK-2.2) that the drift current at the surface will be directed 45° to the right of the wind velocity in the Northern Hemisphere and 45° to the left in the Southern Hemisphere. Actually the angle is

slightly less than 45° between the surface current direction and that of the absolute wind velocity, but since the water velocity is generally much smaller than that of the wind (of the order of 2%), the error can be neglected. With increasing depth it can be seen that the angle increases uniformly and the velocity decreases. This combination can be thought of as forming a spiral staircase downward of smaller and smaller width and presents, in horizontal projection, the familiar Ekman spiral. This is shown in Figure IK-1, where the velocity arrows represent increasing depths, $z = 0, \pi/10a, 2\pi/10a, 3\pi/10a, ---, \pi/a$.

Since the depth of the surface current continues indefinitely based on equations (IK-2.2), it is difficult to determine the exact extent of the wind on the ocean drift currents. However, because the current velocities decrease exponentially, Ekman defined the depth at which the water velocity direction is exactly opposite to that at the surface, which has a magnitude of only about 4% of that at the surface, as the depth of frictional influence, D. Thus,

$$D = \frac{\pi}{a} = \pi\sqrt{\frac{\nu}{\rho_w \omega \sin\varphi}} \qquad \text{(IK-2.4)}$$

One other important result of Ekman's work was that integration of the currents with depth gives

$$S_x = \int_0^\infty u\,dz = \frac{\tau_s}{2\rho_w \omega \sin\varphi}; \quad S_y = \int_0^\infty v\,dz = 0 \qquad \text{(IK-2.5)}$$

where S_x and S_y are the mass transports of water in the two coordinate directions. Thus the resultant transport of water in oceans of depths greater than D is directed 90° to the right of the wind direction. This phenomenon has important applications in the upwelling of cold water along the coasts of California, Peru, Morocco, and Southwest Africa (Sverdrup et al, 1942, IK-47).

Ekman further treated the case of currents caused by a horizontal pressure gradient in water and the earth's rotation. This involves the more practical case of a wind current encountering a continent and causing a slope in the ocean surface to be created. The vertical component of the slope measured from the normal water level represents the wind tide. The current near the bottom is directed 45° to the right of the gradient, and above this the angle of deviation and velocity increase with height. This bottom current is limited to the distance from the bottom in which friction is effective, similar to the depth of frictional influence below the surface. Above the bottom current, the water flows with almost constant velocity and at right angles to the gradient. In depths greater than 2D, it is therefore possible to distinguish three different currents, a bottom current (assumed to be of depth D), a midwater current of uniform velocity, and a surface drift current of depth D.

In the case of an enclosed sea, the total flow in any direction must be zero providing a steady state has been established. Ekman computed the directions and velocities of currents for various depths, assuming the depth of frictional influence at the bottom to be the same as that at the surface. Letting z_o equal the total water depth, Figure IK-2 shows the currents for three different depths: $z_o = 2.5D$, 1.25D, and 0.5D. Only the end points of the currents are shown and connected by a curve. In each case point 1 is at the water surface, point 2 is $0.1z_o$ below the surface, point 3 is $0.2z_o$ below the surface, etc. The current vector for any given depth would consist of an arrow

from the origin to the numbered end point on the curve. It should be noted how nearly the slope of the water surface coincides with the wind direction, regardless of depth. Thus, although the currents may deviate considerably from the wind direction, the earth's rotation has only a small influence on the direction of the gradient in an enclosed sea. Similarly, its influence on the magnitude of the water rise is not large, tending to reduce it only 0.02 for $z_o = 0.5D$, 0.23 for $z_o = 1.25D$, 0.29 for $z_o = 2.5D$, and 0.33 for z_o infinite.

Of importance in the subject of wind tides, is the depth of frictional influence as defined by Ekman. It can be seen from Equation (IK-2.4) that D varies with latitude. Based on the previous derivations, Ekman shows that the slope of the water surface with the wind velocity now parallel to the x-axis is

$$\frac{dz_o}{dx} = \frac{3}{2} \frac{\tau_s}{g \rho_w z_o} \qquad \text{(IK-2.6)}$$

where g is the acceleration due to gravity. Using this equation and observational data from a storm on the Baltic Sea during November 1872 (Colding, 1881, IK-6), Ekman found that

$$\tau_s = .032\ V_{as}^2 \qquad \text{(IK-2.7)}$$

where V_{as} is the surface wind velocity in mps and τ_s is expressed in gm per cm per sec^2. Using this equation and certain approximations, the relationship between D and V_{as} results as

$$D = \frac{7.6}{\sqrt{\sin \varphi}}\ V_{as} \qquad \text{(IK-2.8)}$$

where D will be in meters. From this equation the following approximate values of D may be obtained: 150 m at latitude 45° with $V_{as} = 17$ mps, 60-70 m for $V_{as} = 7$ mps, and 40 m for $V_{as} = 4.5$ mps. Although information as to the bottom friction layer is lacking, it may be assumed that its thickness is of the same order of magnitude as the surface layer.

b. Wind Tides Assuming Laminar Motion:

Equation (IK-2.6) can be derived from another approach using an assumption of laminar flow conditions. This assumption does not agree with observed flow conditions, however, the results will be compared later with derivations assuming turbulent conditions. This derivation, from Hellstrom (1941, IK-22), is based on an analysis of acting forces.

Consider a vertical element of length dx and unit width resting on a horizontal bottom with an origin located as shown in Figure IK-3. The wind and current directions are taken along the positive x-axis. The forces acting on the element (see Figure IK-3) include W, the weight of the water; $\tau_s\,dx$, the wind force on the surface; water pressures, p_1 and p_2; shearing stresses F_1 and F_2; the reaction at the bottom, R; and the bed friction, $\tau_b\,dx$. The sum of components parallel to the x-axis gives

$$\tau_s\,dx + p_1 - p_2 + \tau_b\,dx = 0 \qquad \text{(IK-2.9)}$$

By replacing p_1 and p_2 with

$$p_1 = (1/2)\rho_W g z_o^2 \quad \text{and} \quad p_2 = (1/2)\rho_W g (z_o + dz_o)^2 \qquad \text{(IK-2.10)}$$

and disregarding higher order terms, we get

$$\tau_B = \rho_W g z_o \frac{dz_o}{dx} - \tau_s \qquad \text{(IK-2.11)}$$

If the gradient of the water surface is small, then $F_1 = F_2$; and thus, as a corollary, it follows that $W = R$. Taking moments about the origin gives

$$\tau_s \, dx \left(z_o + \frac{dz_o}{2}\right) + W\frac{dx}{2} - R\frac{dx}{2} + p_1 \frac{z_o}{3} - p_2 \frac{z_o + dz_o}{3} - F_2 \, dx = 0$$

(IK-2.12)

From this expression the surface gradient can be obtained, which results in the following, neglecting higher order terms,

$$\frac{dz_o}{dx} = \frac{2\tau_s}{\rho_W g z_o} - \frac{2}{\rho_W g z_o^2} F_2 \qquad \text{(IK-2.13)}$$

Now only the side shear F_2 remains to be evaluated. Considering forces F_x, F_y, and F_z parallel to the coordinate axes acting on a small plane surface with direction cosines n_x, n_y, and n_z, we have for viscous incompressible flow at constant temperature

$$\left.\begin{aligned}
F_x &= -pn_x + 2\mu n_x \frac{\partial u}{\partial x} + \mu n_y \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \mu n_z \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\
F_y &= -pn_y + \mu n_x \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + \mu 2 n_y \frac{\partial v}{\partial y} + \mu n_z \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\
F_z &= -pn_z + \mu n_x \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) + \mu n_y \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) + 2\mu n_z \frac{\partial w}{\partial z}
\end{aligned}\right\}$$

(IK-2.14)

where μ is the coefficient of viscosity. Following the previous assumptions of flow direction, $v = w = 0$. If the plane is considered as a vertical side parallel to the yz-plane, then the direction cosines are $n_x = 1$, $n_y = n_z = 0$. And finally, since u is independent of x and y, we have that

$$\frac{\partial u}{\partial x} = 0 \quad \text{and} \quad \frac{\partial u}{\partial y} = 0 \qquad \text{(IK-2.15)}$$

Equations (IK-2.14) now reduce to

$$\left.\begin{aligned}
F_x &= -p \\
F_y &= 0 \\
F_z &= \mu \frac{\partial u}{\partial z}
\end{aligned}\right\} \qquad \text{(IK-2.16)}$$

Hence F_2 becomes the integral of F_z, and

$$F_2 = \int_0^{z_o} \mu \frac{\partial u}{\partial z} \, dz \qquad \text{(IK-2.17)}$$

Since u is independent of x and y,

$$\frac{\partial u}{\partial z} = \frac{du}{dz} \qquad \text{(IK-2.18)}$$

so that equation (IK-2.17) reduces to

$$F_2 = \mu V_{sw} \qquad \text{(IK-2.19)}$$

where V_{sw} is the surface water velocity. Substituting this value for F_2 in equation (IK-2.13), we get

$$\frac{dz_o}{dx} = \frac{2\tau_s}{\rho_w g z_o} - 2\mu \frac{V_{sw}}{\rho_w g z_o^2} \qquad \text{(IK-2.20)}$$

To express the surface water slope in terms of the wind stress and depth, it is necessary to obtain an expression for V_{sw} in terms of wind stress. Hellstrom does this based on the assumption that the water surface slopes exactly in the wind direction--which closely approximates that already shown by Ekman, particularly where z_o/D is small (see Figure IK-2). The result is an expression identical with equation (IK-2.6),

$$\frac{\partial z_o}{\partial x} = \frac{3}{2} \frac{\tau_s}{\rho_w g z_o} \quad \text{and} \quad \frac{\partial z_o}{\partial y} = 0 \qquad \text{(IK-2.21)}$$

Having previously derived the velocity components in the form

$$\left.\begin{aligned} u &= \frac{\rho_w g}{\mu} \frac{\partial z_o}{\partial y} \left(\frac{z^2}{2} - z_o z \right) + \frac{\tau_s}{\mu} z \\ v &= \frac{\rho_w g}{\mu} \frac{\partial z_o}{\partial x} \left(\frac{z^2}{2} - z_o z \right) \end{aligned}\right\} \qquad \text{(IK-2.22)}$$

where z is measured positive upward from the bottom; Hellstrom inserts the slope values from (IK-2.21) in the equations (IK-2.22) with the result

$$\left.\begin{aligned} u &= \frac{3\tau_s}{4\mu z_o} z^2 - \frac{\tau_s}{2\mu} z \\ v &= 0 \end{aligned}\right\} \qquad \text{(IK-2.23)}$$

It can be seen that the velocity is greatest at the surface ($z = z_o$), and equals

$$V_{sw} = \frac{\tau_s z_o}{4\mu} \qquad \text{(IK-2.24)}$$

For $z = 0$ and $z = 2/3\ z_o$ the velocity is zero and has a minimum value of

$$V = -\frac{\tau_s z_o}{12\mu} \qquad \text{(IK-2.25)}$$

for $z = 1/3\ z_o$. The curve of velocity distribution with depth is parabolic and is shown in Figure IK-4. Using the value of V_{sw} found in equation (IK-2.24), the surface slope in general terms becomes

$$\frac{dz_o}{dx} = \psi \frac{\tau_s}{\rho_w g z_o} \qquad \text{(IK-2.26)}$$

where ψ is a constant equal to 3/2 for this laminar case.

It is worth noting that if the slope of (IK-2.26) be introduced in equation (IK-2.11), then

$$\tau_B = \frac{\tau_s}{2} \qquad \text{(IK-2.27)}$$

This means that the bed friction in this idealized case would equal one-half of the wind stress at the water surface.

It is well to emphasize that the equations presented here by Hellstrom are based on laminar flow. No account has been taken of turbulent motion, which in fact exists in almost all cases. Thus equations (IK-2.22) show the velocity components as directly proportional to the gradient, whereas it is well known that the velocity varies as the square root of the gradient. Because of this no close agreement can be expected between theoretical results and empirical data, hence the case of turbulent motion must next be considered.

c. Wind Tides Assuming Turbulent Motion

The previous laminar case treated the coefficient of viscosity μ as a constant for any given fluid; it being a molecular property of the fluid. This may be replaced by the eddy viscosity, ν, which combines the molecular and turbulent motions of the fluid. This term agrees with Ekman's virtual coefficient of viscosity. The eddy viscosity may vary with the motion and is not a physical property of the fluid. No exact values have been computed; only the order of magnitude is known (Sverdrup et al, 1942, IK-47). Using this eddy viscosity and following Ekman's definition, we may define the wind stress as

$$\tau_s = \nu \frac{dv}{dz} \qquad \text{(IK-2.28)}$$

where dV/dz represents the shear of the observed velocities. It is evident that the eddy viscosity will be a variable and depend upon the size and intensities of the eddies in the turbulent motion. Because of the greater motion and transfer of momentum involved, the eddy viscosity is normally many times larger than the coefficient of viscosity.

Taylor (1915, IK-48; 1916, IK-49) studied the problem of wind stresses with turbulent motion in the lower layers of the atmosphere. He reasoned that the wind blowing over the earth's surface is dynamically similar to that of a fluid flowing over a small, slightly roughened plate in a laboratory experiment. Using this idea he suggests the empirical relation

$$\tau_s = k\rho_a V_{as}^2 \qquad \text{(IK-2.29)}$$

where τ_s is the surface shearing stress per unit area; k is a wind stress coefficient; ρ_a, the air density; and V_{as}, the wind velocity at the surface. With the components of τ_s designated τ_{sx} and τ_{sy} along the horizontal axes and the gradient wind blowing along the x-axis, Taylor expressed equation (IK-2.28) as

$$\tau_{sx} = \nu \left(\frac{du}{dz}\right)_{z=0} \quad \text{and} \quad \tau_{sy} = \nu \left(\frac{dv}{dz}\right)_{z=0} \qquad \text{(IK-2.30)}$$

where z is measured positive upward from the surface and with the assumption that ν is constant with height above the surface. Substituting these expressions in the equations

$$\left.\begin{aligned} \left(\frac{du}{dz}\right)_{z=0} &= \frac{2\tan\alpha}{1+\tan^2\alpha}\, V_g \,\frac{3/4\pi+\alpha}{H} \\ \left(\frac{dv}{dz}\right)_{z=0} &= \frac{2\tan^2\alpha}{1+\tan^2\alpha}\, V_g \,\frac{3/4\pi+\alpha}{H} \end{aligned}\right) \qquad \text{(IK-2.31)}$$

which are derived from the equations of motion (Lamb, 1932) where α is the angle between the surface and gradient winds, V_g is the gradient wind speed, and H is the height of the gradient wind level; the result is

$$\tau_s = 2\nu V_g \sin\alpha \;\frac{3/4\pi+\alpha}{H} \qquad \text{(IK-2.32)}$$

Replacing τ_s by its value in equation (IK-2.29), the constant k becomes

$$k = \frac{2\nu}{\rho_a} \sin\alpha \;\frac{3/4\pi+\alpha}{H}\;\frac{V_g}{V_{as}^2} \qquad \text{(IK-2.33)}$$

Using this expression and pilot balloon data obtained by Dobson (1914, IK-.2) on Salisbury Plain, Taylor computed the k-values shown in Table IK-1 for three different wind speeds, taken at an elevation of 30 meters.

Table IK-1--Values of the wind stress coefficient, k, for various wind velocities (after Taylor).

Winds	V_{as} m/sec	V_g m/sec	ν/ρ $\frac{cm^3}{gm\text{-}sec}$	α Deg.	H m.	k Non-dimensional
Light	3.3	4.6	28×10^3	13	600	0.0023
Moderate	5.9	9.1	50×10^3	21.5	800	0.0032
Strong	9.5	15.6	62×10^3	20	900	0.0022

The k-values of Table IK-1 appear to have no relation to the wind speed since they remain almost constant over a wide range in velocities; therefore equation (IK-2.29), relating the wind stress to the square of the wind speed, appears to be verified by these empirical data.

Other investigators have computed wind stresses over water, and their results should be given for comparative purposes. Palmen and Laurila (1938, IK-37) found the wind stress to be

$$\tau_s = .0024 \, \rho_a \, V_{as}^2 \qquad \text{(IK-2.34)}$$

while Sverdrup et al (1942, IK-47), give

$$\tau_s = .0026 \, \rho_a \, V_{as}^2 \qquad \text{(IK-2.35)}$$

and Hela (1948, IK-21) found in the Gulf of Bothnia

$$\tau_s = .0019 \, \rho_a \, V_{as}^2 \qquad \text{(IK-2.36)}$$

These results all agree well with Taylor's findings, so that a reliable expression for wind stress on water appears to be known.

One of the chief limitations of the previous work by Taylor is that the eddy viscosity is assumed uniform with height. This restriction is particularly important in the zones immediately above and below the water surface where the eddy viscosities must change by definition as the boundary is approached. Fjeldstad (1929, IK-17) investigated the variation of eddy viscosity with water depth. He suggests a decreasing eddy viscosity with depth expressed by the relationship

$$\nu = \nu_o \left(\frac{z + d}{z_o + d} \right)^n \qquad \text{(IK-2.37)}$$

where ν_o is the eddy viscosity at the surface, d is a very small positive constant, n is a positive number less than one, and z is measured positive upward from the bottom. Sverdrup (1929, IK-46) found this equation gave good agreement with observations when n = 0.5 and d is so small that the eddy viscosity at the bottom becomes at least 100 times smaller than at 2 m above the bottom.

Several studies of wind tides have been made by Palmen (1932a, IK-34; 1932b, IK-35; 1936, IK-36), some of them concerning equation (IK-2.37). Using this relationship he found the water surface slope approximates

$$\frac{dz_o}{dx} = \frac{(3-n)\tau_s}{2\rho_w g z_o} \qquad \text{(IK-2.38)}$$

This corresponds to slope equation (IK-2.26), based on laminar flow conditions, if n = 0, leaving the constant ψ = 3/2. If n = 0.5, ψ = 1.25. It can be seen that the surface slope here depends only upon the variation with depth of ν and not upon its absolute value.

Prandtl (1932, IK-40) and von Karman (1930, IK-55) developed an entirely different solution to the problem of a variable eddy viscosity based upon the idea of a very thin skin friction layer immediately above the surface. This frictional layer was taken to be so thin that motion within it would be controlled entirely by frictional drag. Thus, the horizontal forces of pressure gradient and deflecting forces could be neglected and the frictional drag per unit horizontal area within this layer would be constant in intensity and the direction must coincide with that of the surface wind. Within this layer Prandtl assumed a mixing length, ℓ, which has a finite value at the surface and is a function of the distance from the surface. Expressing this relation as

$$\ell = A\,(z + \delta) \qquad \text{(IK-2.39)}$$

where A is a non-dimensional constant, z is the distance from the water surface, and δ is another constant. At the surface then,

$$\ell = A\delta \qquad \text{(IK-2.40)}$$

and A, according to Prandtl and von Karman, has a value of 0.38. Based on channel tests of sand grains for surface roughness, Prandtl gives

$$\delta = \frac{\epsilon}{30} \qquad \text{(IK-2.41)}$$

where ϵ is the average height of the roughness elements.

Using these theoretical concepts, Rossby (1932, IK-43) and Rossby and Montgomery (1935, IK-44; 1936, IK-45) applied them to the problem of wind tides. The wind stress may be given as

$$\tau_s = \rho_a \ell^2 \left(\frac{dV_{as}}{dz}\right)^2 \qquad \text{(IK-2.42)}$$

Introducing equation (IK-2.39) for ℓ, we obtain

$$\tau_s = \rho_a A^2 (z + \delta)^2 \left(\frac{dV_{as}}{dz}\right)^2 \qquad \text{(IK-2.43)}$$

which when integrated yields

$$V_{as} = \frac{1}{A}\sqrt{\frac{\tau_s}{\rho_a}}\; \ln\left(\frac{z+\delta}{\delta}\right) \qquad \text{(IK-2.44)}$$

Solving this for τ_s gives

$$\tau_s = \rho_a \left(\frac{A}{\ln \frac{z+\delta}{\delta}} \right)^2 V_{as}^2 \qquad \text{(IK-2.45)}$$

This result compares with that of Taylor's, equation (IK-2.29), if his

$$k = \left(\frac{A}{\ln \frac{z+\delta}{\delta}} \right)^2 \qquad \text{(IK-2.46)}$$

Rossby and Montgomery (1936, IK-45) used this approach to determine the depth of the surface drift current, and the relation between its velocity and that of the surface wind. Starting from equation (IK-2.45) they found the depth of the drift current, h, to be

$$h = \frac{3\, k_r^2}{f} \sqrt{\frac{3}{2}}\; V_{sw} \qquad \text{(IK-2.47)}$$

where k_r is a non-dimensional constant, f is the Coriolis parameter, and V_{sw} is the surface current velocity as before. For h to equal Ekman's D-value, in equation (IK-2.4), k_r must equal 0.12. From this same theory, Rossby and Montgomery found the ratio of the surface drift velocity to the wind velocity, known as the wind factor, to be

$$\frac{V_{sw}}{V_{as}} = \frac{\sqrt{k}}{k_r} \sqrt{\frac{2\rho_a}{3\rho_w}} \qquad \text{(IK-2.48)}$$

where all symbols are as previously defined. Taking a mean k-value of 0.0025, corresponding to Taylor's findings (see Table IK-1), $k_r = 0.12$, $\rho_a = 1.26\times10^{-3}$, $\rho_w = 1.025$, equation (IK-2.48) gives a wind factor of 0.0119.

This value for a wind factor may be compared with those of other investigators. Table IK-2 summarizes these values for a latitude of 45° in instances where the author expressed the relation in terms of latitude.

Table IK-2--Comparison of wind factor by various investigators for latitude 45°.

Investigator	Wind factor	How or where determined	Source
Rossby-Montgomery	.0119	Theory	Rossby-Montgomery (1935)
Mohn	.0146	Drift of ship	" "
Nansen	.0180	Ice drift	" "
Dinklage	.0180	Baltic Sea	" "
Witting	.0226	Baltic Sea	" "
Thorade	.0178	Drift of ship	" "

Investigator	Wind factor	How or where determined	Source
Ekman	.0178	Theory	Ekman (1905)
Durst	.0112	Meteorological charts	Durst (1924)
Palmen	.0171	Gulf of Bothnia	Palmen (1931)
von Arx	.03	Bikini Lagoon	von Arx (1948)
Chatley	.035	Theory	Chatley (1950)

Although the values agree reasonably well, Rossby and Montgomery expressed the opinion that the values they presented agree largely by coincidence. They based this on the fact that the results come from many different sources and methods of observation, as indicated in Table IK-2. Nevertheless, for practical purposes, these results indicate that surface current velocities generally equal one to three percent of the average surface wind speed.

Hellstrom (1941, IK-22) derived an expression for the slope of the water surface similar to that for laminar flow, equation (IK-2.26), but based on turbulent motion conditions. Beginning with the Euler-Navier equations and introducing the eddy viscosity, ν , in place of the coefficient of viscosity, μ , the first equation takes the form

$$\rho_w\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho_w F_x - \frac{\partial p}{\partial x} + \left[\frac{\partial}{\partial x}\left(\nu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)\right] \qquad \text{(IK-2.49)}$$

where F_x is the component of the extraneous force along the x-axis. The bottom friction may be expressed in a form first suggested by Boussinesq (1877, IK-1; 1897, IK-2) as

$$\tau_B = \rho_w g B V_b^2 \qquad \text{(IK-2.50)}$$

where B is a constant depending upon the nature of the bottom surface (Boussinesq dealt with wide, open channels of constant depth and gradient) and V_b is the mean velocity at the bottom of the channel. Then putting the eddy viscosity equal to (also from Boussinesq)

$$\nu = \frac{\rho_w g}{\theta} \sqrt{B}\, z_o V_b \qquad \text{(IK-2.51)}$$

where θ is a constant depending upon the nature of the fluid. At the water surface, where $z = z_o$, the shear becomes

$$\frac{du}{dz} = \frac{\tau_s}{\nu} \qquad \text{(IK-2.52)}$$

Equation (IK-2.49) can then be reduced to

$$\frac{dz_o}{dx} = \psi \frac{\tau_s}{\rho_w g z_o} \qquad \text{(IK-2.53)}$$

where

$$\psi = \frac{3}{2} \frac{(\theta \sqrt{B} + 2)}{(\theta \sqrt{B} + 3)} \qquad \text{(IK-2.54)}$$

Although θ and B may vary over a wide range, it can be seen that ψ remains nearly constant. For example, as $\theta\sqrt{B} \rightarrow 0$, $\psi \rightarrow 1$ and as $\theta\sqrt{B} \rightarrow \infty$, $\psi \rightarrow 3/2$. Equation (IK-2.53), based on turbulent flow conditions, expresses the slope of the water surface in the same form as equation (IK-2.26) does for laminar motion. Therefore, the derivations based on laminar and turbulent flow conditions give very nearly the same results for the wind tidal water surface slope.

Having developed equations for water surface slope due to wind stresses, Hellstrom applied these to idealized basins of given shapes. He first considered a basin with a plane, horizontal bottom of length, L. Introducing a system of coordinates to represent the equation of the water surface in cross-section, let the x-axis parallel the wind direction and let the z-axis be positive upward (see Figure IK-5). Designating the ordinate to the water surface as z_o, equation (IK-2.53) then expresses the slope of the water surface. Integrating, we get

$$z_o^2 = \frac{2\psi \tau_s}{\rho_w g} (x + C_1) \qquad \text{(IK-2.55)}$$

where C_1 is an integration constant. The water surface thus has a parabolic shape as shown in Figure IK-5. If the calm water depth is z_{os}, then the wind tide may be represented by s, where

$$s = z_o - z_{os} \qquad \text{(IK-2.56)}$$

Substituting this value in equation (IK-2.55), we get

$$s = \sqrt{\frac{2\psi \tau_s}{\rho_w g} (x + C_1)} - z_{os} \qquad \text{(IK-2.57)}$$

If the depth is large in relation to the wind tide, we may regard the depth as a constant under the influence of wind and equation (IK-2.53) becomes

$$z_o = \psi \frac{\tau_s}{\rho_w g z_{os}} x + C_2 \qquad \text{(IK-2.58)}$$

where C_2 is another constant. From this we get

$$z_o = \psi \frac{\tau_s}{\rho_w g z_{os}} x + z_{os} - \psi \frac{\tau_s L}{2 \rho_w g z_{os}} \qquad \text{(IK-2.59)}$$

and

$$s = \psi \frac{\tau_s}{\rho_w g z_{os}} \left(x - \frac{L}{2}\right) \qquad \text{(IK-2.60)}$$

In this case the water surface assumes a straight line and the wind tide at the ends of the basin has the value

$$s = \pm \psi \frac{\tau_s L}{2 \rho_w g z_{os}} \qquad \text{(IK-2.61)}$$

where the sign depends upon the windward (-) or leeward (+) ends of the basin.

The second basin that Hellstrom analyzed was one having a plane, sloping bottom. The coordinate system and symbols previously used will be repeated, and z_1, the ordinate to the bottom of the basin, will be added. Let

$$\tan \beta = \frac{z_1}{x} \qquad \text{(IK-2.62)}$$

so that equation (IK-2.53) becomes

$$\frac{dz_o}{dx} = \psi \frac{\tau_s}{\rho_w g (z_o - x \tan \beta)} \qquad \text{(IK-2.63)}$$

Integration gives the following equation for the water surface

$$x = \frac{1}{\tan \beta} \left(z_o - \frac{\psi \tau_s}{\rho_w g \tan \beta}\right) + C_3 \exp\left[-\frac{\rho_w g \tan \beta}{\psi \tau_s} z_o\right] \qquad \text{(IK-2.64)}$$

where C_3 is a constant that can be solved for by a trial-and-error solution based on the fact that the water volume remains constant. For $C_3 = 0$, it can be seen that the surface becomes plane and parallel to the bottom. When $C_3 > 0$ the surface is convex upward, and when $C_3 < 0$ the surface is concave upward. Figure IK-6 illustrates a typical case of $C_3 > 0$.

One of the more recent theoretical studies of wind tides has been made by Langhaar (1949, IK-29). He derived wind tidal expressions by methods similar to those of Hellstrom and applied them to specific water bodies. Rectangular and elliptical lakes and converging channels were analyzed. He concluded that wind tides are of almost equal magnitudes in the rectangular and elliptical cases.

The discussion heretofore has neglected two factors which, for the sake of completeness, should be mentioned. These are the effect of a non-homogeneous water body and the time relationships in developing wind tides. The first factor is important where water exists in stratified layers due to temperature and/or salinity variations, while the time relations enter if one is interested in forecasting wind tides or is dealing with a variable wind force.

Hellstrom has investigated the problem of stratified fluids along lines similar to the homogeneous cases previously covered. Because of the complexity of the derivations and the limited applications thereof, only his results for the case of specific gravity proportional to depth will be included here. Letting γ represent specific gravity and γ_0 and γ_1 the surface and bottom values, respectively, then

$$\gamma = \gamma_1 - \frac{\gamma_1 - \gamma_0}{z_0} z \qquad \text{(IK-2.65)}$$

represents the directly proportional relationship with depth if z is measured upward from the bottom. The slope of the water surface becomes

$$\frac{dz_0}{dx} = \psi \left(1 + \frac{\gamma_1 - \gamma_0}{\gamma_1 + 3\gamma_0} \right) \frac{\tau_s}{\gamma_m z_0} \qquad \text{(IK-2.66)}$$

where γ_m is the average specific gravity. Since $\gamma_1 > \gamma_0$ the quantity in the parentheses is greater than one; therefore the gradient of the water surface is steeper than in the homogeneous case. This statement is true in general because closed circulation systems tend to develop in each layer of a stratified fluid. This creates the same effect as a shallower bottom on wind tides, so that for a given water depth, wind tides will be higher in stratified water.

Based upon the transfer of energy from wind to water, Hellstrom also computed an estimate of the time required to develop a steady state of wind tides. He assumed the energy for steady motion, E, was approached asymptotically by the actual energy, E', so that for

$$E = E', \ t \rightarrow \infty \qquad \text{(IK-2.67)}$$

Then the actual energy could be expressed by the form

$$E' = \left(1 - \exp\left[\frac{-t}{T}\right]\right) \qquad \text{(IK-2.68)}$$

where t is the time variable and T is the time required for a given energy to attain the same value as the actual energy for steady motion. This time may be given in integral form as

$$T = \frac{\rho_w g}{2\tau_s \int_0^L V_{sw} dx} \left[\frac{1}{g} \int_0^L \int_0^{z_0} V^2 dx dz + \int_0^L s^2 dx \right] \qquad \text{(IK-2.69)}$$

where all symbols are as previously defined. If we use z_{os}, a constant as an estimate of z_0 and let $\psi = 3/2$, then equation (IK-2.69) can be reduced to

$$T = \frac{\rho_w g z_{os} V_{sw}}{15 g \tau_s} + \frac{3}{32} \frac{\tau_s L^2}{\rho_w g z_{os}^2 V_{sw}} \qquad \text{(IK-2.70)}$$

Using this equation and the previous assumption of development rate, approximately 95% of the steady state energy will be developed at time = 3T. Jeffreys (1923, IK-26) made some rough estimates of time required for wind tides to reach a steady state in a deep ocean 200 km long.

He calculated, for example, that hurricane winds over this entire area would develop a full wind tide in about 3 hours, whereas with lighter winds about 12 hours would be required. He also considered the same ocean, but having a depth of only 30 m, and estimated a wind of roughly 25 mph would develop a full wind tide in this shallow water in only 4.5 hours. Observational data of wind tides in enclosed seas indicate the time lag between winds and wind tidal development to be small. As a first approximation, particularly in relatively shallow water areas, it may be assumed that wind tides vary directly as the wind force generating them.

3. Laboratory Investigations:

There appear to be only two published results on laboratory experiments of wind tides. The first of these is found in Hellstrom's comprehensive treatise (1941, IK-22) and the second in a recent study by Keulegan (1951, IK-27) of the U.S. National Bureau of Standards. The laboratory equipment and results will be briefly summarized since these studies constitute the only controlled measurements available on wind tides.

Hellstrom used a covered glass channel 3 meters long, 0.5 meters high, and 0.12 meters wide connected to a fan at one end. By regulating the fan varying wind speeds could be created through the channel, ranging from 5 meters per second to 16 meters per second. The water depth in the channel was kept between 7 and 40 millimeters and water levels were measured at different points by observing water levels in glass tubes connected by rubber tubing to holes in the bottom of the channel. The pressure drop of the air current was measured with water level manometers and the air velocity by means of a small anemometer located halfway between the top and bottom of the channel. Before beginning experiments, the anemometer was calibrated and the velocity distribution within the channel was determined. Experiments were performed with varying water depths and wind velocities. Points showing the water surface profile were plotted and then fitted by a curve using equation (IK-2.26) with τ_s as a variable and $\psi = 3/2$. His experimental points in almost all cases followed the theoretical parabolic curves very closely, indicating excellent agreement. From the values of τ_s found, Hellstrom was able to relate this wind stress to the air speed, measured in meters per second, by the equation

$$\tau_s = m\, V_{as}^{3} \qquad \text{(IK-3.1)}$$

where m is a coefficient depending upon the roughness of the water surface. He found that m varied linearly with the still water depth, z_{os}, as

$$m = 1.42 \times 10^{-6}\, z_{os} + 0.7 \times 10^{-5} \qquad \text{(IK-3.2)}$$

when z_o is measured in millimeters. Thus as the depth approaches zero, the wind stress tends to a constant value as the water surface becomes a smooth surface. Hellstrom's experiments in general confirm

the previous theoretical treatments, but no positive conclusions can be made regarding his wind stress versus wind speed relation. His cubic equation (IK-3.1) is not in agreement with previous reasoning--equations (IK-2.7) and (IK-2.29).

Keulegan (1951, IK-27) used a similar laboratory setup for his experiments. His channel measured 20 m long, 0.28 m high, and 0.11 m wide, and was connected to a blower at one end and a Venturi nozzle at the other. The blower was operated by a constant speed motor, so that air speeds in the channel were controlled by varying the intake area to the blower with a damper. The air speeds were measured by an inclined manometer connected to the Venturi nozzle. This was calibrated by simultaneous readings with an anemometer and an Ott current meter. The wind distribution within the channel was also determined. Keulegan conducted two sets of experiments in this channel--those with waves and those without waves. He found that by adding soap or a detergent to the water that waves would not form in the channel no matter how high the wind speed.

In the non-wave case Keulegan found the wind tide slope could be represented by

$$\frac{dz_o}{dx} = 3.30 \times 10^{-6} \frac{V_m^{\;2}}{gz_{os}} \qquad \text{(IK-3.3)}$$

where V_m is the average channel wind velocity. Keulegan also investigated the ratio of surface current velocity to wind speed, previously defined as the wind factor, and found that the equation

$$\frac{V_{sw}}{V_m} = 7.6 \times 10^{-4} \left(\frac{V_{sw}\, z_{os}}{\nu}\right)^{1/2} \qquad \text{(IK-3.4)}$$

fitted the data well for a Reynolds number below 1000. Above that the wind factor tended to approach a constant value, indicating the effect of eddy viscosity to be small. The limiting value was estimated to be 0.033. It was also found that the wind factor varied as stated regardless of whether waves existed or not in the channel--in other words the presence of waves had no apparent effect on the surface water velocity. Keulegan found that his non-wave results fitted wind tide theories very well. He found the wind stress to be

$$\tau_s = 0.00185\, \rho_a V_m^{\;2} \qquad \text{(IK-3.5)}$$

which is only slightly lower than values found by Taylor (1916, IK-49) (see Table IK-1) and other investigators. These experiments, therefore, verified the previous theoretical premises that the wind stress is independent of the depth and eddy viscosity of the water and are in agreement with fluid flow experiments over roughened plates.

In the second case studied by Keulegan, wind tides in the presence of waves, the slope was found to fit the equation

$$\frac{dz_o}{dx} = 3.30 \times 10^{-6} \frac{V_m^2}{gz_{os}} + 2.08 \times 10^{-4} \frac{(V_m - V_o)}{gz_{os}} \left(\frac{z_{os}}{L}\right)^{1/2} \qquad \text{(IK-3.6)}$$

where V_o is a velocity defined as the "formula characteristic velocity". Keulegan discussed this velocity in relation to a minimum critical velocity for the formation of waves. That is, below a certain small wind speed the water surface is relatively unbroken by waves. Munk (1947, IK-30) also investigated this phenomenon. Keulegan found that the formula characteristic velocity to be independent of the channel length and related to the eddy viscosity by

$$V_o = 21.0 \left(\frac{g \rho_w \nu}{\rho_a}\right)^{1/3} \qquad \text{(IK-3.7)}$$

4. Wind Tide Observations:

In an effort to determine empirically the relationship between wind stress and wind velocity, Hellstrom collected several sets of observations of actual wind tides. These observations, together with their source and persons responsible for them, are mentioned in the following paragraphs. From each set of observations Hellstrom computed a wind stress, using equation (IK-2.26) with $\psi = 3/2$, and obtained a wind velocity value which existed over the water surface at the time of the wind tide. He expressed the wind stress in units of kg per sq m as compared with gm per cm per sec^2 used in results given by equations (IK-2.7, 2.29, 2.34, 2.35 and 2.36) of other investigators.

a. Lake Erie:

In 1911 the Carnegie Institution of Washington began an extensive study of evaporation on the Great Lakes. In connection with this study Hayford (1922, IK-20) investigated water level variation phenomena, including barometric pressure effects, seiches, and wind tides. He not only collected and analyzed wind and water level data for wind tides, but also developed a formula for computing wind tides based on Chezy's open channel flow formula. His derivation and results are not incorporated herein because of the marked disagreement with other investigators. Hayford assumed the transport of water due to wind stresses to be proportional to the wind velocity only, regardless of depth, which is contrary to the original concepts of Ekman. Hellstrom discussed Hayford's work and pointed out the various discrepancies in Hayford's reasoning as compared with his own.

Of more interest are some of the wind and water level data during storms on Lake Erie collected by Henry (1900, IK-23; 1902, IK-24). Hellstrom analyzed these data and computed wind tides for the same periods. He compared wind tides at Amherstburg and Buffalo, on the west and east ends of the lake, respectively, to show the contrasting differences in elevation. Figure IK-7 shows water surface profiles under the influence of east and west winds of 10 mps and $\tau_s = 0.02$ kg per sq m, the bottom profile, and an outline map of Lake Erie. In Table IK-3 are listed the observed and computed maximum wind tides for several storms during 1899 and 1900. It should be noted that the sums of maximum wind tides observed at the two stations do not necessarily equal the maximum differences observed. This is due to a time lag effect, particularly at Amherstburg, which is located in a relatively sheltered position on the lake (see Figure IK-7). The wind stresses are computed for the maximum wind tides at the two stations and for the maximum difference between the two. The average of these three values is shown in the far right column of Table IK-3.

The six-hour wind speed corresponding to this average wind stress is also shown. It can be seen that computed wind tides by Hellstrom agree well with those actually observed. The wind stress values shown in parentheses are somewhat out of line with the others and Hellstrom accounted for these discrepancies as being caused by seiches.

Table IK-3--Wind tides on Lake Erie during storms of 1899 and 1900 (after Hellstrom).

Date of Storm	Wind tide at Buff., m		Wind tide at Amherst., m		Max. difference in elev., m		Av. wnd vel. mps.	Wind stress kg/m^2
	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.		
March 5, 1900	0.43	0.38	0.33	0.30	0.71	0.63	8.7	0.021
March 6, 1900	0.61	0.55	0.43	0.39	1.02	0.92	12.5	0.031
March 25-26, 1900	0.37	0.33	0.36	0.32	0.71	0.63	6.2	0.021
Dec. 12, 1899	1.58	1.49	0.92	0.85	2.38	2.22	11.8	(0.074)
Sept. 12, 1900	1.72	1.64	0.58	0.54	1.82	1.69	14.2	(0.064)
Nov. 21, 1900	2.10	1.99	1.90	1.81	3.98	3.78	18.4	(0.122)
Feb. 24-25, 1900	0.35	0.32	0.40	0.38	0.75	0.70	11-12	0.023

b. Lake Geneva:

Wind tide observations were made by Forel (1895, IK-18) on Lake Geneva, Switzerland, during a storm of December 19-21, 1877. Hellstrom analyzed these data and computed wind stresses of 0.030 kg per sq m with a wind velocity of 10 mps, 0.048 with 18.5 mps, and 0.084 with 25 mps. These wind values are averages based on increasing and decreasing winds and are only approximate because of the rather crude wind data available.

c. Lake Hornborga:

Estimates of wind stress have also been made for Lake Hornborga in southwestern Sweden by Hellstrom. This lake is roughly 10 km long and 2-3 km wide and has a depth of only 0.2-0.4 m. Because of its extreme shallowness, wind tides become quite pronounced. It is not uncommon to have wind tides to 0.4 m at one end of the lake, while at the other end a large area of the bottom is exposed. According to rough measurements, Hellstrom found a wind stress of 0.05 kg per sq. m to be developed with a wind of 18 mps.

d. North Sea:

A rough estimate of wind stress was made by Rossby (1925, IK-42) during a voyage on the North Sea. Based on an estimate of the friction between layers of air near the surface, he calculated a value of 0.0052 kg per sq m with a wind velocity of 5 mps.

e. Baltic Sea:

One of the earliest investigations of wind tides was that made by Colding (1881, IK-6) on the Baltic Sea. He collected observations from a large storm over the area during November 12-14, 1872. Hellstrom examined his comprehensive data of winds, wind tides, and atmospheric pressures and computed wind stresses of 0.115 kg per sq m with an average wind of 23.5 mps and 0.07 with an 18.5 mps wind.

f. Lake Okeechobee

Probably some of the largest wind tides in the world occur during the passage of hurricanes over Lake Okeechobee, Florida. The importance of this lake and the observational program now in progress there are discussed in a later section of this report; however, to complete the material of wind tide observations and wind stresses, the observations during two early hurricanes will be included here.

Hellstrom collected information on the hurricanes of September 18, 1926 and September 16, 1928, which passed over the lake. Exact data are scarce because of the destructive natures of the storms, particularly the 1926 storm. Water level measurements were obtained after the storms from high water marks on trees, ridges, and levees. Figure IK-8 shows a cross-section of the lake and the water surface profile under the influence of a northerly wind during the first phase of the 1928 hurricane as computed by Hellstrom. The observed high water marks confirm his computed surface curve at the south end of the lake. At the northern end the computations indicate an area several miles wide of exposed bottom. To verify this Hellstrom quoted from a letter by a resident engineer of the area as follows, "In reference to the bottoms of Lake Okeechobee on the windward wide of the storm which were exposed by wind effect, I have no accurate data. The duration of the first phase of the storm, which was from the northward, was much longer than that of the second phase, which was from the southward. People from Okeechobee City told me that persons had walked out on the bottoms of the lake a half mile or more and picked up fish which had been stranded by the recession of the water". This statement appears to confirm, at least qualitatively, the surface curve by Hellstrom.

Wind stress values of 0.24 and 0.28 kg per sq m at wind velocities of 37 and 39 mps, respectively, were found from the 1928 hurricane data, and a value of 0.17 kg per sq m at 31 mps from the 1926 storm.

g. Summary of a-f:

The various wind stresses mentioned in the preceding paragraphs are summarized in Table IK-4 together with the locations where the data were obtained and the corresponding wind velocities. In Figure IK-9 the wind stresses have been plotted against wind speed. The numbered points refer to the listing in Table IK-4. Hellstrom has drawn a curve through these points which has the equation

$$\tau_s = 0.00037\, V_{as}^{\;1.8} \qquad \text{(IK-4.1)}$$

based, as in all of his previous computations, on ψ = 3/2. This equation provides an empirical relation covering many localities and conditions between wind stress and speed. It can be noted that this equation differs slightly from the original parabolic conception of the relation, namely

$$\tau_s = k \rho_a V_{as}^{\ 2} \qquad \text{(IK-4.2)}$$

However, only further wind tidal data can determine whether the empirical relation should more closely conform to the theoretical one.

Table IK-4--Summary of wind stress and wind velocity values from various locations (after Hellstrom).

Location	Reference Number	Wind velocity mps	Wind stress kg/m^2
Lake Erie	1	8.7	0.021
Lake Erie	2	12.5	0.031
Lake Erie	3	6.2	0.021
Lake Erie	4	11.5	0.023
Baltic Sea	5	23.5	0.115
Baltic Sea	6	18.5	0.07
Lake Geneva	7	10	0.030
Lake Geneva	8	18.5	0.048
Lake Geneva	9	25	0.084
Lake Hornborga	10	18	0.05
North Sea	11	5	0.005
Lake Okeechobee	12	37	0.24
Lake Okeechobee	13	39	0.28
Lake Okeechobee	14	31	0.17

h. Gulf of Bothnia:

Palmen and Laurila (1938, IK-37) collected extensive data on water levels in the Gulf of Bothnia of the Baltic Sea. The most illustrative features of their study were maps showing the contours of the sea surface together with the isobaric pattern of atmospheric pressure during a storm over the area. One of these maps, for 1400h,

October 4, 1936, is reproduced in Figure IK-10 to show observed wind tides over a large area. The water level observations have been corrected for variations due to atmospheric pressure. At the time of this map, a low pressure center was located over the Baltic States immediately south of the map area. The surface winds were blowing roughly from west to east at an angle of 15-30° to the left of the isobars (Petterssen, 1940, IK-39). It can be seen therefore, that the water surface contour lines are everywhere almost perpendicular to the surface wind direction. Thus, Ekman's original hypothesis that the surface drift current and the water surface gradient closely parallel the surface wind direction in an enclosed shallow sea (which the Gulf of Bothnia approximates) is verified by these observations.

5. Lake Okeechobee Project:

The Corps of Engineers, Department of the Army, has initiated an extensive investigational project at Lake Okeechobee for the measurement and study of wind tides. This project, entitled "Waves and Wind Tides in Inland Waters, Lake Okeechobee, Florida", was authorized for the purpose of obtaining "information on the inter-relationships between wind velocities, wind tides, and lake bottom topography in relatively shallow inland lakes" (Corps of Engineers, U.S. Army, 1950a, IK-7). Present plans call for the design and construction of many miles of levees around the lake for flood control and protection purposes, so that this project will govern design features representing many millions of dollars. In addition, data obtained from this project will be applicable to related engineering problems at Lake Pontchartrain, Louisiana, and to numerous other small lakes and coastal areas.

Lake Okeechobee, situated in south-central Florida, has an area of about 730 sq mi. It is a shallow, saucer-shaped lake with a bottom elevation of zero mean sea level. It is 25 miles wide in an east-west direction and 30 miles long in a north-south direction. The maximum depth is only 12 to 15 feet. The land surrounding the lake is relatively flat and is farmed extensively. Levees have been built in the past around the entire south shore to a crown elevation of 32.5 feet msl and for a portion of the way around the north shore to the same height. A map of Lake Okeechobee is shown in Figure IK-11.

The instrumentation setup on the lake in 1949 consisted of seven shore stations and three lake stations. Locations of these are indicated in Figure (IK-11). The typical shore station includes an anemograph, barograph, and water-stage recorder, for recording wind velocity, atmospheric pressure, and water level, respectively. The lake stations, built on steel-girder pylons, include an anemograph, wave-height recorder, and water-stage recorder. All instruments are battery operated and are visited weekly for servicing and collecting records. Since 1949 additional instrument stations have been installed.

Lake Okeechobee is an ideal body of water for detailed wind tide investigations. It is large enough and unprotected by surrounding terrain to be fully exposed to surface winds. Its geographic location places it in the center of the hurricane belt, so that it is frequently exposed to high winds. Its extreme shallowness makes it subject to wind tides of greater magnitude than would be experienced by lakes

of comparable size but of greater depth. During the period 1947-1949 the lake experienced three major hurricanes, one in each of the three years. With the instrumentation on the lake during this period, detailed measurements were made of wind tides and meteorological conditions. The records of these measurements constitute probably the most precise and complete data ever available on the subject. Because of the interest in and importance of these data, the Corps of Engineers has published the results of these measurements for the three hurricanes in three project bulletins (Corps of Engineers, U.S. Army, 1950b, IK-8; 1951a, IK-9; 1951b, IK-10). Thus, the detailed measurements of wind tides and winds during these three major storms are available to all interested investigators. It is believed that when these data have been analyzed, important new contributions will be made to the subject of wind tides.

To illustrate the results of the measurements, a typical collection of data is shown in Figure IK-12. The water level elevations and water surface contours have been taken from Project Bulletin No. 2 (Corps of Engineers, U.S. Army, 1950b, IK-8), while the wind directions and velocities are from a special wind study published as Hydrometeorological Report No. 26 by the U.S. Weather Bureau (1951, IK-53). This figure represents conditions at 0100EST, August 27, 1949. The dashed arrows indicate the wind direction over the lake, and the short dashed lines are isotachs, lines of constant 10-minute-average wind speeds. The solid lines are water surface contours. The conditions shown in Figure IK-12 existed during the height of the hurricane and it can be seen that a portion of the lake in the west, which is marshy and very shallow, has been exposed by the wind action. Reports at this hour also indicate a large overflow from the lake occurred in the flat areas along the northwest side. A maximum difference in elevation of 11.5 feet in a distance of 20 miles existed between the western lake station and the shore station at the northern end of the lake.

For the design of levees and shore protection installations, wave heights must be computed for maximum winds and added to wind tidal elevations. For example, using standard wave forecasting procedures (U. S. Navy Hydrographic Office, 1951, IK-52), O'Brien (1949, IK-31) has estimated a hurricane on Lake Okeechobee could develop waves 8 feet high. For coastal areas bordering an open ocean, this value would be even larger.

6. Conclusions:

The various theoretical aspects of wind tides seem to be fairly well defined. Studies based on turbulent flow conditions, as outlined previously, have indicated the relationships between winds and the water surface. Balloon ascents, model studies, and observations on water bodies have contributed empirical data for applying the theoretical results. Although discrepancies still exist between theory and observation, further studies should tend to eliminate many of these. Analyses of the Lake Okeechobee data and contemplated model studies hold promise for the future. The end result of this research, of course, is to be able to estimate wind tides at any given location for design purposes, and also to be able to forecast wind tides for military and civilian needs.

It is well to point out some of the damaging effects of wind tides along open coastal areas. Many references are available on wind tides in enclosed seas or lakes, primarily because of the ease of observation or larger magnitudes found in shallow water areas. However, wind tides are capable of achieving large heights along coastal zones as well, particularly where the ocean bottom slopes gradually away from the coast. The most disastrous wind tide known occurred at Galveston, Texas, in connection with the hurricane of September 8, 1900. This city is located on a long, low sand island paralleling the Texas coast and was exposed to the full force of the wind tide. The city was inundated to a depth of from 15 to 20 feet by the high water (Cline, 1900, IK-5) and some 6000 persons lost their lives in a city of 38,000 population. A similar occurrence happened during the 1938 hurricane that struck the Connecticut coast, although fortunately the loss of life was considerably less. Recently, during a cyclonic disturbance along the Atlantic Coast, wind tides flooded many low-lying areas in and around New York. Figure IK-13 illustrates graphically the effect of wind tides on air bases located in coastal zones. This New York storm occurred at the end of November and was not a hurricane, hence the damage there could have occurred at many other coastal cities or locations.

These wind tidal effects have been mentioned to stress their importance in the design and location of shore installations and in the protection of existing works. For amphibious landing operations, adequate meteorological data and personnel for the detection of major storms which might cause excessive wind tides become essential. The probability of occurrence of a destructive wind tide at any given location, other than in the common hurricane belts, is usually small, but this does not justify ignoring it. The damage resulting from large wind tides is considerably larger than the small investment required for providing warnings and protection against wind tides.

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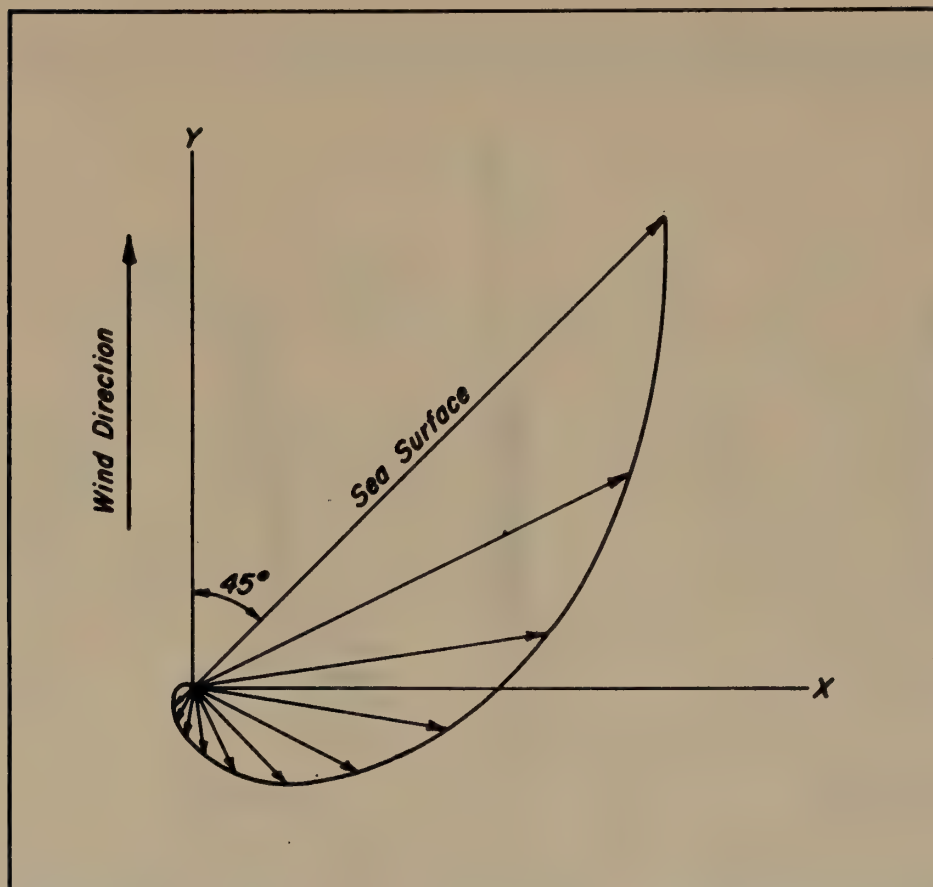

FIG. IK-1 - Ekman spiral of current velocities for a surface wind blowing along the positive y-axis

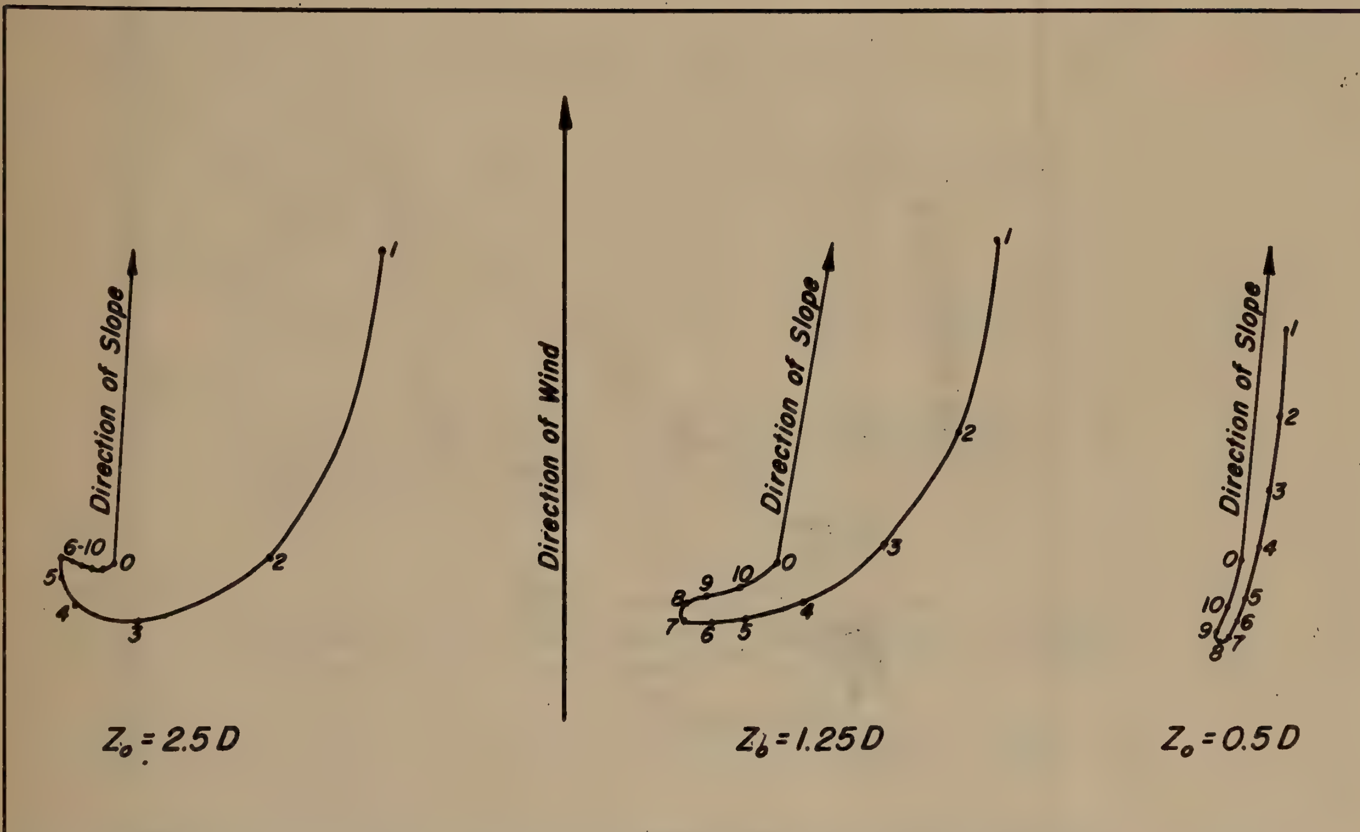

FIG. IK-2 - Current velocity curves for enclosed seas of different depths (after Ekman)

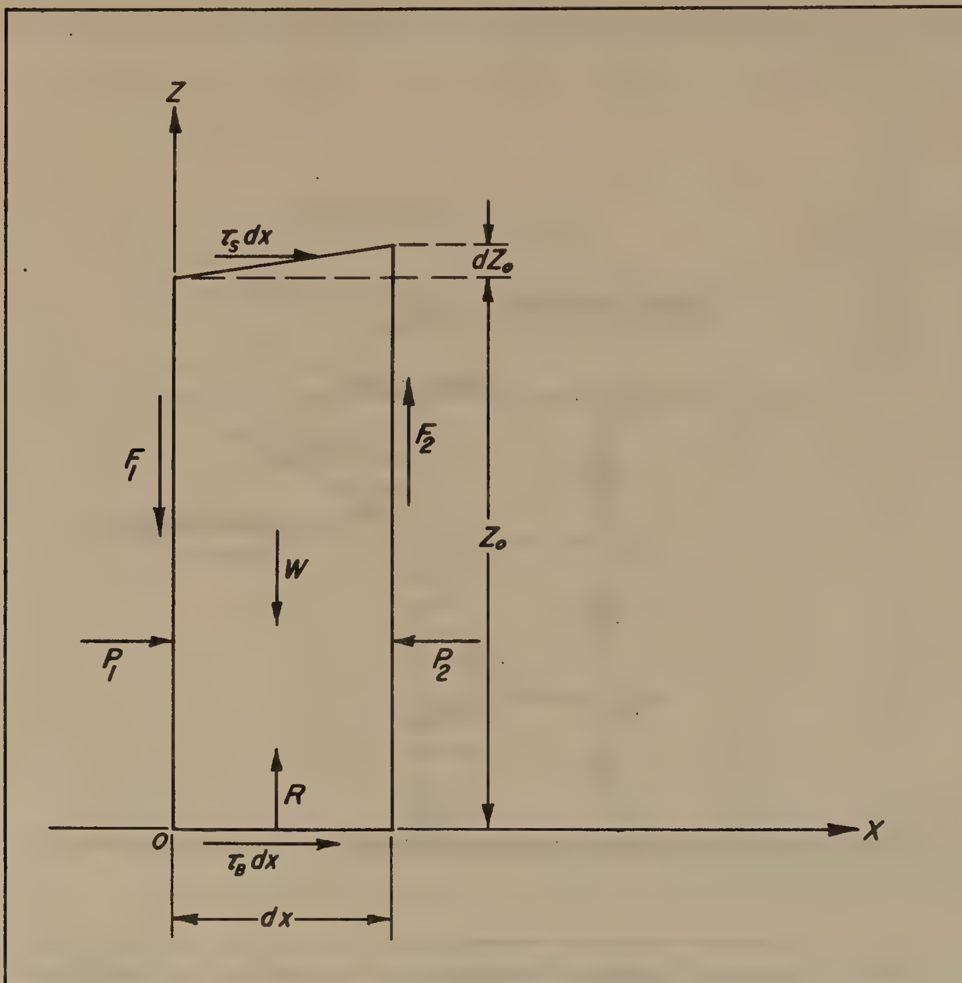

FIG. IK-3 - Diagram of forces acting on a vertical water element of unit width.

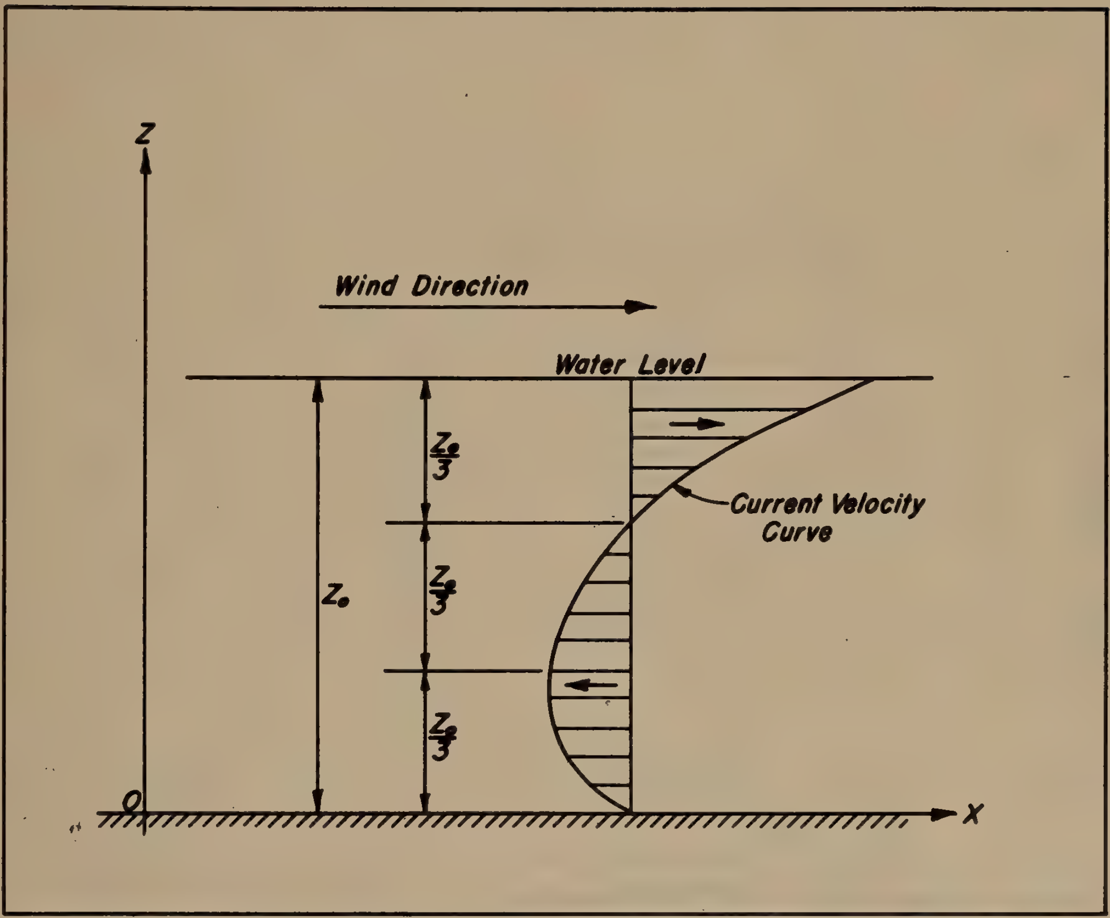

FIG. IK-4 - Curve of velocity distribution (according to Hellstrom)

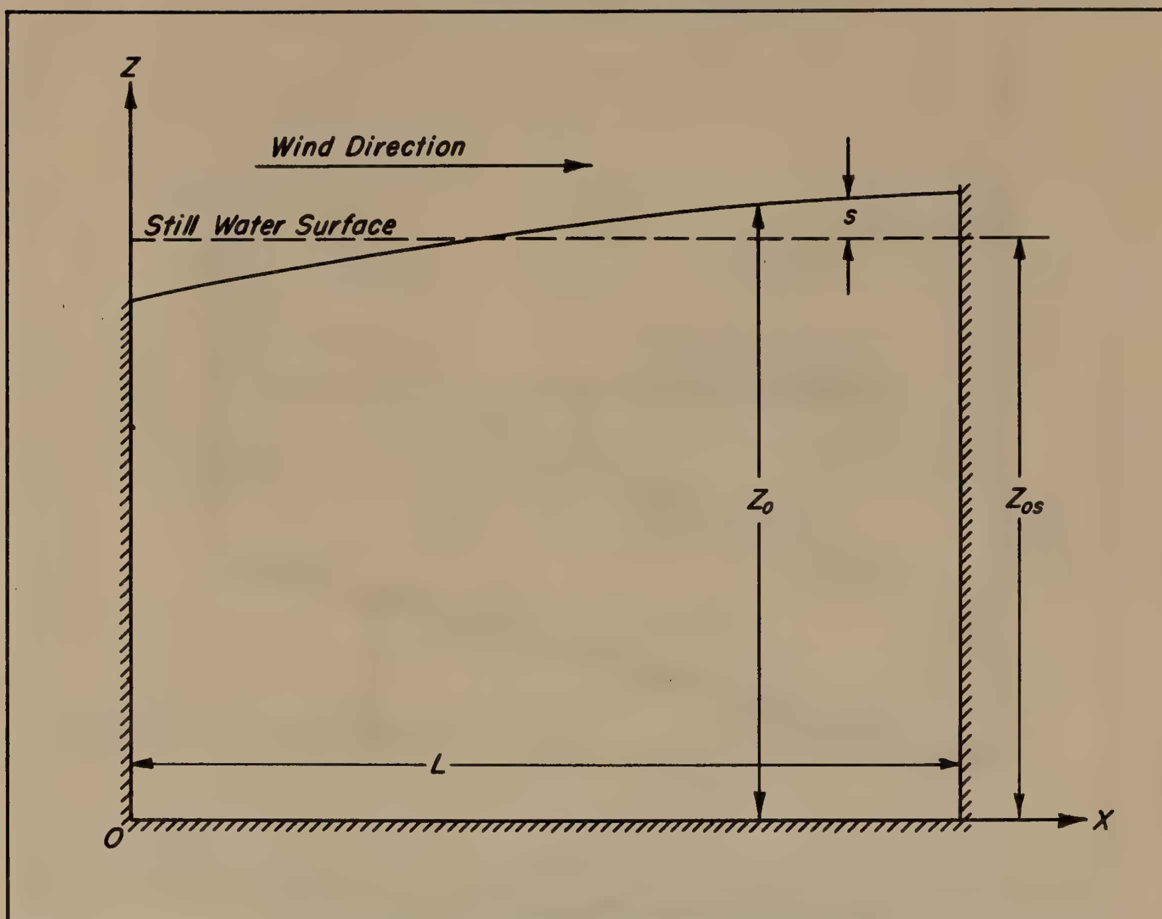

FIG. IK-5 - Cross-section of basin with horizontal bottom showing wind tide (after Hellstrom)

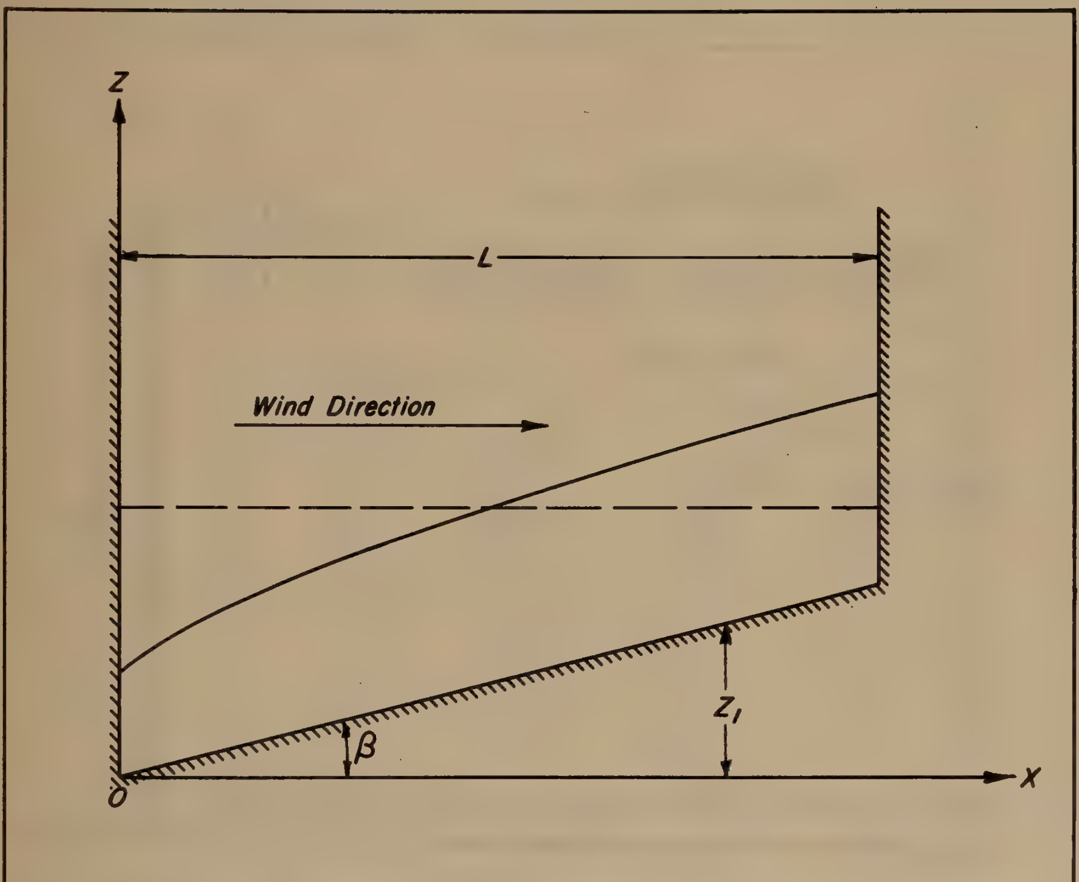

FIG. IK-6 - Cross-section of basin with sloping bottom showing wind tide with $C_3 > 0$ (after Hellstrom)

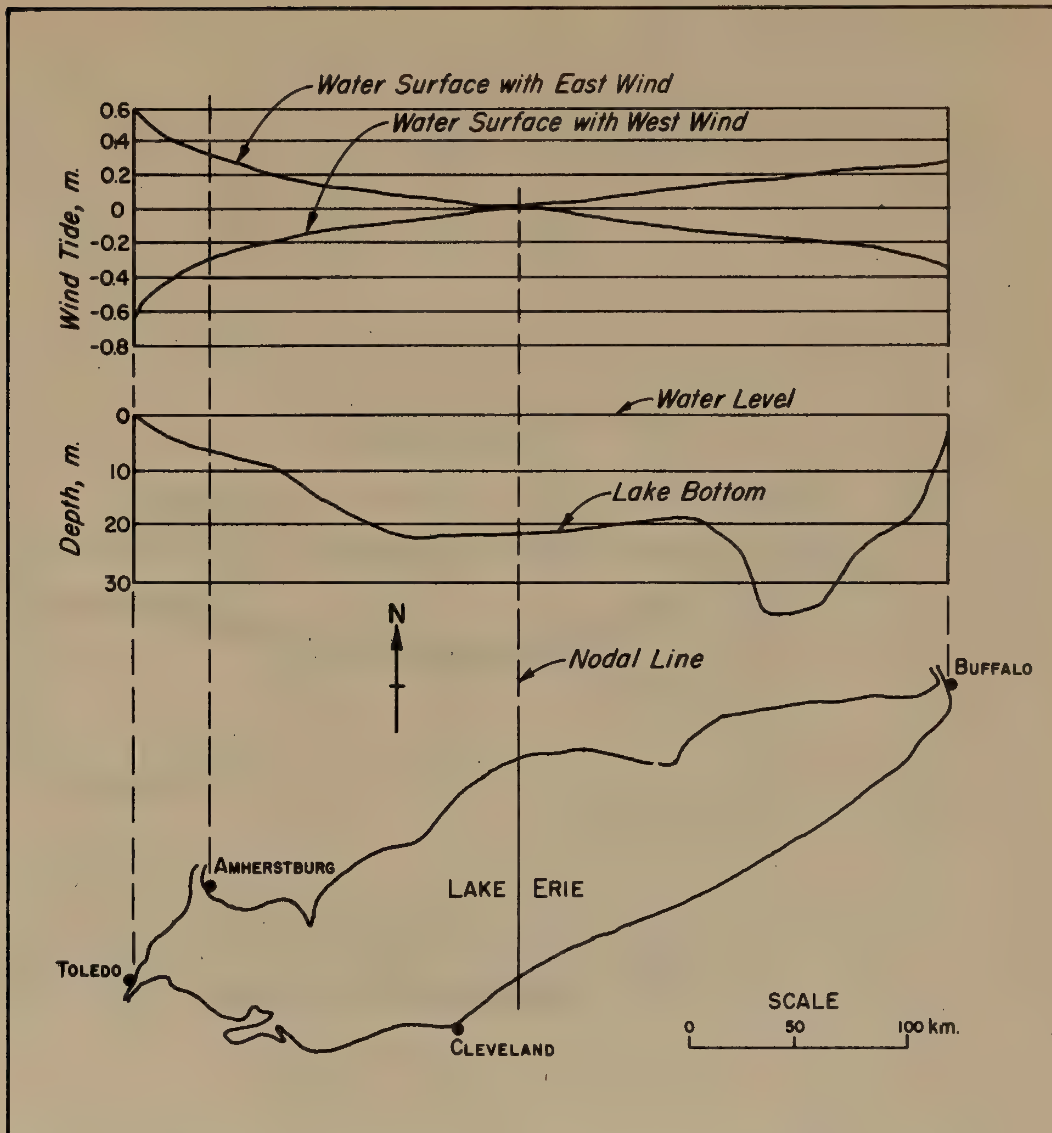

FIG. IK-7 - Map of Lake Erie, showing bottom profile and wind tides with east and west winds (after Hellstrom)

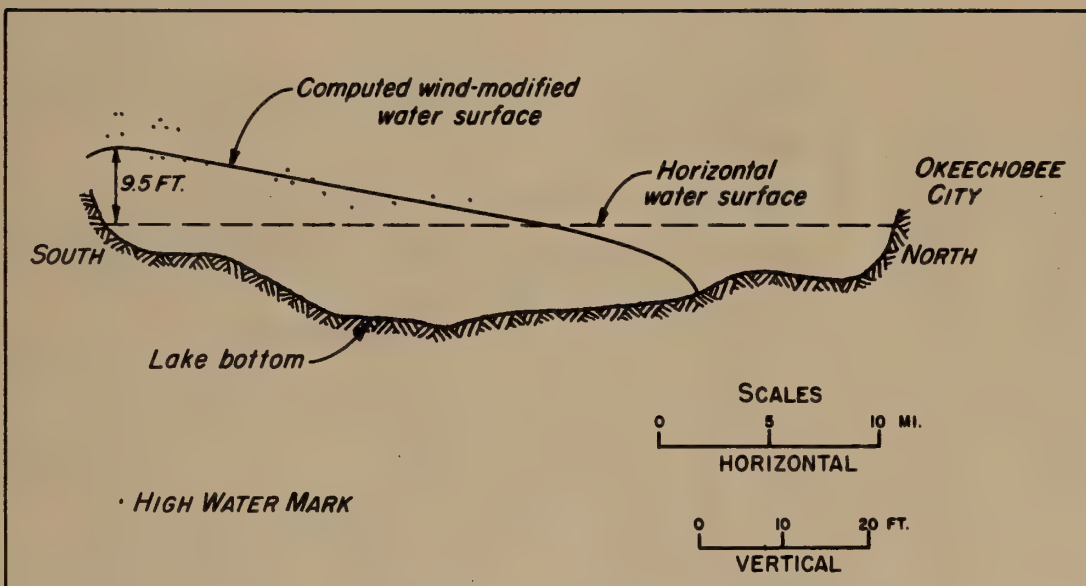

FIG. IK-8 - Profile of Lake Okeechobee from north to south during first half of September, 1928, hurricane (after Hellstrom)

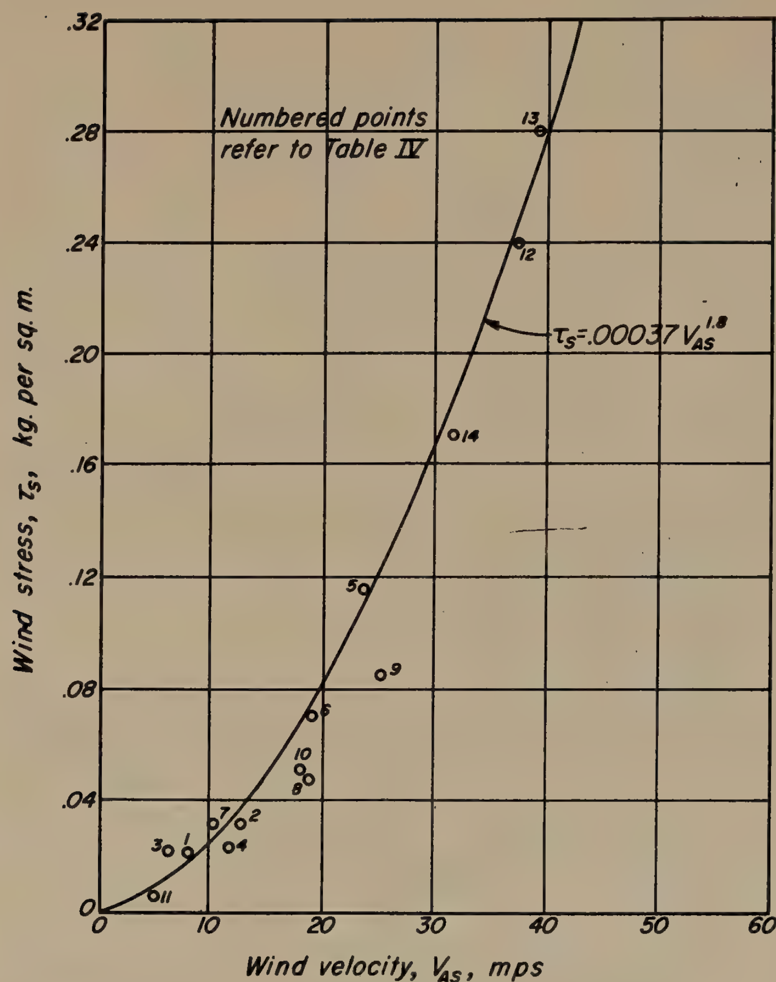

FIG. IK-9 - Relation between wind stress and wind velocity (after Hellstrom)

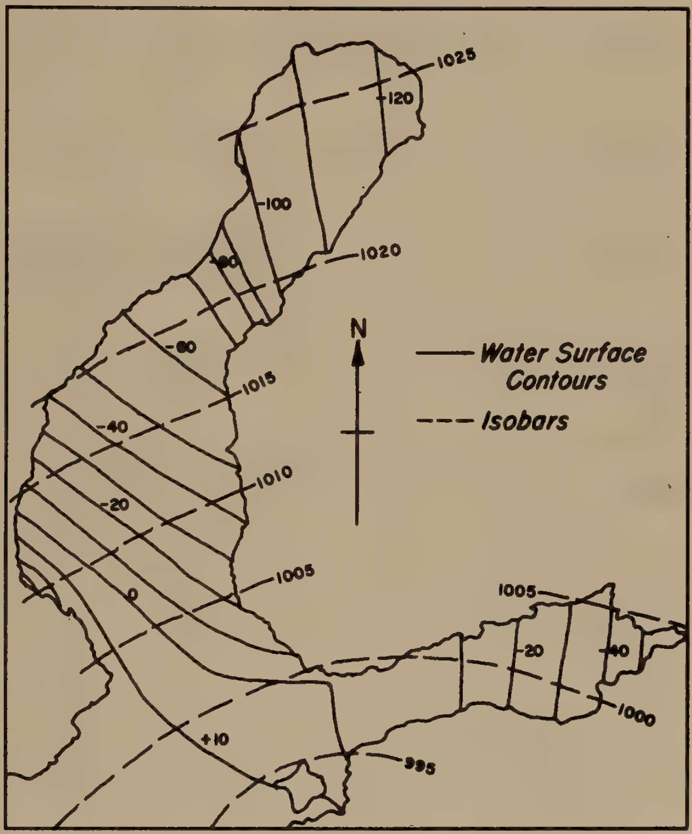

FIG. IK-10 - Map of Gulf of Bothnia showing isobars, mb, and water surface contours, cm, at 1400h, 4 October 1936 (after Palmen)

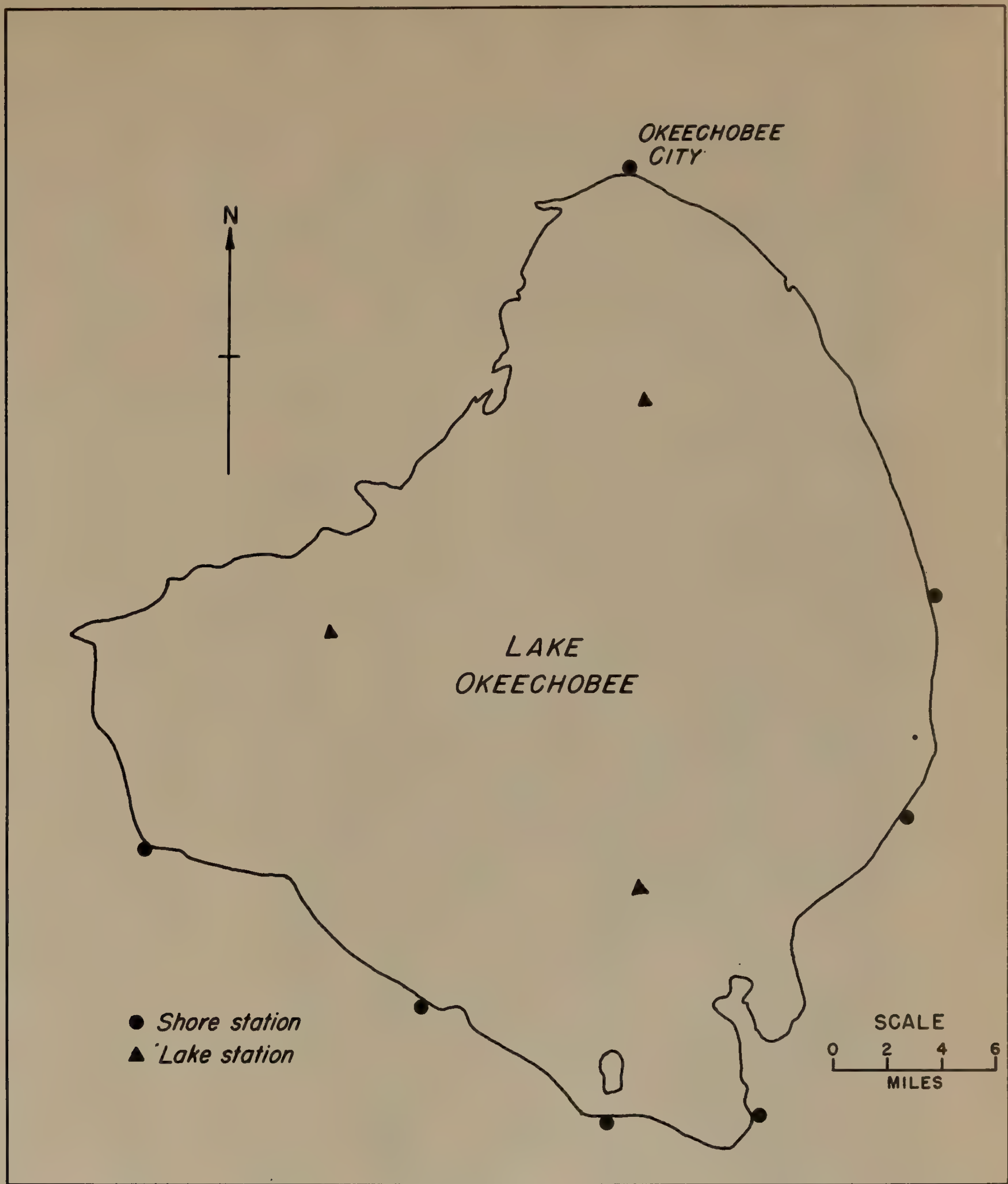

FIG. IK-11 - Map of Lake Okeechobee, showing locations of shore and lake instrument stations for measuring wind tides.

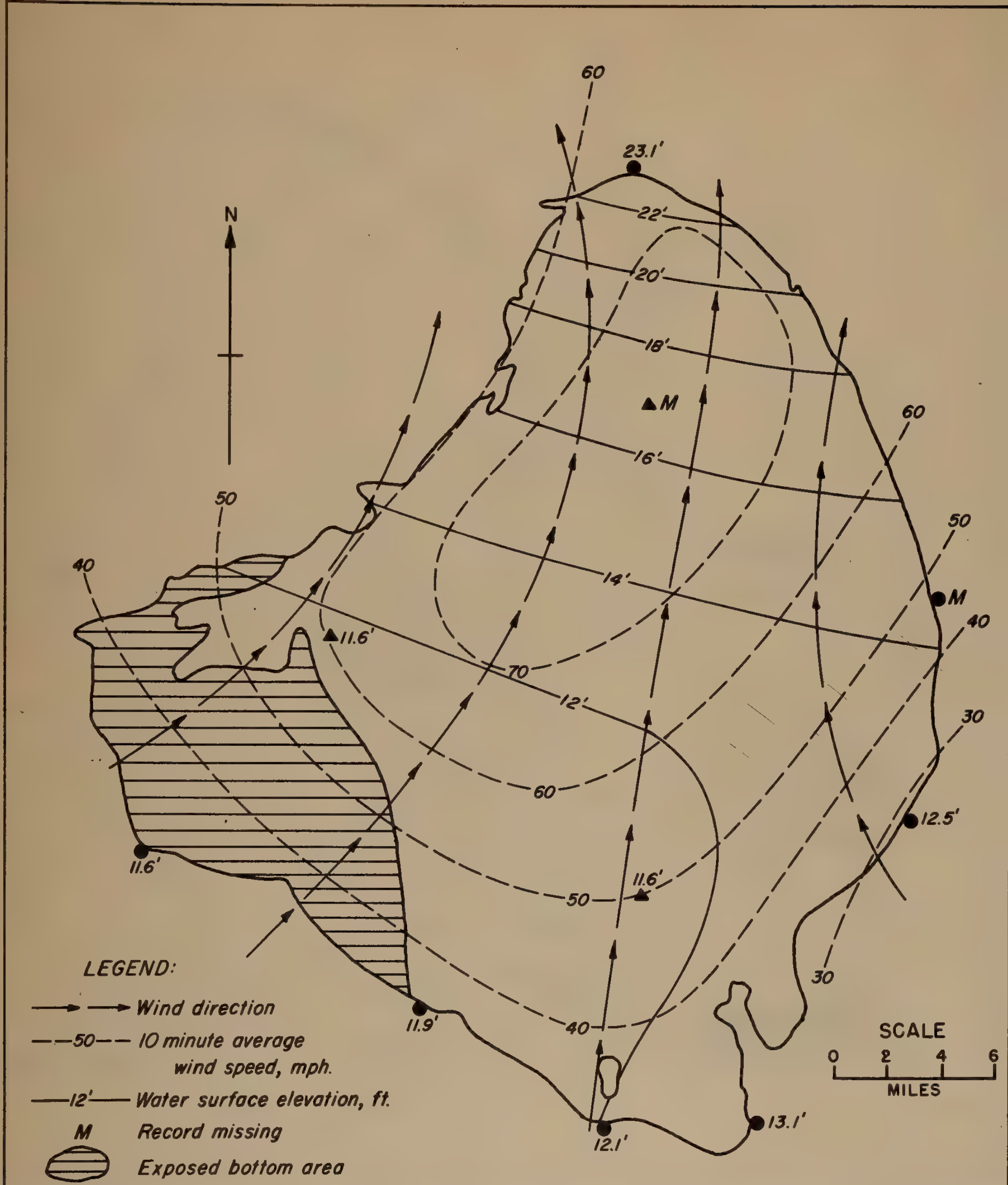

FIG. IK-12 - Wind and water surface conditions at Lake Okeechobee at 0100EST, 27 August 1949, during a hurricane.

FIGURE IK-13 - Wind tides on LaGuardia Airport, New York, under the influence of a 90 mph wind (Courtesy Time, Incorporated)

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION I - WAVES, TIDES AND BEACHES

L. TIDES

BY

W. N. BASCOM

Table of Contents

L. TIDES

1. Introduction:

The matter of our planet moves in response to the gravitational attraction of the other members of the solar system, principally the sun and moon. The atmosphere, the waters, and even the solid earth itself are each subject to constantly changing forces as the earth spins through space. Earth and atmospheric tides however, are scarcely measurable and have been so overshadowed by the importance of the tides of the seas as to have been practically forgotten. This discussion will follow that example.

On all seacoasts there is a rhythmic rise and fall of the water which is called the tide; associated with this vertical movement of the water surface are horizontal motions of the water called tidal currents. Together these are known as the tides. Little is known or cared about tidal changes in the deep water which makes up most of the Earth's surface. However, in shallow water near land where small differences in water depth are of importance to navigation, man's knowledge of the tides has reached a high state of development. Tides can now be accurately predicted long in advance for most of the navigable ports and entrances of the world. These predictions can be made because the water changes its position as the relationships in space between earth, sun, and moon change; relationships which are well known. A study of the tides therefore requires a knowledge of the motions of these three bodies in space which result in the tide-producing forces. A detailed study of these forces requires long and complicated mathematics which are beyond the scope of this writing, but an attempt will be made here to explain simply the major forces which produce tides. For the complete mathematical treatment the reader is referred to the bibliography, particularly the work of Schureman (Reference IL-11). The height of the tide is measured vertically from some fixed datum or arbitrary reference mark - usually the plane below which depths are given on charts of the area and usually related to mean sea level. Mean sea level is a plane about which the tide oscillates and is determined by averaging the tabulated hourly heights of the water over a period of several years. A rising tide is said to be flood tide; at high-water the height of the tide remains constant for a short period of time and the horizontal water motions are small; this is high slack. After the tide has passed its peak, the water recedes and the level lowers as the tide ebbs; at low tide there is a low slack. The height of the tide is the vertical distance between the water level and the datum plane. The range of tide is the difference in level between successive high and low water; it varies from tide to tide. High and low waters usually occur about twice a day, the intervals between the successive high water being about 12½ hours; such tides are semi-diurnal (half daily). If the two high or low waters are unequal in height, this is referred to as diurnal inequality. For several days during every fortnight the high waters are much higher and the low waters lower than usual; these are called spring tides and occur a few days after the moon is either full or new. Also for a few days in each fortnight when the moon is in the first and last quarters the high waters are lower and the low waters higher than usual; these are the neap tides. Since the times of high (or low) waters falls 50 minutes later every day, it follows that they may occur at any time of day, however, for any given place the highest spring tide is at approximately the same time of day. The intervals between springs and neaps is 14.3 days; this is called the fortnightly periodicity of the tides. Twice a year there are extra high spring tides at the times of the vernal and autumnal equinoxes called the equinoctial spring tides. All the regular, predictable changes in sea level are caused by the gravitational influences of the elements of the solar system; there are also

changes in sea level caused by atmospheric conditions which are called "meteorological tides" or "storm tides". Although often of considerable hazard to man and his works, these will not be considered here.

2. History of Tidal Knowledge:

The existence of the tides was probably known to the earliest men and it seems reasonable that coastal dwellers who undoubtedly got some of their food from the inter-tidal zone realized vaguely that a connection existed between the moon and the tides. The first written record of which we know, that connected the tides with the phases of the moon, was made by Pytheas of Masilla in 325 BC after he had ventured beyond the Gates of Hercules into the Atlantic and sailed to England where the tides are large. The civilizations whose histories we know best, the Greeks, Egyptians, Hebrews, lived on the shores of the Mediterranean where the tides are small; consequently there are few references to the tides in their writings.

In 77 A.D. Pliny's "Natural History" appeared in which the relation between the phases of the moon and the height of the tides was described and the influence of the sun was recognized. Virtually no changes in mans thinking about the tides occurred for the next 1500 years, although, Johan Kepler did suggest a magnetic attraction of some sort between the moon and the earths waters; Galileo called this preposterous.

In 1687, Isaac Newton published "Principia" and, for the first time, the connection between moon and tide received a rational explanation. Newton showed that the tides are a necessary consequence of the law of gravitation which states that two bodies attract each other with a force directly proportional to their masses and inversely proportional to the square of the distance between them. Newton described the principal action of the tides in his "Equilibrium Theory", which, although and over simplification, is the foundation on which all subsequent tidal work is based.

In 1774 Pierre S. Laplace tood up the work and in "Mechanique Celeste"; he formulated the equations of continuity and the dynamical equations; he also established the principle of forced oscillations which is basis of the harmonic methods.

The connection between the moon, sun and tides was so obvious that tidal predictions founded on empirical methods were regularly made and published long before mathematicians devoted their attention to the subject. Holden's Tide Tables for Liverpool, based on twenty years of observation by the harbormaster there was in use around 1800 and the first automatic tide gauge came into use about 1830. Since then, many well known physicists and mathematicians (including Airy, Lord Kelvin, and R. A. Harris) have worked on the problem of reducing the errors in prediction by adding new components to the calculation. Tides can now be predicted for most places to within a few hundredths of a foot and tide tables and tidal current tables are available for all of the world's ports.

3. Astronomical Background *

The study of the tides requires some knowledge of the motions of the sun and moon with respect to the earth for it is the movement and changes of these three bodies which creates the tides. The principal motions to be considered are the rotation of the earth on its axis, the revolution of the moon around the earth, and the revolution of the earth around the sun (or the apparent revolution of the sun around the earth).

The earth rotates on its axis once each day. There are, however, several kinds of days - the sidereal day, the solar day, and the lunar day - depending upon the object used as a reference for the rotation. Since the stars are the most nearly fixed objects we have for comparison, the sidereal day, which is the time between two successive passages of the same star across any given meridian of the earth, is usually considered as the true period of the earth's rotation. The solar and lunar days are the time between two successive transits of the sun and moon respectively over a given meridian; they vary a little in length because of the lack of uniform motion of the earth and moon in their orbits and so the average value of each is taken as the unit of measure. The mean solar day is the one which is used for the ordinary calendar day; each day is divided into 24 equal parts called hours. For convenience we describe the lunar day as being 24.84 solar hours.

The moon revolves around the earth with a eliptical orbit. Its changes in position with respect to the sun and earth are called phases; these are described and diagrammed later.

The period of revolution of the moon around the earth is called a month. The month is designated as sidereal, tropical, anomalistic, nodical or synodical according to whether the revolution is relative to a fixed star, the vernal equinox, the perigee, the ascending node, or the sun. The calendar month is a rough approximation of the synodical month.

It is customary to refer to the revolution of the earth around the sun, although, it may be more accurately stated that they both revolve around their center of gravity; but if one imagines the earth as fixed, the sun will describe an apparent path around the earth which is exactly the same size and form as that of the earth around the sun, and the effect upon the tides would be just the same. The period of the earth around the sun is one year and, of course, there are several kinds of years - however since, the declination of the sun, and consequently its seasons, depend upon its relation to the equinox, the tropical year with which the calendar must agree.

* Abstracted from Schureman (Reference IL-11)

1. Tide-Producing Forces *

As the sun and moon are similar in their action in the production of the tide, the force of either may be considered by itself, and the resulting forms of expression may then be readily adapted to the other.

The tide-producing force of the moon is that portion of its gravitational force which is effective in changing the water level on the earth's surface. This effective force is the difference between the attraction for the earth as a whole and the attraction for the different particles which constitute the yielding part of the earth's surface; or, if the entire earth were considered to be a plastic mass, the tide-producing force at any point within the mass would be the force that tended to change the position of a particle at that point relative to a particle at the center of the earth. That part of the earth's surface which is directly under the moon is nearer to that body than is the center of the earth and is therefore more strongly attracted since the force of gravity varies inversely as the square of the distance. For the same reason the center of the earth is more strongly attracted by the moon than is that part of the earth's surface which is turned away from the moon.

The purpose of the following discussion is to illustrate directions and relative magnitudes of the tide producing forces. Consider Figure IL-la, in which O represents the earth's center, C the moons center, and P and P^1 random points on the surface of the earth. The line P C is the direction of the attractive force of the moon on point P; if the length of P C is allowed represent the magnitude of the earths attraction at point P, it becomes a vector. Since the attraction of gravitation varies inversely as the square of the distance the length of the line P Q can be allowed to represent the magnitude of the moon's attraction for a particle at O if it is selected so that $PQ : PC = PC^2 : PQ^2$.

The direction of P Q is of course, parallel to the direction of the attraction of the moon for O. The line Q C is the resultant and, at the same scale as the first two vectors, represents the magnitude and direction of the differential force at P which tends to move P away from O.

A similar example using P^1 on the opposite side of the earth from the moon shows why the tidal forces raise the water on the side away from the moon. The vectors are laid out as before; the length of P^1 C represents the moon attraction for P^1 (on a different scale from the previous example) and if a vector $P^1 Q^1$ is laid off on a scale commensurate with that of P^1 C (ie. so that $P^1Q^1 : P^1C = P^1C : P^1Q^1$) parallel with the direction of the attraction for O it will represent the moons pull on O. Accordingly, the triangle of forces can be closed with the resultant Q^1 C which, on this new scale, shows the magnitude and direction of the force on P^1.

When laid off at P^1 it shows the tide producing force to be directed <u>away</u> from the moon on the far side of the earth. Although this may seem paradoxical at first, it will be noted that the moon tends to separate O from P^1 but as O is the point of reference the resulting force is considered as being applied at P^1 only.

The resultants which represent the total tide producing forces at the respective locations can be further resolved into vertical components which tend to raise the water and horizontal components which tend to move the water horizontally. Schureman has derived expressions for the value of the tide producing forces at any point in or on the earth (dependent on the cube of the moons parallax)

Vertical component $= \frac{u\,Mr}{d^3}(3\cos^2\theta - 1)$

Horizontal component $= 3/2\,\frac{u\,Mr}{d^3}\sin 2\theta$

Where: u = attraction of gravitation between unit masses at unit distance apart

M = mass of the moon

r = distance of the point from the center of the earth

d = distance from the center of the earth to the center of the moon

θ = angle at the center of the earth between a line to the point and a line to the center of the moon

Expressed in terms of gravity the forces become

Vertical component $5.6 \times 10^{-8}\,(3\cos^2\theta - 1)\,g$

Horizontal component $8.4 \times 10^{-8}\,(\sin 2\theta)\,g$

Figure IL-1b indicates the directions and relative magnitudes of the tide producing forces; it is evident from this pattern that the tides are bulges of the freely-moving ocean waters which form on either side of the earth along the line of the direction of the moon's attraction. The heaping up of water by these forces on an ideal earth uniformly covered with an ocean such as we have considered in this example would amount to a foot or two at most. In other words, the shape of the ocean basins and the configuration of the coast line account for the great tides that will be discussed a little later. Figure IL-4a illustrates these bulges (greatly exaggerated) for a time when the moon is not on the equator. Suppose that the earth rotates (very slowly and without friction) around its N-s axis. The tidal bulges will remain where they are and the earth slips around beneath them so that a wave will appear to travel around the earth with a period of half a day (because there are two bulges). An observer at point A would notice that the two tide waves that passed him in a day were of unequal height because the generating forces are not at right angles to the earths axis. This is the explanation of the diurnal inequality. For simplicity this discussion has been confined to the moons tide producing forces; the action of the sun is the same, only the magnitudes differ.

The values of the tide producing forces of the sun can be obtained by the use of the same expressions by simply substituting the values of the sun's mass and distance. Tide producing forces are very small in comparison with the total attraction because they represent differences in attraction of particles which are relatively near together. If the forces acting on each particle were equal and parallel, there would be no tendency to change their relative positions and no tide producing force. The sun forces are readily seen to be much closer to this situation than the moon forces. Although the attraction of the sun for the earth is 200 times that of the moon its tide producing force is less than half as great.

$$\frac{\text{mass of sun}}{\text{mass of moon}} \times \frac{(\text{mean distance to moon})^3}{(\text{mean distance to sun})^3} = 0.46$$

By similar reasoning it can be shown that none of the other heavenly bodies can produce significant tides on earth; the closest planets are too small, the large ones are too far away.

5. Variations in the Tide-Producing Forces

The sun, the earth, and the moon are continuously changing their relative positions and this of course changes the directions and magnitudes of the tide producing forces. There are three principal types of changes which are: alignment (phase), distance from the earth and angle with respect to the earths equinoctial plane (declination). A short discussion of each of these types should serve to explain the changes in the height of tides caused by the moons phase and by the season.

a. Phase:

It has been shown that the effect of either the sun's or moon's gravitational attraction is to raise tide bulges on either side of the earth along the line of attraction. It is evident then that when the sun, earth, and moon are aligned, the independant bulges of the sun and moon are superimposed. This condition is called syzygy and is illustrated in Figure IL-2a where the horizontal line represents the plane of the eliptic of the earth's orbit around the sun. Then, neglecting declination, the moon is shown in opposition and conjunction, the conditions that exist at the times of the full and new moon respectively. This situation causes spring tides. (see Figure IL-4c)

In Figure IL-2b the three bodies form nearly a right angle and the moon is said to be in quadrature, a condition that exists in the first and last quarters of the moon. It can be seen that the sun will tend to produce high tides at a place on the earth where the moon produces low tides and visa versa. Because the moon's tide producing power is so much greater than that of the sun its forces dominate and the high tides are due to the moon alone; these are naturally much lower than the combined tides and are called neaps. (see Figure IL-4b)

b. Distance:

The forces acting on the water particles are proportional to their distance from the mass which causes the attraction. Because the orbits of the moon about the earth and the earth about the sun are elipses, the distance between their centers is always changing and with it, the tide producing forces. At the point where the moon is nearest the earth it is said to be in perigee and the tide forces are a maximum; when it is furthest away, it is in apogee. Perigee or apogee may occur at any time relative to the phases of the moon or to the times of greatest or least declination of the moon.

The corresponding positions of the sun and earth are known as perhelion (sun nearest the earth) and aphilion (furthest away). The major axis of the elipse is a line that passes through the points of maximum and minimum distances and is called the line of apisides.

c. Declination:

The angle that the sun and moon make with the plane of the earths equator (equinoctial plane) is called the declination and it describes the alignment of the three bodies in a plane perpendicular to that considered before. At the time of the solstice, the sun reaches its maximum declination ($23\frac{1}{2}^{o}$) and the plane of the moon's orbit around the earth is inclined at an angle of 5^{o} to the ecliptic. At such a time, when the moon

is in syzygy, ordinary spring tides exist. When the earth is at the equinoxes, the declination of the sun is nearly zero (a condition that lasts several days). Also, for a day or two in every fortnight the declination of the moon is nearly zero. At such times when the moon is in syzygy, the combined gravitational fields most directly effect the earth's equator and high or equinoctial springtides occur.

d. Summary:

Motions Causing the Various Tidal Forces *

Tidal Period	Associated Astronomical Phenomena
Semi diurnal	Rotation of the earth
Diurnal	Rotation of the earth and declination of the sun and moon
Fortnightly	Revolution of the moon in its orbit; syzygy and quadrature; declination of the moon
Monthly	Revolution of the moon; apogee and perigee
Half yearly	Revolution of the earth in its orbit; suns varying declination
Yearly	Revolution of the earth; perihelion and aphelion
18.6 years	Rotation of the line of nodes
1600* years	Absolute maxima in the potential force
$4\frac{1}{2}$, 9, 19, 84 to 93 years	Subsidiary maxima

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* Johnstone (Reference (IL-7)

Most Important Components of the Tide-Producing Forces **

Partial tide	Symbol	Period in hours
Semi-diurnal		
Principal lunar	M_2	12.42
Principal solar	S_2	12.00
Larger lunar eliptic	N_2	12.66
Luni-solar	K_2	11.97
Diurnal		
Luni solar	K_1	23.93
Principal lunar	O_1	25.82
Principal solar	P_1	24.07
Long period		
Lunar fortnightly	M_f	327.86
Lunar monthly	M_m	661.30
Solar semi-annual	S_{sa}	2191.43

** Sverdrupetal (Reference IL-12)

6. Types of Tides:

Tides vary considerably from place to place although the previous discussion showed that there are rather small differences between the tide producing forces.

It might be expected that the semi-diurnal forces would be greatest at the equator and least at the poles and the diurnal forces would become stronger toward the poles. However, the shapes of the ocean basin influence the motion of the tidal waters so greatly that the tides differ markedly with little regard for latitude. For example, off the Northeast coast of the U. S., within the space of a few hundred miles are places on the Bay of Fundy with tides of forty feet and Nantucket Island with tides of about a foot. Within Chesapeake Bay, the tide at different places varies in time by as much as 12 hours.

Tides may be divided into three general classes on the basis of the appearance of their daily tide curves. See Figure IL-6.

(i.) Semi diurnal: Two tides a day of about equal heights. This type prevails in the Atlantic Ocean and on the East coast of the United States.

(ii.) Diurnal: Only one tide per tidal day of 24 hours and 50 minutes. Such tides are uncommon but exist in certain regions of the Gulf of Mexico, Alaska and the Phillipine Islands.

(iii.) Mixed: Two high and two low waters each day of differing heights. Most of the Indian and Pacific Oceans have mixed tides. The inequality may take several forms: (a) the high waters may be unequal in height (Honolulu); (b) the low waters may be unequal in height (See Figure IL-5 of Seattle); (c) the inequality may be about evenly divided between the high and low water (San Francisco).

Because the diurnal inequality varies principally in response to the varying declination of the moon, when the moon is on the equator the mixed tides are about equal like those of the semi-diurnal type. When the declination of the moon is large the inequality may become so large that a high tide is actually lower than the preceeding low and the tide curve appears to be diurnal.

7. Tidal Currents:

Accompanying the rise and fall of the tides are important horizontal motions of the water known as tidal currents. Like the vertical changes, they have very little significance in the open sea or offshore; in narrow waters, straights, harbors and estuaries, however, they are of considerable practical importance. On the rising tide, tidal currents are said to be flooding; on the falling tide, they ebb; maximum flow in either direction is called strength and when there is no flow, the current is slack. The direction of flow is called the set of the current. "The relation of current to tide is not constant, but varies from place to place and the time of slack water does not generally coincide with the time of high or low water nor does the maximum velocity of the current coincide with the time of the most rapid change in the vertical height of the tide. At stations located on a tidal river or bay, the time of slack water may differ from one to three hours from the time of high or low water" (Quoted from Reference IL-15). Tidal currents vary upwards to as much as 12 knots (Seymour Narrows) and in much-used shipping channels like that at the Golden Gate (San Francisco), a four knot current is not unusual. (See Figure IL-8) These currents which are intermittent and vary greatly in direction and velocity are obviously a considerable hazard to vessels and cause many harbor engineering problems. In a way though, they compensate for these difficulties by maintaining the channel at navigational depth; this is discussed in some detail in the paragraph on tidal prism.

Tidal currents are caused by the same periodic forces that produce the vertical rise and fall of the tide. It is therefore possible to represent these currents by harmonic expressions similar to those used for the tide and with slight modification, the tide predicting machine can be used to predict the tidal currents. (Amplitudes are expressed in knots and ebb and flood directions show as negative and positive numbers.) There are two general types of tidal currents known as the reversing type and the rotary type. The reversing type of current is that which is of greatest interest since it is found in estuaries, tidal bays, and harbor areas; it flows alternately in opposite directions. The rotary type is found offshore and the velocities are likely to be small; it changes its direction continually, and during a tidal cycle will set in all directions. Currents may be likened to the type of tide with which they occur; that is, diurnal currents can be expected to accompany diurnal tides, etc. In addition, if there is much diurnal inequality in the heights of the tide, there will be about the same inequality in the velocity of the currents.

There are other currents in the ocean which must be distinguished from the tidal currents; currents off the mouths of rivers, oceanic currents like the Gulf Stream, and wind currents may occur with the tidal currents. The actual current acting at any point is the resultant of the combination of these forces. On open coasts and beaches and in wide mouthed embayments, tidal currents are usually of negligible magnitude.

Reversing currents or "ordinary" tidal currents are characteristic of narrow channels through which the rising and falling tidal water must pass. A tidal wave moving up an estuary or into a bay has a relatively high velocity at the restricted entrance, particularly in the center of the channel where the water is deep. The discussion of Tides is Estuaries describes the importance to commerce of bodies of water in which tidal currents play an important part; even the great "Queen Mary" waits for slack tide to come to her pier in New York.

Rotary tidal currents are found offshore, away from the immediate influence of e coast; instead of setting in one direction for six hours and the opposite rection for the following six hours, they change their direction continually so at in 12.4 hours they set in all directions of the compass. Figure IL-9 shows .ots of two rotary currents made from data taken in lightships. Each of the rays is vector representing the current direction and velocity at hourly intervals through-it the day; the points of the rays have been connected with a smooth curve to show ie rotary nature of the change. The figures indicate the times relative to the tage of the tide, i. e., LL + 2^h indicates the current at two hours after lower low ater. A characteristic feature of rotary currents is the absence of slack water hich would be indicated by the curve touching the center. The examples given in igure IL-9 are of days when there was considerable diurnal inequality; when the ides are equatorial (moon declination = zero) the curves of the two tide changes ill be about the same.

8. Tidal Theories:

a. Equilibrium Theory

The Equilibrium Theory of tides was first developed by Newton and deals with the situation on an ideal earth which has no continental barriers and is uniformly covered with water of considerable depth. It further assumes that these waters respond instantly to the tide producing forces of the sun and moon and move around the earth without viscosity or friction. Of course, the movement of the tidal bulges cannot take place without the water masses changing position but the water motion is completely disregarded. The actual conditions that exist on the earth differ greatly from the ideal conditions assumed for the equilibrium theory, that the tides predicted by its use can be expected to differ from observed tides. Schureman (Reference IL-11) has noted some of the agreements and difference between the predicted and observed tides:

(i.) "Generally two high waters and two low waters occur during each day, but the high waters do not necessarily occur when the moon and sun are on the meridian. The interval between a transit of the moon and the occurrence of a high water varies in different parts of the earth without any apparent regard for the equilibrium theory, and high water may occur at any hour between successive transits of the moon; but for any particular place the interval between the time of transit and the time of high water remains approximately constant."

(ii.) "Usually the alternate high waters or the alternate low waters are nearly equal in height when the moon is near the Equator and have an increasing diurnal inequality as the moon's declination increases north or south of the Equator. According to the equilibrium theory there should be a diurnal inequality in the high waters only, and with any given declination this inequality should depend upon the latitude. As an actual fact we find that at many places there is much larger inequality in the low water heights than in the high water heights, and that the magnitude of the inequality apparently has no direct relation to the latitude of the place."

(iii.) "By the equilibrium theory the diurnal tides would be expected only in latitudes near the poles, but observations show that stations near the Equator as well as those near the poles have diurnal tides.

Although it is recognized that any calculations of the tide based solely upon this theory may give results entirely at variance with the real tide, because the actual conditions on the earth differ so much from the assumed ideal conditions, yet such a formula is very useful, inasmuch as we may introduce into it certain factors and differences determined from actual observations of the tide at any place and obtain a corrected formula which will generally represent very satisfactorily the true height of the tide at that place for any desired time."

b. The Dynamic Theory:

The following is quoted from Sverdrup, et. al. (Reference IL-12)

"The dynamic theory is based on the fact that only the horizontal tide-producing forces are of importance to the movement of the water. The vertical tide-producing forces are unimportant because they can be considered as consisting of small periodical variations in the acceleration of gravity. Where the depth of water is about 5,000 meters the variations in gravity due to the tide-producing forces cause a maximum variation of only 0.85 cm; this is far too small to be considered. The problem then consists in determining what types of motions in the oceans arise under the influence of the periodically varying horizontal forces that are distributed over the ocean in a given manner. From this point of view the tides must be considered as waves that are induced in rhythmical forces and therefore have the same periods as the forces."

"The general equations of the dynamic theory, which were developed by Lagrange, lead to problems mathematically so difficult that they have not yet been solved so far as the tides of the oceans are concerned. Instead, the application of the dynamic concept has followed two different lines. In the first place, the theory of tides in basins of defined geometrical shape has been developed, particularly by Proudman and Doodson; in the second place, tides in natural basins have been studies mainly by Sterneck and Defant by methods of numerical integration of the hydrodynamic equations. Both methods have helped toward an understanding of the observed phenomena, but here only the latter approach will be dealt with, because it leads to direct comparisons between theoretical results and observed conditions, and because it does not require any lengthly mathematical presentation."

"Tides in relatively small bodies of water that are in communication with the open sea have much in common with seiches, and can therefore be discussed in much the same manner. The tides, however, differ from the seiches in the respect that they are forced oscillations of periods which must coincide with the periods of the impulses by which they are maintained, whereas the periods of the free oscillations depend only upon the geometrical shape of the bay."

"The similarities and differences between tides and seiches are brought out by considering the oscillations in a long, rectangular bay of constant depth. Any wave in such a bay must fulfill the equations of motion and continuity.

These equations must always hold, and in addition, certain boundary conditions must be satisfied. These conditions depend upon whether one considers a free oscillation of the bay, or the co-oscillation in which the bay oscillates with the same period as that of the body of water with which it communicates. These equations show that if the ratio between the periods of the free and tidal oscillations, has the values 1, 3, 5, ___, the amplitudes of the co-oscillating tide become infinite and resonance occurs."

"For the independent tide which is produced directly by the tidal forces, it is necessary to add a periodic force to the equation of motion; again resonance occurs at the same values. In all cases the character

of the forced oscillation depends on the relation of the period of the forced oscillation to that of the free."

"The preceding remarks apply to the simplest case, a rectangular bay of uniform depth whose long axis is parrallel to the tide-producing forces. The difficulty of applying the theory to basins of irregular shape, orientation and depth is readily visualized. Moreover, several other major complications must be considered; principally the effect of friction and the rotation of the earth. The effect of friction may be so great that the amplitude at the closed end is zero and the co-oscillating tide has the character of a progressive wave subject to extreme damping. The effect of the rotation of the earth (which is even more serious than the friction effect) let Lord Kelvin to define a wave, the Kelvin wave, which proceeds along a channel in such a manner that the amplitudes on the right hand side are much greater than those on the left (in the Northern Hemisphere). At high water when the current flows in the direction of progress, the wave crest slopes down from right to left, and the component of gravity acting down slope is exactly balanced by the deflecting force of the earth's rotation acting in the opposite direction. On the ebb the directions of slope and current are reversed and the forces again balance each other.

These considerations have been applied to the Atlantic and to "adjacent seas" such as the Adriatic Sea, the Red Sea, and the North Sea; in the other oceans, applications of the dynamic theories presents enormous difficulties and has not been attempted."

See the section on tides in the open ocean for the "progressive wave" theory and a discussion of cotidal diagrams.

9. Harmonic Analysis: *

Harmonic analysis as applied to the tides is a process by which the actual observed tide at any place is separated into a number of partial or constituent tides of which it is composed, the rise and fall of each partial tide being a simple harmonic function of time.

Harmonic prediction of the tides consists in reuniting the partial tides in accordance with the relations which will prevail at the time for which the predictions are to be made.

The partial tides are called components and are usually represented by letters either with or without subscripts, as M_2 K_1 Mm, and Sa. Theoretically the tides consist of innumerable components of various magnitudes, but only a comparatively few are of sufficient size to be of practical importance in the prediction of the tides. The prediction machine used by the Coast and Geodetic Survey is designed to take account of a maximum of 37 components.

Each component represents an elementary periodic cause producing or affecting the tide. The principal component, designated as M_2, represents the mean effect of the moon. Another component S_2, represents the mean effect of the sun. Other components take account of the various inequalities in the motions of the moon and the sun, such as changes in parallax and declination, and also inequalities resulting from shallow water and seasonal meteorological changes.

A simple harmonic function is a quantity that varies as the cosine of an angle that increases uniformily with time. In the equation $y = A \cos at$, y is an harmonic function of the angle at, in which a is a constant and t represents time as measured from any initial epoch.

The harmonic constants are the numerical values of the amplitudes and epochs of the components for any place. The determination of these constants from the records of tidal observations is the purpose of the harmonic analysis.

The rise and fall of the tide may be graphically represented by a curve, with the ordinates representing the height of the tide and the abscissas the time. The tidal record as traced by an automatic tide gauge is such a curve. The general equation of this curve, giving the height of the tide as a function of time, is usually written in the form

$$y = H_0 + A \cos (at + \alpha) + B \cos (bt + \beta) + C \cos (ct + \gamma)$$

in which y is the height of the tide at any time t, H_0 is a constant depending upon the datum from which the heights are reckoned, and each cosine term represents the height of a component tide.

* Abstracted from Schureman (Reference IL-11)

10. Tide Predicting Machines:

The invention of the tide predicting machine has made simple the solution of two basic formulas which were formerly worked out laboriously. The first tide-predicting machine was designed by Sir William Thomson (afterwards Lord Kelvin) and was made in 1873 under the auspices of the British Association for the Advancement of Science. This was an integrating machine designed to compute the height of the tide. It provided for the summation of ten of the principal components, and the resulting predicted heights were registered by a curve automatically traced by the machine. This machine is described in Part 1 of Thomson and Tait's Natural Philosophy, edition of 1879. Several other tide-predicting machines designed upon the same general principles, but providing for an increased number of components were afterwards constructed.

The first tide-predicting machine used in the United States was designed by William Ferrel, of the U. S. Coast and Geodetic Survey. This machine, which was completed in 1882, was based upon modified formulas and differed somewhat in design from any other machine that has ever been constructed. No curve was traced, but both the times and heights of the high and low waters were indicated directly by scales on the machine. The intermediate heights of the tide could be obtained only indirectly.

Another machine designed by the U. S. Coast and Geodetic Survey is known as the tide-predicting machine No. 2. This machine not only sums simultaneously the terms of the tide formulas but register the successive heights of the tide graphically and indicates the time of high and low water. It has been continuously used since 1912 to make prediction for the U. S. Tide Tables and takes into account 32 short period and five long period components.

11. Datum Planes:

A datum is a plane from which heights and depths are reckoned; it has zero elevation and zero depth. Every nautical chart has its datum clearly marked and tides are calculated and shown in the tide tables as being heights above that plane. (The datum is marked at the bottom of every page in the USC and GS Tide Tables) Because tide types and local customs vary from place to place there are a number of datum planes in use. Since the purpose of charts and of tide tables is to assist the navigation of ships it is reasonable that the principal datum planes in use are some average of the low water elevations; as a result, the depth of water is nearly always greater than that shown on the chart (except during minus tides). For example, Mean Lower Low Water, the datum which is used extensively on the west coast of North America, is the average of the lower of the two low waters of the day (Figure IL-75). Other similar reference planes are Mean Low Water, Mean Low Indian Springs, Mean Low Water Springs, Tropic Lower Low Water; these are generally abbreviated by the use of their capital initials.

Sea level is the height that the surface of the sea would assume if undisturbed by the rise and fall of the tides or the influences of waves and weather. Because these disturbances do exist, the concept of sea level applies only to an instant and in order to have a satisfactory reference plane, the technique of averaging all the sea levels has been adopted. The result is mean sea level. Usually it is calculated on a long term basis (19 years) in order to include changes in the general level caused by the rotation of the moon's mode. Mean Sea Level is generally used in surveys ashore and the low water datums are used for nautical charts; this means that the relation between the two must be known. At Santa Barbara, California for example, MSL is 2.56 feet above MLLW. In England the original tide datum was the level of the Old Dock sill at Liverpool; the sill is long since gone but the Ordinance datum used throughout Britain is taken to be 4.67 feet above this old mark. In the U. S., the Coast and Geodetic Survey maintains an overland network of level lines which continually checks on the relationships between sea and land at the various tidal stations. In this manner, it is possible to tell whether the land is sinking or rising with respect to the sea.

12. Special Situations:

a. Tides in the Open Ocean:

There is no direct evidence as to the action of the tide away from land because there are no definite points from which measurements can be referenced. The real data that exists are the heights and times of tides along the ocean shores and on an occasional island. Since it is evident that the shoal waters near shore influence the moving water considerably, the factual foundation is a bit shaky. Since these data can be combined in more than one way there are several theories, each of which try to give a consistent interpretation. Most of the work has been done on the semidiurnal tides of the Atlantic Ocean by Whewell, Airy, Sterneck and Defant who have attacked the problem mathematically. Defant's solution indicates that the semidiurnal tide of the Atlantic is composed of two standing oscillations having a phase difference of three lunar hours (1/4 period). The observed values which he used were obtained at Tristan da Cunha and the Azores and the theory has been checked by comparison with the tides at other islands and with the results of the current studies made by the Meteor expedition. No comparable work has been done in the Pacific where the free tides are probably of much greater importance but there are more complexities.

The most favored method of indicating tides in the open ocean is a map of cotidal lines. Cotidal lines join points having high water at the same time, referred to Greenwich or some other standard meridian. Figure 10 is a cotidal map of the Atlantic for a partial (semi-diurnal) tide. Notice that just south of Greenland there is an amphidromic point which is a theoretical point where the tide vanishes (location not certain). It may be caused by the effect of the earth's rotation or by the interference of two tidal waves. Cotidal diagrams of the total tide appear in certain Hydrographic Office publications but are principally intended to indicate the times of tide along the oceanic borders. In the example shown of the cotidal lines in the Atlantic, it can be seen that the time of high tide moves from south to north. The observation of this phenomena led to the "progressive wave" or "southern ocean theory" which, although no longer in favor, contained the beginnings of the Dynamic Theory. According to Johnstone IL-7 it goes like this.

The Antarctic Ocean is a belt of water which extends completely around the earth 600 miles wide at its narrowest point. This seems to be sufficient water for a tidal wave and was seized upon as a practical solution to the limitations of the equilibrium theory. A tidal wave sweeps around the "Southern Ocean" as a forced oscillation kept going by the tide-producing forces; its main constituents have periods of 12.4 and 24.8 hours. The Atlantic, Pacific, and Indian Oceans may be regarded as gulfs opening out from the Southern Ocean; at the equator in the Atlantic there is an obstruction (the configuration of South American and Africa) which does not exist in the other two oceans. The forced wave, then, which sweeps around the earth in the Southern Ocean initiates free waves as it passes the mouths of the oceanic gulfs. These waves travel up the Pacific and Indian Oceans; in the Atlantic they travel as far as the obstruction

and then initiate a new free wave in the North Atlantic. These waves are not quite "free" because of the tide waves already in these oceans; they do, however, give periodic impetus to the waves already there and modify them considerably. The cotidal diagram, all types of which agree to some extent, is regarded as a chart of the wave advance at the times indicated. Discrepancies are plausibly explained by attributing them to land outlines or bottom complexities--an argument that is hard to refute.

Harris Marmer has mapped open ocean tides as "oceanic oscillatory systems" and developed the "stationary wave" theory. It states that the tide producing forces of the sun and moon sweep over the oceans and maintain a stationary wave oscillation in the open ocean systems, the rise and fall being greatest at the ends and least at the nodal lines. These oscillations give rise to the dominant semi-diurnal tides of the world; openings in the coast line and irregularities in the depth cause the stationary waves generated in these ocean basins to send off progressive waves into coastal tidal waters or other parts of the sea. Although there has been considerable criticism of this theory it explains very nicely several puzzling tidal features which are not well understood (such as the large range of tide on the Atlantic Coast compared to that at the Bahamas and Puerto Rico). The Dynamic Theory is generally accepted today, however.

b. <u>Tides in Lakes and Seas:</u>

Even the largest lakes are much too small to have appreciable tides; in Lake Superior for example, the tide range is only about 5 cm (two inches). Seiching, or the free oscillation of standing waves in a partly enclosed body of water is fairly common. If the dimensions of the lake are such that its natural period is in resonance with the tidal period, the measured range of the tide may be much larger than that predicted from astronomical data. This is particularly true if the lake is long in the east-west dimension. A comparative water motion may be observed of one tries to carry water in a shallow tray. According to Kuenen (Reference II-8), forced oscillations of seiches by the tidal forces cause "tides" of 8 cm in Lake Erie as compared with larger Lake Baikal where the tides are only 1.5 cm. This is because the natural period of Erie is 14 hours (not far from the tidal period) and that of Baikal is 4.6 hours. In the Baltic Sea the tidal range is about 2 cm, in the Black Sea, 8 cm (a circulating tidal wave) and in the Mediterranean the maximum is about 30 cm (one foot).

Fairly large seas which have an appreciable opening to the ocean often have complex tides. In the Red Sea a tidal wave inters from the Indian Ocean, passes the full length, and is reflected from the other end. The tides at the ends of the sea have a range of about two feet but Port Sudan in the center is at a nodal point and there the water level remains constant. Accurate and detailed studies of the tides in the former Zuider Zee by Lorenz showed that there the Coriolis force alone raised the water 20-30 cm.

c. Tides in the Bay of Fundy: *

The Bay of Fundy lies between Nova Scotia and New Brunswick in southeast Canada; its tides are legendary and their astonishing dimensions are worth reviewing. The bay is some 80 miles wide at the mouth and is about 150 miles long, tapering considerably in the upper reaches. Into its area of about 6000 square miles the average flood tide (22 feet) brings 3½ trillion cubic feet of water.

The range of tide increases toward the head of the bay varying from 10 feet at Bass Harbor, Maine to 40 feet at Folly Point on the north west shore; while on the south east shore the range is 14.0 feet at Yarmouth and 44 feet at Noel Bay. The greater heights on the south or right hand (with respect to the flood) shore are accounted for by the fact that moving bodies in the Northern hemisphere are deflected to the right. The ebb tide is deflected to the west (its right hand shore), and consequently the low tide on that shore is relatively high. This phenomena sometimes called the "Kelvin Wave". On spring tides the rise of tide is about 14 per cent greater than the average figures just given.

The tremendous amount of water flowing in and out every 12 hours suggests that the currents at the mouth are very swift; however, calculations checked by measurements show that the velocity of the tidal current in the center of the entrance is only 1½ knots.

At Noel Bay in the northernmost part of Fundy, the rate of rise of the tide was measured and found to reach a maximum of 11.2 feet per hour where the total range was 49.7 feet. The reason that tides are so large in the Bay of Fundy is that it has a natural period of oscillation of about 11½ hours which is very near that of the tidal period. The tide wave arrives just in time to reinforce the natural oscillation. In addition, there is some effect of construction and of shoaling which gives the very high tides at the upper end.

d. Tides in Estuaries and Rivers:

Tidal estuaries are arms of the sea which usually consist of a river which opens near its mouth to an embayment that is partly separated from the sea by a rocky barrier or sand spit. Its important characteristic is that the tidal waters of the sea move in and out of the enclosed waters through a relatively narrow tidal entrance. Many of the world's great ports (New York, Philadelphia, San Francisco, Liverpool, Antwerp, Bordeaux) are located in estuaries and as a result, the times and heights of the tide and the direction and velocities of the tidal currents are generally better known there than on the open coast. London, for example, is 50 miles from the mouth of the Thames; Hamburg is 75 miles up the Elbe. Usually tide gauges are located in estuaries where there is natural damping of the water surface fluctuations; these give a continual record of the actual water level which may vary from the predicted level because of unusual fresh-water or other reasons. Tide tables may give the time and

*Abstracted from Marmer (Reference IL-10)

height of tides at many locations inside large bays (like that at San Francisco) and current tables sometimes have elaborate instructions for the timing of ship passage to take advantage of the optimum current.

There are several features of tidal flow in estuaries which are worth considering briefly. The tide wave must move in and out through a narrow opening; this requires time and so there is a time difference between the arrival of the tide crest at the entrance and at points upstream. The further inside the greater the lag; on the ebbing tide, the same is true; the tide falls near the entrance first so that low tide is reached there while the tide is still fairly high upstream. This may result in the tide flooding at the entrance at the same time that it is ebbing upstream (depending on the length of the estuary and the duration of slack water). Because of the damping effect of the entrance, the tide range is less inside and in long estuaries eventually goes to zero.

(1) Tidal Prism:

The volume of water enclosed by the planes which are the average of higher high waters and the average of lower low waters is called the tidal prism. During the average tidal cycle (12.4 hrs.) this water passes out through the tidal entrance and returns; in so doing it maintains a channel whose smallest vertical area is proportional to the volume of the tidal prism. As shown in Figure IL-11, this ratio has been established by O'Brien for the principal harbors of the U. S. Pacific Coast. It is nearly hopeless to try to maintain an entrance section greater or smaller than that which will be maintained by the flow of tidal waters. This is often tried however, and public clamor for a boat harbor has led to many ill-advised channel dredging projects; because the volume of the tidal prism is too small the dredged entrance rapidly silts up. Since the depth of the channel is the principal concern of the vessels using the entrance, it is often necessary to construct jetties which maintain the channel within certain horizontal limits (causing the flowing water to deepen rater than widen the channel).

In long river channels where the bottom slopes slightly upward from the sea and there is some fresh water flow, there is a difference in the duration of the rise and fall of the tide. The period of rise is shortened and that of the fall, lengthened. Marmer has shown such a tide curve for the Hudson River at Albany, New York where the difference is markedly greater when the fresh water flow increases in the spring.

(2) Bores:

In some rivers or narrow estuaries the change of the tide from ebb to flood is quite spectacular. Instead of the calm change in direction of low velocity currents such as occurs on most of the U. S. coast, the flooding tide arrives as a wall of water many feet high. This wall of water, which is much like a translatory wave in the surf zone, is called a tidal bore and is often announced by a hissing rushing sound something like that

of a distant train or of breakers on a beach.

There are two principal requirements for the existence of a bore: (i) there must be a large range of tide; (ii) there must be a considerable area of shoal water near the estuary mouth which holds the flooding water back for a while at the turn of the tide. Bores usually arrive about midway between low and high water (when the rise of tide is taking place at its maximum rate-about three hours before high water) and move with speeds up to 13 knots. According to Carson (Reference IL-1) the bore in the Amazon travels as much as 200 miles upstream and as many as five bores may be present in the river at one time. Near the entrance, the Amazon bore has been observed as much as 5 meters high. In the Tsientan Kiang which empties into the South China Sea, bores ten feet high are seen daily and during spring tides heights of 25 feet are said to occur.

The importance of the shape of the entrance shoal is indicated by the fact that the French rivers Siene and Gironde both had impressive bores until the intrances were dredged. The Tsientan bore apparently has varied considerably in the last few hundred years as the shoals have shifted.

The effect of bottom friction and river current is to increase the period of falling tide and decrease that of rising tide making the tide curve quite assymetrical. Tide records of bores show an instantaneous increase in water level followed by a very rapid smooth rise of the tide.

(3) Tides at Inchon, Korea:

Although Inchon (or Jinsen Ko as the Japanese called it) is one of the regular tidal reference stations for the Yellow Sea, certain difficulties were experienced in obtaining tidal information for the planning of the amphibious operation there. As is now well known, a U. S. amphibious fleet ventured up the narrow and crooked estuary and landed against Wolmi Island and Inchon Harbor. The tides in the area have a spring range of over 30 feet and the only suitable landing zones could be reached by boat or ship at the highest tide. It was therefore necessary to know the height and time of the high flood tide very accurately. Three tide tables (U. S., Japanese, and Korean) existed for the port and each was different in both height and time; little information could be found about the details of how the tidal currents behaved although the U. S. had used the port until the previous summer. The questioning of refugees who knew the harbor well disclosed the fact that occasionally offshore winds of about 30 knots blew and reduced the height of the water over two feet below the predicted level. This made the total possible error as much as three and half feet - - - enough to seriously impair the operation if the luck was bad on both counts. As it turned out all went well, but the intelligence offices had some bad hours; landings were scheduled for a little ahead of the flood so that if the times were wrong the craft would

eventually have water enough to beach and unload. On the ebb the water disappeared so rapidly that many vessels were trapped in embarrassing positions; LSTs were left high and dry on wharves, and and LCM was balanced on the side of an overturned merchant ship 20 feet above the mud flats; and one ship actually was caught grounded when the high spring tide receded and was held in the mud for ten days until the next spring tides floated it off again. The worst hazard was the lack of fire-fighting water on the landing ships which were high and dry during the low tide stages.

e. Tides on Beaches:

Although the tide tables give the times and heights of tides at various points on the open coast (compared to the main reference stations which are inside estuaries) There is some evidence that on ocean beaches the actual average water level deviates from the predicted value. (Tide tables are intended for the use of mariners whose object is to avoid shoals and beaches.) These differences in water level are not easy to see or measure because the water surface is roughened by wave action; however, instruments which measure average water level or craft which alternately float and touch bottom, prove they exist. There are several types of causes of water level change which may or may not be truly tidal - - their effect is the same. The most obvious disturbing force is that of the wind which is particularly effective if the nearshore water is shallow. It actually drags the water along by friction tending to raise the water level if the wind is onshore, decrease it if offshore (see discussion of wind tides). The other causes are more difficult to assign. (i) The variability of the surf which often consists of a series of maximum waves arriving on the beach with low waves inbetween, tends to raise and lower the water level at regular intervals of perhaps 2 - 4 minutes. (ii) There is some seiching even in fairly open water. Since the periods of seiches depends on the length and depth of the specific body of water, it is sometimes difficult to determine exactly what part of the ocean basin is causing them. Although the amplitude of these waves is generally small in moderately deep water, the horizontal motion may be rather large and in very shallow water on a sloping beach the vertical changes may be appreciable. Marmer (IL-10) has measured seiches of a foot or more and periods of 15 minutes at Atlantic City, New Jersey. (This may be the oscillation of the basin between Cape Hatteras and Nantucket brought about by wind and barometric pressure variations.) The author has often observed "surging" or seiching in the harbor at Monterey which has a period of about three minutes; on a flat beach inside the harbor the vertical change in water level seems to be about five to ten centimeters. (Presumably this is the natural period of Monterey Bay and is caused by forces other than wind.) (iii) Experiments at Carmel Beach in the summer of 1946 by the Waves Investigation Group of the University of California, disclosed that the elevation of the water level in a pair of open but heavily damped tubes in the surf zone (compared to USC & GS bench marks) was nearly always 0.5 ft. higher than the predicted tide level and on two occasions was 1.5 ft. above the predicted level. A time lag or lead was also apparent in these studies.

It can be seen that the water level on the beach is not a satisfactory reference elevation, nor can it be regarded as being the same height as the average water level in deeper water. Repeated attempts to match land surveys with hydrographic surveys have shown this to be so. It is evident that an unfortunate combination of these long-period waves and variations in the height of sea level may cause an error in depths of water inshore which are obtained by measuring below the water surface. In order to make accurate surveys to determine the elevation of subsurface features near a beach, it is necessary to carry the elevation at sea by leveling from land

at frequent intervals.

The amphibious implications are fairly obvious. Depths determined by aerial photographic methods (especially the waterline method or the wave methods in heavy surf) should be adjusted to take into account possible variations in the datum plane which can only be determined from the tide tables. The seiching or surging which causes a variation in the water level is often helpful to the smaller landing craft which can be "bumped" in and out across longshore bars on the crests of surges even after the tide has fallen too low for them to (theoretically) make the transit.

(1) Tides at Tarawa: *

"The question of tides and absence of sufficient water over the reef to float landing craft at Tarawa will probably never be resolved satisfactorily.---Tarawa was attacked in the neap tide period, and (since the assault for logistical reasons and because of tactical commitments in the South Pacific could not be delivered earlier than it was) the question arises: why not wait seven days and land with a spring tide? The answer is that --- during late November 1943, the spring tide was coming either in darkness or relatively late in the day. Tactically, a landing at either of these times was unacceptable.---While tidal and reef data for Tarawa was the best that could be complied, it was full of inaccuracies.---The calculation of water over the reef turned out to be faulty, and according to the estimate of eye-witness, low water increased casualties fifty percent.---On the basis of the best calculation possible, the most favorably timed neap tide for late November would begin to flood rapidly at roughly half-past eight o'clock on the morning of November 20. The entire operation was planned around that moment. Most of the British acquainted with the Gilberts promised five feet of water over the reef by the time the non-amphibian craft were wanted on the beach. Since four feet would be adequate to float the landing craft, then there would be (if the data and the over-whelming majority of the people experienced in navigating the waters of the Gilberts were right) twelve extra inches of depth over the reef, and the assault would carry through with sustained momentum."

"One British officer who stood out against his compatriots, however, happened to be right. He was Major F.L.G. Holland of the New Zealand Army who had lived on Tarawa for fifteen years. He predicted something he called a "dodging tide" for around November 20. The United States Coast and Geodetic Survey, however, is unfamiliar with the term "dodging tide." Holland had been around Tarawa long enough to have sensed, at least, that the tidal and reef data being used was unreliable; but he had at his disposal none that was better. Him ominous prediction for November 20 created grave doubts in the minds of Julian Smith and others, and caused them to drill the marines in what to do in case of low water. As far as men can ever be mentally prepared, they were

*Quoted from Isely and Crowl (Reference IL-6)

prepared for the contingency that the boats would ground at the edge of the reef and some of them would have to wade in. But there was no other choice than to plan the time of the landing on the most highly scientific basis possible, and to hope that it was correct. Had the accurate hydrographic data complied after the capture of Tarawa been available, it would have been obvious that there would not be enough water over the reef at high neap tide to allow non-amphibian landing craft to reach the beach."

(2) Tide Predicting Fish:

Numerous marine animals have adapted themselves to take advantage of the tides. Among these are the Grunion which spawn on sandy southern California beaches between March and August. Somehow these fish are able to predict the highest tides of the month and time their arrival on the beach so that just after the turn of the highest tide they lay their eggs in the sand. The eggs require two weeks to incubate and during this time they are out of reach of the waves. Then, just as the eggs are ready to hatch, the next spring tide arrives and frees the little grunion which swim out to sea. (From Carson, Reference IL-1).

f. Tides on Islands:

Generally tides on Pacific island shores are low; this may be because the island positions are near nodal points in the deep ocean or it may be that islands, which often rise abruptly from deep water, are unable to modify the masses of water moved by the tide. According to Kuenen (Reference IL-8) the tides throughout the Indonesia range from one-half to one meter and the curves are exceedingly complex because of the interference of the many islands.

The Island groups of Hawaii, Tuamotu, Society, and the Mariannas all have tide ranges of less than 2 feet. It is said that at Tahiti where the tide seems to be entirely solar and semi-diurnal, one can tell the time by looking at the beach; high tides occur at noon and midnight.

g. Tides at the Panama Canal:

The Atlantic and Pacific ends of the Panama Canal are only about 30 airline miles apart but there is a great difference in the tides at the two terminals. At Colon on the Atlantic side the tide is generally diurnal and the range averages about a foot. At Balboa on the Bay of Panama tides are semi-diurnal with an average range of 14 feet and the locks are built to withstand spring tides reaching as much as 21 feet. The original proposal for the Canal (in 1879) was that it be at sea level -- a scheme that has been reconsidered again and again. In view of the possible wartime destruction of the very vulnerable dams (which would drain the canal) the matter is still under study, for a sea level canal, even if the tide locks were badly damaged, could be operated for about a third of each day.

h. Large Tides of the World: *

(Average Range)

Bay of Fundy, Canada	44 feet
Port Gallegos, Argentina	36 feet
Frobisher Bay, Davis Straight	35 feet
England, West Coast	33 feet
Koksoak River, Hudson Straight	30 feet
Cook Inlet, Alaska	30 feet
Magellan Straight, Chile	30 feet
France, North Coast	28 feet
Ashe Inlet, Hudson Bay	23 feet
Collier Bay, Western Australia	23 feet
Arabian Sea, India	23 feet
Colorado River, Mexico	22 feet
Chemulpo, Korea	21 feet
St. Croix River, Maine	20 feet

The spring tides are as much as 20% considerable higher.

* (From Marmer, Reference IL-9)

13. Tide Gauges:

In order to examine tides as they really are so that the actual height of water can be compared with the predicted height it is necessary to make a continuous measurements of the level of the water surface. As shown by many of the illustrations herein, the most convenient way of describing a tide is to show it as a curve in which the height of the water is plotted against time. Most of the tide gauges in use today automatically plot such a curve. Clockwork runs graph paper through the machine at a fixed speed; at the same time a float in a well (arranged to damp out wave action) rises and falls on the tide and through a suitable arrangement of wires and pulleys causes the proportional movement of a stylus which traces a line on the paper. The product of the machine is a continuous curve which is not exactly like the predicted curve because it records changes in the water surface due to winds, barometric pressure, seiching, and other non-astronomical causes.

Although practically all gauges now in use operate on such a mechanical float system, it is possible to make a pressure actuated guage, in which the weight of the water over a bottom-located pressure head could be used to unbalance a bridge or otherwise alter an electrical flow in a cable leading ashore. This intelligence could then be recorded on paper much as in the present gauges. No such instrument is now owned by the U. S. C. & G. S. although the method has been considered; present types of wave recorders can be damped to operate this way. The advantage of such a device would be that it could be used to record tides where the necessary structure for a mechanical gauge could not be maintained.

14. Tide and Current Tables: *

Tide Tables for the use of mariners have been published by the United States Coast and Geodetic Survey since 1853. For a number of years these tables appeared as appendixes to the annual reports of the superintendent of the survey, and consisted of more or less elaborated means for enabling the mariner to make his own prediction of tides as occasion arose.

The first tables to give the predicted tides for each day were those for the year 1867. They gave the times and heights of high waters only, and were published in two separate parts, one for the Atlantic coast and the other for the Pacific coast of the United States. Together they contained daily predictions for 19 stations and tidal differences for 124 stations. A few years later, predictions for the low waters were also included, and for the year 1896 the tables were extended to include the entire maritime world, with full predictions for all tides at 70 ports and tidal differences for about 3000 stations. Present tide table publications contain, in addition to the times and heights of high and low tides and directions for determining the height of the tide at any time, a number of other tables useful to mariners. These tables include; local civil time of sunrise and sunset; reduction of civil time to standard time; moonrise and moonset; Greenwich civil time of the moons phases, apogee, perigee, zero and the greatest north and south declination; and the time of the solar equinoxes and solstices.

The British Admiralty, London and the Deutsche Seewarte, Hamburg also prepare tables for the use of mariners by means of similar machines. In some ports, particularly on the far east, empirical tables are still in use at harbors where the tides are large. It must be remembered that in the preparation of the tide tables the best available material is used, but the predictions and tidal differences are necessarily of unequal merit for different parts of the globe, owing to a lack of properly distributed observations upon which to base conclusions. Because tide recorders are more easily maintained in protected waters, the times and heights given for reference stations may be considerably out-of-phase with nearby points on the open coast. For example, the tide on the San Francisco bar leads that at the tide station in the bay by 40 minutes. Means of calculating the time of tides at coastal points and the height of the tide at any time from the data given for the reference station are carefully explained in the tide tables and need not be reiterated here.

* Abstracted from USC & GS Tide Tables (Reference IL-B)

Current tables for the use of mariners which give the times and velocities of maximum flood and ebb and the time that slack begins have been published by the Coast and Geodetic Survey since 1890. Generally a number of measurements are made of the currents for each of the various stages of the tide; these are used together with calculated average values to determine the average velocity of the current at any location. Inasmuch as at any instant the current velocity varies considerably over short horizontal distances, the currents given are those on the surface in the main channel. The current tables also include some information on currents due to winds and general data on coastal currents as well as diagrams for the use of vessels who wish to navigate the currents inside our larger bays as efficiently as possible

a. Approximate Method for Drawing Tide Curves: *

If the height of the tide is required for a number of times on a certain day the full tide curve for the day may be obtained by the one-quarter, one-tenth rule. The procedure is as follows:

(i.) On cross section paper, plot the high and low water points in the order of their occurrence for the day, measuring time horizontally and height vertically. These are the basic points for the curve.

(ii.) Draw light straight lines connecting the points representing successive high and low waters.

(iii.) Divide each of these straight lines into four equal parts. The halfway point of each line gives another point for the curve.

(iv.) At the quarter point adjacent to high water draw a vertical line above the point and at the quarter point adjacent to low water draw a vertical line below the point, making the length of these lines equal to one-tenth of the range between the high and low waters used. The points marking the ends of these vertical lines give two additional intermediate points for the curve.

(v.) Draw a smooth curve through the points of high and low waters and the intermediate points, making the curve well rounded near high and low waters. This curve will closely approximate the actual tide curve, and heights for any time of the day may be readily scaled from it.

* Abstracted from Tide Tables, USC & GS (Reference IL-B)

b. Tidal Illumination Diagrams:

Figure IL-2, a typical tidal illumination diagram, is a means of representing the relationships between the character of the tide and the state of illumination. They were first prepared for the use of the forces landing amphibiously against northern Africa in 1944 and have now become a standard piece of intelligence presentation. An office of the unit which devised these diagrams describes them thus: "The upper part of the diagram shows the amount of rise and fall of successive tides as well as daily range variations. The times of occurrence of high and low tides are shown by a series of connected points in order that the user might visualize the sequence throughout the month. Days of the month are represented by vertical lines covering the period from noon through midnight to noon of the next day.

In the diagrams the daily times of the three types of twilight are indicated so as to bring out the graduation in light from sundown through darkness, as well as from darkness to sunrise. Thus, in the evening, Civil Twilight begins with sunset and ends when the sun is 6° below the horizon. During this period objects can be readily distinguished and a newspaper can be read without the aid of artificial light. At the end of Civil Twilight the brightness of the sky is approximately twenty times that of the full moon at its zenith. The following period of Nautical Twilight ends when the sun is 12° below the horizon. During this time all the brighter stars are visible, the horizon is generally indistinct but general outlines of objects not too distant can be distinguished. Astronomical Twilight follows next and ends when the sun is 18° below the horizon. During this period the amount of illumination is still distinctly brighter than during complete solar darkness.

Variations of moonlight are illustrated by periods of bright and dim moonlight. Thus, during moonlight, taken as the period four days before and after the appearance of full moon, the intensity of light varies between the brightness of the full moon at its zenith and about one-third of this value. The period of dim moonlight which includes four days before the beginning of moonlight and four days after its end, marks those nights during which the illumination intensity varies from about one-third to about one-tenth of the brightness of the full moon at zenith.

15. Tidal Friction: *

The movement of the tide obviously is accompanied by friction, arising chiefly from the movement of the water over the beds of the oceans, seas and rivers. This friction consumes energy, and it can be demonstrated that this energy must come from the rotational energy of the earth. That is, tidal friction acts as a sort of brake on the rotating earth tending to reduce its velocity of rotation. It follows, therefore, that tidal friction tends to make the day longer. The loss of energy due to tidal friction, however, is so small when compared with the earth's stock of energy, that it is only by a minute quantity that the day is lengthened by this cause - - something like the thousandth part of a second in a century.

On investigating the matter mathematically it is found that the effect of tidal friction is not confined to the earth, but makes itself felt also on the moon. Besides decreasing the rotational velocity of the earth, tidal friction also tends to make the moon recede from the earth and at the same time to lengthen the period of the lunar month. These changes, like the increase in the length of the day, are at an exceedingly slow rate, but they have been operating over such a long period of time that the cumulative effects must by this time be of relatively large magnitude.

The consideration of the effects of tidal friction makes inescapable the conclusion that if we travel backward in time, the day must become shorter and the moon come nearer the earth. But when the moon was nearer the earth the tides must have been of greater range than at the present time, for we have found that the tide-producing power of a heavenly body varies inversely as the cube of its distance. And quite apart from the increased friction due to greater tides, tidal friction also varies as the cube of the moon's distance from the earth. Hence the efficiency of tidal friction in increasing the length of the day and the distance between moon and earth varies inversely as the <u>sixth</u> power of the distance. Thus, when the moon was one-quarter her present distance from the earth the effects of tidal friction were at a rate $4^6 = 4096$ times as great as now.

Starting with these considerations, Sir George Derwin developed an exceedingly interesting theory of the evolution of the earth-moon system, from which it appears probable that the moon was at one time part of our earth. And since the effects of tidal friction are still continuing, it follows that the gradual change in the lengths of the day and month will continue until they have the same period, while the tides will become smaller because of the increasing distance of the moon. When day and month become equal, the lunar tide will cease completely, since the earth will no longer rotate relatively to the moon.

* Abstracted from Marmer (Reference IL-10)

16. Glossary of Tidal Terms:

Age of the Tide: the average interval in days between new or full Moon and the next following spring tide

Amphidromic Point: a point where cotidal lines meet and where the tide vanishes. (no rise and fall of the sea level)

Apogee: condition when the moon is furthest from the earth

Bore: the front of flooding tide as it races up an estuary; bores usually have a steep front and are succeeded by a general rise in water level. (also called mascaret)

Cotidal Line: a line connecting points which have high or low water at the same time

Datum: a plane or level used as a basis for reckoning heights and depths. A number of tidal datums are in use, some of which are Mean Low Water, Mean Lower Low Water, Mean Low Indian Springs, Mean Low Water Springs, Tropic Lower Low Water.

Diurnal: daily

Diurnal Inequality: two high and two low tides daily, each of which are unequal in height

Ebb: the movement of tidal water away from shore or out of an estuary

Equatorial Tide: when the moon is on the equator (declination nearly zero) the heights of successives highs and lows are nearly equal and the tides are said to be equaltorial

Equinoctial: the plane of the earth's equator prolonged out into space

Equinoctial Spring Tides: when the declinations of the sun and moon are both nearly zero (near the equinoxes) their combined gravitational field when the moon is in syzygy gives unusually high spring tides.

Establishment of the Port: the average value of the lunitidal interval on the days of the new and full moon for any specific location.

Flood: the movement of tidal water towards shore or up an estuary

Half Tide Level: exactly half way between mean high water and mean low water; it usually differs from mean sea level

Height of the Tide: the difference between the datum plane and the water level - - - a continuously changing figure

High Water Interval: see lunitidal interval

Low Water Interval: the length of time between the moons crossing the meridian and the next low water at the same place.

Lunitidal Interval: the time between the moon crossing of the meridian at any point and the next high water at that point.

Maelstrom (Malstrom): a whirlpool or eddy often caused by strong tidal currents passing between islands

Mean High Water: average of the height of all high tides

Mean Low Water: average height of all low tides

Mean Lower Low Water: average height of the lower of the two low tides caused by diurnal inequality

Mean Sea Level: the average level of the sea at any point referred to some datum. This figure is usually the average of a great number of measurements over a period of at least 18.6 years

Meteorological Tide: changes in sea level caused by a meteorological factors, such as wind or barometric pressure. These tides are independent of gravitational forces and cannot be predicted

Neap Tide: minimum tides which occur when the moon is in quadrature

Overfalls: obstructions to the tidal current, principally by the bottom configuration in narrow channels, diverts the tidal stream upward; these show as up-boiling water called overfalls

Partial Tide: tide waves can be broken down into periodic constituents or harmonic oscillations: each of these is a partial tide

Perigee: condition when the moon is nearest the earth

Quadrature: situation when the lines joining the centers of earth, sun and moon form a nearly right angle

Range of the Tide: the difference in level between successive high and low waters

Rip Tide: see tide rip

Slack: a state of rest of the water when the tidal currents are nearly zero. The time of slack does not always coincide with the time of high or low water

Spring Tide: maximum tides which occur at full or new moon (moon in syzygy)

Syzygy: situation when the lines joining the centers of earth, sun and moon from nearly a straight line

Tidal Currents: horizontal motions of water which accompany the rise and fall of the tide; these usually flow in the same direction as the tidal wave itself

Tidal Entrance: the mouth of an estuary or bay through which the water must ebb and flow with the changing tide

Tidal Wave: the tide is a true wave whose crest is dragged around the earth by the gravitational pull of the moon. The term tidal wave is rarely used in connection with tides because of the confusion with tsunamis or earthquake generated waves which are referred to as "tidal waves" by the press and public

Tide: the alternating vertical motions of the water of the sea in response to the gravitational pull of the sun and moon

Tide Curve: a plot of the height of the water against the time of day

Tide Gauge: a device for automatically measuring and recording the height of the tide with respect to time. A tide staff is also a simple form of gauge which consists of a graduated scale set at some datum from which the height of the water can be read directly

Tide Race: very bold headlands which divert the coastal tidal currents and cause narrow swift tidal streams. If the stream is opposed by a strong wind, a very choppy sea may result

Tide Rip: a fast and narrow tidal current or the boundary between two currents with different directions

Tide Tables: tables of data published in advance by various maritime powers which contain predictions of the heights and times of high and low tides

Tropic Tide: the greatest diurnal inequality is displayed when the moons declination is at a north or south maximum; these tides are called tropic tides

Vanishing Tide: when the diurnal inequality becomes so great that a high tide is actually lower than the previous low tide there is a period of several hours on an out going tide when the water level remains nearly constant before ebb resumes. A high and low tide have vanished from the day and the semi-diurnal tide apperts diurnal

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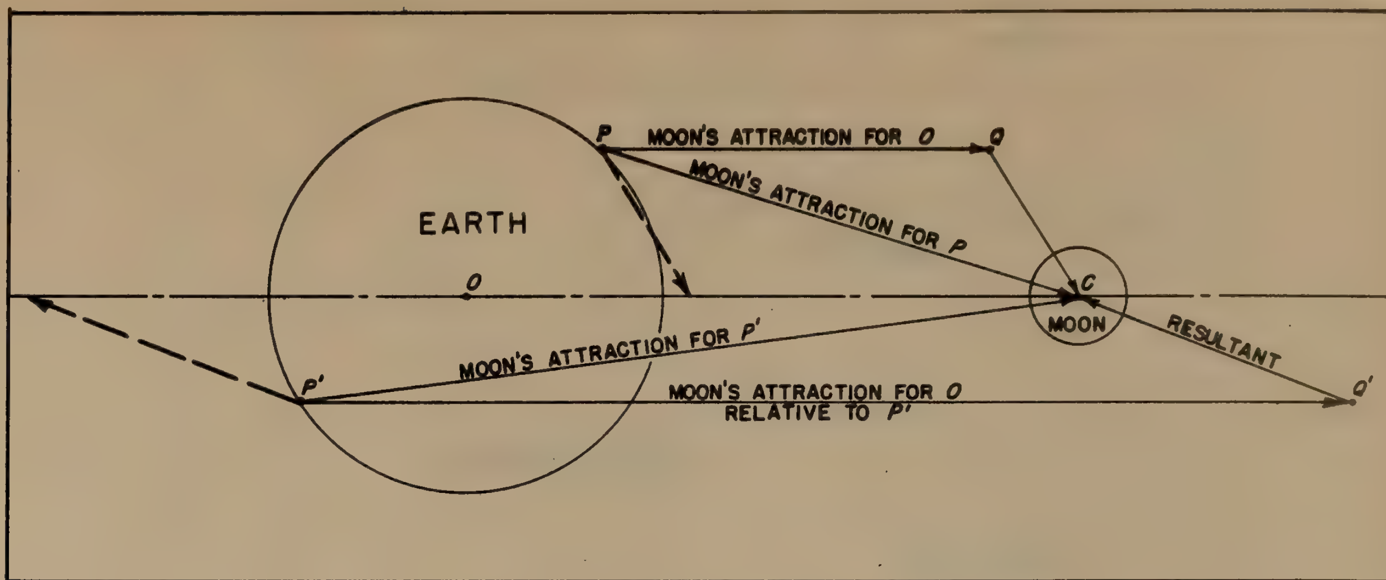
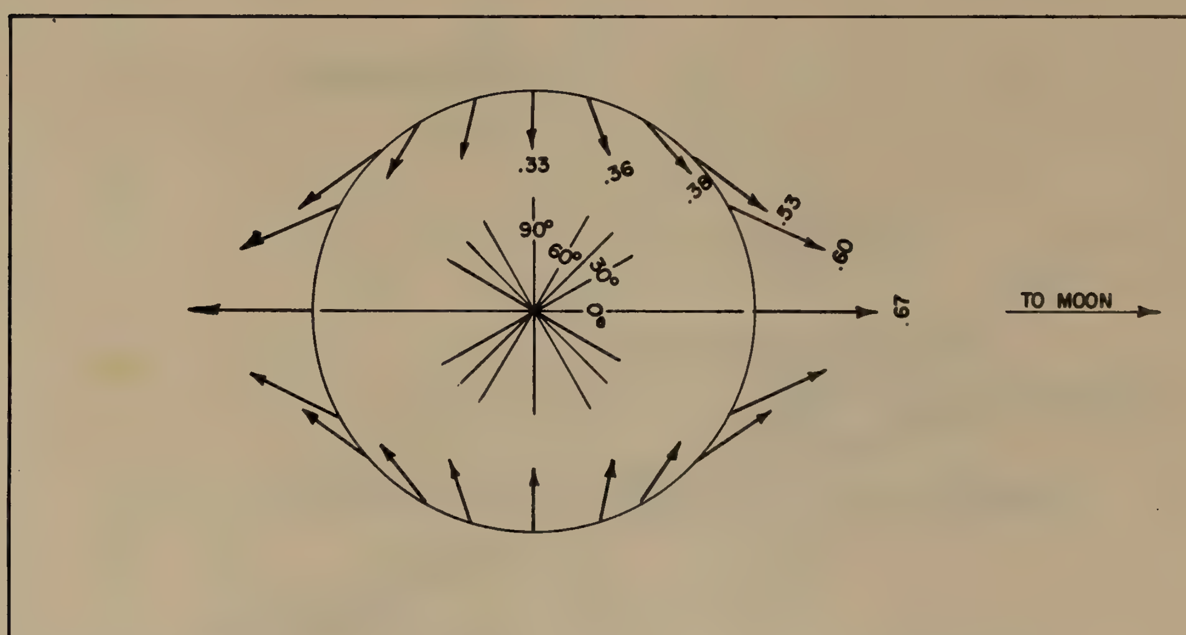
No. 121. Coastal Currents Along the Pacific Coast of the U. S., 1926
No. 135 Tidal Datum Planes, 1927
No. 196 Manual of Tide Observations, 1941
No. 215 Manual of Current Observations, 1938
No. 228 Tide and Current Glossary, 1941
No. 230 Coastal Currents along the Atlantic Coast of the U. S., 1942
(Also Numbers 111, 115, 123, 127, 142, 150, 162, 174, 180, 208, 211 which deal with tides and currents at specific locations on the U. S. coasts.)



a. The differential attraction of the moon for the points P, P' & O, shown as vectors. The heavy dashed lines are the forces that tend to move P & P' relative to O. (from Schureman)



b. Direction and relative magnitude of the tide-producing forces (from *Encyclopedia Britannica*)

FIGURE IL-1

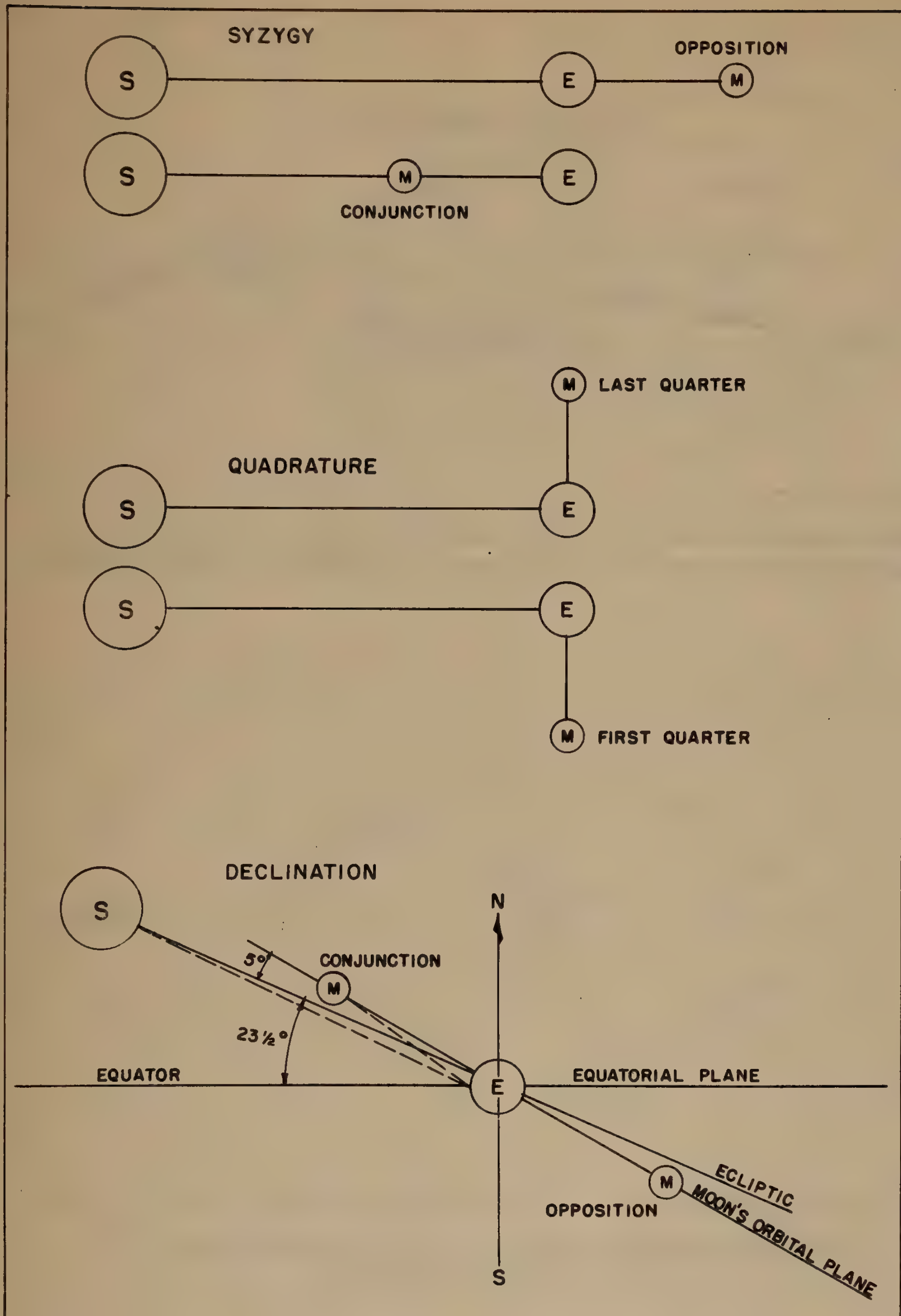
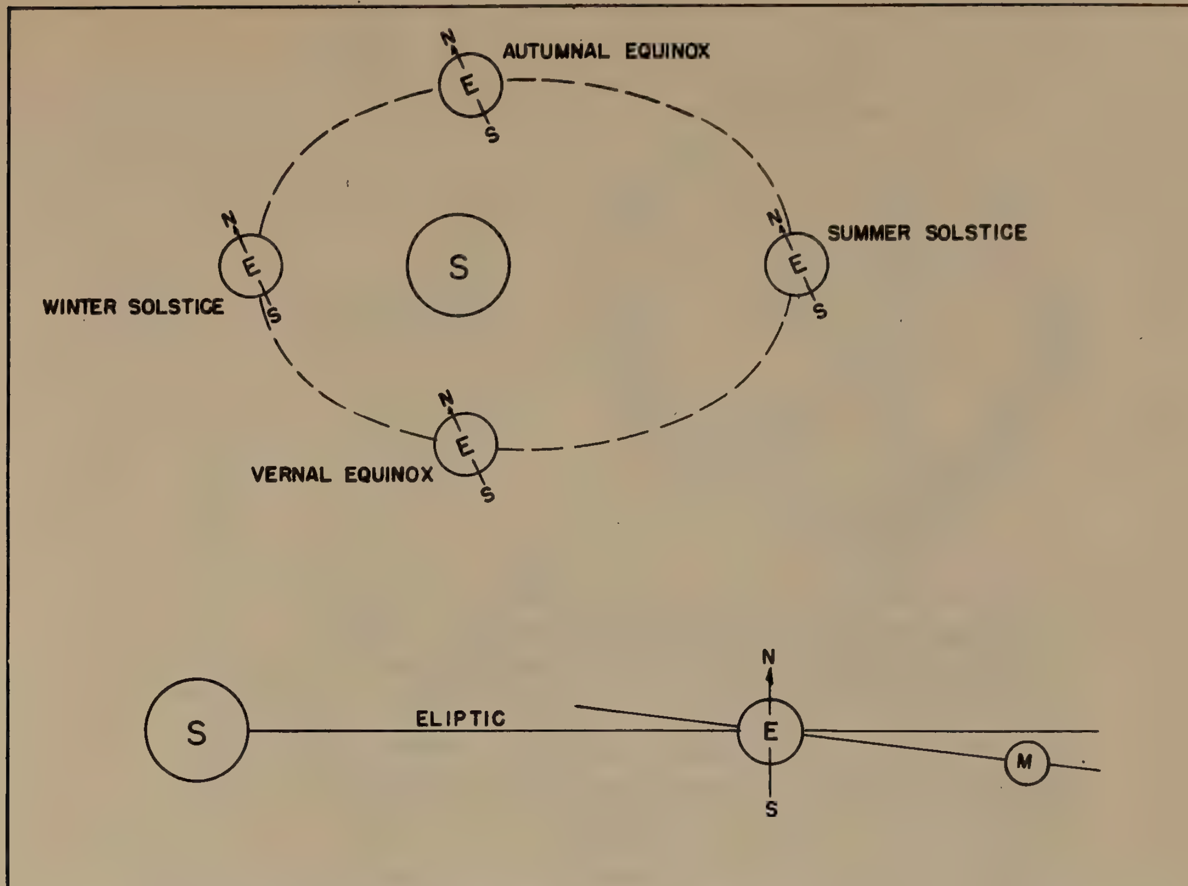


FIG. IL-2 -- LIMITING CONDITIONS - SUN, EARTH, MOON



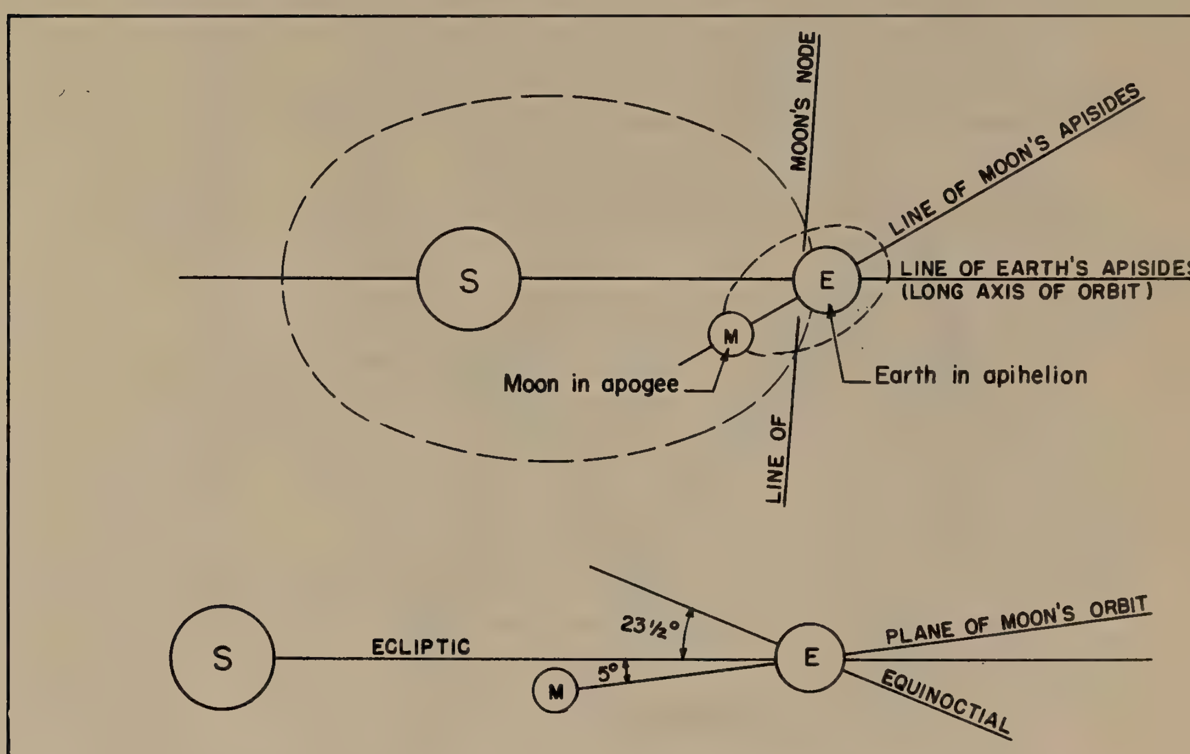
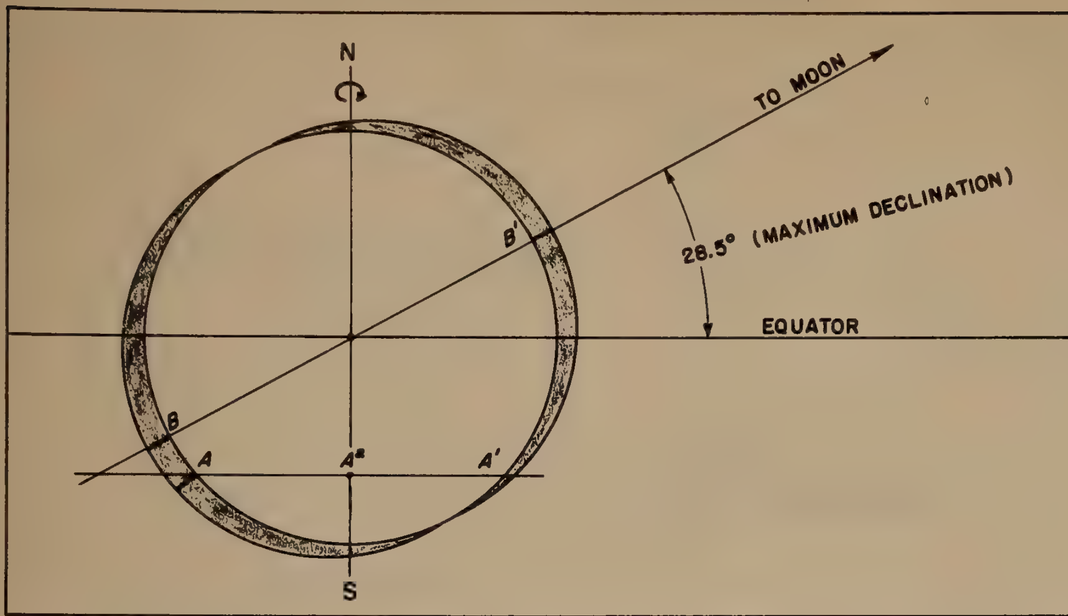
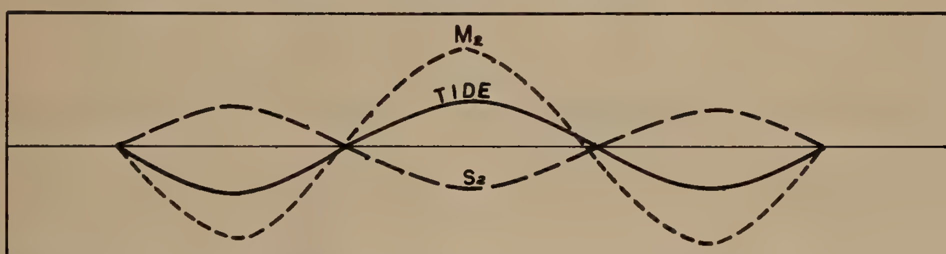


FIG. IL-3 - ORBITS OF THE EARTH AND MOON

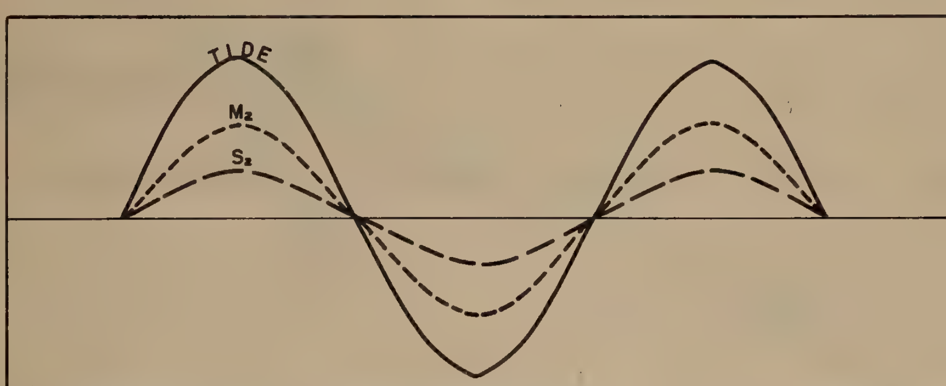


a. Diurnal inequality caused by the declination of the moon

A	0 hrs., 24.8 hrs.	higher high water	
A'	12.4 hrs.	lower high water	tropic tides
A²	6.2 hrs., 18.6 hrs.	low water	
B - B'	Tides of equal height on line with the moon		



b. Neap tides; sun and moon out-of-phase at quadrature



c. Spring tides; moon in syzygy forces of sun and moon are additive

FIGURE IL-4

ENVELOPE OF TIDE CURVES, SEATTLE, WASHINGTON, 1 NOVEMBER TO 6 DECEMBER 1949

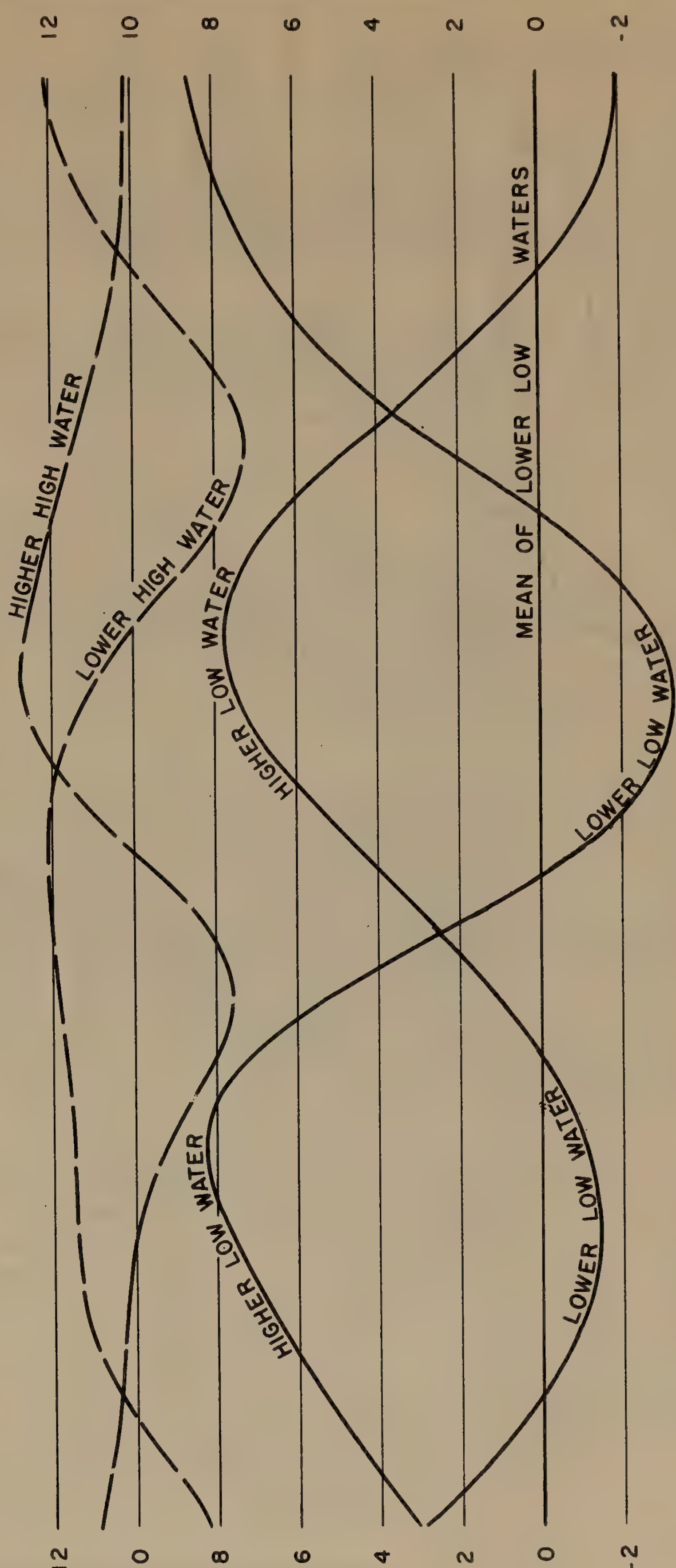


FIG. IL-5 - EXTREME DIURNAL EQUALITY

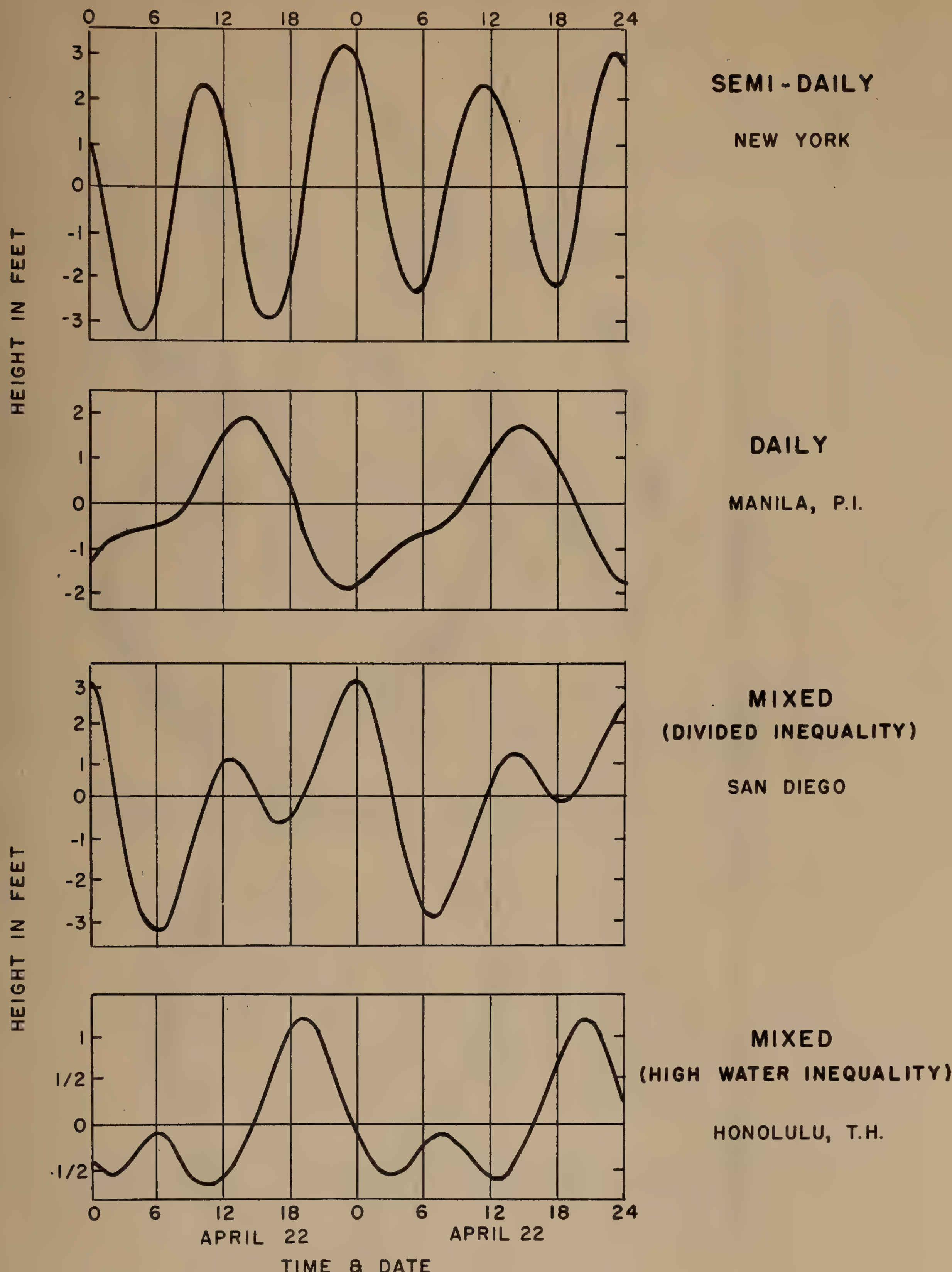
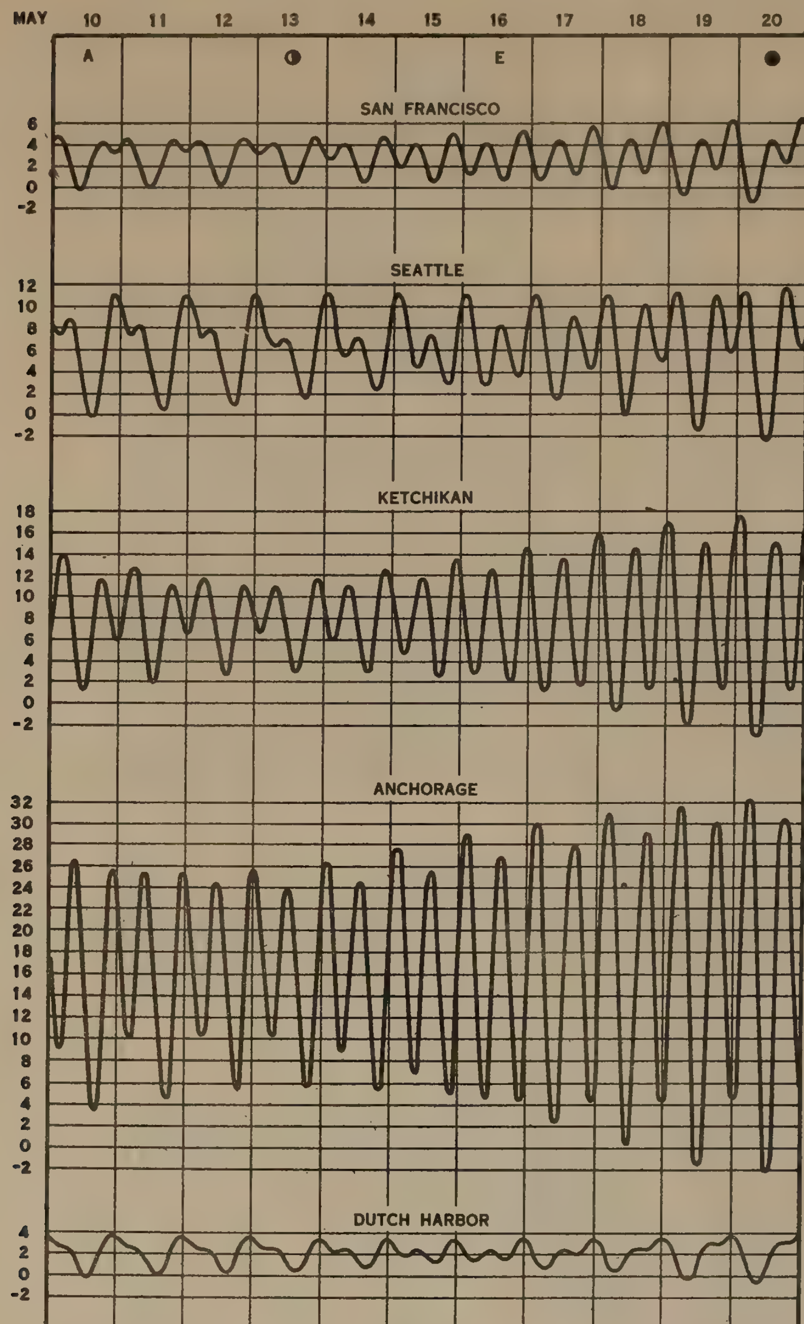


FIG. IL-6 - TYPES OF TIDES (from Marmer)



A Apogee E Moon on equator

Note the diurnal tide at Dutch Harbor

FIG. IL-7 - TYPICAL TIDE CURVES
(from U.S.C.&G.S. Tide Tables)

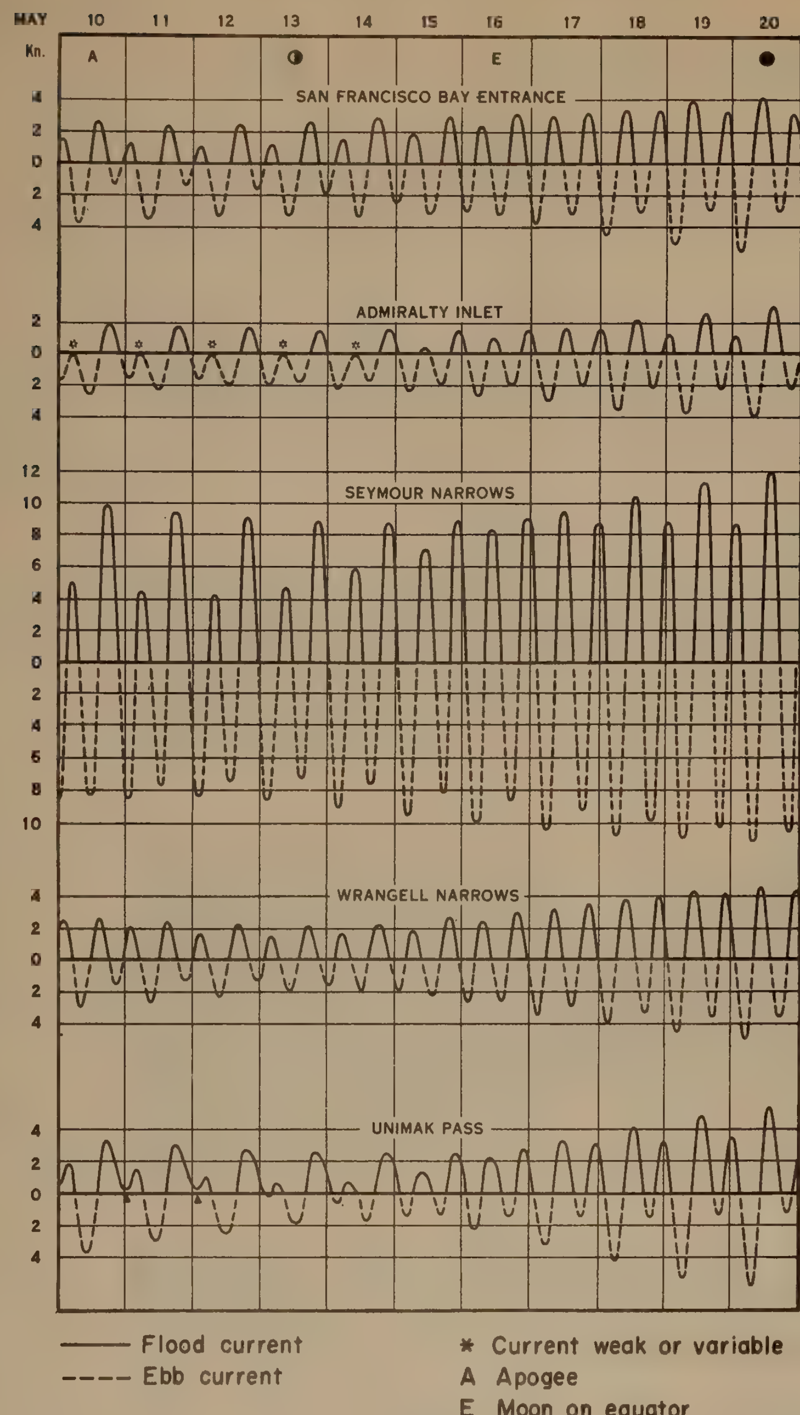
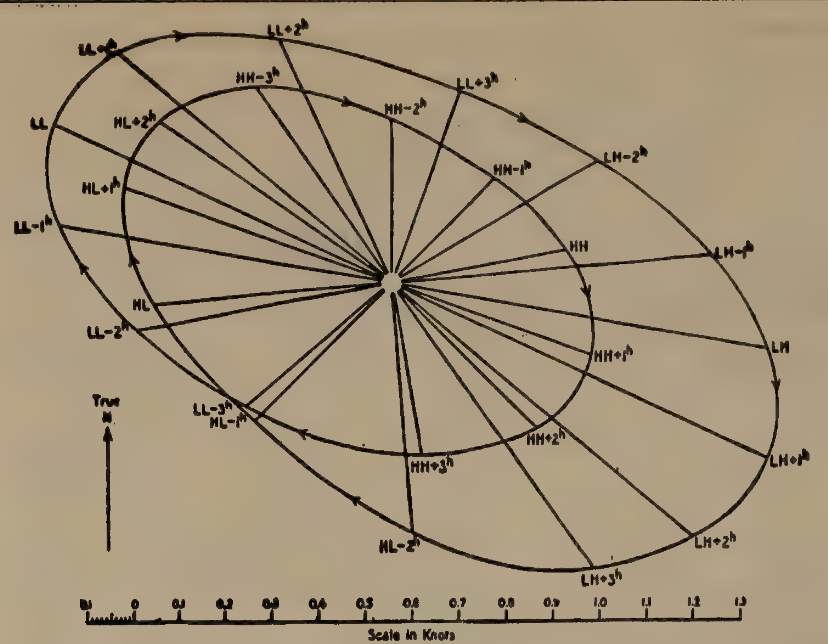
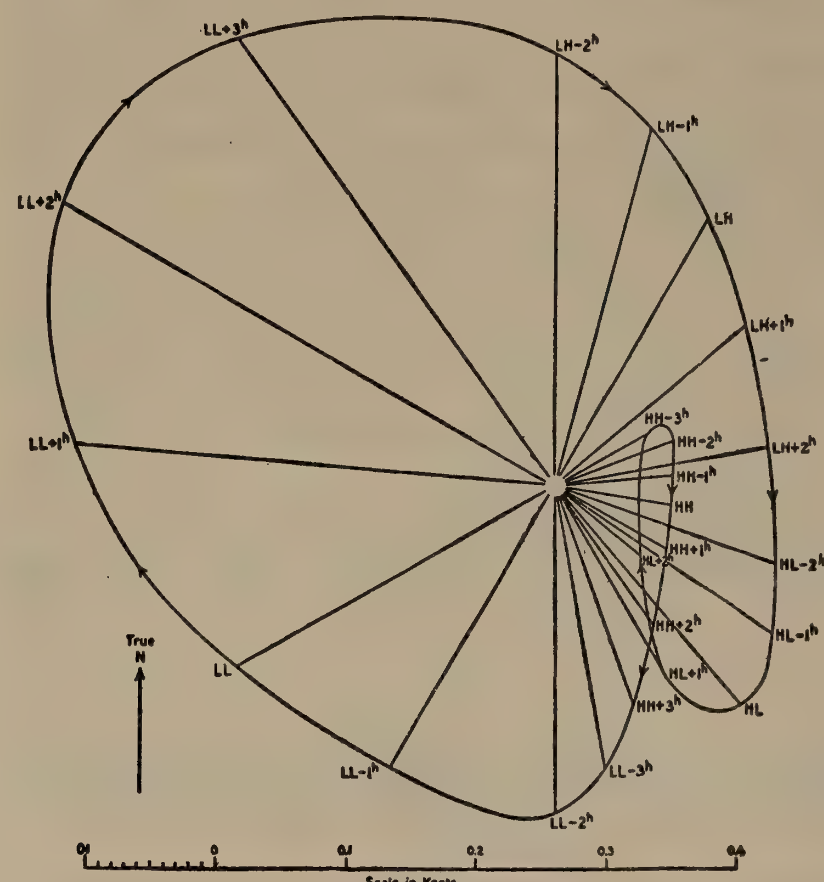


FIG. IL-8 - TYPICAL CURRENT CURVES
(from U.S.C.&G.S. Current Tables)



Tidal Current Curve, Swiftsure Bank Lightship. Referred to predicted time of tide at Astoria, Oreg.



Tidal Current Curve, San Francisco Lightship
Referred to predicted time of tide at San Francisco (Golden Gate), Calif.

FIG. IL-9 - ROTARY TIDAL CURRENTS
(from U.S.C.&G.S. Current Tables)

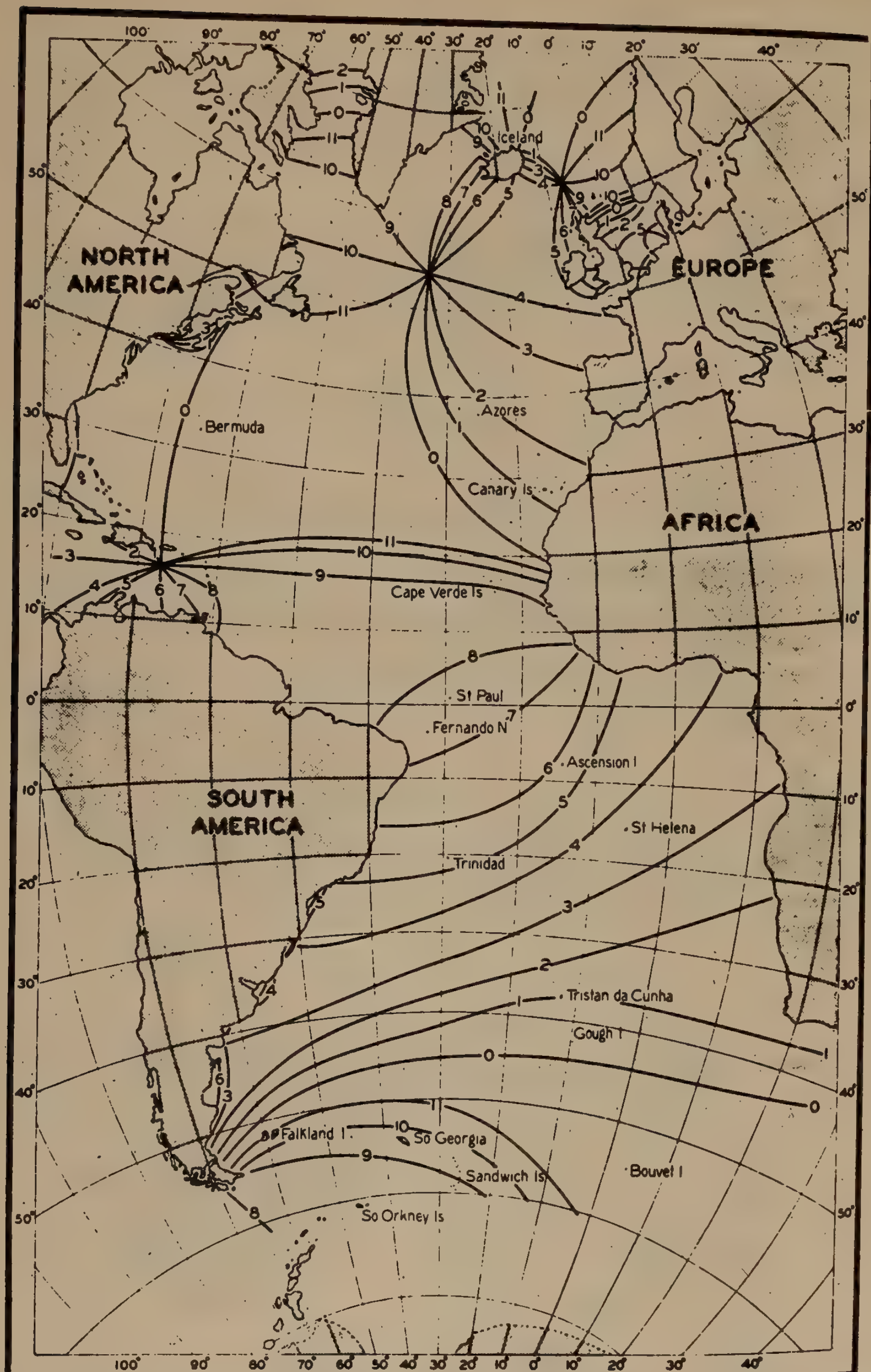


FIG. IL-10 - Cotidal lines of the semidiurnal tide in the Atlantic Ocean (Sterneck)

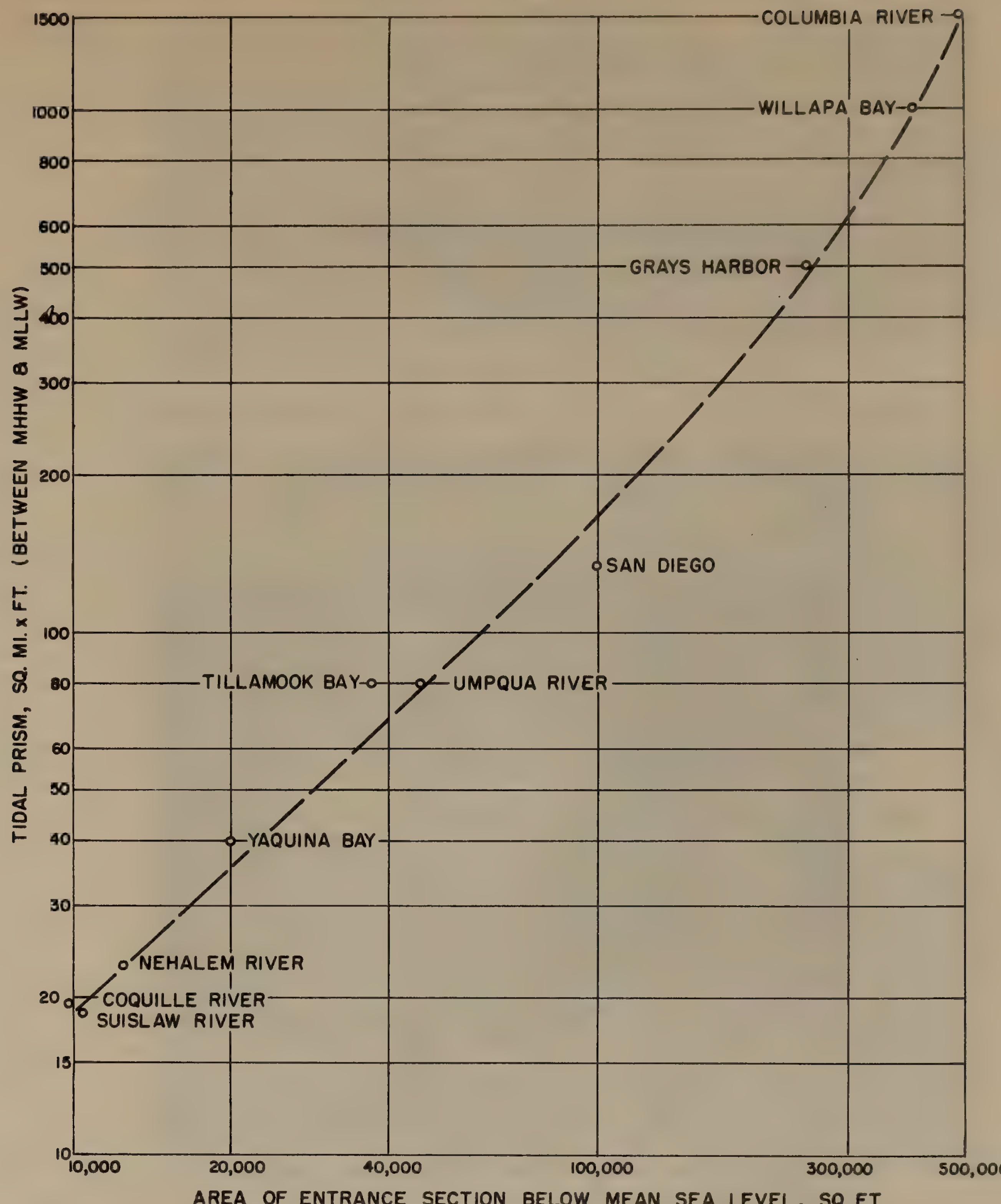


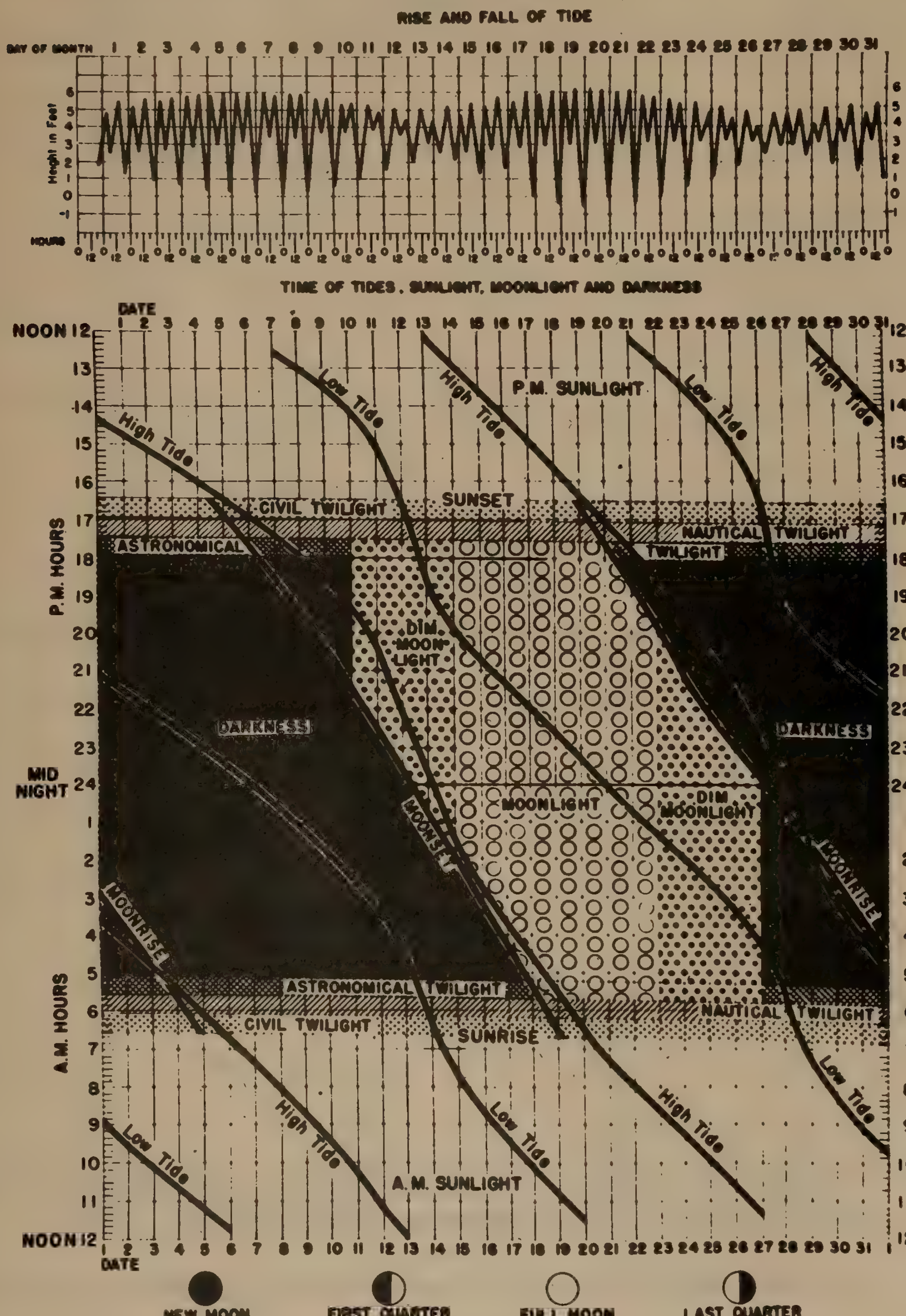
FIG. IL-11 - RELATIONSHIP BETWEEN TIDAL PRISM AND ENTRANCE SECTION
(from O'Brien)

TIDAL-ILLUMINATION DIAGRAM

YOKOHAMA

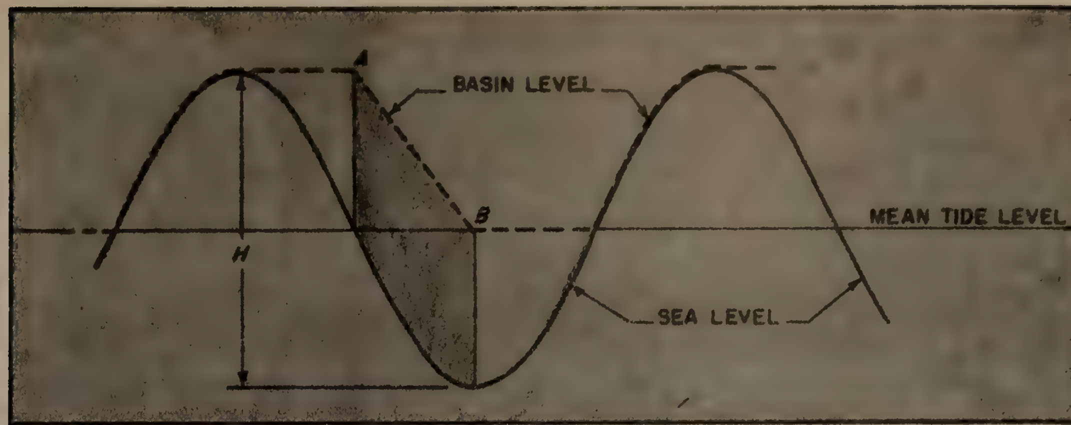
LAT. 35° 27' N - LONG. 139° 38' E
TIME MERIDIAN 135° E

DECEMBER 1945

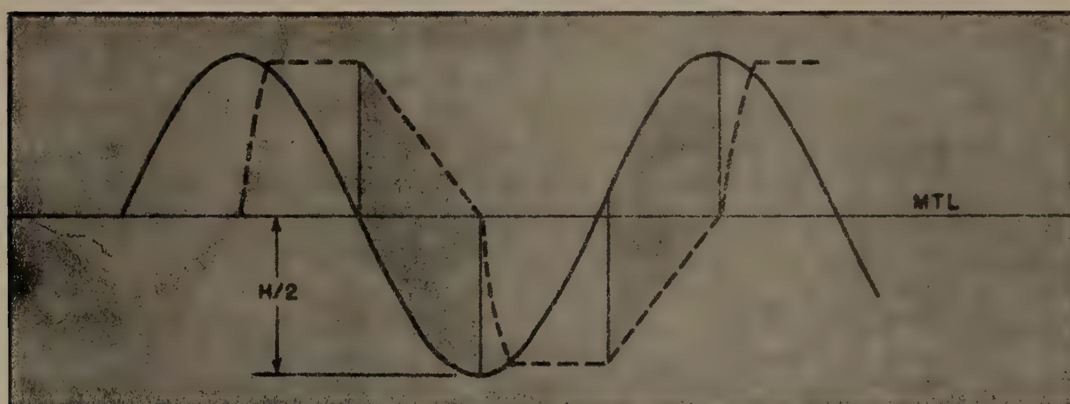


6211 BEACH INTELLIGENCE UNIT
HQ. USAFMIDPAC APO 958

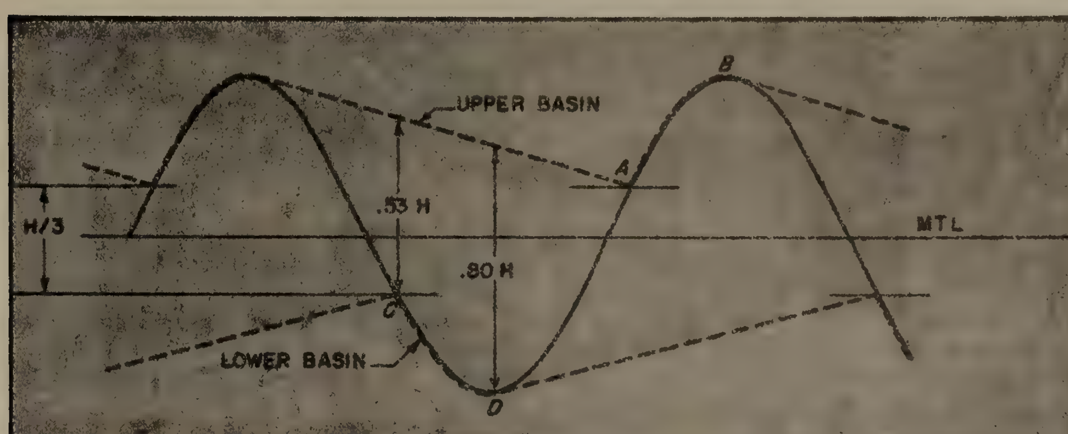
FIGURE IL-12



a. Power extracted as water flows from basin to sea



b. Power extracted by water flowing both into and out of basin



c. Power extracted as water flows between two basins whose levels are kept above and below 1/3 points of the tide range.

FIG. IL-13 - DIAGRAMS OF TIDAL POWER SCHEMES
(from *Encyclopedia Brittanica*)

C68075

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION II: WAVE THEORY, USEFUL GRAPHS AND TABLES OF FUNCTIONS

A. WAVE THEORY

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Many helpful discussions with the other members of the waves project are sincerely appreciated.

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION II: WAVE THEORY, USEFUL GRAPHS AND TABLES OF FUNCTIONS

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MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION II. WAVE THEORY, USEFUL GRAPHS AND TABLES OF FUNCTIONS

A. THEORY

by

R. A. FUCHS

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MANUAL

of

AMPHIBIOUS OCEANOGRAPHY

SECTION II

WAVE THEORY, USEFUL GRAPHS

and

TABLES OF FUNCTIONS

1. Introduction:

Aside from the earliest qualitative observations of sea waves the first studies of wave motion on water were theoretical. It was only after the establishment of rather accurate theoretical descriptions of wave motion that controlled experiments were run more for the sake of verification of the existing theory than to predict new events. Many of the qualitative phenomena associated with wave motions were known to Leonardo da Vinci (1452-1519), some of whose information was of an even earlier origin (see Avicenna, 980-1037).

In a certain sense the first wave theory was given by Newton (1687), although it was little more than a crude analogy. Assuming that oscillations of the water behaved like that of water in a U-tube, he deduced that the wave velocity of periodic progressive waves must be proportional to the square root of their length. It was of course known to Newton as well as his predecessors that the orbits of the water particles were nearly circular, at least in relatively deep water.

The first real theoretical treatment of wave motion after the establishment by Euler (1755) of the equations of fluid dynamics was due to Laplace (1776). His solution is today recognized as the periodic solution of the exact linear theory of waves in water of arbitrary constant depth. It is an approximate theory in the sense that terms of the order of $(H/L)^2$ are neglected in comparison with those of order (H/L) thus leading to use of the term "waves of infinitesimal amplitude". This solution will be discussed in detail later. Suffice it to say, however, that for many practical purposes, Laplace's solution with only slight adaptations is quite adequate.

A particularly convenient approximate theory for shallow water waves was discovered by Lagrange (1781). The pressure was assumed to be entirely hydrostatic, and the horizontal velocity component to be independent of the depth coordinate. The surface elevation was found to satisfy the classical wave equation $\eta_{tt} = (gd)\eta_{xx}$ for waves of permanent form moving with velocity $c = \sqrt{gd}$. This theory can be thought of as a limiting case of Laplace's theory for $d/L \rightarrow 0$.

An exact solution of the problem was first proposed by Gerstner (1802). The principal objection to it was first raised by Stokes (1845) who pointed out that the motion was rotational and hence according to a result of Lagrange (1781) and Cauchy(1815) could not be produced in a liquid at rest by the action of conservative forces. Moreover, in order to generate such waves it was necessary to have imposed initially in the water a velocity distribution opposite in direction to the direction of wave motion and decreasing exponentially with depth.

The initial value problem of waves generated by a localized initial disturbance at the surface of deep water was first treated by Cauchy (1815) and Poisson (1815). Apart from the obvious practical importance, this problem is of considerable interest for the notion of dispersive media and the propagation of wave groups in such media had their origin in its study. The essentials of the two papers of Cauchy and Poisson are concealed in reams of analysis and the details of the calculations are seldom carried out. As a consequence, they were often regarded as obscure and in the words of Scott Russell, who contributed much to the experimental side of wave problems, "mathematical exercises".

Stokes can be considered to be the founder of modern wave theory. He succeeded in showing that the theory of surface waves is a problem in potential theory with a non-linear free surface condition, the problem being "in the small" since the surface is assumed to depart only slightly from the original undisturbed form. Moreover, he succeeded in determining the periodic wave motions to a rather high order of approximation. His methods have inspired and continue to inspire numerous investigations along the same or similar lines.

In spite of the fact that the equations of motion of perfect fluids have been known for over two hundred years there are relatively few completely satisfactory solutions of the problems of wave motion. The problems of most interest from the point of view of applications are those involving waves of non-permanent form, such as waves generated by localized disturbances and waves which peak up and plunge in shoaling water. The theoretical solutions with relatively few exceptions have been concerned with wave trains which are periodic in time and in cases most satisfactory from a purely mathematical point of view also periodic in space. A noteworthy exception is the treatment of localized initial disturbances due to Cauchy and Poisson. This analysis as well as most others is based on the approximate linear theory for waves of infinitesimal amplitude.

2. Fundamental Equations of Fluid Dynamics:

a. Euler's Equations

We shall deal in general with continuous and homogeneous distributions of matter, called fluids, which are characterized by the property that when they are not in motion, the force on any surface in the continuum is normal to that surface. More particularly we shall be almost exclusively concerned with fluids called "perfect" which are incompressible and non-viscous, the latter implying that the tangential force on an interior surface is zero even though the fluid is in motion.

It will be convenient in the derivations to follow, not to restrict ourselves to any one coordinate system and we shall therefore use the methods of vector analysis. The fluid motion can be described from either of two different points of view, both due to Euler. The Lagrangean representation considers the path of the individual fluid particles describing them as functions of time. In the Euler representation attention is confined to a particular fixed point in space and the velocity components are given as functions of position and time. We shall find it convenient to denote by appropriate subscripts the derivatives in the Euler system. The transition from the Eulerean to the Lagrangean representation is effected by solving the equations for the velocity components along the path.

The fundamental equations of fluid dynamics are based on the principle of the conservation of mass, and on Newton's second law of motion. We shall assume that we are dealing with continuous motions, that is, we assume that the velocity components vary continuously throughout the fluid and their space derivatives are finite. If we consider a closed surface drawn within the fluid, then in continuous motion the increase in mass of fluid within the surface within any time interval δt must be equal to the excess of mass flowing in over the mass flowing out. Now for a

closed surface S bounding a volume V:

$$\frac{\partial}{\partial t}\left\{\iiint_V \rho \; dV\right\}\delta t = \text{increase in mass inside } S \text{ in time } \delta t$$

$$= \text{excess of flow in over flow out across } S \text{ in time } \delta t$$

$$= -\iint_S \rho \, q_n \, dS \, \delta t$$

$$= -\iiint_V \text{div } (\rho \vec{q}) \, dV \delta t \text{ (by Green's theorem)}$$

where $\vec{q}$ is the vector particle velocity and q_n is its component normal to the surface S. If this is to hold for all V the integrands of the volume integrations must be equal and hence since ρ is constant

$$\text{div } \vec{q} = 0 \qquad \text{(II A-2.1)}$$

This restricts the arbitrariness of the velocity components. In rectangular coordinates this continuity equation is

$$u_x + v_y + w_z = 0 \qquad \text{(II A-2.2)}$$

Suppose now that the fluid is subjected to an external body force, $\vec{F}$ per unit mass. The total force acting on a portion of the fluid occupying a volume V bounded by a closed surface S is

$$\iiint_V \rho \; \vec{F} \; dV + \iint_S - p \; \vec{n} \; dS$$

where $\vec{n}$ is the outward normal to S. The total momentum of the fluid within V is on the other hand equal to

$$\iiint_V \rho \vec{q} \; dV$$

By Newton's law the time rate of change of momentum in V is equal to the total force acting on the fluid within V; that is

$$\frac{d}{dt}\iiint_V \rho \vec{q} \, dV = \iiint_V \rho \vec{F} \, dV + \iint_S - p \vec{n} \, dS$$

Now according to Gauss' theorem

$$\iint_S p \; \vec{n} \, dS = \iiint_V \text{grad } p \, dV$$

where grad p is a vector having rectangular components p_x, p_y, and p_z. Substituting

we find

$$\iiint\limits_V \rho \frac{d\vec{q}}{dt}\, dV = \iiint\limits_V (\rho \vec{F} - \operatorname{grad} p)\, dV$$

For arbitrary V the continuity implies that the integrands must be equal, that is

$$\rho \frac{d\vec{q}}{dt} = \rho \vec{F} - \operatorname{grad} p \qquad \text{(II A-2.3)}$$

or in rectangular coordinates

$$\begin{aligned} u_t + u\, u_x + v\, u_y + w\, u_z + \frac{1}{\rho} p_x &= X \\ v_t + u\, v_x + v\, v_y + w\, v_z + \frac{1}{\rho} p_y &= Y \\ w_t + u\, w_x + v\, w_y + w\, w_z + \frac{1}{\rho} p_z &= Z \end{aligned} \qquad \text{(II A-2.4)}$$

where X, Y, Z are the components of the external body force. These are Euler's dynamical equations. We notice immediately the non-linearity of Euler's equations, arising from the convective terms

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + v \frac{\partial u}{\partial z} \quad \text{etc.}$$

The presence of these terms makes it impossible to use the principle of superposition of solutions according to which more general solutions are found by linear addition of elementary solutions. We shall be interested later in approximate solutions which result from dropping the non-linear terms.

Only in the case of a special type of flow, called irrotational, is it possible to find a first integral of Euler's equations which greatly simplifies the solutions.

b. Irrotational Flow

A particularly important result of the fluid dynamic equations due to Kelvin (1869) is the conservation of circulation. If we let C be an arbitrary closed curve moving with the fluid then the circulation associated with C is defined by the following line integral of the projection $q \cos\alpha$, of the fluid velocity $\vec{q}$ in the direction of the path multiplied by the line element of the path d S.

$$\Gamma = \oint\limits_C q \cos\alpha\, dS = \oint\limits_C \vec{q} \cdot d\vec{S} = \oint\limits_C u\,dx + v\,dy + w\,dz \qquad \text{(II A-2.5)}$$

Kelvin succeeded in showing that for a perfect fluid the circulation remains constant as the contour C moves. A flow for which Γ is identically zero for all such curves in the flow region is called an irrotational flow. A flow which starts from rest is such that $\Gamma = 0$ initially and hence must remain zero, the flow being irrotational. If Γ is to be identically zero for each closed path it is necessary that $\vec{q} \cdot d\vec{S}$ be the exact differential of some function ϕ, called the velocity potential, i.e.,

$$d\phi = \vec{q} \cdot d\vec{S} = u\,dx + v\,dy + w\,dz \qquad \text{(II A-2.6)}$$

In this case the velocity components u, v, w, can be written as space derivatives of the single function ϕ, i.e.,

$$u = \phi_x, \quad v = \phi_y, \quad w = \phi_z \qquad \text{(II A-2.7)}$$

In general the component of velocity in some direction is the space derivative of ϕ in that direction.

A streamline of the flow is by definition a curve in the fluid at an instant of time whose tangent at each point is in the direction of the resultant fluid velocity at this time and these points. A path line is by definition the actual trajectory of a fluid particle. If the motion is in a steady state, so that the velocity components at each position are independent of time, the streamlines have a permanent form and moreover coincide with the path lines. In such a case the velocity is simply the derivative of ϕ along the streamline, i.e., $q = \frac{d\phi}{dS}$. The stream function ψ for two-dimensional flows is defined in such a way that the streamlines are the lines ψ = constant. The continuity equation for two-dimensional flow is

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} \qquad \text{(II A-2.8)}$$

This is, however, the condition that the quantity $-v\,dx + u\,dy$ be the exact differential of some function, which in this case is the stream function ψ, i.e.,

$$d\psi = -v\,dx + u\,dy \qquad \text{(II A-2.9)}$$

It is readily shown that

$$\psi(x,y) = \int_{(a,b)}^{(x,y)} (-v\,dx + u\,dy) \qquad \text{(II A-2.10)}$$

represents the amount of fluid crossing, per unit time, any curve joining (a,b) with (x,y). Thus we can write

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \qquad \text{(II A-2.11)}$$

and hence ϕ and ψ satisfy the celebrated Cauchy-Riemann equations

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad \text{(II A-2.12)}$$

One can regard ϕ and ψ as the real and imaginary parts, respectively, of some analytic function $f\,(x+iy)$, that is

$$f\,(x+iy) = \phi\,(x,y) + i\psi(x,y)$$

where $i = \sqrt{-1}$. The study of analytic functions is one of the most important chapters in mathematics and it is for this reason that the study of two dimensional irrotational flows has been cultivated so diligently by mathematicians. These problems are essentially those of finding analytic functions which satisfy certain boundary conditions. The continuity equation is the celebrated Laplace's equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \text{(II A-2.13)}$$

which occurs in many branches of applied mathematics.

The equations of motion admit of an important first integral for irrotational flow called Bernoulli's equation. We note that for a particular velocity component, say u

$$\begin{aligned} \frac{du}{dt} &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad \text{(II A-2.14)} \\ &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} \\ &= \frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}(u^2+v^2) \end{aligned}$$

Multiplying the two equations of motion by dx, dy, respectively, and adding we find on integrating the result that

$$\frac{p}{\rho} + \frac{\partial \phi}{\partial t} + U + \frac{q^2}{2} = f\,(t) \qquad \text{(II A-2.15)}$$

where $f\,(t)$ is an arbitrary function of t and the body forces per unit mass are assumed to be derivable from a potential U, so that its components are

$$X = -\frac{\partial U}{\partial x}, \quad Y = -\frac{\partial U}{\partial y}$$

Bernoulli's equation can similarly be shown to hold for three-dimensional flows. Bernoulli's (1738) original derivation of this equation was based on energy considerations along a stream tube of flow (Lamb, p.21, 1945).

3. Linear Theory

a. Periodic Progressive Waves in Water of Constant Depth

We consider here only the solution of a boundary value problem for wave trains which are infinite in extent and of permanent form. We shall assume that the only external body force acting is that of gravity. The terminology used is indicated in Figure II A-1.

The differential equation describing the irrotational flow of fluid is the equation for the conservation of mass, namely

$$\nabla^2\phi = \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \qquad \text{(II A-3.1)}$$

as boundary conditions we require that

(1) at the rigid bottom the normal velocity be zero, i.e.,

$$\phi_y = 0 \quad \text{for } y = -d \qquad \text{(II A-3.2)}$$

(2) the pressure be zero at the free surface, i.e.,

$$\frac{p}{\rho} = -gy - \frac{\partial\phi}{\partial t} - \frac{1}{2}(\phi_x^2 + \phi_y^2 + \phi_z^2) = 0 \qquad \text{(II A-3.3)}$$

(3) a particle originally on the surface stays on the surface, i.e.,

$$p_t + u\,p_x + v\,p_y + w\,p_z = 0 \quad \text{on } y = \eta \qquad \text{(II A-3.4)}$$

In the linear theory we consistently neglect terms of the square of the ratio, H/L of wave-height to wave-length, in the velocity potential. Since the velocity potential ϕ must begin with terms of the order of H/L we shall neglect terms of the order of ϕ^2. Applying Bernoulli's equation to the free surface we find by (IIA-3.3)

$$-g\eta = \phi_t(x, o, z, t) + \phi_{ty}(x, o, z, t)\eta + \text{----}$$

where the right hand side represents the Taylor series expansion of $\phi_t(x, \eta, z, t)$ about $y = 0$. Neglecting second order terms we find

$$\eta = -\frac{1}{g}\,\phi_t(x, o, z, t) \qquad \text{(II A-3.5)}$$

By (II A-3.3) and (II A-3.4)

$$\phi_{tt} + \phi_{tty} + g(\phi_y + \phi_{yy}\eta) = 0$$

where all derivatives are to be evaluated at $y = 0$. In the linear theory this furnishes the surface condition

$$g\phi_y = -\phi_{tt} \quad \text{on } y = 0 \qquad \text{(II A-3.6)}$$

Our problem then reduces to the solution of Laplace's equation with the boundary conditions given on straight lines $y = 0$ and $y = -d$.

We assume that the waves travel with a constant velocity c in the positive x-direction so that the wave motion reduces to a steady state in a coordinate system moving in the positive x-direction with this velocity. In this new system, related to the original by $x' = x - ct$, equations (II A-3.1) and (II A-3.2) remain unchanged in form while equations (II A-3.5) and (II A-3.6) become

$$\eta = \frac{c}{g} (\phi_{x'})_{y=0} \qquad \text{(II A-3.7)}$$

$$g \phi_y = -c^2 \phi_{x'x'} \quad \text{on } y = 0 \qquad \text{(II A-3.8)}$$

Separating variables in equation (II A-3.1) we find typical solutions of physical interest of the form

$$\phi = A\, e^{ry} \begin{matrix}\sin\\ \cos\end{matrix}(m x') \begin{matrix}\sin\\ \cos\end{matrix}(n z)$$

where $$r^2 = m^2 + n^2$$

A typical solution satisfying (II A-3.2) is of the form

$$\phi = B \cosh r (d + y) \cos m x' \cos n y$$

Then by (II A-3.7) and (II A-3.8)

$$\eta = -\frac{c\, B}{g} m \cosh r d \sin m x' \cos n z$$

$$c^2 = \frac{g\, r}{m^2} \tanh r d \qquad \text{(II A-3.9)}$$

In terms of the wave amplitude $a = -\dfrac{c\, B\, m \cosh r d}{g}$

$$\eta = a \sin m (x - ct) \cos nz \qquad \text{(II A-3.10)}$$

$$\phi = -\frac{g\, a}{cm} \frac{\cosh r (d + y)}{\cosh r d} \cos m (x - ct) \cos nz \qquad \text{(II A-3.11)}$$

$$r^2 = m^2 + n^2 \qquad \text{(II A-3.12)}$$

The quantities m and n are related to the wave lengths L and L' in the x and z-directions respectively by $m = \frac{2\pi}{L}$, $n = \frac{2\pi}{L'}$.

Furthermore

$$\frac{p}{\rho g} = -y + \eta \frac{\cosh r\,(d+y)}{\cosh r d} \qquad \text{(II A-3.14)}$$

$$u = \phi_x = \frac{g\,a}{c} \frac{\cosh r\,(d+y)}{\cosh r d} \sin m\,(x - ct) \cos nz \qquad \text{(II A-3.15)}$$

$$v = \phi_y = -\frac{g\,a\,r}{cm} \frac{\sinh r\,(d+y)}{\cosh r d} \cos m\,(x - ct) \cos nz \qquad \text{(II A-3.16)}$$

$$w = \phi_z = \frac{g\,a\,n}{cm} \frac{\cosh r\,(d+y)}{\cosh r d} \cos m\,(x - ct) \sin nz \qquad \text{(II A-3.17)}$$

Waves for which L and L' are approximately equal are called short-crested, a term first employed by Jeffreys (1924). Waves for which L' is much larger than L are called long-crested. Such waves have relatively long straight crests characteristic of swell which has travelled a long distance. It has been observed that waves in deep water are most frequently short-crested but that as they move up on a sloping beach they tend to align themselves into long-crested waves. The formulas for long crested waves can conveniently be obtained from those given above by simply substituting $n = 0$.

The velocity of propagation of cylindrical waves ($n = 0$, $L' = \infty$) is given by

$$c_{cyl}^2 = \frac{g}{m} \tanh md = \frac{gL}{2\pi} \tanh \frac{2\pi}{L} d \qquad \text{(II A-3.18)}$$

Since

$$c^2 = \frac{gr}{m^2} \tanh rd \geq \frac{g}{m} \tanh md \qquad \text{(II A-3.19)}$$

we see that the cylindrical wave propagates more slowly than a short-crested wave having the same wave length L in depth d. In terms of wave lengths

$$c^2 = \frac{gL}{2\pi} D \tanh \frac{2\pi d}{L} D \qquad \text{(II A-3.20)}$$

where

$$D = \sqrt{1 + \left(\frac{L}{L'}\right)^2}$$

The relation between wave velocity, period and depth is plotted in Figure II A-2. using the formula $L = CT$. Note that the wave lengths are measured through lines of equal phase and not through successive lines of crests, the two being the same only in the limit as L' becomes infinite.

The orbital paths of the water particles are obtained from the velocity components (II A-3.15, 3.16, 3.17) by integration and subsequent expansion about a position of equilibrium. One finds that the orbits are ellipses with one axis in the direction of wave propagation and the plane of the ellipse making with a vertical plane an angle β given by

$$\tan \beta = \frac{n}{r} \tan nz \coth r\,(d+y) \qquad \text{(II A-3.21)}$$

For any particular vertical plane parallel to the direction of wave propagation the ellipses have their smallest inclination to the vertical at the still water level and become entirely horizontal at the bottom. Typical ellipses are shown in Figure II A-3. One finds that the length of the semi-axes of the ellipses in the x-direction is

$$A = \frac{a\,m}{r}\,\frac{\cosh r\,(d+y)}{\sinh rd}\cos nz \qquad \text{(II A-3.22)}$$

while the semi-length of the inclined axis is

$$B = a\,\frac{\sinh r\,(d+y)}{\sinh r\,d}\cos nz\,\sec\beta \qquad \text{(II A-3.23)}$$

If the waves are long-crested the ellipses will all be in vertical planes parallel to the direction of wave motion, ellipses at the same depth being congruent. They become more eccentric as one approaches the bottom reducing there to a straight line. The particles move in such a way that they are going forward in the direction of wave motion when the crest passes and in the opposite direction when the trough passes. The ratios

$$\frac{A_o}{a} = \frac{\cosh \frac{2\pi}{L}(d+y)\,D}{\sinh \frac{2\pi}{L}\,Dd}$$

and

$$\frac{B_o}{a} = \frac{\cosh \frac{2\pi}{L}(d+y)\,D}{\sinh \frac{2\pi}{L}\,D\,d}$$

which are equal respectively to $\frac{A}{a}$ sec nz D and $\frac{B}{a}$ sec nz cosβ are shown in Figures II A-4 and II A-5. These ratios essentially furnish the ratios of the horizontal and vertial amplitudes of particle oscillation to the wave amplitude. For $L' = \infty$, $D = 0$ and these plotted quantites are equal to the ratios of amplitudes of particle oscillations to wave amplitude.

A quantity of considerable interest in interpretation of readings of underwater pressure recorders is the bottom pressure variation beneath a regular periodic wave train. We find from equation II A-3.14 that this pressure variation Δp is given by

$$\frac{\Delta p}{\rho g} = \frac{\eta}{\cosh rd} \qquad \text{(II A-3.24)}$$

Knowing the pressure response factor $K = \operatorname{sech} \frac{2\pi d}{L} D$ and Δp the surface height of such an ideal wave train can be computed. Since, however, one can only obtain L from a single record one would expect to obtain different results depending on the value chosen for L'. This is illustrated in Figure II A-6, where K is plotted versus d/L for several typical values of L/L'. Since $D \geq 1$, $\cosh \frac{2\pi}{L} d\,D \geq \cosh \frac{2\pi d}{L}$ and we see from equation II A-3.24 that if the usual pressure response factor $K_{cylind} = \operatorname{sech} \frac{2\pi d}{L}$

is applied to a record resulting from a train of short-crested waves the wave height will be underestimated. The wave heights of ocean waves are usually underestimated from underwater pressure records although there are many contributing factors such as those due to the combination of wave trains of different frequencies. We shall return to this question in a later section. As an example, suppose $d/L = 0.25$, then the actual surface elevation will be 4/3 of its estimated value if L' is approximately equal to $1.5L$, a not unreasonable condition for ocean waves.

In deep water, for which d/L is large (usually for purposes of wave velocities $d/L > 1/2$) the formulas simplify considerably. We find for example, that

$$\frac{p}{\rho g} = -y + \eta e^{ry}$$

$$u = \frac{ga}{c} e^{ry} \sin m(x - ct) \cos nz$$

$$v = -\frac{g}{c}\frac{a\,r}{m} e^{ry} \cos m(x - ct) \cos nz \qquad \text{(II A-3.25)}$$

$$w = \frac{g}{c}\frac{a\,n}{m} e^{ry} \cos m(x - ct) \sin nz$$

$$c^2 = \frac{gL}{2\pi}\sqrt{1+\left(\frac{L}{L'}\right)^2}$$

The pressure and particle velocities decrease very rapidly with depth, the dependence on depth being given by $e^{+\frac{2\pi y}{L}D}$. This decrease is much more rapid for short-crested waves than for long-crested waves having the same L. The orbits are only circular for long-crested waves.

The mean potential energy per unit surface area is

$$E_p = \frac{\frac{1}{2}\rho g}{L\,L'} \int_0^L \int_0^{L'} \eta^2 \, dx\, dz$$

$$= \frac{1}{8} g \rho a^2 \qquad \text{(II A-3.26)}$$

For small oscillations the kinetic energy is equal to the potential energy and consequently the total energy is

$$E = \frac{1}{4} g \rho a^2 \qquad \text{(II A-3.27)}$$

The corresponding total energy of cylindrical waves is twice as great, namely

$$E_{cyl} = \frac{1}{2} g \rho a^2 \qquad \text{(II A-3.28)}$$

The theory of progressive wave motions in heterogeneous heavy fluids has been investigated by Love (1891) and Burnside (1889) . Under the assumption that the equilibrium value of the density is a function of the vertical depth coordinate only it can be shown that (see Lamb, 1945, p. 378)

$$\sigma^2 = g\,k \text{ or } c^2 = \frac{g\,L}{2\pi} \tag{II A-3.29}$$

which is the same relation between period and wave length found to hold for homogeneous liquids of infinite depth.

b. Standing Waves:

Standing waves can be obtained by superimposing two progressive waves having the same characteristics but travelling in opposite directions. The surface profile is of the form

$$\eta = a \cos mx \cos nz \cos \sigma t \tag{II A-3.30}$$

It owes its name to the fact that the wave profile does not progress horizontally; for example, the nodal lines are fixed.

The pressure variations on the bottom beneath standing waves can be estimated using a simple device due to Longuet-Higgins and Ursell (1948). Consider one of the regions bounded by vertical planes through successive maxima and minima, parallel and perpendicular to the x and y axes. The mass of water in this region always contains the same particles since there is no flow across these vertical planes. Summing the equations of motion for each particle of mass, m, in this region and cancelling internal forces we find that the vertical component of the total external force acting on the mass is

$$F = \sum m \frac{d^2y}{dt^2} = \frac{1}{g} \frac{d^2E_p}{dt^2} \tag{II A-3.31}$$

The forces across the vertical boundaries contribute nothing to F; the pressure p_o at the free surface contributes a downwards force of $L\,L'\,p_o$ and gravity a constant force $L\,L'\,\rho\,g\,d$. Hence $F = L\,L'\,(p - p_o - \rho g\,d)$, where p is the mean pressure on the bottom.

Now

$$E_p = \frac{1}{2}\,g\rho \int_0^L \int_0^{L'} \eta^2\,dx\,dz$$

$$= \frac{1}{2}\,g\rho\; a^2\,\frac{L\,L'}{4} \cos^2 \sigma t \tag{II A-3.32}$$

By equation II A-3.1 we find

$$\frac{p - p_0}{\rho} = g\,d - \frac{1}{4}\;a^2 \sigma^2 \cos 2\sigma t \tag{II A-3.33}$$

This differs from the formula of Longuet-Higgins and Ursell by a factor of 1/2 in the last term. This result shows that one has an unattenuated pressure on the bottom beneath standing waves which has a frequency 2σ of twice that of the surface waves. This has been advanced as the cause of microseisms and has been discussed in considerable detail by Longuet-Higgins (1950). Experimental confirmation has been given by Cooper and Longuet-Higgins (1951). In the general case in which two long-crested waves of equal length but of different amplitudes a_1 and a_2 encounter one another travelling in opposite directions the unattenuated pressure variation is given by

$$\Delta p = 2\,\rho\, a_1\, a_2\, \sigma^2 \cos 2\sigma t \tag{II A-3.34}$$

which agrees with the previous result for long-crested waves when $a_1 = a_2 = \frac{a}{2}$. Cooper and Longuet-Higgins suggest that this formula can be used to find the reflection coefficient, R, of various objects including a sloping beach by measurements of Δp, for in terms of the amplitude, a, of the incident wave one can write

$$\Delta p = 2\rho\, R\, a^2\;\sigma^2 \cos 2\sigma t \tag{II A-3.35}$$

They find experimentally that the reflection coefficient is nearly unity for vertical plane reflectors, and decreases steadily with the inclination to the horizontal having values of 0.87 for 45° and 0.08 for 15°. These experimental results are important in establishing the validity of the assumption commonly used in theoretical treatments of waves on sloping beaches, namely that the waves are progressive, all the energy being dissipated at the shore in breaking and friction.

c. Waves at the Common Surface of Two Fluids:

The oscillations of two superimposed fluids were first investigated by Stokes (1847). The extension of the theory to the case of any finite number of superimposed fluids has been investigated by Webb (1884) and Greenhill (1887). Suppose as an illustration of the methods that the lower fluid is of a constant finite depth d and the upper fluid is unlimited. Taking the origin in the mean level of the interface we have

$$\begin{aligned} \phi &= A \cosh k\,(d + y)\cos kx\; e^{i\sigma t} \\ \phi' &= A'\; e^{-ky} \cos kx\; e^{i\sigma t} \end{aligned} \tag{II A-3.36}$$

the primes referring to the upper fluid. If the free surface profile has the equation

$$\eta = a \cos k\,x\; e^{i\sigma t} \tag{II A-3.37}$$

we find, using the free surface condition $\frac{\partial \eta}{\partial t} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0}$, that

$$k A \sinh kl = -k A' = i\sigma a \qquad \text{(II A-3.38)}$$

Continuity of pressure at the interface implies that

$$\rho\left(-\frac{\partial \phi}{\partial t} - gy\right) = \rho'\left(-\frac{\partial \phi'}{\partial t} - gy\right) \qquad \text{at } y = 0$$

Substituting for ϕ and ϕ' we find

$$\rho(-i\sigma A \cosh kd - ga) = \rho'(-i\sigma A' - ga) \qquad \text{(II A-3.39)}$$

Eliminating A and A' by means of equation (II A-3.3) the wave velocity is found to be

$$c = \sqrt{\frac{g}{k} \tanh kl \frac{\rho - \rho'}{\rho + \rho' \tanh kl}} \qquad \text{(II A-3.40)}$$

The effect of the upper fluid is to decrease the wave velocity of waves of given length in water of given constant depth in the ratio

$$\sqrt{\frac{1 - \frac{\rho'}{\rho}}{1 + \frac{\rho'}{\rho} \tanh kd}} \qquad \text{(II A-3.41)}$$

which increases as the water becomes shallower. It is interesting to notice that the tangential velocity changes sign at the interface; in other words the interface is a vortex sheet. The decay of the discontinuity due to viscosity is discussed by Harrison (1908); the vortex sheet is replaced by a vorticity film.

When the two fluids are moving parallel to each other with velocities of U and U' we can treat the motion approximately as one of small oscillations about a steady state (Lamb, 1945, p. 373). Writing the velocity potentials in the form $\phi = -Ux + \phi_1$, $\phi' = -U'x + \phi_1'$, a solution can be found in a manner similar to that previously discussed. The velocity of propagation is

$$c = \frac{\rho U + \rho' U'}{\rho + \rho'} \pm \sqrt{\frac{g}{k} \frac{\rho - \rho'}{\rho + \rho'} - \frac{\rho\rho'}{(\rho + \rho')^2} (U - U')^2} \qquad \text{(II A-3.42)}$$

The interface is unstable for

$$(U - U')^2 > \frac{g}{k} \frac{\rho^2 - \rho'^2}{\rho\rho'} \qquad \text{(II A-3.43)}$$

which includes that of very small wave lengths. In such a case slight disturbances would cause waves to form and grow. The case $\rho = \rho'$, $U = U'$ requires a separate investigation. This latter situation is of interest in connection with the flapping of sails and flags. A slight disturbance will grow with amplitude increasing proportionally to the time. For a complete discussion see Rayleigh (1879).

d. Linear Superposition of a Finite Number of Waves. Simple Groups:

For waves of infinitesimal steepness velocity potentials and wave heights can be simply added due to the approximate linearity of the equations of motion. As an example consider two sinusoidal waves having the same amplitude and slightly different wave lengths and phase angles. For such waves we write

$$\eta_1 = a \cos (k_1 x - \sigma_1 t + \theta_1)$$
$$\eta_2 = a \cos (k_2 x - \sigma_2 t + \theta_2) \qquad \text{(II A-3.44)}$$

and assume that $k_1 \approx k_2$, $\sigma_1 \approx \sigma_2$, $\theta_1 \approx \theta_2$, the latter being constant phase angles. If we let $k_1 = k - \alpha$, $k_2 = k + \alpha$, $\sigma_1 = \sigma - \beta$, $\sigma_2 = \sigma + \beta$, $\theta_1 = \theta - \delta$, $\theta_2 = \theta + \delta$ we find the resultant surface elevation is

$$\eta = \eta_1 + \eta_2 = 2a \cos (\alpha x - \beta t + \delta) \cos (kx - \sigma t + \theta) \qquad \text{(II A-3.45)}$$

If the wave lengths are nearly equal α and β are small and the first factor $2a \cos (\alpha x - \beta t + \delta)$ can be considered to be a slowly varying amplitude function. The surface profile has the appearance of a rapidly varying sinusoidal function with a slowly varying amplitude alternating between $\pm 2a$ and 0. Since the successive groups are separated by bands of nearly still water the consecutive groups can be considered to be nearly independent of each other. The group velocity of the combination is equal to the wave velocity of the amplitude function, namely,

$$c_g = \frac{\beta}{\alpha} \approx \frac{d\sigma}{dk} \text{ if } \alpha \text{ and } \beta \text{ are small.} \qquad \text{(II A-3.46)}$$

Since

$$c^2 = \frac{g}{k} \tanh k d \qquad \text{or}$$

$$\sigma^2 = g k \tanh k d$$

we find

$$c_g = \frac{1}{2} c \left(1 + \frac{2kd}{\sinh 2kd}\right) \qquad \text{(II A-3.47)}$$

In deep water $c_g = \frac{1}{2} c$ so that the individual elevations making up the group are observed to run through the group with twice the group velocity increasing in height until the maximum amplitude 2a is attained and then dying out in front of the disturbance. Thus, since one always has a range of wave lengths present in nature one must calculate the time of arrival of a particular narrow frequency

band of swell as if it travelled not with the wave velocity appropriate to its period, but with exactly 1/2 of this velocity. This explanation seems to have originated with Stokes (1876) after the occurrence of the phenomenon was pointed out to him a few years earlier by Froude (1873). The group velocity appears then to be of particular importance and even in acoustics and optics the wave velocity is of importance chiefly because it coincides with the group velocity.

The simple group is generated by superimposing two sinusoidal wave trains of neighboring wave lengths and frequencies and of the same amplitude. For short-crested waves we have

$$\eta = a \cos (m_1 x - \sigma_1 t) \cos n_1 z + a \cos (m_2 x - \sigma_2 t) \cos n_2 z$$

which can be written in the form

$$\frac{\eta}{a} = 2 \cos (\alpha x - \beta t) \cos (mx - \sigma t) \cos \gamma z \cos nz + 2 \sin (\alpha x - \beta t) \sin (mx - \sigma t) \sin \gamma z \sin nz$$

where

$$m_1 = m - \alpha,\ m_2 = m + \alpha,\ \sigma_1 = \sigma - \beta,\ \sigma_2 = \sigma - \beta$$

$$n_1 = n - \gamma,\ n_2 = n + \gamma$$

The amplitude factors $2\,a \cos (\alpha x - \beta t) \cos \gamma z$ and $2\,a \sin (\alpha x - \beta t) \sin \gamma z$ are slowly varying functions of x and z giving rise to a two dimensional beat pattern. As an example take $m = n, \alpha = \gamma, \sigma^2 = \sqrt{2\ gm}, \alpha = \frac{m}{10}$. Figure II A-7 illustrates a typical set of cross sections of the free surface.

Two waves with different lengths travel with different group velocities and therefore separate from each other more and more as time goes on. Since the intermediate wave lengths remain the same the number of waves increases with time. The waves produced by a distant storm of small duration and observed at a distant point should be rather regular in height and period over a small time interval, such as 20 minutes, with a period slowly decreasing throughout a longer interval. The waves appear to run through the group gradually disappearing at the front of the group while fresh waves arise in the rear. The particular situation is not general but depends upon the condition that the velocity increases with wave length. That other conditions can occur will become evident if one considers capillary waves, where surface tension rather than gravity is the controlling factor. The wave velocity of capillary waves as given by Lamb (1945, p.457) is

$$c = \left(\frac{2\pi T_1}{\rho + \rho'}\right)^{\frac{1}{2}} L^{-\frac{1}{2}} \qquad \text{(II A-3.48)}$$

where T_1 is the stress between two adjacent portions of the surface per unit boundary length. The group velocity is then

$$c_g = c - L \frac{dc}{dL} = \frac{3}{2} c \qquad \text{(II A-3.49)}$$

In general, for combined capillary and gravity waves, the group velocity is greater or less than the wave velocity, according as $L \lessgtr L_m$, where

$$L_m = 2\pi\sqrt{\frac{T_1}{(\rho - \rho')g}} \qquad \text{(II A-3.50)}$$

For a water surface L_m = 1.8cm≈2/3 in. showing that only for small ripples will the group outrun the individual waves. A situation in which both gravity and capillary type waves occur simultaneously is found when a small object, such as a fish line, is moved through still water. On the upstream side the wave length is small and capillary waves are present, while on the downstream side the waves are longer and gravity waves occur. In order for both sets of waves to maintain a fixed position relative to the object they must move with the same velocity relative to the water. Due to capillarity waves have in addition to a minimum wave velocity also a minimum group velocity. Thus for a ring-shaped disturbance there is smooth water near the origin. The minimum wave velocity on water is 23 cm/sec. The minimum group velocity is 18 cm/sec, the corresponding wave velocity being 28 cm/sec, and the wave length 4.6 cm.

Consider now a surface record obtained for a definite point in space when two wave trains are added. The character of the surface depends upon the relative size of the wave period (Manley, 1945). If one simply adds two sinusoidal wave forms having the same period one obtains another sinusoidal wave of the same period. When the periods are not the same it is of interest and of considerable importance to study the two separate cases where the periods are nearly the same and where they are very different. The first case corresponds to almost pure swell which has very nearly separated out into its component wave trains at a considerable distance from the storm. The second situation arises when relatively short period local waves are superimposed on a long swell.

When the frequencies of the waves are approximately equal the phenomenon of beating takes place as is described above for the simple wave groups. One has an approximately sinusoidal wave form with a slowly varying sinusoidal form of amplitude. Such a wave form can be easily recognized and analyzed by using the rule (Manley, 1945) that:

> "The resultant wave has the same apparent frequency of the major component (i.e., that with the greater amplitude) and its amplitude varies between the sum and the difference of the component amplitudes, the beat frequency being the difference between the frequencies of the components."

To determine whether the minor component is of higher or lower frequency we have the rules:

(1) If the minor component is of higher frequency than the major, the distance between successive peaks at the bulge is less than the corresponding distance at the waist.

(2) If the minor component is of lower frequency than the major, the distance between successive peaks at the bulge is greater than the corresponding distance at the waist.

When there are only two components the envelope of the crests is the mirror image in the center line of the envelope of the troughs.

When the frequency of one wave is much greater than that of the other a new type of wave form results. The characteristics of this type are:

(1) The high frequency components appears in the resultant wave as a "ripple" superimposed on the low frequency component.

(2) Both envelopes represent, in amplitude and phase, the low frequency component.

(3) The width of the envelope strip is equal to the amplitude of the high frequency component.

The rules in more general situations are too complicated to be of much practical use.

The rules may prove convenient in the interpretation of those underwater pressure instrument records which seem to consist of only two harmonic components. As a check of some existing methods of determining surface profiles suppose we start from a surface profile formed by adding two sine waves of given amplitude. The theoretical pressure response curve is then easily obtained. This curve can be analyzed by existing methods to obtain a surface profile which can then be compared with the correct profile in order to assess the errors involved in the methods. Consider then a time history of the surface at a particular point which is given by

$$\eta = a \cos \sigma_1 t + b \cos \sigma_2 t \qquad \text{(II A-3.51)}$$

The pressure response on the bottom in water of depth d is

$$\frac{\Delta p}{\rho g} = \frac{a}{\cosh k_1 d} \cos \sigma_1 t + \frac{b}{\cosh k_1 d} \cos \sigma_2 t \qquad \text{(II A-3.52)}$$

By the rules given above one could compute σ_1, σ_2, $a/\cosh k_1 d$, $b/\cosh k_2 d$ and hence a and b thus determining the original surface profile. However, if this procedure were not followed the methods used for example, by Putz, (1950) and Snodgrass and Stiling (1950) for the given underwater pressure record would lead to errors. As a numerical example, suppose we consider the case in which $d = 50$ ft., $L_1 = 250$ ft., $L_2 = 150$ ft. and a and b are both unity. The surface profiles and the underwater pressure record are shown in Figure II A-8. The methods of Putz and Snodgrass lead to quite erroneous surface histories. For example, one of the known crests is not predicted by either approximate method. The method used by Snodgrass in employing an average pressure response factor for the entire record leads to an underestimate of the wave heights.

e. Energy Transmission:

This subject was first considered by Reynolds (1877). Progressing waves usually transmit energy as in the wave moving along a rope one end

of which is fixed and the other is rapidly wiggled, in which case all of the energy is transmitted. Waves caused by wind on a wheat field, on the other hand, transmit no energy. Reynolds concluded that for waves of permanent form in which the particles revolve in circles kinetic energy cannot be transmitted since the wave velocity is constant. The potential energy is transmitted for the particles move forward when they are above their mean position and backward when they are below.

For a travelling disturbance having the form of a wave packet the group velocity must be equal to the rate of energy transmission so that the group may be maintained, for in an elementary wave the energy is proportional to H^2. If we assume that the energy density at any position and time is proportional to the square of the surface height we see that by squaring the surface height for a wave pocket and taking the mean value we obtain a result equal to the square of the amplitude function. The work, dW done at a surface element dy.l in time dt is simply

$$dW = p\,u\,dy\,dt = p\frac{\partial \phi}{\partial x}\,dy\,dt$$

The work performed at the cross-section $x = 0$ during one period T represents the flow of energy through this cross-section in time T. Now this is

$$S = \int_0^T dt \int_{-\infty}^{\eta} p\frac{\partial \phi}{\partial x}\,dy = -\int_0^T dt \int_{-d}^{\eta} dy\left(\rho\frac{\partial \phi}{\partial t}+\ldots\right)\frac{\partial \phi}{\partial x} \qquad \text{(II A-3.53)}$$

The terms $\rho g\, y +$ const omitted in the expression for p when multiplied by the period factor $\frac{\partial \phi}{\partial x}$ vanish when the integration with respect to t is carried out.

Now for a simple long-crested wave train in water of constant depth

$$\phi = \frac{ga}{\sigma}\,\frac{\cosh k(d+y)}{\cosh kd}\,\cos(kx-\sigma t) \qquad \text{(II A-3.54)}$$

Since $\cosh k(d+\eta) = \cosh kd + O\left(\left(\frac{H}{L}\right)^2\right)$ we find on consistently neglecting terms of order $(H/L)^2$

$$S = \frac{\rho g^2 a^2 k}{\sigma \cosh^2 kd}\int_0^T dt \int_{-d}^{0}\cosh^2 k(d+y)\,\sin^2\sigma t\,dy$$

$$= \frac{\rho g^2 a^2 k}{\sigma \cosh^2 kd}\,\frac{\pi}{\sigma k}\left(\frac{kd}{2}+\frac{\sinh 2kd}{4}\right)$$

$$= \frac{1}{4}g\rho a^2 L\left(1+\frac{2kd}{\sinh 2kd}\right) \qquad \text{(II A-3.55)}$$

The total energy per unit wave length and unit crest distance has already been shown to be

$$E = \frac{1}{2}g\rho a^2 L$$

Thus

$$\frac{S}{E} = \frac{1}{2}\left(1 + \frac{2\ kd}{\sinh 2\,kd}\right) \qquad \text{(II A-3.56)}$$

From previous work this is also equal to $\frac{c_g}{c}$. The rate of energy transmission is then equal to the group velocity.

This identification of the rate of energy transmission with the group velocity is generally valid for a group of waves (Lamb, p.383). Let P be the center of a group, and Q that of the relatively quiet region ahead of P. During a time τ which extends over several periods, but which is small compared to the transit time of a group, the center of the group will have moted to P', such that P P' = $c_g\tau$ and the space between P and Q will have gained a corresponding amount of energy.

The energy flux between neighboring orthogonals a distance apart of $\Delta \ell$ is

$$\frac{1}{4}\, g\, \rho\, a^2\, L\left(\frac{m}{r}\right)\left(1 + \frac{2\ kd}{\sinh\ 2\,kd}\right)\ \Delta \ell \qquad \text{(II A-3.57)}$$

For conservation of energy flux we must have

$$\frac{a}{a_o} = \sqrt{\frac{c_o}{c}\ \frac{\bar{n}_o}{\bar{n}}}\ \sqrt{\frac{\left(\frac{m_o}{r_o}\right)^2}{\left(\frac{m}{r}\right)^2}}\ \sqrt{\frac{\Delta \ell_o}{\Delta \ell}} \qquad \text{(II A-3.58)}$$

$$= \frac{\bar{H}}{\bar{H}_o}\ K_D$$

$$\bar{n} = \frac{1}{2}\left(1 + \frac{2kd}{\sinh 2kd}\right)$$

where $K_D = \sqrt{\frac{\Delta \ell}{\Delta \ell_o}}$ indicates the extent to which the wave height is modified by refraction, and where $\frac{H}{H_o}$ indicates the ratio of wave heights for waves approaching at right angles to the depth contours. This latter quantity is plotted in Figure II A-9 vs. d/L_o together with the ratio c/c_o. Suquet (1949) has discovered by plotting a/a_o and c_g/c_o vs. d/L_o the curious fact that a minimum of a/a_o corresponds to a maximum of c_g/c_o.

f. Energy Transmission over a Submerged Rectangular Breakwater:

The energy transmitted forward in a wave incident on a submerged barrier is partially transmitted and partially reflected. The transmitted energy is reformed into a wave motion subject to shoaling over the barrier. The wave length will decrease over the barrier and lengthen to the original value after leaving the barrier if the period is assumed to be constant. We exclude the cases of non-regular transition where the period changes abruptly at the edge of the barrier.

The energy transmitted forward with the incident wave motion per unit time and below a depth d-h is (Lamb, 1945, p. 383)

$$E_R = +\int_d^{d-h} p \frac{\partial \phi}{\partial x}\, dy$$

where

$$p = \rho\left(-\frac{\partial \phi}{\partial t} - g\, y\right)$$

$$\phi = \frac{g\, H}{2\sigma} \frac{\cosh k\,(y + d)}{\cosh kd} \cos\,(kx - \sigma t)$$

$$k = \frac{2\pi}{L}$$

$$\sigma = \frac{2\pi}{T}$$

Substituting and carrying out the integration we find for the time average of E_R

$$\overline{E_R} = \frac{1}{16}\, g\rho\, H^2\, c\, \frac{\sinh 2\,kh}{\sinh 2\,kd}\left(1 + \frac{2\,kh}{\sinh 2\,kh}\right)$$

The total average energy transmitted per unit time in the incident wave is obtained by setting $h = d$ or

$$\overline{E_i} = \frac{1}{16}\, g\rho H_s^{\,2}\, c\left(1 + \frac{2\,kd}{\sinh 2\,kd}\right)$$

The average energy transmitted per unit time over the barrier is

$$\overline{E_T} = \frac{1}{16}\, g\rho\, H_s^{\,2} c\left(1 + \frac{2\,kd}{\sinh 2\,kd}\right)\left[1 - \frac{\sinh 2\,kh}{\sinh 2\,kd}\; \frac{1 + \dfrac{2\,kh}{\sinh 2\,kh}}{1 + \dfrac{2\,kd}{\sinh 2\,kd}}\right] \qquad \text{(II A-3.59)}$$

There are two cases to be considered here, if the wave period is assumed to remain constant. The wave length and wave velocity depend upon the depth according to the relations

$$k = \frac{\sigma^2}{g} \coth kd$$

$$c = \frac{g}{\sigma} \tanh kd \qquad \text{(II A-3.60)}$$

Case 1. The transmitted wave is measured in water of depth d shoreward of the barrier. In this case k and c are equal to their original values for the incident wave. Since $\overline{E_T}$ must have the same form as $\overline{E_i}$ the transmission coefficient is

$$\left(\frac{H_i}{H_s}\right)_d = \sqrt{1 - \frac{\sinh 2\,kh}{\sinh 2\,kd}\; \frac{n\,(h)}{n\,(d)}} \qquad \text{(II A-3.61)}$$

where

$$n\ (h) = \frac{1}{2}\left(1+\frac{2\ kh}{\sinh\ 2\ kh}\right)$$

Case 2. The transmitted wave is measured over the barrier as for a reef of type 1. The transmitted wave is subject to shoaling over the barrier, its height and length varying in such a wave that energy and wave period are conserved. The average energy transmitted per unit time over the barrier can be written in the form

$$\overline{E_T} = \frac{1}{16}\, g \rho\, H_i^{\,2}\, c_a \left(1+\frac{2\ k_a\ a}{\sinh\ 2\ k_a\ a}\right) \qquad \text{(II A-3.62)}$$

where the subscript a refers to conditions in water of depth a = d-h. Comparing II A-3.59 and II A-3.62 we find for the transmission coefficient

$$\left(\frac{H_i}{H_s}\right)_a = S\left(\frac{H_i}{H_s}\right)_d$$

$$S = \text{shoaling factor} = \sqrt{\frac{c_d}{c_a}\frac{n_d}{n_a}\frac{(d)}{(a)}}$$

The shoaling factor can be readily determined from the publication of Wiegel (1948). It gives the relative wave height in a wave moving from depth d to depth d-h without reflection.

Two important special cases merit individual attention. For shallow water (kd $\ll$ 1) one finds

$$\left(\frac{H_i}{H_s}\right)_d \cong \sqrt{1-\frac{h}{d}}$$

$$\frac{H_i}{H_s}\,{}_d \cong \sqrt[4]{1-\frac{h}{d}}$$

For deep water (kd $\gg$ 1) one finds

$$\left(\frac{H_i}{H_s}\right)_d \cong \sqrt{1-4\,e^{-2kd}\,(\sinh\ 2\ kh)\ n\ (h)}$$

which is nearly equal to unity unless the barrier is near the free surface in which case

$$\left(\frac{H_i}{H_s}\right)_d \cong \sqrt{1-e^{-2ka}}$$

For shallow water and for deep water in case the barrier is near the surface, the transmission coefficients are the same as if it is assumed initially that a fixed fraction of the total energy associated with the incident wave is reflected.

A comparison of the transmission coefficient with experimental data is given in Figure II A-10 as taken from a report by Johnson, Fuchs and Morison (1951).

4. Waves of Finite Height:

a. Periodic Waves

When the wave steepness, H/L, is no longer very small the approximations made in the preceding section are poor representations of the wave motion. This is of particular significance for waves in shallow water. The slowness of convergence of the Stokes' series approximations has lead to the use of other methods of approximation discussed in later sections. The determination of the higher Stokes approximations is straightforward but tedious. One continues the expansion of the velocity potential about the still-water level $y = 0$. Using previously stated conditions one obtains a non-linear surface condition for the potential on the plane $y = 0$, consisting of an infinite series containing partial derivatives of ϕ. The solution of this equation subject to the bottom condition, proceeds by successive approximations. One additional approximation has been carried out for short-crested waves in a recent report by Fuchs (1951). The calculations show that ϕ has the form

$$\begin{aligned}\phi &= A \cosh r\,(d + y) \cos m\,(x - ct) \cos nz \\ &+ B \cosh 2r\,(d + y) \sin 2m\,(x - ct) \cos 2\,nz \\ &+ D \cosh 2m\,(d + y) \sin 2m\,(x - ct) \end{aligned} \qquad \text{(II A-4.1)}$$

where

$$D = \frac{A^2}{8\,cm} \frac{4n^2 \cosh^2 rd - 3r^2}{2 \cosh 2\,md - \frac{m}{r} \frac{\sinh 2\,md}{\tanh rd}}$$

$$B = -\frac{3\,A^2}{16\,cm} \frac{r^2}{\sinh^2 rd}$$

The wave profile has the equation

$$\begin{aligned}\eta &= a \sin m\,(x - ct) \cos nz - \frac{a^2 r}{4 \sinh 2\,rd}\left(2 \sinh^2 rd - 1 + \frac{3 \cosh 2\,rd}{\sinh^2 rd}\right) \\ &\cos 2m\,(x - ct) \cos 2\,nz - \frac{a^2\,(m^2 - n^2 \cosh 2\,rd)}{4\,r \sinh 2\,rd} \cos 2\,nz \\ &- \frac{a^2}{4\,r \sinh 2\,rd}\left[\frac{(3r^2 - 4n^2 \cosh rd)}{\cosh 2md - \frac{m}{2r}\frac{\sinh 2\,md}{\tanh md}}\right. \\ &\left. - m^2 \cosh^2 rd + 3r^2 \sinh^2 rd + n^2 \cosh^2 rd\right] \cos 2m\,(x - ct) \end{aligned} \qquad \text{(II A-4.2)}$$

A comparison is given in Figure II A-11 between the coefficients of the second harmonic terms in the surface profile of long and short-crested waves in terms of the same initial steepness in deep water, the transformation being given in Section 3. This figure shows that for a mixture of long and short-crested waves having the same steepness in deep water the long-crested wave will be predominant in shallow water. The predominant second harmonic term for short-crested waves is proportional to $\cos 2\, mx \cos 2\, nz$.

This expression (II A-4.2) reduces essentially to Stokes' classical second order formula for the surface elevation when $n = 0$. It is of course not possible to express this surface elevation by a simple superposition of two cylindrical waves of second order meeting at an angle even though this was possible to the first order. A pictorial sketch of the free surface is given in Figure IIA-3 for conditions made clearer in Section 3.

The particle motion can be determined by expanding the particle coordinates about an equilibrium position, neglecting third order terms and integrating the resulting formulae for particle velocity components. The resulting motion is quite complicated. One finds that in addition to the oscillatory motion the particles are propagated in the direction of the wave motion with a constant velocity

$$U = \frac{a^2\, cm^2 \coth^2 rd}{4r^2}\left[r^2(\tanh^2 rd + 1) + \cos 2\, nz\, (r^2 \tanh^2 rd + m^2 - n^2)\right] \qquad \text{(II A-4.3)}$$

Thus U is a maximum for lines in the direction of propagation passing through crests and troughs and a minimum for lines midway between these. These maximum mass transport velocities coupled with the large translational velocities of the short-crested waves breaking on these lanes give rise to eddy cells in the near shore circulation. One would expect then that the separation between lanes of maximum current would vary with the crest wave length being large for long-crested waves and small for short-crested waves. This behavior has been noted by Shepard and Inman (1950) in their studies of rip currents.

The recognition of a regular short-crested system of waves should be of importance for operators of small craft traversing a surf zone. Those craft which keep to lanes parallel to the direction of wave motion and midway between crests will encounter the smallest wave action.

The approximations have been carried out even farther for cylindrical waves in water of constant finite depth. Stokes proceeded to a third approximation, neglecting terms of the order of $(H/L)^4$. The steepening of the crests and broadening of the troughs becomes more pronounced than that indicated previously especially in shallow water. The principal difference between this approximation and that given earlier, neglecting $(H/L)^2$, is that the wave velocity, in a system of reference in which the mean horizontal wave velocity is zero, depends upon the wave steepness according to the formula

$$c^2 = \frac{g}{m} \tanh md \left[1 + \left(\frac{\pi H}{L}\right)^2 \frac{2\cosh^2 2\, md + 2\cosh 2\, md + 5}{8 \sinh^4 md}\right] \qquad \text{(II A-4.4)}$$

The correction to the first order result increases rapidly as $d/L \rightarrow 0$. For deep water

$$c^2_{deep} \cong \frac{g}{m}\left[1+\left(\frac{\pi H}{L}\right)^2\right] \qquad \text{(II A-4.5)}$$

For shallow water

$$c^2_{shallow} \cong gd\left[1+\frac{9}{32}\left(\frac{H}{d}\right)^2\frac{1}{m^2d^2}\right] \qquad \text{(II A-4.6)}$$

Stokes (1846) pointed out that it is possible to have any value for the wave velocity depending on one's reference system. In deriving formula IIA-4.4 Stokes assumed that the velocity is that with which the profile propagates in a system of referencein which the mean horizontal velocity in time of the water particles is zero at every point. The second definition of velocity suggested by Stokes involves a reference system in which the horizontal projection of the velocity of the center of gravity of the mass of fluid contained between two sufficiently distant planes, normal to the direction of propagation, is zero in the mean with respect to time. This latter definition has been employed by Biesel (1951) who finds that in this system of reference.

$$c^2 = \frac{g}{m}\tanh md\left[1+\frac{1}{2}\left(\frac{H\pi}{L}\right)^2\left(\frac{\cosh 4\,md+2\cosh 2\,md+6}{8\sinh^4 md}-\frac{1}{2\sinh^2 md}-\frac{\coth md}{md}\right)\right] \qquad \text{(II A-4.7)}$$

This definition is of importance for waves propagating in a limited canal, the center of gravity of the fluid mass contained in this canal being fixed in the mean.

Rayleigh (1876) was the first one to show on general principles that a mass transport is necessarily associated with a two-dimensional irrotational motion. This question has been investigated rigorously by Levi-Civita (1922) with the same conclusions. He has shown moreover that the mass transport velocity is of the order $(H/L)^2$ but that it can nevertheless be computed from the formulas of the well-known linear theory. He finds that the mean transport velocity, γ, averaged over the depth of water, is given by

$$\frac{\gamma}{c} = \frac{1}{2}\left(\frac{a}{d}\right)^2\frac{gd}{c^2} \qquad \text{(II A-4.8)}$$

The mass transport is assumed to vanish for the lower layers. In connection with this Biesel (1949) has shown that for such permanent periodic swell the mean value of the rotation is equal to the derivative of the mean velocity in the lower layers with respect to depth. If the motion is irrotational it follows that the mean velocity is a constant which can be taken to be zero by appropriate choice of the reference system. Conversely it can be shown that if this mean velocity is zero, the movement is irrotational.

For long-crested waves the series approximations used by Stokes have been continued in order to determine surface profiles in shallow water. Following Stokes (1880) we choose ϕ and ψ as independent variables rather than x and y. In the new coordinate system the region of interest is bounded by two straight lines $\psi =$ constant. Expanding x and y in series we have

$$x = -\phi + \sum_{i=1}^{\infty} A_i \cosh i\,(\psi + k) \sin i\,\phi$$

$$y = -\psi + \sum_{i=1}^{\infty} A_i \sinh i\,(\psi + k) \cos i\,\phi \qquad \text{(II A-4.9)}$$

where the surface streamline is $\psi = 0$ and the bottom streamline is $\psi = -k$. Bernoulli's equation becomes

$$\frac{p}{\rho} = -g\,(y + C) - \frac{1}{2S} \qquad \text{(II A-4.10)}$$

where C is an arbitrary constant and

$$S = \left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial x}{\partial \psi}\right)^2 \qquad \text{(II A-4-11)}$$

Substituting from (1) and (2) and setting $p = C$, $\psi = 0$ one obtains on collecting terms of the form $\cos \frac{i\,m\,\phi}{c}$ a system of equations from which the coefficients C, A_1, A_2 ——, can be calculated. The solution of these equations is facilitated by choosing related coefficients in such a way that these coefficients are separated, i.e., the coefficients can be determined successively from succeeding equations. The coefficients have been calculated numerically in certain cases neglecting terms of the order of $(H/L)^7$ and higher. A comparison of surface profiles, as determined by various orders of approximation, is given in Figure II A-12 for shallow water conditions and high relative steepness. It appears that for $d/L = 0.0513$ the wave is very near its limiting steepness (i.e., beyond this the wave will break at the crest) when H/L is approximately 0.00564. For this value of d/L Miche's (1944) approximate formula for the limiting steepness $(H/L)_{max} = 0.142 \tanh kd$ gives $(H/L)_{max} = 0.044$. Similar numerical calculations have been carried out to the 6th order for infinitely deep water. The surface profiles given in Figure II A-13 indicate that a parameter value of $a/L = 0.1$ is too large for a reasonable wave using this order of approximation, a secondary crest appearing in the trough. For the value of $a/L = 1/14$, $H/L = 0.154$. The approximations we used here give a reasonable wave although other investigations indicate that in deep water the limiting wave should have a ratio closer to $H/L = 0.142$.

Since the calculations are simpler to perform and the convergence of the series approximations is most rapid for water of infinite depth many investigations have been concerned with long-crested cylindrical waves of appreciable steepness for which the bottom effects can be neglected. The earliest such investigations appear to be due to Stokes (1880) who found

$$\eta = a \cos mx' - \frac{1}{2}\left(ma^2 + \frac{17}{24} m^3 a^4\right) \cos 2 mx' \qquad \text{(II A-4.12)}$$

$$+ \frac{3}{8} m^2 a^4 \cos 3 mx' - \frac{1}{3} m^3 a^4 \cos 4 mx' + \text{-----}$$

$$c^2 = \frac{g}{m}\left(1 + m^2a^2 + \frac{5}{4} m^4 a^4 + \text{-----}\right)$$

Platzmann (1947) has given a new method of performing the successive approximations for deep water. He finds that the wave velocity, c, is given by

$$c^2 = \frac{gL}{2\pi}\left(1 + \alpha^2 + \frac{1}{2}\alpha^4 + \frac{1}{4}\alpha^6 - \frac{22}{45}\alpha^8 + \text{---}\right) \qquad \text{(II A-4.13)}$$

where $\alpha = H/L$. Since the maximum value of H/L in deep water is (0.1418), according to Havelock (1918) the extreme partial sums of the series are 1.000, 1.198, 1.218, 1.220, 1.219, --- the proper value being 1.220 for the steepest wave. Thus the error in c in using the approximate formula $c = \sqrt{gL/2\pi}$ is a maximum of nearly 10% for the steepest waves. The error in using the formula

$$c = \sqrt{\frac{gL}{2\pi}}\left(1 + \frac{1}{2}\left(\frac{\pi H}{L}\right)^2\right)$$

is less than 1% for the steepest waves.

Platzmann (1947) also investigates the energy partition for deep water waves. He finds for the kinetic energy

$$E_k = \frac{\pi \rho g}{2k^3}\left(\beta^2 - \frac{1}{2}\beta^4 - \frac{4}{3}\beta^6 - \frac{251}{144}\beta^8 + \text{---}\right) \qquad \text{(II A-4.14)}$$

where $1 + \beta^2 = \frac{kc^2}{g}$ and for the potential energy

$$E_p = \frac{\pi \rho g}{2k^3}\left(\beta^2 - \beta^4 - \frac{5}{6}\beta^6 - \frac{257}{144}\beta^8 + \text{---}\right) \qquad \text{(II A-4.15)}$$

Thus the result that the total energy is equally divided between kinetic and potential is only approximately true for the difference between them is of the fourth order in the wave steepness, i.e.,

$$\frac{E_k - E_p}{E_p} = \frac{1}{2}\alpha^2 + \frac{1}{4}\alpha^4 + \frac{7}{12}\alpha^6 + \frac{2033}{1440}\alpha^8 + \text{---} \qquad \text{(II A-4.16)}$$

For the steepest waves the partial sums of the series in II A-4.16 are 9.93%, 10.92%, 11.37%, 11.59%. Hence although the convergence of this series is quite slow it appears that the maximum value of $\frac{E_k - E_p}{E_p}$ is about 1/8.

The energy transmitted by waves of finite height in deep water has been investigated by Starr and Platzmann (1948). They find that the transmitted energy

E_t is related to the total energy E_k+E_p by

$$\frac{E_t}{E_k + E_p} = \frac{1}{2} + \frac{5}{2}\ \frac{E - p}{E\quad p} > \frac{1}{2} \tag{II A-4.17}$$

Since for the steepest waves $E_k-E_p \simeq \frac{1}{8}p$, we have

$$\frac{E_t}{E_k + E_p} < \frac{11}{17} \simeq 0.65$$

The ratio is 1/2 if terms such as $(H/L)^4$ are neglected.

For waves of appreciable steepness the series approximations converge very slowly. One is naturally led to inquire as to the maximum value of H/L for which the series converges and so represents a possible wave motion. This is a difficult problem to solve since the calculation of successive series terms, though straightforward, is very tedious. Observers on the other hand frequently voiced their skepticism that anything similar to a periodic system of waves of permanent form with constant height and period could exist, especially in shallow water.

The question of the convergence and validity of the series approximations seems to first have been raised by Burnside (1916). Rayleigh (1917) attempted to answer the question by carrying out calculations for water of infinite depth to a 6th approximation. He concludes that it is very probable that the series does in fact converge by showing that if all other conditions are satisfied the condition of constant pressure at the free surface can be satisfied to a very good degree of accuracy by taking sufficiently small values of H/L. Investigations of the limiting value of H/L beyond which the basic assumptions of irrotational flow and constant pressure at the surface are no longer valid have been made by Wilton (1913), Michell (1893), Gwyther (1900), Havelock (1918), Miche (1944), and Jeffreys (1945), for infinite depth. They found that the limiting steepness is given by H/L = 0.141.

Levi-Civita (1905) has rigorously established the existence of the periodic irrotational wave in the case of infinite depth. He utilizes a method of conformal representation in transforming the original problem into th[illegible] of determining, in a circle of unit radius, an analytic function $\omega = \theta + i\tau$ which satisfies the condition

$$\frac{d\tau}{d\sigma} = p\, e^{-3\tau} \sin\theta \tag{II A-4.18}$$

on the circumference of the circle. The solution is obtained by a development of the unknowns in powers of a small parameter, μ, and the convergence is demonstrated by the method of majorants. A similar method has been employed by Struik (1926) in generalizing the previous existence proof to the case of finite depth.

One finds for the height of a crest above the still water level

$$a' = \frac{L}{2\pi}\left(\mu + \frac{1}{2}\mu^2 + \frac{2}{2}\mu^3 + \frac{13}{6}\mu^4 + \text{---}\qquad\right) \tag{II A-4.19}$$

and for the depth of a trough

$$a'' = \frac{L}{2\pi}\left(\mu - \frac{1}{2}\mu^2 + \frac{3}{2}\mu^3 - \frac{13}{6}\mu^4 + \cdots \right) \qquad \text{(II A-4.20)}$$

from which

$$a' - a'' = \frac{L}{2\pi}\left(\mu^2 + \frac{13}{3}\mu^4 + \cdots \right) \qquad \text{(II A-4.21)}$$

Thus the height of the crest is always greater than the depth of a trough by terms beginning with $\frac{H^2 \pi^2}{2L}$ since,

$$\frac{H}{L} = \frac{1}{\pi}\left(\mu + \frac{3}{2}\mu^3 + \cdots \right) \qquad \text{(II A-4.22)}$$

The numerical calculations given by Struik contain errors which have been corrected by Wolff (1944). The existence proof has been given by Kotchine (1927) for irrotational waves at the surface separating two different fluids. This type of calculation is not the only method of proving existence as has been shown by Neumann (1930) and Lichtenstein (1931) who applied a method of successive approximations to a system of non-linear integral equations describing the problem.

According to Stokes (1880) it appears that the series will remain convergent until a singularity appears at the boundary of the fluid. He discusses the occurrence of a singular point at which two branches meet at a finite angle 2α. Introducing polar coordinates with origin at the crest and $\theta = 0$ vertically downwards we have

$$\psi = C\, r^m \cos m\theta \qquad \text{(II A-4.23)}$$

Since the surface is the streamline $\psi = 0$, the limiting angle there is given by $m\alpha = \frac{\pi}{2}$. The particle velocity there is

$$q = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = -\, m\, C\, r^{m-1} \qquad \text{(II A-4.24)}$$

Since the particle velocity vanishes at a crest its value at a neighboring point on the surface is

$$q^2 = 2\, gy = 2\, gr \cos \alpha \qquad \text{(II A-4.25)}$$

Compari the exponents of r in the two expressions for q we find $m = \frac{3}{2}$. This implies $\alpha = \frac{\pi}{3}$ or that the angle between the limiting branches is 120°.

The extreme form of a radially symmetric elevation can be investigated in a manner similar to Stokes' discussion for plane wave motions. The corresponding stream function for the radial motion is

$$\psi = \frac{1}{n+1}\, r^{n+1}(1-\mu^2)\,\frac{dp_n}{d\mu}, \quad \mu = \cos\theta \qquad \text{(II A-4.26)}$$

P_n being the Legendre polynomial of order n. At the free surface $\frac{d\,P_n}{d\,\mu} = 0$. The particle velocity is

$$q = \frac{1}{r^2}\ \frac{\partial \psi}{\partial \mu} = -\,n\ r^{n-1}\ P_n\,(\mu). \qquad \text{(II A-4.27)}$$

Applying Bernoulli's theorem to the free surface

$$q^2 = 2\ gy = 2\ gr\ \mathrm{fcn}\ (\theta) \qquad \text{(II A-4.28)}$$

Comparing exponents of r gives as before n = 3/2. The limiting angle is therefore determined by the condition

$$\frac{d\ P_{3/2}}{d\ \mu} = 0 \qquad \text{(II A-4.29)}$$

The crest angle is found to be 130.5°.

The surface particle velocities can be described by use of Bernoulli's theorem in a steady state flow. On the surface of the fluid

$$q^2 + 2\ gy_{\ell} = \text{const.} = q_o^2 \qquad \text{(II A-4.30)}$$

where q_o is the fluid velocity at the points where the surface meets the mean water level. Since y_{ℓ} is measured positively upwards we see that the minimum speed of the surface particles occurs at the crest and the maximum at a trough. Now by definition of the mean surface level

$$\int_0^L y_{\ell}\ dx = 0 \qquad \text{(II A-4.31)}$$

Therefore by II A-4.30 and II A-4.31

$$\int_0^L q^2\ dx = q_o^2\ L \qquad \text{(II A-4.32)}$$

Consider the fluid bounded by vertical planes through two successive crests, the free surface and a horizontal plane at depth –h at which the velocity is essentially horizontal and equal to c. The total vertical mass acceleration is zero, since the flux of vertical momentum across the boundaries is zero. Letting p_s, p_h be the pressures at the surface and at depth h respectively, we find

$$\int_0^L (p_h - p_s)\ dx = g\rho \int_0^L (h + y_{\ell})\ dx = g\rho\ h\ L \qquad \text{(II A-4.33)}$$

Comparing the pressures in the same vertical

$$p_h - p_s = g\rho\,(h + y_{\ell}) + \tfrac{1}{2}\rho\,(q^2 - c^2) \qquad \text{(II A-4.34)}$$

Therefore

$$\int_0^L q^2\, dx = c^2 L \qquad \text{(II A-4.35)}$$

Comparing with (3) we find $q_o = c$, i.e., the velocity at points where the surface profile intersects the mean water level is equal to the wave velocity, c.

b. Non-linear Superposition of Waves:

In analyzing the motion of the sea one is obliged to decompose the complex wave records currently being obtained into a spectrum, probably continuous, of amplitudes and frequencies. Such an analysis presumably depends upon a theory for the law of combination of two or more swells of different wave lengths and amplitudes. If the individual waves and their resultant sum are of infinitesimal steepness the individual motions can be simply added since the linear theory which applies to this case allows simple addition. However, when the waves are of finite height, the case of most interest, simple addition of wave heights is not valid and one must investigate the dependence of the resultant surface form on height by proceeding to higher order terms in the series approximations for combinations of waves. The same basic assumptions will be made as for the single wave train with irrotational flow.

For simplicity we consider only the case of the motion of long-crested waves in deep water. Calculations have in addition been carried out for water of finite depth but the results are not as tractable.

The wave is not now of permanent form so that we cannot reduce the wave motion to a steady state as before. The calculations can however, be easily carried out by a similar method of successive approximations without eliminating the dependence on time. It is readily shown that the problem for the second approximation reduces to the solution of Laplace's equation with the boundary condition

$$-g\,\phi_y - \phi_{tt} = 2\,\phi_x\,\phi_{xt} + 2\,\phi_y\,\phi_{yt} - \frac{\phi_t\,\phi_{tty}}{g} - \phi_t\,\phi_{yy} \qquad \text{(II A-4.36)}$$

on the still water level $y = 0$. Having determined ϕ, the surface elevation is given by

$$-\eta = -k + \frac{\phi_t}{g} - \frac{\phi_t\,\phi_{ty}}{g^2} + \frac{1}{2g}\left(\phi_x^{\,2} + \phi_y^{\,2}\right) \qquad \text{(II A-4.37)}$$

We consider now the simplest case where we have to a first approximation two sinusoidal waves whose velocity potentials can be simply added, i.e., we assume

$$\phi_o = A\,e^{ky}\cos(kx - \sigma t) + B\,e^{my}\cos(mx - \gamma t) \qquad \text{(II A-4.38)}$$

where $\sigma^2 = gk$, $\gamma^2 = gm$.

Substituting into the non-linear terms of (1) we find that

$$-g\,\phi_y - \phi_{tt} = 2\,AB\,mk\,(\sigma - \gamma)\,\sin\left[(k - m)\,x - (\sigma - \gamma)\,t\right] \qquad \text{(II A-4.39)}$$

For $k = m$, we have essentially a single wave and our formula confirms a result given by Stokes that a first order velocity potential for a single wave train in deep water is also exact to the second order and involves only neglecting terms of order $(H/L)^3$. The surface profile, however, does contain second order terms as has been shown. We find after some calculation

$$\phi = \phi_o + \frac{2\,AB\,mk\,(\sigma - \gamma)e^{(k-m)y}}{(\sigma - \gamma)^2 - g(k-m)} \sin\left[(k-m)\,x - (\sigma - \gamma)\,t\right] \quad \text{(II A-4.40)}$$

for $k > m$ and in the particular case in which the first order amplitudes $\frac{A\sigma}{g}$ and $\frac{B\gamma}{g}$ are equal the surface elevation is of the form

$$\frac{\eta}{a} = -\frac{k}{a} + \sin(kx - \sigma t) + \sin(mx - \gamma t) - \frac{1}{2}(ka)\cos 2(kx - \sigma t)$$
$$- \frac{1}{2}(ma)\cos 2(mx - \gamma t) - \frac{1}{2}(k+m)a\cos\left[(k+m)x - (\sigma+\gamma)\,t\right]$$
$$- \frac{(k-m)a}{2}\cos\left[(k-m)x - (\sigma - \gamma)\,t\right] \quad \text{(II A-4.41)}$$

Representative surface profiles are plotted in Figures II A-14 and II A-15 for $H/L = 0.1$ and two different ratios of wave lengths. In Figure II A-14 the wave lengths have a relative difference of 0.1 which would lead in a first approximation to a simple group or beat pattern. In this case the non-linear last term in II A-4.41 has a negligible influence on the value of η. In Figure II A-15 the behavior is more complex, the surface rapidly changing form with time. The maxima appear to move in this latter case with a velocity $c_g \cong 0.785\sqrt{\frac{g}{m}}$. If one simply assumed that the maximum would move with 1/2 the wave velocity associated with the average wave length instead of 0.785 one would have 0.613. As an example of the effects occurring because of the non-linearity of the superposition we have compared in Figure II A-16, the surface profiles assuming that the surface elevation to the second order of approximation of the two individual waves could be simply added with the correct superposition given in Figures II A-14 and II A-15.

e. The Solitary Wave:

We have seen that for small values of d/L the periodic irrotational waves approach the form of a succession of sharply peaked crests separated by relatively broad flat troughs. This suggests an investigation to determine if it is possible to have a wave motion consisting of a single elevation propagating without change of form and with constant velocity in water of constant depth.

Scott Russell made extensive observations of waves of this type which he called "solitary waves". Since the form of approximations used by Stokes for periodic waves are unsuitable the theory of solitary waves has proceeded independently along somewhat different lines. Up to the present time it has been found impossible to treat these waves rigorously as was the case for periodic waves.

The first approximate theories were given independently by Boussinesq (1871) and Rayleigh (1876). In Rayleigh's method the velocity

components are expanded in powers of the distance y above the bottom. Using the conditions that the flow is irrotational and incompressible he finds the following series expansions for the functions ϕ and ψ:

$$\phi = F - \frac{y^2}{2!} F'' + \frac{y^4}{4!} F^{IV} - \text{---} \qquad \text{(II A-4.42)}$$

$$\psi = yF' - \frac{y^3}{3!} F''' + \text{---}$$

where F is an arbitrary function of x and the primes denote differentiation with respect to x. The function F is determined by successive approximations from the free surface condition

$$\frac{1}{2}(u^2 + v^2) + gy = \frac{1}{2} c^2 + gd \qquad \text{(II A-4.43)}$$

Rayleigh finds that

$$c^2 \cong g\,(d + \eta_0)$$

where η_0 is the maximum surface height above the undisturbed water level $y = d$. This formula was earlier adopted by Russell as fitting his observational data. The surface profile above $y = d$ is given by

$$\eta = \eta_0 \operatorname{sech}^2 \frac{1}{2} \frac{x}{b} \qquad \text{(II A-4.44)}$$

if the crest is located on the y-axis and $b^2 = \frac{d^2(d+\eta_0)}{3\eta_0}$. The extent of the wave can be estimated by noting that $\eta / \eta_0 = 0.1$ when $x/b = 3.636$. By superimposing a velocity $- c$ in the x-direction a progressive wave with velocity c is obtained. If η_0/d is sufficiently small the path of each particle is practically a parabolic arc with vertical axis and concave downwards. The height and length of the solitary wave are not independent as was the case for the periodic wave motions previously considered but one finds that the height decreases as the length increases. The approximations consist in neglecting the fourth power of $\frac{d+\eta_0}{2b}$.

McCowan (1891) found it convenient to assume that

$$\psi + i\phi = -c\,(y + ix) + \sum_{n=0}^{\infty} a_{2n+1} \tan^{2n+1} \frac{1}{2} m(y + ix) \qquad \text{(II A-4.46)}$$

with $my < \pi$. This satisfies all but the free surface condition, which one uses to determine a_1, a_3, --- . As a first approximation, involving neglect of terms of the order of η_0^4 and higher, he found that in the progressive wave motion

$$u = c\,ma \frac{1 + \cos my \cosh mx}{(\cos my + \cosh mx)^2}$$

$$v = c\, ma \frac{\sin my \sinh mx}{(\cos my + \cosh mx)^2} \tag{II A-4.47}$$

$$c^2 = \frac{g}{m} \tan md$$

where m and a are determined from the equations

$$m\, a = \frac{2}{3} \sin^2 m \left(d + \frac{2}{3} \eta_o\right)$$

$$\eta_o = a \tan \frac{1}{2} m \,(d + \eta_o) \tag{II A-4.48}$$

The surface profile is given by

$$\eta = a \frac{\sin m\,(d+\eta)}{\cos m\,(d+\eta) + \cosh mx} \tag{II A-4.49}$$

$$\simeq \frac{a \sin md}{\cos md + \cosh mx} + \frac{a^2}{2!} \frac{d}{d(d)} \left(\frac{\sin^2 md}{(\cos md + \cosh mx)^2}\right) + \text{---}$$

A particle starting from the level y returns to the same level after attaining a maximum elevation ζ_o given by

$$\zeta_o = a \tan \frac{1}{2} m \,(y + \zeta_o) \tag{II A-4.50}$$

The maximum displacement of a particle is

$$\delta = 2a \left[1 + ma \frac{\sin my - my \cos my}{\sin^3 my} \right]$$

$$\simeq 2a \left[1 + \frac{1}{3}\, ma \right] \tag{II A-4.51}$$

The path of the particle is approximately that section of the parabola

$$(\xi - a)^2 + 2a\, \zeta \cot my = a^2 \tag{II A-4.52}$$

lying above the level y where ξ and ζ are the displacement in the x and y directions respectively of a particle initially at the point (x, y).

The total energy, E, is approximately equally divided between kinetic and potential and one finds

$$E = \frac{2}{3} g \rho\, m\, a^2 d^2 \left(1 + \frac{7}{10}\, ma \right) \tag{II A-4.53}$$

For the extreme wave we have approximately

$$md \simeq 1.1 \quad , \quad m\eta_o \simeq 0.9$$

The calculations involving McCowan's solitary wave theory are facilitated by the use of table II A-1 for $\frac{a}{d}$ and md vs $\frac{\eta}{d}$ as given by Bagnold (1946).

For solitary waves near the limiting form McCowan (1894) finds that the maximum wave height is $\eta_o = 0.78d$, which agrees with the experimental value of 0.75d determined by McCowan. The corresponding wave velocity is $c_m = \sqrt{1.56\ gd}$. For regions fairly far from the crest the surface elevation is approximately

$$\frac{\eta}{d} = 1.04\ e^{-x/d} - 0.44\ e^{-2x/d} + \cdots \qquad \text{(II A-4.54)}$$

The corresponding velocity components are approximately

$$\begin{aligned} u &= -1 + 1.24\ e^{-x/d} \cos y/d \\ v &= 1.24\ e^{-x/d} \sin y/d \end{aligned} \qquad \text{(II A-4.55)}$$

The crest is formed by two surfaces, equally inclined to the bottom, meeting at an angle of 120°. The surface profile as given by McCowan is shown in Figure II A-17.

The solitary wave problem has been studied by Weinstein (1926) employing the method of Levi-Civita (1925). Writing the complex velocity in the form $w = u - iv = c\ e^{\tau} e^{-i\theta}$ (τ, θ real) the problem reduces to the determination of regular functions $\omega = \theta + i\tau$ in the band of the $\phi + i\psi$-plane limited by the straight lines $\psi = 0$ and $\psi = 1$ which satisfy the conditions

$$\begin{aligned} &\theta = 0, \quad \text{for } \psi = 0 \\ &\frac{d\tau}{d\phi} + p\ e^{-3\tau} \sin\theta = 0\ , \quad \text{for } \psi = 1 \end{aligned} \qquad \text{(II A-4.56)}$$

where

$$p = \frac{gd}{c^2}$$

As an approximate solution Weinstein finds

$$c^2 = gd\left(1 + \frac{\eta_o}{d} - \frac{21}{20}\left(\frac{\eta_o}{d}\right)^2\right) \qquad \text{(II A-4.57)}$$

which differs from the formula of Rayleigh by less than 4% in ordinary cases (assuming $\eta_o/d \leq 1/5$).

Gwyther (1900) has examined the direct reflection of a solitary wave by a vertical wall. He finds that the incident wave is reflected nearly unchanged in form for waves of moderate size.

d. Cnoidal Waves:

The term cnoidal (in analogy with sinusoidal waves) was coined by Kortweg and de Vries (1895) to describe certain periodic waves of permanent form in shallow water. They resemble the Stoke's waves in shallow water but the analysis is carried out independently in a different fashion. Their analysis begins, following Rayleigh, with an expansion of the velocity components in power series in the coordinate y measured from the bottom. Having satisfied all the other conditions the free surface condition is satisfied by successive approximations. The equations of the first approximation are those of the shallow water theory. In this approximation progressive waves of arbitrary form travel unchanged in form. In a second approximation terms such as $\frac{\partial \eta}{\partial x}$. $\frac{\partial^2 \eta}{\partial x^2}$ and $\left(\frac{\partial \eta}{\partial x}\right)^3$ are neglected in comparison with $\eta \frac{\partial \eta}{\partial x}$ and $\frac{\partial^3 \eta}{\partial x^2 \partial t}$ in comparison with $\frac{\partial \eta}{\partial t}$. In an approximately steady state Kortweg and de Vries have shown that

$$\frac{\partial \eta}{\partial t} = \frac{3}{2}\sqrt{\frac{g}{d}}\,\frac{\partial}{\partial x}\left(\frac{1}{2}\eta^2 + \frac{2}{3}\alpha\eta + \frac{1}{3}\sigma\frac{\partial^2 \eta}{\partial x^2}\right) \qquad \text{(II A-4.58)}$$

where α is a small but arbitrary constant, which is closely connected with the exact velocity of the uniform motion given to the liquid, and where $\sigma = 1/3\ d^3$ (more generally $\sigma = 1/3\ d^3 - Td/\rho g$ where T is the capillary tension).

Possible stationary waves, which do not change form, are solutions of the equation $\partial \eta/\partial t = 0$. If we take d as the smallest depth of the fluid the surface profile is found to be given by

$$\eta = a\ \mathrm{cn}^2\, x\sqrt{\frac{a+k}{4\sigma}} \quad \left(\text{mod. } M = \sqrt{\frac{a}{a+k}}\right) \qquad \text{(II A-4.59)}$$

where cn is the Jacobian elliptic function of modulus M. This type of wave is called cnoidal. These waves are periodic with a wave length which increases as k decreases. For k = 0 this wave length becomes infinite, and the equation reduces to

$$\eta = a\ \mathrm{sech}^2\ x\sqrt{\frac{a}{4\sigma}} \qquad \text{(II A-4.60)}$$

which we have seen represents the solitary wave. These waves are of positive elevation, the case of negative waves arising only when surface tension becomes predominant. For water at 20° C. it is shown that these negative waves occur only for water which is less than 0.47 cm. in depth and thus are not practically important.

The velocity components are

$$u = \sqrt{gd} - \sqrt{\frac{g}{d}} \left\{ \eta + \frac{1}{2}(k - a) - \frac{\eta^2}{4d} + \frac{1}{d}\left[(a - k)\eta + \frac{1}{2}ka - \frac{3}{2}\eta^2 \right] \right\}$$

$$+ \frac{1}{2\sigma}\sqrt{\frac{g}{d}} \left\{ (a - k)\eta + \frac{1}{2}ka - \frac{3}{2}\eta^2 \right\} y^2 + \text{----} \qquad \text{(II A-4.61)}$$

$$v = \sqrt{\frac{g\eta(a - \eta)(k + \eta)}{d\,\sigma}}\; y$$

The wave velocity of the solitary wave is found to be

$$c = \sqrt{gd}\left(1 + \frac{a}{2d}\right) \qquad \text{(II A-4.62)}$$

which was first derived by Rayleigh. For cnoidal waves Kortweg and de Vries essentially adopt Stokes' second definition of wave velocity defining it as the velocity of propagation of the wave form when the horizontal momentum of the liquid has been reduced to zero by the addition of a uniform motion. Retaining only terms of the first order compared with η, a, and k they find that

$$c = \sqrt{gd}\left(1 + \frac{k + a}{2d} - \frac{k + a}{d}\frac{E(K)}{K}\right) \qquad \text{(II A-4.63)}$$

The wave length is given by

$$L = \frac{2K\sqrt{\sigma}}{\sqrt{a + k}} \qquad \text{(II A-4.64)}$$

If we let x_o, y_o be the initial coordinates of a particle and $x' = x_o + \xi'$, $y' = y_o + \varphi'$ its coordinates at time t then

$$\xi' = -\frac{(a + k)L}{2\,Ki}\left[Z\left(\frac{2K\,(x_o + \sqrt{gd}\,t)}{L}\right) - Z\left(\frac{2K\,x_o}{L}\right)\right]$$

$$\varphi' = -\frac{a\,y}{d}\left[cn^2\,(x_o + \sqrt{gd}\,t)\,\frac{a + k}{4\sigma} - cn^2\,x_o\sqrt{\frac{a + k}{4\sigma}}\right] \qquad \text{(II A-4.65}$$

where $Z(u) = \left(1 - \frac{E(K)}{K}\right) - M^2\int_0^u sn^2\, u du.$

Since all fluid particles with the same y describe congruent paths, these formulae may be simplified by supposing $x_o = 0$.

Several cases of the deformation of stationary waves are discussed. For a solitary type wave whose surface is given by

$$\eta = a \operatorname{sech}^2 p x \qquad \text{(II A-4.66)}$$

it is found that:

(i) the wave is stationary if $a = 4\sigma p^2$, which is the solitary wave discussed previously;

(ii) the wave becomes steeper in front if $a > 4\sigma p^2$;

(iii) the wave becomes steeper in the rear if $a < 4\sigma p^2$.

Now as we have seen earlier in connection with the solitary wave the "effective" wave length, as given by the condition that $\eta / a = 0.1$, is

$$L = \frac{3.636}{p} \qquad \text{(II A-4.67)}$$

The approximate condition that this wave plunges would then be that

$$\frac{a}{d} > 5\left(\frac{d}{L}\right)^2$$

or

$$a > 5\,\frac{d^2}{gT^2} \qquad \text{(II A-4.68)}$$

We see that a solitary wave which is less steep than the stationary solitary wave of the same height will tend to plunge. Moreover a wave peaking up more rapidly than waves which are stable in that condition will tend to plunge. For a cnoidal wave of the form

$$\eta = a \operatorname{cn}^2 p x \qquad \text{(II A-4.69)}$$

the waves are stationary for $a = 4\sigma n^2 p^2$ and for $a > 4\sigma n^2 p^2$ tend to become steeper in front. Here p is inversely proportional to the height and one finds that for a given height and modulus, the wave tends to plunge if its wave length is larger than that required for the stationary wave of this modulus and height. For sinusoidal waves $n = 0$, so that these waves will always plunge.

For a train of sine waves of the form

$$\eta = a \sin \frac{2\pi}{L} x$$

it is found explicitly that

$$\frac{d\eta}{dt} = \frac{3\, q_0 \pi a^2}{2\, d\, L} \sin \frac{4\pi x}{L} \qquad \text{(II A-4.70)}$$

Stoker (1948) has shown on the basis of the non-linear shallow water theory that sine pulses would plunge in water of constant depth. This solution together with an approximate treatment is given in the section on the shallow water theory.

Carrying out the calculations to a higher order of approximation Kortweg and de Vries (1895) find

$$\eta = \left[1 - \frac{3(a-k)}{4d}\right]\eta_1 + \frac{3}{4d}\eta_1^2 + \cdots$$

$$\eta_1 = a\,\mathrm{cn}^2\,\frac{1}{2}\left(1 - \frac{5(a-k)}{8d}\right)x\sqrt{\frac{3(a+k)}{d^3}} \qquad \text{(II A-4.71)}$$

$$M = \left(1 - \frac{3k}{8d}\right)\sqrt{\frac{a}{a+k}}$$

For the solitary wave, when k = 0

$$\eta = \left(1 - \frac{3a}{4d}\right)\eta_1 + \frac{3}{4d}\eta_1^2$$

$$\eta_1 = a\,\mathrm{sech}^2\,\frac{1}{2}\left(1 - \frac{5a}{8d}\right)x\sqrt{\frac{3a}{d^3}} \qquad \text{(II A-4.72)}$$

A systematic search for stationary waves in shallow water has been presented by Keller (1948). The solution depends upon an expansion, due to Friedrichs (1948), of the solution of Euler's equations for irrotational flow in power series in $\sigma = (d/R)^2$, R being the radius of curvature at some point on the free surface. Inserting these series into the equations and boundary conditions, various orders of approximation are obtained by equating coefficients of successive powers of σ to zero. The zero order terms are the equations of non-linear shallow water theory. The only solutions of these equations which are of stationary form are the piecewise constant (shock type) solutions. The equations of the second approximation (coefficients of σ) yield waves of permanent form the result being quite similar to those of Kortweg and de Vries. Among the differences is that the velocity component u is found to depend upon x and y. The pressure is found to be given by the hydrostatic formula. The surface profile of a periodic wave, as given by Keller, is shown in Figure II A-18. This profile is unfortunately somewhat misleading since it is shown for d/L = 0.5 although the theory only applies to quite shallow water. The considerable difficulties in actually computing a single wave profile for given characteristics H, L, d precludes presenting any further profiles here. The surface profile of a solitary wave, as given by Keller, is presented in Figure II A-19. For the actual equations of the state of motion the reader is referred to Keller's original report.

e. <u>Gerstner's Trochoidal Waves:</u>

In 1802 Gerstner determined the only steady state wave motion in which the lines of constant pressure coincide with the streamlines of the flow. An

alternative derivation due to Rankine (1864) and Froude (1862) is based on the assumption that the water particles revolve in circular orbits with constant angular velocity. This latter assumption leads to a simple derivation of the velocity of propagation of such waves. In a reference system in which the motion is reduced to steady state we assume that the circular orbit of radius r is described with frequency σ. The speed of the water particle at the wave crest is $U_1 = c - r\sigma$ and at the trough is $U_2 = c + r\sigma$. Bernoulli's equation applied to the free surface at which p = 0 results in $U_2^2 - U_1^2 = 2\,gH = 4\,gr$. Thus the wave velocity is

$$c = \frac{g}{\sigma} = \sqrt{\frac{gL}{2\pi}} \qquad \text{(II A-4.73)}$$

since $c = \frac{\sigma}{k}$.

Gerstner's original derivation has been presented in a more compact form by Lamb (1895, p. 416). A somewhat different version is obtained by finding the horizontal and vertical components of the acceleration of a water particle. Suppose for convenience that we choose the origin of coordinates at a minimum of a streamline at which point the velocity is q_o. In terms of the differential of arc length dS and normal dn we find

$$\begin{aligned} x'' &= \frac{1}{\rho}\frac{dp}{dn}\frac{dy}{dS} = \frac{dp}{\rho v dn}\frac{dy}{dt} \\ y'' &= -g - \frac{1}{\rho}\frac{dp}{dn}\frac{dx}{dS} = -g - \frac{dp}{\rho v dn}\frac{dx}{dt} \end{aligned} \qquad \text{(II A-4.74)}$$

the primes indicating differentiation with respect to time. For integration along a streamline the quantity $\sigma = -\,dp/\rho\,d\psi$, where $q = \partial\psi/\partial n$, is by hypothesis a constant. Therefore the equations can be written in the following form:

$$\begin{aligned} x'' &= -\sigma y' \\ y'' &= -g + \sigma x' \end{aligned} \qquad \text{(II A-4.75)}$$

Integrating we find that

$$x = \frac{g}{\sigma}t + \beta\sin\sigma t, \quad y = \beta(1 - \cos\sigma t) \qquad \text{(II A-4.76)}$$

where $\beta = q_o\sigma - g/\sigma^2$ = constant assuming t = 0 as the particles pass the origin.

We now attempt to satisfy the equation of continuity with the equations

$$x = \frac{g}{\sigma}t + \beta\sin\sigma t, \quad y = b - \beta\cos\sigma t \qquad \text{(II A-4.77)}$$

obtained by a shift of the origin of y.

Consider two points p and p' on adjacent streamlines which are in the same phase of motion. The projections of $\overline{p\,p'}$ on the coordinate axes are

$$\delta\beta \sin\sigma t \text{ and } \delta b - \delta\beta \cos\sigma t$$

The flux across a fixed segment parallel to the instantaneous position of $\overline{p\,p'}$ is

$$\delta\psi = -\frac{g}{\sigma}\,\delta b + \sigma\beta\delta\beta + \left(\frac{g}{\sigma}\,\delta\beta - \sigma\beta\,\delta b\right)\cos\sigma t \qquad \text{(II A-4.78)}$$

This is independent of time if

$$\frac{\delta\beta}{\beta} = \frac{\sigma^2}{g}\,\delta b \qquad \text{(II A-4.79(}$$

so that $\beta = K\,e^{kb}$, $k = \sigma^2/g = 2\pi/L$

Thus we finally have

$$x = ct + K\,e^{kb}\sin\sigma t, \quad y = b - K\,e^{kb}\cos\sigma t \qquad \text{(II A-4.80)}$$

where $c = \sigma/k = g/\sigma = \sqrt{gL/2\pi}$. Since b is merely a constant, fixing the origin of y, we may suppose that for the limiting cycloidal path b = 0, which implies that K = 1/k. For a more general origin for t we write ct + a for ct, where a is arbitrary. A progressive wave motion in a coordinate system fixed in space is obtained by superimposing a velocity c in the negative x direction. The particle paths are given then by

$$x = a + \frac{1}{k}\,e^{kb}\sin(ka + \sigma t)$$

$$y = b - \frac{1}{k}\,e^{kb}\cos(ka + \sigma t) \qquad \text{(II A-4.81)}$$

where a and b characterize the individual particles.

These paths can be generated by rolling circles of radius 1/k on the under side of the lines b + 1/k, the distances of the travelling points from the centers is e^{kb}/k. The lines of constant pressure are the streamlines b = constant which are shown in Figure II A-20 by solid lines. The dotted lines represent the position of particles which lay in a vertical line when the crest or trough of the wave passes. Since the particles revolve in circles the mass transport velocity is zero. Any one of the streamlines can be chosen as the surface profile since they are all lines of constant pressure.

According to Gerstner the wave was considered to break when the rotational motion of the water particles becomes greater than the transverse motion. The extreme cycloidal wave, corresponding to b = 0 has a cusp at the crest with an included angle of 0°. The limiting steepness of $H/L = 1/\pi$ is much larger than the value for irrotational flow and also larger than any observed value.

The rotation or vorticity is found to be

$$\omega = - \frac{k\ c\ e^{2\ kb}}{1 - e^{2\ kb}} \qquad \text{(II A-4.82)}$$

which is greatest at the surface, and diminishes rapidly with depth. The sense of the rotation is opposite to that of the revolution of the particles in their circular orbits. If we suppose that by a properly applied system of pressures applied to the free surface, the flow can be reduced to flow in horizontal lines then the velocity of this flow is

$$\mu' = c\ e^{2\ kb} \qquad \text{(II A-4.83)}$$

Thus, if these waves can be generated from rest by a system of conservative forces it is necessary that the water be moving horizontally in the direction opposite to the direction of propagation of the wave motion with a rapidly decreasing magnitude as given by equation II A-4.83 where b is a function of the vertical coordinate y' at which μ' is calculated given by

$$y' = b - \frac{1}{2k}\ e^{2\ kb} \qquad \text{(II A-4.84)}$$

These waves when established have zero momentum. The total energy E associated with the trochoidal wave motion is found to be equally divided between kinetic and potential and to be given by

$$E = \frac{1}{8} \rho L\ H^2 \left(1 - \frac{H^2\ k^2}{8}\right) \approx \frac{1}{8} \rho L\ H^2 \qquad \text{(II A-4.85)}$$

The superposition of two trochoidal waves of equal length has been discussed geometrically by Rankine (1864). If the waves are travelling in the same direction, the amplitude a of the resulting wave is given by

$$a^2 = a_1{}^2 + a_2{}^2 + 2a_1\ a_2 \cos \frac{2\pi D}{L} \qquad \text{(II A-4.86)}$$

where D is the distance between the crests of the component waves. The resulting wave has the same length and velocity as the component waves, and the particles move in circular orbits. If the component waves are running in opposite directions, the orbits of the resultant wave are elliptical. When the crests coincide with each other the major axes of the ellipses are vertical and when a trough and crest coincide the major axes are horizontal. The resultant wave travels in the direction of the component wave of greatest amplitude.

The Gerstner type waves have taken on added interest in view of a recent investigation by Ursell (1950) which considers the effects of the earth's rotation on the theoretical form of ocean swell. In addition to the conservative force which generates Stokes' waves the non-conservative Coriolis' force on a rotating earth must be taken into account. It is shown that if a mass transport is present in the wave motion a steady state motion is not possible. The effect of the earth's curvature is neglected. In the general case Ursell indicates that each particle moves approximately in a horizontal circle of inertia as well as in the nearly vertical Gerstner motion; the diameter of the inertia circles is greatest at the surface, where it may be several hundred meters, and the period of revolution is half a pendulum day (equal to 12 cosec ϕ hours, where ϕ is the latitude).

f. <u>More General Rotational Wave Motion:</u>

Rotational wave motions have been investigated by Cisotti (1914-1915) and Vergne (1911). The existence of an infinity of such waves, taking as a particular case the irrotational wave and, for infinite depth, the wave of Gerstner, was established by Dubreil-Jacotin (1933). The fluid domain is transformed into a circle for infinite depth and an annulus for finite depth. The method of conformal representation as used by Levi-Civita and Struik for these problems in irrotational flow is no longer applicable for the rotational flows. The problem is reduced to the study by successive approximations of a system of integro-differential equations.

Lagrange's theorem and Kelvin's theorem show that the wave motion in water should be nearly irrotational. Miche (1944) has investigated the wave motions in which the current of mass transport if it exists is horizontal and of the second order in the wave height. Thus, we shall suppose there are no large scale currents independent of time such as those occurring when waves cross tidal currents or flowing rivers. The first order solution is then irrotational and the previous equations apply. In a system of Lagrangean coordinates

$$\phi = \frac{ga}{\sigma} \frac{\cosh m(d+y)}{\cosh md} \sin(mx - \sigma t)$$

x = x-component of particle displacement

$$= x_o - \frac{a \cosh m(d+y_o)}{\sinh md} \sin(mx_o - \sigma t)$$

y = y-component of particle displacement

$$= y_o + \frac{a \sinh m(d+y_o)}{\sinh md} \cos(mx_o - \sigma t)$$

$$\frac{p}{\rho g} = -y_o - \frac{2a \sinh m y_o}{\sinh 2md} \cos(mx_o - \sigma t) \qquad \text{(II A-4.87)}$$

u = x-component of particle velocity

$$= \frac{ga}{c} \frac{\cosh m(d+y_o)}{\cosh md} \cos(mx_o - \sigma t)$$

v = y-component of particle velocity

$$= \frac{ga}{c}\frac{\sinh m(d+y_o)}{\cosh md}\sin(mx_o - \sigma t)$$

$$\eta = a\cos(mx_o - \sigma t)$$

Note that one could replace the Lagrange variables x_o, y_o (the equilibrium coordinates of the particle) by the coordinates x, y and still obtain the solution, to the approximation which involves neglecting the terms of order a^2. It is rather remarkable to note that these equations describe exactly a possible wave motion in deep water namely the Gerstner rotational wave motion. Gaillard supposed erroneously that these equations were exact for an arbitrary constant depth. This case of mistaken identity continues to be known in some circles as the "reduced trochoidal theory".

Miche (1944) carries the approximations one step further admitting a second order rotation proportional to $(H/L)^2$ with an arbitrary depth dependence. He finds that the pressure is given by

$$\frac{p}{\rho g} = -y_o - \frac{2a\sinh my_o}{\sinh 2md}\cos(mx_o - \sigma t) - \frac{a^2 m\sinh my_o}{4\sinh^2 md}\left\{3\cos 2(mx_o - \sigma t)\left[\frac{\cosh m y_o}{\sinh^2 md} - \frac{2\cosh m(d+y_o)}{\cosh md}\right] + \frac{2\cosh m(d+y_o)}{\cosh md}\right\} \qquad \text{(II A-4.88)}$$

The mean rotation is

$$\omega = \frac{1}{2}(v_x - u_y) = -\frac{a^2}{2}\left[-\frac{\partial \nu(y_o)}{\partial y_o} + \frac{m^2\sigma\sinh 2m(d+y_o)}{\sinh^2 md}\right]$$

(II A-4.89)

where $a^2\nu(y_o)$ is an arbitrary function of y_o representing the velocity of mass transport. For irrotational swell $\omega = 0$ and we find on determining the constant so that the mean mass transport across a vertical plane is zero

$$a^2\nu(y_o) = \frac{a^2\sigma m}{2\sinh^2 md}\left[\cosh 2m(d+y) - \frac{\sinh 2md}{2md}\right] \qquad \text{(II A-4.90)}$$

This behavior is represented in Figure II A-21.

In deep water

$$a^2 \nu(y_o)_{deep} = a^2 m\sigma e^{2my_o} = \pi^2\left(\frac{H}{L}\right)^2 c\, e^{2my_o} \qquad \text{(IIA-4.91)}$$

The maximum value of the mass transport current is approximately c/5, c being the wave velocity.

Miche has introduced the term normal rotational swell for waves having the same dependence of mass transport velocity on parameters as the irrotational waves but having a different magnitude. For these special wave motions he sets

$$\omega(y_o) = \frac{a^2\sigma m}{2\sinh^2 md}\sinh 2m(d+y_o)$$

$$a^2\nu(y_o) = (\mu+1)\frac{a^2\sigma m}{2\sinh^2 md}\left[\cosh 2m(d+y_o) - \frac{\sinh 2md}{2md}\right] \qquad \text{(II A-4.92)}$$

μ being a constant. For $\mu = 0$ the rotation takes place in the same sense as that of the particles on their orbits. It could be obtained physically by having a violent wind blowing in the direction of wave motion. The irrotational waves are obtained by setting $\mu = 0$. For $-1<\mu<0$ the rotation is negative and the mass transport current has the same sense as for $\mu = 0$ but is of less intensity. The Gerstner type swell is characterized by $\mu = -1$, for which the mass transport velocity is zero and the rotation negative. For still smaller values of $\mu < -1$ both the mass transport velocity and rotation are negative. This could be obtained physically for a prolonged wind blowing in the direction opposite to the wave motion.

The mean surface level is found to be at a distance above the still water level of

$$\frac{\pi a^2}{L}\coth\frac{2\pi d}{L}\left(1+\frac{3}{2\sinh^2 md}\right) \qquad \text{(II A-4.93)}$$

as compared with the value usually given of

$$\frac{\pi a^2}{L}\coth\frac{2\pi d}{L} \qquad \text{(II A-4.94)}$$

The new value may be several times the old value as for example when L/d = 8, the value is tripled.

g. <u>Standing Waves:</u>

Miche has also treated the case of standing waves in which the mass transport velocity is taken to be zero in order to represent for example the wave motion before vertical breakwaters. He now finds that for a standing wave of amplitude 2a

$$\frac{p}{\rho g} = -y_o - \frac{4a \sinh m y_o \sin m x_o \sin \sigma t}{\sinh 2 md}$$

$$- \frac{a^2 m \sinh m y_o}{2 \sinh^2 md} \left\{ \cosh m (2d + y_o) \left[\frac{\cos 2m x_o}{\cosh^2 md} + 4 \sin^2 \sigma t \right] \right.$$

$$+ 4 \tanh md \sinh m (2d + y_o) \left[1 - 3 \sin^2 \sigma t \right] \qquad \text{(II A-4.95)}$$

$$\left. + 3 \cos 2m x_o \cos 2\sigma t \left[\frac{\cosh m y_o}{\sinh^2 md} - \frac{2 \cosh m (d + y_o)}{\cosh md} \right] \right\}$$

One finds that the pressure does not decrease exponentially with depth in deep water as was the case for progressive waves. The mean value of the pressure on the bottom for a complete oscillation is

$$\frac{p_m}{\rho g} = \frac{1}{L} \int_0^L \frac{p}{\rho g}\, dx = d + 2m\, a^2 \tanh md \cos 2\sigma t \qquad \text{(II A-4.96)}$$

which has been essentially derived earlier for deep water. One finds moreover, that for deep water

$$\frac{p}{\rho g} + y_o = 2m\, a^2 \left(1 - e^{2a\, y_o}\right) \cos 2\sigma t \qquad \text{(II A-4.97)}$$

5. Waves Generated by Localized Initial Disturbances

a. The Cauchy-Poisson Wave Problems; Wave Groups

We shall consider disturbances in water which are at some initial instant of time in a definite state and are acted upon henceforth only by the force of gravity in a basin of effectively infinite horizontal extent. Such a disturbance we believe will represent a simple storm of small extent and short duration. It should be possible to approximately represent a storm of wide extent and long duration as the superposition of appropriate simple storms. According to the simplest view as given by Stokes (1870), the motion in a storm area though complicated, may be regarded as made up of different series of regular waves superimposed with different amplitudes and wave lengths. Since the long waves outrun the shorter they should arrive first at a distant shore and one ought to observe a slow decrease of the mean period. Such an effect was noticed by Stokes (1870)[2] and has been reported on more recently by Barber and Ursell (1948).

By superimposing more and more sine waves in an appropriate way, we can approximate an arbitrary reasonable initial disturbance in a given finite interval as closely as we please. Since the individual terms of the resulting series are periodic the result cannot be used to represent a localized disturbance for there is always a disturbance at great distances no matter how many terms of a series are taken although the approximation becomes better as the number of terms increases. In the limit we must consider waves with infinite wave lengths and the separated harmonic frequencies become a continuous sequence.

We consider then the infinite superposition of elementary solutions with continuously varying amplitudes, i.e., for propagation in one direction

$$\eta(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{i\,(kx - \sigma t)}\, dk \qquad \text{(II A-5.1)}$$

where the real part of the integral is to be considered. $A(k)$ represents the wave number spectrum of the disturbance.

If $A(k) = 0$ for k outside of an interval of width $2\,\Delta k$ about k_o as center we say that η represents a wave packet. Such a packet contains only a small range of wave lengths. In this case

$$\eta(x,t) = \int_{k_o - \Delta k}^{k_o + \Delta k} A(k)\, e^{i\,(kx - \sigma t)}\, dk \qquad \text{(II A-5.2)}$$

The evaluation of a wave packet is carried out by expansion of the quantities involved about $k = k_o$. Thus we have

$$\sigma(k) = \sigma(k_o) + \left(\frac{\partial \sigma}{\partial k}\right)_{k=k_o} (k - k_o) + \cdots$$

The condition that higher order terms shall be negligible imposes the restriction on Δk that

$$\frac{\left(\dfrac{d^2\sigma}{dk^2}\right)_{k=k_o} \Delta k}{\left(\dfrac{d\sigma}{dk}\right)_{k=k_o}} \ll 1$$

and (II A-5.3)

$$\frac{\left(\dfrac{d^2\sigma}{dk^2}\right)_{k=k_o} (\Delta k)^2}{\sigma(k_o)} \ll 1$$

Therefore

$$kx - \sigma t = k_o x - \sigma_o t + (k - k_o)\left[x - \left(\frac{d\sigma}{dk}\right)_{k=k_o} t\right] + \cdots$$

and

$$\eta = \eta_o\, e^{i\,(k_o x - \sigma_o t)} \qquad \text{(II A-5.4)}$$

where

$$\eta_o = \int_{k_o - \Delta k}^{k_o + \Delta k} A(k)\, e^{i(k - k_o)\left[x - \left(\frac{d\sigma}{dk}\right)_{k=k_o} t\right]}\, dk$$

which may be called the packet amplitude. Thus amplitude is constant over the surfaces

$$x - \left(\frac{d\sigma}{dk}\right)_{k=k_0} t = \text{const.} \qquad \text{(II A-5.5)}$$

which means that the wave pocket propagates with the group velocity

$$c_g = \left(\frac{d\sigma}{dk}\right)_{k=k_0} \qquad \text{(II A-5.6)}$$

a result derived earlier for the special case of two waves of slightly different wave lengths having the same amplitudes. The particular wave length associated with the stationary point has a constant value.

The group velocity can be constructed for a given wave velocity from the dispersion curve by noting that

$$c_g = \frac{d(ck)}{dk} = c + k\frac{dc}{dk} = c - L\frac{dc}{dL} \qquad \text{(II A-5.7)}$$

as shown in Figure II A-22, where $\tan\alpha = \pm\frac{dc}{dL}$.

The above expression for the group velocity depends only on the fact that the waves travelling with different velocities are gradually sorted out and that in the neighborhood of a point travelling with the group velocity the wave length is stationary. Since L is a function of x and t

$$\frac{dL}{dt} = 0$$

for a point moving with the group velocity, i.e.,

$$\frac{\partial L}{\partial t} + c_g\frac{\partial L}{\partial x} = 0 \qquad \text{(II A-5.8)}$$

Now for a point moving with the waves

$$\frac{\partial L}{\partial t} + c\frac{\partial L}{\partial x} = L\frac{\partial c}{\partial x} = L\frac{dc}{dL}\frac{\partial L}{\partial x} \qquad \text{(II A-5.9)}$$

Combining II A-5.8 and II A-5.9 we are led again to formula II A-5.7

According to Fourier's integral theorem, if the initial conditions are

$$\eta(x,o) = f(x)$$
$$\left(\frac{\partial\eta}{\partial t}\right)_{(x,o)} = 0 \qquad \text{(II A-5.10)}$$

then

$$A(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\mu)e^{-ik\mu}\,d\mu \qquad \text{(II A-5.11)}$$

In order to study the diffusion of a limited initial disturbance which has an apparently constant wave length consider the case of a finite sine wave given by

$$f(x) = 0 \quad \text{for } |x| > \frac{W}{2} \qquad \text{(II A-5.12)}$$

$$f(x) = \cos k_o x \quad \text{for } |x| \leq \frac{W}{2}$$

Then we find

$$A(k) = \frac{1}{\pi} \frac{\sin \frac{(k_o - k)}{2} W}{k_o - k} \qquad \text{(II A-5.13)}$$

which is an oscillating function with a maximum value at k_o of $A(k_o) = W/2\pi$ and with zeros at $k_o - k = \frac{2\pi}{W} n$ for $n = \pm 1, \pm 2, - - -$. We see that as W is increased, thereby increasing the extent of the disturbance, the packet is decreased in width about $k = k_o$. On the other hand if the initial disturbance represented by $f(x)$ is relatively confined then the disturbance at a later time will be considerably diffused and cannot be considered to be represented as a wave packet. The group velocity, in this case, will lose the significance as a velocity of the disturbance as a whole.

For given initial conditions the rate of diffusion will be determined by the behavior of $\sigma(k)$. If $\sigma(k)$ is a slowly varying function of k then in the immediate neighborhood of the values of k actually present the packet will diffuse quite slowly. This situation occurs for shallow water. Actually the diffusion from the form of a wave packet is measured by the curvature of the curve σ vs. k, i.e., approximately by $d^2\sigma / dk^2$. A graphical analysis of this situation has been given by Lamb (1945, p.397).

The width of the wave packet can be estimated by noting that for a slowly varying quantity η_o one has constructive interference at a fixed time for

$$|k - k_o| \Delta x < 1 \qquad \text{(II A-5.14)}$$

but for larger values of k destructive interference. Now $A(k)$ is large only for $|k - k_o| < \Delta k$. Thus destructive interference is important for

$$\Delta x \, \Delta k > 1 \qquad \text{(II A-5.15)}$$

The width of the pocket can be estimated roughly from the condition

$$\Delta x \, \Delta k \simeq 1 \qquad \text{(II A-5.16)}$$

Thus a localized disturbance is associated with a wide range of wave lengths and periods.

To show how the derivative $\left(\frac{d^2\sigma}{dk^2}\right)_{k=k_o}$ affects the spread of the packet consider

$$\eta = \int_{-\infty}^{\infty} f(k - k_o) \, e^{i(kx - \sigma t)} \, dk \qquad \text{(II A-5.17)}$$

Now

$$\sigma(k) = \sigma_o + c_g\,(k - k_o) + \frac{1}{2}\alpha\,(k - k_o)^2 + - - - -$$

where

$$\sigma_o = \sigma(k_o)\ , \qquad c_g = \left(\frac{d\sigma}{dk}\right)_{k=k_o}$$

$$\alpha = \left(\frac{d^2\sigma}{dk^2}\right)_{k=k_o}$$

k_o being the value of k for which the phase is stationary. With $k - k_o = K$ we find

$$\eta = \exp\left\{i\,(k_o x - \sigma_o t)\right\}\int_{-\infty}^{\infty} f(K)\,e^{iK(x-c_g t)-\frac{i\alpha K^2 t}{2}}\,dK$$

If $\alpha = 0$ the pulse would not change shape. Such a situation takes place in shallow water. Consider now

$$f(K) = e^{-K^2/2(\Delta K)^2} \qquad \text{(II A-5.19)}$$

then

$$\eta = \exp i\,(k_o x - \sigma_o t)\int_{-\infty}^{\infty} e^{iK(x-c_g t) - \frac{K^2}{2}\left(i\alpha t + \frac{1}{(\Delta k)^2}\right)}\,dK$$

Completing the square in the exponential we find

$$= e^{i\,(k_o x - \sigma_o t)}\sqrt{\frac{2\pi(\Delta K)^2}{1 + i\alpha(\Delta K)^2\,t}} \qquad \text{(II A-5.20)}$$

$$e^{-\frac{(\Delta K)^2}{2}\,\frac{(x - c_g t)^2}{1 + t^2(\Delta K)^4\alpha^2}}\quad e^{\frac{i\alpha t\,(\Delta K)^4}{2}\,\frac{(x-c_g t)^2}{1+\alpha^2 t^2(\Delta K)^4}}$$

The amplitude factor is proportional to

$$I = e^{-\frac{(\Delta K)^2}{2}\,\frac{(x - c_g t)^2}{1 + t^2(\Delta K)^4\alpha^2}} \qquad \text{(II A-5.21)}$$

which is a Gaussian distribution centered at $x = c_g t$ in agreement with the calculation of a group velocity. The mean width of the distribution (where I falls to e^{-1} of its maximum value) is

$$\delta x = \frac{\sqrt{2}}{\Delta k}\sqrt{1 + \alpha^2 t^2 (\Delta k)^4}$$
$$= \delta x_o\sqrt{1\ \ \frac{64\,\alpha^2 t^2}{(\delta x_o)^4}} \qquad \text{(II A-5.22)}$$

For times so short that $\alpha^2 t^2 (\Delta k)^4 \ll 1$ we have approximately $\delta x = \frac{2\sqrt{2}}{\Delta k} \simeq \delta x_o$ where x_o is the spread at $t = 0$. The packet begins to spread appreciably only when $t > \frac{1}{\alpha(\Delta k)^2}$.

For initial conditions $\eta(x,o) = f(x), \left(\frac{\partial\eta}{\partial t}\right)_{(x,o)} = g(x)$ a solution can be obtained by superimposing plane wave solutions in the form

$$\eta = \int_{-\infty}^{\infty} e^{ikx} (N \cos\sigma t + \frac{M}{\sigma} \sin\sigma t) \; dk \qquad \text{(II A-5.23)}$$

and applying Fourier's integral theorem. Thus we find

$$N = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\alpha) \, e^{-ik\alpha} \, d\alpha$$

and (II A-5.24)

$$M = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\beta) \, e^{-ik\beta} \, d\beta$$

For convenience we shall assume that f(x) is an even function of x and g(x) = 0. Then we can write

$$\eta = \frac{1}{2\pi} \int_0^{\infty} \phi(k) \cos(kx - \sigma t) \, dk + \frac{1}{2\pi} \int_0^{\infty} \phi(k) \cos(kx + \sigma t) \, dk \qquad \text{(II A-5.25)}$$

where

$$\phi(k) = \int_{-\infty}^{\infty} f(\mu) \cos k\mu \, d\mu \, .$$

The first integral in η represents a disturbance travelling in the positive x-direction, while the second integral represents a disturbance travelling in the negative direction. After a sufficient time the two disturbances will have separated. Since we shall be interested only in the single disturbance travelling in one direction we shall confine our attention to the first integral. Initially the waves making up the localized disturbance are in phase near the origin and one has appreciable elevations while at points removed from the origin the phase differences cause destructive interference and zero surface displacement. Subsequently each component wave moves ahead with the wave velocity appropriate to the wave length and period. The waves at later times do not have the same phase at any point and so one finds smaller surface heights. The waves having longer wave lengths travel most rapidly while those with shorter wave lengths drop behind.

A case of considerable interest is that for which

$$\eta(x,o) = f(x) = \begin{cases} \cos k'x & \text{for } |x| \leq (2n + \frac{1}{2}) \frac{\pi}{2k'} \\ 0 & \text{elsewhere} \end{cases} \qquad \text{(II A-5.26)}$$

This case, first discussed by Havelock (1914), represents a limited train of harmonic waves extending over a region whose length is equal to $(2n + \frac{1}{2})$ wave

lengths about the origin. For this $f(x)$ we find

$$\phi(k) = 2\int_0^{(2n+\frac{1}{2})\frac{\pi}{k'}} \cos k'\mu \cos k\mu \, d\mu = \frac{2k'}{k'^2-k^2} \cos\,(2n+\tfrac{1}{2})\,\frac{\pi k}{k'}$$

Thus

$$\eta(x,t) = \frac{k'}{\pi}\int_0^{\infty} \frac{\cos\,(2n+\frac{1}{2})\frac{\pi k}{k'}}{k'^2 - k^2} \cos\,(kx - \sigma t) \quad dk \qquad \text{(II A-5.27)}$$

As we observed previously there is no point for $t > 0$ where all the component waves are in phase but there may be a point for which many of the waves are in phase. Such a point occurs for stationary values of the phase. Whether the predominant effect in the immediate neighborhood of the point of stationary phase is correctly represented by the method of stationary phase remains to be seen.

The phase $\theta = kx - \sigma t$ is stationary for $k = k_o$ when $\left(\frac{\partial \theta}{\partial k}\right)_{k_o} = 0$, i.e., $\frac{x}{t} = \left(\frac{d\sigma}{dk}\right)_{k_o}$. Since $\sigma^2 = g\,k \tanh kd$ we find $c_g = \frac{x}{t} = \frac{1}{2}\sqrt{[illegible]}\left(1 + \frac{2\,k_o d}{\sinh\,2k_o d}\right)$.

(II A-5.28)

The value of η is evaluated according to the method of stationary phase by expanding the integrand in IIA-5.27 about the value of k_o which satisfies IIA-5.28 In general we have to deal with the transcendental equation IIA-5.28 but in the particular cases of deep or shallow water the work simplifies. For deep water

$$k_o = \frac{gt^2}{4x^2}$$

Now

$$kx - \sigma t = k_o x - \sigma_o t + \frac{(k-k_o)^2}{2}\,\frac{d^2}{dk^2}\,(kx - \sigma t)\Big|_{k=k_o} + \dots$$

$$= k_o x - \sigma_o t + \frac{(k-k_o)^2}{2}\left(\frac{1}{4}\,\frac{\sqrt{g}}{k_o^{3/2}}\;t\right) +$$

$$= -\frac{gt^2}{4x} + \frac{(k-k_o)^2\,x^3}{gt^2} \qquad \text{(II A-5.29)}$$

If k deviates from k_o by an amount ϵ and $\phi(k)$ in this interval remains approximately equal to $\phi(k_o)$ then

$$\eta = \frac{k'}{\pi}\,\frac{\cos\,(2n+\frac{1}{2})\frac{\pi g t^2}{4k'x^2}}{k'^2 - \left(\frac{gt^2}{4x^2}\right)^2}\int_{k_o-\epsilon}^{k_o+\epsilon} \cos\left[\frac{gt^2}{4x} - \frac{(k-k_o)^2\,x^3}{gt^2}\right] dk \,.$$

(II A-5.30)

Let

$$\sigma^2 = \frac{(k - k_o)^2 x^3}{gt^2}, \text{ then } dk = \frac{\sqrt{g}\ t}{x^{3/2}} d\sigma$$

Since one gets destructive interference outside of the region of integration one can approximate to the integral by taking the limits of integration from $-\infty$ to ∞. Expanding the integrand and noting that

$$\int_{-\infty}^{\infty} \cos \sigma^2\, d\sigma = \int_{-\infty}^{\infty} \sin \sigma^2\, d\sigma = \sqrt{\frac{\pi}{2}}$$

we find

$$\eta = \frac{k'\sqrt{g}\ t}{\sqrt{\pi}\, x^{3/2}} \quad \frac{\cos\left(2n+\frac{1}{2}\right)\frac{\pi g t^2}{4k' x^2}}{k'^2 - \left(\frac{gt^2}{4x^2}\right)^2} \cos\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right) \qquad \text{(II A-5.31)}$$

Introducing $\tau = \frac{1}{2}\sqrt{\frac{g}{k'}}\ \frac{t}{x}$ this becomes

$$\eta = \frac{2}{\sqrt{\pi k' x}} \frac{\tau}{1 - \tau^4} \cos\left(2n+\frac{1}{2}\right)\pi\tau^2 \cos\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right) \qquad \text{(II A-5.32)}$$

The time history of the wave envelope at a fixed point is given by the plot in Figure II A-23 of the amplitude factor

$$A(\tau) = \frac{\tau}{1 - \tau^4} \cos\left(2n+\frac{1}{2}\right)\pi\tau^2 \qquad \text{(II A-5.33)}$$

vs. τ. Surface profiles for selected values of t are given in Figure II A-24. One easily shows that A (τ) has its maximum value when $\tau = 1$ and

$$\lim_{\tau \to 1} A(\tau) = \left(2n+\frac{1}{2}\right)\frac{\pi}{2}$$

Thus the maximum of the disturbance moves with a group velocity associated with the original wave length $2\pi/k'$. Moreover the wave is approximately sinusoidal in the neighborhood of $\tau = 1$ for

$$\eta\,(\tau = 1) = \frac{\left(2n+\frac{1}{2}\right)\pi}{\sqrt{\pi k' x}} \cos\left(k'x - \frac{\pi}{4}\right). \qquad \text{(II A-5.34)}$$

Jeffreys (1934) has remarked that a short-crested wave may be considered as the resultant of two long-crested waves moving in different directions. Waves in a storm area will consist largely of short-crested waves and as these waves leave the storm area they will tend to separate into long-crested components. As we have seen this is true to a first approximation for infinite wave trains since a product of the form cos mx cos ny can always be written in the form of a sum of simple cosines. A fundamental question then is to what extent does this

separation take place for localized disturbances. This is by no means answered by the simple remark above.

In order to attempt to answer this question we shall solve the initial value problem for which the surface displacement has the form

$$\zeta(x,z,o) = f(x,z) \tag{II A-5.35}$$
$$\frac{\partial \zeta(x,z,o)}{\partial t} = g(x,z)$$

by superposition of plane waves using Fourier's integral theorem. One finds that

$$\zeta = \iint_{-\infty}^{\infty} e^{i(kx+lz)}\left(N\cos\sigma t + \frac{M}{\sigma}\sin\sigma t\right) dk\, dl$$
$$N = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} f(x,z)\, e^{-i(kx+lz)} dx\, dz \tag{II A-5.36}$$
$$M = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} g(x,z)\, e^{-i(kx+lz)} dx\, dz$$

We consider for convenience the case $g(x,z) = 0$ and we suppose the water is infinitely deep so that

$$\sigma^2 = g\sqrt{k^2+l^2}$$

Thus ζ can be represented as a variety of plane waves moving in opposite directions as follows:

$$\zeta = \frac{1}{2}\iint_{-\infty}^{\infty} N(k,l)\, e^{i(kx+lz-\sigma t)} dk\, dl$$
$$\frac{1}{2}\iint_{-\infty}^{\infty} N(k,l)\, e^{i(kx+lz+\sigma t)} dk\, dl \tag{II A-5.37}$$

Now ζ can be evaluated approximately by applying the method of stationary phase to each integral separately. Introducing polar coordinates r, θ, one finds that stationary values of the phases occur for

$$k_o = \frac{gt^2}{4r^2}\cos\theta, \quad l_o = \frac{gt^2}{4r^2}\sin\theta \tag{II A-5.38}$$

Expanding the phase about these stationary points one finds on making the usual approximations that

$$\zeta = \frac{N(k_o, l_o)}{t\sqrt{B^2-AC}} \cos\left(\frac{gt^2}{4r}\right) \qquad \text{(II A-5.39)}$$

where A, B, C are the derivatives σ_{kk}, σ_{kl}, σ_{ll} evaluated at k_o, l_o. As a particularly important case consider that of an initial elevation of the form cos ax cos az for a square region of length $2n+\frac{1}{2}$ wave-lengths about the origin and zero outside. This generalizes Havelock's (1914) model for cylindrical waves. Then

$$N(k,l) = \frac{1}{\pi^2 a^2} \frac{\cos\left\{(2n+\frac{1}{2})\frac{\pi k}{a}\right\} \cos\left\{(2n+\frac{1}{2})\frac{\pi l}{a}\right\}}{\left(1 - \frac{k^2}{a^2}\right)\left(1 - \frac{l^2}{a^2}\right)} \qquad \text{(II A-5.40)}$$

Substituting, one finds

$$\zeta = \frac{gt^2}{2\sqrt{2}\pi a^2 r^3} \phi(\tau^2 \cos\theta)\phi(\tau^2 \sin\theta) \cos\left(\frac{gt^2}{4r}\right)$$

$$= \frac{4\sqrt{2}}{\pi a r} \tau^2 \phi(\tau^2 \cos\theta)\, \phi(\tau^2 \sin\theta) \cos\left(\frac{gt^2}{4r}\right) \qquad \text{(II A-5.41)}$$

where

$$\phi(\tau^2) = \frac{\cos\left\{(2n+\frac{1}{2})\pi\tau^2\right\}}{1-\tau^4}$$

and (II A-5.42)

$$\tau^2 = \frac{gt^2}{4r^2 a}$$

These are waves moving radially outward from the origin with a variable amplitude factor $A(\tau, \theta) = \tau^2 \phi(\tau^2 \cos\theta)\phi(\tau^2 \sin\theta)$ for fixed r. Since τ is proportional to t, $A(\tau, \theta)$ gives essentially the time history of the motion.

The wave length and period are

$$L = -2\pi \Big/ \frac{d\alpha}{dr} = \frac{8\pi r^2}{gt^2}$$

$$T = 2\pi \Big/ \frac{d\alpha}{dt} = \frac{4\pi r}{gt} \qquad \text{(II A-5.43)}$$

$$\alpha = \text{phase angle} = \frac{gt^2}{4r}$$

Hence

$$\text{group velocity} = \tfrac{1}{2}\text{ wave velocity} = \frac{gT}{4\pi} = \frac{1}{2}\sqrt{\frac{gL}{2\pi}} \qquad \text{(II A-5.44)}$$

In general one finds that the group velocity for any depth d is

$$= \tfrac{1}{2}\,(\text{wave velocity})\left(1 + \frac{2\ rd}{\sinh 2\ \ rd}\right) \qquad \text{(II A-5.45)}$$

The wave length at the point of maximum amplitude is easily determined for $\theta = 0, \pm \frac{\pi}{4}, \pm \frac{\pi}{2}, - - -$. For $\theta = 0$, $A(\tau, \theta)$ is a maximum for $\frac{gt^2}{4r^2} = a$ or $L = \frac{2\pi}{a}$. The maximum of the disturbance for $\theta = 0$ propagates with a group velocity associated with the original wave length $\frac{2\pi}{a}$. For $\theta = \pi/4$ the maximum amplitude occurs when $\tau^2 = \sqrt{2}$ which implies $L\pi/4 = L_o/\sqrt{2}$. This is the wave length of a long-crested component of an infinite train of the short-crested waves given initially. After a fixed time the distances travelled by the maximum disburbance will be in the ratio of the corresponding group velocities which implies that the distance along $\theta = 0$ is $\sqrt[4]{2}$ times the distance along $\theta = \pi/4$. The maximum disturbance occurs along $\pi/4$ and one finds for fixed r

$$\frac{\text{maximum amplitude at } \theta = \pi/4}{\text{maximum amplitude at } \theta = 0} = \sqrt{2}\ (2n + \tfrac{1}{2})\ \frac{\pi}{2} \qquad \text{(II A-5.46)}$$

which is quite large for a large number of initial waves.

The amplitude factors $A(\tau, \theta)$ for $\theta = 0, \pi/4$ are plotted in Figure II A-25 for $4\frac{1}{2}$ complete wave-lengths of the initial disturbance about the origin. The principal effect occurs in the group moving with the group velocities corresponding to the initial wave lengths. In advance of these oscillations, however, are several other groups of appreciable amplitude with wave lengths of $9/2L_o$, $9/4L_o$, $9/6L_o$ for $\theta = 0, \pi/2, \ldots$ where L_o is the initial wave length $2\pi/a$. These groups moving with the corresponding group velocities will arrive first at a given place. After the arrival of the maximum disturbance one will find smaller disturbances having wave lengths of $9/12\,L_o$, $9/14\,L_o$, $9/16\,L_o$, . . . for $\theta = 0$, $\pi/2$, ... In any particular group the period decreases with time at a given position like $1/t$. This leads to a broadening of the frequency spectrum toward shorter wave lengths as observed by Barber and Ursell (1948).

The behavior of the amplitude factor should be regarded only as indicating the trend of the amplitude as a function of time. Inherent in the use of the group approximation is the condition that at large distances the waves appear to be due to a concentrated point displacement of infinite amplitude. In order to secure amplitudes which are of the same order of magnitude as the assumed initial amplitude one must go to distances large compared with the dimensions of the initial disturbance. This may be neither convenient nor possible in actual situations. The solution of these difficulties requires a more refined asymptotic treatment of integrals such as II A-5.36.

One should note that the last factor of η, in II A-5.34, namely $\cos\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right)$ is independent of the initial displacement, depending only on the validity of the method of stationary phase and the assumption that the

water is infinitely deep. Assuming that the amplitude factor is slowly varying over the region of several oscillations of the last factor we find that the state of the motion is approximately reproduced when the increment in $\frac{gt^2}{4x}$ equals 2π. At a fixed point we have with $\Delta t = T$, the period of the oscillation,

$$T = \frac{4\pi}{g}\frac{x}{t} \tag{II A-5.47}$$

At a fixed instant of time we have with $x = -L$, L being the wave length,

$$L = \frac{8\pi}{g}\frac{x^2}{t^2} \tag{II A-5.48}$$

The wave velocity is, therefore

$$c = \frac{L}{T} = \frac{2x}{t} = \sqrt{\frac{gL}{2\pi}} = 2\,c_g \tag{II A-5.49}$$

Suppose we have a localized storm whose distance from the point at which the waves are recorded is D and the time it takes the first noticeable waves to reach the recorder is t_o. Although the method adopted implies that the disturbance should propagate instantaneously, the first waves noticed will have a definite period which is approximately the same as the period within the storm area. If one had a single disturbance the observed period would decrease like $1/t$ and when the maximum disturbance occurs the period would be equal to its original value in the storm area. For disturbances of equal intensities the longest period waves are generated initially. If the wind continues to set up disturbances for a length of time Δt the observed periods will be limited and

$$\frac{4\pi}{g}\frac{D}{t_o+\Delta t} \leq T \leq \frac{4\pi}{g}\frac{D}{t_o} \tag{II A-5.50}$$

The upper limit thus is fixed and the lower limit increases as the storm continues. The width of the period spectrum increases like

$$\Delta T = \frac{4\pi}{g} D\left(\frac{1}{t_o} - \frac{1}{t_o+\Delta t}\right)$$

$$\approx \frac{4\pi}{g}\frac{D\,\Delta t}{t_o^{\,2}} \tag{II A-5.51}$$

if $t_o \gg \Delta t$. Thus we are led to conclude that as long as the storm lasts the width of the period spectrum increases approximately linearly with time, has a fixed upper limit and the mean period decreases. Since the mean period is

$$\bar{T} = \frac{4\pi}{2g} D\left(\frac{1}{t_o} + \frac{1}{t_o+\Delta t}\right) \tag{II A-5.52}$$

we find

$$\Delta T = \frac{8\pi D}{g t_o} - 2\,\overline{T}$$

$$\simeq 2\,T_o - 2\,\overline{T} \qquad \text{(II A-5.53)}$$

Plotting the period range ΔT, or the standard deviation which is proportional to it, vs. the mean wave period we get a family of straight line segments of small extent, since Δt is small, and of negative slope. The intercepts are proportional to the upper limit of the period spectrum and depend upon D, t_o and hence indirectly to some extent on the wind velocity in the storm area. This trend is clearly visible in the data presented by Putz (1951) for the wave period variability of twenty five records if one connects successive points computed for a particular location. This trend does not show up in Putz's analysis since no account is taken of the possible dependence of the results on parameters of interest such as D, wind velocity, etc.

The approximation used in the standard formula in the method of stationary phase depends on $\frac{d^2}{dk^2}(kx - \sigma t)$ being large enough so that $e^{i(kx-\sigma t)}$ can be replaced by

$$e^{i(k_o x - \sigma_o t)} \; e^{\frac{1}{2} t \frac{d^2}{dk^2}(kx - \sigma t)\big|_{k_o} (k-k_o)^2}$$

until it is small compared with the value at the saddle point. This is possible for t sufficiently large if $\frac{d^2}{dk^2}(kx - \sigma t)\big|_{k_o}$ is not equal to zero. If this second derivative is zero the behavior will depend on the third derivative terms. Even if k is near the value for which the second derivative is zero one would need a much larger value of t than would be necessary to secure a good approximation elsewhere.

As an example Jeffreys and Jeffreys (1950) treat the initial disturbance

$$\eta(x,o) = \begin{cases} 1 & \text{for } -h < x < h \\ 0 & \text{otherwise} \end{cases} \qquad \text{(II A-5.54)}$$

They find that at points where $x = \sqrt{gd}\,t$, the amplitude decreases with time like $t^{-1/3}$ instead of $t^{-1/2}$ as in deep water.

> "The front of the gravity wave therefore becomes more and more the most prominent feature of the disturbance. The disturbance for $x > \sqrt{gd}\,t$ falls off rapidly and smoothly. The maximum elevation is a little behind the place where $x = \sqrt{gd}\,t$, and is followed by a train of waves becoming smaller and shorter."

b. Frontal Velocities of Semi-infinite Wave Trains:

Since the Fourier integral of a nonterminating wave train has no meaning the case of a wave motion which is at rest up to a certain time and then consists of a uniform sequence of sinusoidal vibrations of unlimited duration requires special consideration. The convergence difficulties associated with the Fourier representation of such wave motions, called signals, have been avoided by Sommerfeld (1914) by deforming the path of integration in the complex plane. This integral has been put into the usual form of a Laplace transform and discussed by Stratton (1941). The signal velocity with which the principal part of the wave motion propagates must be distinguished from the frontal velocity of the initial disturbance. It is shown that the signal does not preserve its form but that at a distance the first effect is a very weak wave motion, the forerunner, which gradually builds up to its full intensity. It is an essential result of Brillouin's work (1914) that this signal velocity practically coincides with the group velocity and exactly so if there is no strong absorption. The frontal velocity is shown to be identical with the phase velocity. Though an individual wave has a velocity appropriate to its length, the waves do not travel with constant velocity.

A similar investigation was made independently by Jeffreys (1945). He considers the propagation of a train of waves initially of semi-infinite extent. He finds that the extent of the smudged front due to dispersion is effectively confined to a distance of the order of the geometric mean of the distance travelled and the wave-length which is quite small in most cases of practical interest.

c. Ring Waves:

We consider the disturbance radiated from an initial elevation at rest having circular symmetry about some point on the still water surface. The typical solution for this case in deep water is

$$\phi = \frac{g \sin \sigma t}{\sigma} e^{ky} J_o (kr) \qquad \text{(II A-5.55)}$$

$$\eta = \cos \sigma t \; J_o (kr)$$

where $\sigma^2 = gk$. By superposition the most general solution for which $\eta(x_{z},o) = f(r)$, $\left.\frac{\partial \eta}{\partial t}\right|_{t=0} = 0$ is

$$\phi = g \int_0^\infty A(k) \frac{\sin \sigma t}{\sigma} e^{ky} J_o (kr) \, k \, d k \qquad \text{(II A-5.56)}$$

$$\eta = \int_0^\infty A (k) \cos \sigma t \; J_o (kr) \, k \, d k \qquad \text{(II A-5.57)}$$

where A (k) is determined by the Fourier-Bessel integral theorem

$$A (k) = \int_0^\infty f (\alpha) \; J_o (k\alpha) \, \alpha \, d \alpha \qquad \text{(II A-5.58)}$$

The treatment of this essentially one-dimensional problem is similar to that for propagation along a straight line. For large kr the function $J_o(kr)$ has the approximate asymptotic representation

$$J_o(kr) \cong \sqrt{\frac{2}{\pi kr}} \sin\left(kr + \frac{\pi}{4}\right) \qquad \text{(II A-5.59)}$$

Replacing J_o (kr) in II A-5.55 and II A-5.56 by this asymptotic representation we have a problem treated earlier.

For a stationary phase $\frac{r}{t} = \left.\frac{d\sigma}{dk}\right|_{k_o}$ or

$$k_o = \frac{gt^2}{4r^2} \qquad \text{(II A-5.60)}$$

The interpretation of this result is exactly the same as for straight line propagation since the form of k_o is the same in both cases.

6. Waves on Sloping Beaches:

For wave motion in a region of unlimited extent the superposition of two standing wave solutions, obtained by separation of variables, which are 90° out of phase leads to a progressive wave. Introducing a sloping plane barrier, representing a sloping beach, complicates the situation for it is no longer evident how two such standing waves can be obtained. As an example consider a vertical barrier at x = 0. One standing wave solution of the linearized problem is $\eta = a \cos mx \cos nz \cos \sigma t$. A second standing wave solution which is 90° out of phase with the first one is difficult to visualize unless perhaps this second solution has a singularity at the barrier which is what actually happens.

The one type of regular standing wave was first investigated by Hanson (1926) for cases in which the bottom slopes at angles $\pi/2n$, with n a positive integer.. Two types of standing wave solutions were first constructed by Bondi (1943) and Miche (1944) again for bottom slope angles of $\pi/2n$. For the pure standing wave of regular form Miche has shown that the ratio of the amplitude at the shore to the amplitude in deep water is

$$\frac{H_{shore}}{H_{deep}} = \sqrt{\frac{\pi}{2\alpha}} \qquad \text{(II A-6.1)}$$

where α is the angle the beach makes with the horizontal. In the case of a beach for which $\alpha < 90°$ the amplitude at the shore is always greater than the amplitude in deep water and increases rapidly with decreasing beach slopes. The situation is reversed for an overhanging cliff the ratio being a minimum equal to $\sqrt{2}/2$ for a dock.

The vertical component of particle velocity at the shore is

$$v_{shore} = -2a\sigma\sqrt{\frac{\pi}{2\alpha}} \cos \sigma t \qquad \text{(II A-6.2)}$$

the amplitude of the standing wave in deep water being 2a. This velocity component is zero at points of extreme elevations and a maximum when the fluid passes through the equilibrium position. A similar situation exists for the quantity u. The velocity parallel to the bottom at the shore line is

$$w = \frac{v_{shore}}{\sin\alpha} \qquad \text{(II A-6.3)}$$

which increases rapidly with decreasing α thus leading to increased frictional losses.

In order to form a standing wave on a beach the limiting steepness of the generating wave must be

$$\left(\frac{H}{L}\right)_{max} = \sqrt{\frac{2\alpha}{\pi}}\,\frac{\sin^2\alpha}{\pi} \qquad \text{(II A-6.4)}$$

As an example one finds that for values of α of 90°, 45°, 30°, 15°, 5° one has values of $(H/L)_{max}$ of 0.159, 0.112, 0.046, 0.0088, 0.0006 respectively. One concludes that the formation of a standing wave of appreciable steepness is impossible on a beach of small slope.

A new method of treating progressive wave motions was presented by Lewy (1946) and used later by Stoker (1947) and others. A differential equation for the velocity potential is derived using the given boundary conditions. This equation permits explicit integrations for cases where $\alpha = \frac{p\pi}{2n}$ in which p is any odd integer and n is any integer such that $2n > p$ the problem being two dimensional. In all cases there are two types of standing waves: the one type has finite amplitude all the way to shore, the other type has an amplitude which becomes logarithmically infinite at the shore. The solution is uniquely determined if one prescribes the wave length and amplitude in deep water and requires that the wave is entirely progressive with no reflection at the shore back to deep water. The results appear to be described with remarkable accuracy by an approximate theory described later for which the height and wave length are essentially the quantities predicted by the well-known theory assuming conservation of power and constancy of period.

A combination of a reduction method, similar to that used by Stoker, and the method of separation of variables or eigenvalue method has been employed by Weinstein (1949) in treating quite simply progressive wave motion in deep water bounded on one side by a vertical cliff. Since the solution is rather typical of more general applications we shall indicate here how the method applies for wave crests which are initially parallel to the cliff. The case, treated by Weinstein, of wave crests which at large distances make an arbitrary angle with the shore line requires only minor modifications. We assume that the motion is periodic so that the velocity potential can be written in the form

$$\Phi(x,y,t) = e^{i\sigma t}\phi(x,y)$$

The boundary value problem has the form:

$$\text{differential equation} \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \quad \text{for } x > 0,\ y < 0$$

$$\text{boundary conditions} \quad \frac{\partial\phi}{\partial y} = \frac{\sigma^2}{g}\phi \quad \text{for } y=0,\ x>0 \qquad \text{(II A-6.5)}$$

$$\frac{\partial\phi}{\partial x} = 0 \quad \text{for } x=0,\ y<0$$

Note that the quantity

$$\chi = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} - \frac{\sigma^2}{g}\right)\phi \qquad \text{(II A-6.6)}$$

vanishes for both boundaries according to (II A-6.5). In terms of the new variable χ the boundary value problem has the following form in polar coordinates:

$$\text{differential equation} \quad \frac{\partial^2\chi}{\partial r^2} + \frac{1}{r}\frac{\partial\chi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\chi}{\partial\theta^2} = 0$$

$$\text{(II A-6.7)}$$

$$\text{boundary conditions} \quad \chi = 0 \quad \text{for } \theta = 0,\ -\frac{\pi}{2}$$

Separating variables, by assuming $\chi(r,\theta) = R(r)\Theta(\theta)$, we find

$$\Theta'' + \lambda\Theta = 0$$

$$\text{(II A-6.8)}$$

$$R'' + \frac{1}{r}R' - \frac{\lambda}{r^2}R = 0$$

the prime denoting differentiation with respect to the appropriate variable. The boundary conditions $\Theta = 0$ for $\theta = 0, -\frac{\pi}{2}$ implies that a particular solution is

$$\Theta_n = \sin 2n\theta \quad , \quad \lambda = 4n^2$$

$$\text{(II A-6.9)}$$

$$R_n = A_n r^{2n} + B_n r^{-2n}$$

with arbitrary real constants, A_n and B_n. The only solution regular everywhere is $\chi = 0$ while a function which is regular at ∞ is of the form $\chi = r^{-2}\sin 2\theta$, the weakest singularity being taken to somewhat mitigate the contradiction to the original assumption of small amplitudes. Integrating the resulting differential equation for ϕ one obtains two standing waves 90° out of phase which can be combined into a standing wave. The velocity potentials of the two standing waves are of the form

$$\Phi_1 = \pi\, e^{i\sigma t}\, e^{my} \cos mx$$

$$\Phi_2 = e^{i\sigma t}\, e^{my}\left[\cos mx \int_{\infty}^{mx} \frac{\cos \xi}{\xi}\, d\xi + \sin mx \int_{\infty}^{mx} \frac{\sin \xi}{\xi}\, d\xi + \pi \sin mx\right] \qquad \text{(II A-6.10)}$$

Their surface behavior is illustrated in Figure II A-26 as given by Stoker (1947) together with the function $\pi \sin mx$, which yields the asymptotic behavior of Φ_2 on the surface. Since $\eta = -\frac{1}{g}\, \phi_t\, (x,o,t)$ these curves differ from the corresponding surface elevations only by a phase shift and a constant multiplier.

If we assume that the power transmitted with the waves is conserved and the wave period remains constant we find as shown in Section 3 that

$$\frac{a}{a_o} = \sqrt{\frac{1}{2n\; c/c_o}}$$

$$n = \tfrac{1}{2}\left(1 + \frac{2\,md}{\sinh 2\,md}\right) \qquad \text{(II A-6.11)}$$

$$\frac{L}{L_o} = \tanh \frac{2\pi d}{L}$$

Since these quantities are continuous functions of depth it is natural to take as the equation of the wave profile

$$\eta = a \sin (\lambda (L) + \sigma t) \qquad \text{(II A-6.12)}$$

where the local wave length $L = 2\pi / \frac{d}{dx} (\lambda (L))$ since this would be the wave length if $\lambda (L)$ were proportional to x and the wave accordingly was purely sinusoidal. It is interesting to note that this formula for η has been derived by Friedrichs (1948) as an asymptotic form for small slope angles of the solution given by Stoker. A comparison is given in Friedrichs' paper between the profiles calculated using the asymptotic formula and those from the exact linear theory as given by Stoker for an angle α of 6^o. The approximation is quite adequate there for bottom slopes of less than about 1/10.

The quantity $\lambda (L)$ is found to be

$$\lambda (L) = 2\pi \int_0^x \frac{dx}{L}$$

$$= \frac{1}{\alpha} \int_0^L \frac{\tanh^{-1} \frac{v}{L_o}}{v}\, dv + \frac{1}{\alpha L_o} \int_0^L \frac{dv}{1 - \left(\frac{v}{L_o}\right)^2} \qquad \text{(II A-6.13)}$$

$$\simeq \frac{1}{\alpha}\left[\tanh^{-1} \frac{L}{L_o} + \frac{L}{L_o} + \frac{1}{3^2}\left(\frac{L}{L_o}\right)^3 + \frac{1}{5^2}\left(\frac{L}{L_o}\right)^5 + \cdots\right]$$

The surface elevation relative to the amplitude in deep water is plotted in Figures II A-27a and 27b for $t = 0$. The curves show that to this approximation, which is really only the first order expression for waves on sloping beaches, the waves do not change appreciably in height or shape until they enter very shallow water. Figure II A-27b indicates that in very shallow water the waves increase in height and become steeper on the forward face than on the rear face. This is the type of behavior one observes for waves which eventually break by plunging. Quite recently Biesel (1951)[2] has claimed to have carried the expansions in powers of steepness and beach slope one step farther and shown that the waves actually plunge and break as has been observed.

Yoshida (1951)[1] has derived these formulas for low waves provided the depth varies gradually within the limits of a wave length. His method proceeds directly from the linearized equations of the problem and involves expansion of the potential in a power series in the depth coordinate.

In addition to the familiar formulas he finds that the magnitude of the horizontal velocity of the water particles at the bottom is

$$|u|_{bottom} = \frac{g\,k\,a}{\sigma \cosh kd} \qquad \text{(II A-6.14)}$$

This result is exactly that for uniform waves in constant depth where now however, a is a slowly varying function whose behavior has been described previously.

7. Shallow Water Waves

a. Linear Theory; Approximate Results for Waves of Finite Height

When the wave length is long compared to the depth of water it is frequently possible to make approximations which greatly simplify the mathematical analysis. The resulting theories form the basis of description of tidal waves and tsunamis in addition to the shorter period wind waves in very shallow water. Beginning with the exact equation describing the vertical motion

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial y} - g\rho \qquad \text{(II A-7.1)}$$

we find on integration with respect to y

$$p - p_o = g\rho(y_o+\eta - y) + \rho\int_y^{y_o+\eta} \frac{dv}{dt}\, dy \qquad \text{(II A-7.2)}$$

Now the integral involved is surely less than βd, where β is the maximum value of the vertical particle acceleration $\frac{dv}{dt}$. The time taken for the portion of a wave between two consecutive nodes to pass a particle is $\frac{L/2}{c}$ and hence if the steepness is small the vertical velocity will be of the order of $\eta\frac{c}{L/2}$ and the vertical acceleration of the order $\eta\left(\frac{c}{L/2}\right)^2$ where η is the maximum deviation or depression. Thus βd will be small compared with $g\eta$, provided $(d/L)^2$ is small. Since the horizontal velocity will be shown to be of the order of $\eta\frac{c}{d}$ the ratio of vertical to horizontal velocity components is of the order of d/L.

Assuming then that the pressure is hydrostatic, i.e.,

$$p - p_o = g \rho \, (y_o + \eta - y) \qquad \text{(II A-7.3)}$$

we find

$$\frac{\partial p}{\partial x} = g \rho \frac{\partial \eta}{\partial x} \qquad \text{(II A-7.4)}$$

It follows from this that the horizontal velocity u depends only on x and t. The equation of horizontal motion for infinitely small motions becomes

$$\frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x} \qquad \text{(II A-7.5)}$$

This assumption is distinct from the initial one, which may be fairly good even if the moti n is not small. In a progressive wave $\frac{\partial u}{\partial t} = \pm c \frac{\partial u}{\partial x}$ and hence neglecting $u \frac{\partial u}{\partial x}$ in comparison with $\frac{\partial u}{\partial t}$ implies that $u \ll c$. Since $\frac{u}{c} \simeq \frac{\eta}{d}$ this latter ratio must be small. From the equation of continuity we have

$$v = \int_0^y \frac{\partial u}{\partial x} dy = -y \frac{\partial u}{\partial x} \qquad \text{(II A-7.6)}$$

taking for the moment the origin at the bottom. At the free surface we have $y \doteq d + \eta$, $v = \partial \eta / \partial t$ and thus neglecting small quantities of higher order

$$\frac{\partial \eta}{\partial t} \simeq -d \frac{\partial u}{\partial x} \qquad \text{(II A-7.7)}$$

By II A-7.5 and II A-7.6

$$\frac{\partial^2 \eta}{\partial t^2} = gd \frac{\partial^2 \eta}{\partial x^2} \qquad \text{(II A-7.8)}$$

When d is no longer constant the equation can be written generally in the form

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{\partial^2 (g\, d\, \eta)}{\partial x^2} \qquad \text{(II A-7.9)}$$

Equation (II A-7.8) is the well-known wave equation in one dimension describing disturbances propagating with a velocity $c = \sqrt{gd}$. For two dimensional propagation we have

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{\partial^2 (g\, d\, \eta)}{\partial x^2} + \frac{\partial^2 (g\, d\, \eta)}{\partial y^2} \qquad \text{(II A-7.10)}$$

If d is constant it is easily shown that the waves propagate without change of form, the general solution being

$$\eta = f\,(x - ct) + g\,(x + ct) \qquad \text{(II A-7.11)}$$

in two dimensions, where f and g are arbitrary twice differentiable functions.

This solution suggests the term wave, for the function f (x - ct), is constant at points for which x - ct = const., i.e., for points moving with a velocity $\frac{dx}{dt} = c$. The function g (x+ct) represents a wave travelling in the negative x-direction with a constant velocity c.

The initial value problem for the wave equation is readily solved by elementary means. For example, suppose the surface elevation at t = 0 is given by $\eta(x,o) = f(x)$ and the vertical surface velocity is zero. Then it is easily shown that the surface elevation at any later time is represented by

$$\eta(x,t) = \tfrac{1}{2}\left[f(x - ct) + f\,(x + ct)\right] \qquad \text{(II A-7.12)}$$

We see that the disturbance splits into two equal parts which travel in opposite directions with the same velocity, c, as shown in the isometric drawing of Figure II A-28.

Certain approximate relations can be obtained independently for shallow water. Let us assume that the horizontal velocity component is independent of the depth coordinate and approximately equal to the particle velocity q. For surface profiles which approach a constant elevation above the bottom equal to the stream depth d, the continuity equation becomes

$$q\,(d + \eta) = c\,d \qquad \text{(II A-7.13)}$$

where c is the velocity, in the steady motion, for $\eta = 0$. Bernoulli's equation furnishes the relation

$$g\,d\,(1 + \frac{\eta}{d}) = -\tfrac{1}{2}\,c^2\,(1 + \frac{\eta}{d})^{-2} \qquad \text{(II A-7.14)}$$

For small η/d we find an approximate wave velocity

$$c = \sqrt{gd} \qquad \text{(II A-7.15)}$$

By II A-7.15 an approximate particle velocity is

$$q = c\,(1 - \frac{\eta}{d}) \qquad \text{(II A-7.16)}$$

Relative to the undisturbed water the particle velocity is $c\,\frac{\eta}{d}$ in the direction of wave motion. Carrying the approximation for c one step further we find

$$c \simeq \sqrt{gd}\,(1 + \frac{3}{2}\frac{\eta}{d}) \qquad \text{(II A-7.17)}$$

a formula due essentially to Airy (1845). This result can be applied to trace the progress of a single sinusoidal pulse whose elevation at x = 0 is

given by

$$\eta(o,t) = A \sin \sigma t \qquad \text{(II A-7.18)}$$

for $t = 0$ and $x > 0$, $\eta = 0$ and the particle velocity u_o is constant. We find that the distance x_b which the wave travelled in still water until it has a vertical face is

$$x_b = \frac{2\ c_o^3}{3\ g\ A\sigma} \qquad \text{(II A-7.19)}$$

the corresponding time being

$$t_b = \frac{2\ c_o^2}{3\ g\ A\sigma} \qquad \text{(II A-7.20)}$$

These formulae have been derived by Stoker (1948) employing the methods of gas dynamics, the phenomenon of breaking being analogous to the development of a discontinuous shock wave in a compressible gas. It also follows that the amplitude of the waves does not change but they become more and more asymmetric. This extension of the theory to include the height dependence of such waves is not strictly valid, however, for if one considers higher order terms in η/d in the approximation one violates the initial assumptions upon which the theory is based. The more exact theory to be given later predicts that initial pulses should not break but should continue indefinitely as a rapidly diffusing wave group. Qualitatively our above results state that a wave breaks because the upper portions of the wave move with greater velocities than the lower portions until eventually the wave has a vertical face, at which time it is assumed to break. This qualitative explanation occurs quite frequently in the literature, for example in an appendix written by Jeffreys (1934) to a book by Cornish. Presumably a similar method could be applied to sloping beaches if one uses in addition the principle of conservation of the (transmitted) power. The above formulas are interpreted by Stoker to indicate that the phenomena of breaking depends upon the quantity A/T, T being the period in deep water. The usual scatter of points in the curves given in H. O. 234, Breakers and Surf (1944) where one plots H_b/H_o' vs. H_o'/L_o and d_b/H_o' vs. H_o'/L_o is attributed to the fact that the amplitude ratios should be relatively independent of the initial steepness especially for the smaller beach slopes.

b. Underwater Obstacles in Shallow Water:

For sufficiently low waves simple wave solutions may be added in order to represent more complex situations. Thus for a vertical barrier one can secure the condition of zero horizontal velocity at the barrier by superimposing the wave travelling towards the barrier with its image in the barrier. The wave profile is of the general form

$$\eta = F\left(t - \frac{x}{c}\right) + F\left(t + \frac{x}{c}\right) \qquad \text{(II A-7.21)}$$

For a wave travelling in one direction the horizontal particle velocity is

$$u = \pm \frac{c}{d}\eta = \pm \frac{g}{c}\eta$$

the + sign being associated with waves of the form $\eta = f\left(t - \frac{x}{c}\right)$, travelling in the positive x - direction with velocity $c = \sqrt{gd}$ and the - sign for waves of the form

$\eta = g\left(t + \frac{x}{c}\right)$, travelling in the negative x-direction.

Waves travelling over a shelf are partially reflected and partially transmitted (Lamb, p.262). Taking the origin of the discontinuity we have for $x < 0$ corresponding to the seaward side of the barrier

$$\eta_1 = F\left(t - \frac{x}{c_1}\right) + f\left(t + \frac{x}{c_1}\right),$$

$$u_1 = \frac{g}{c_1} F\left(t - \frac{x}{c_1}\right) - \frac{g}{c_1} f\left(t + \frac{x}{c_1}\right) \qquad \text{(II A-7.22)}$$

and for $x > 0$

$$\eta_2 = \phi\left(t - \frac{x}{c_2}\right), \quad u_2 = \frac{g}{c_2} \phi\left(t - \frac{x}{c_2}\right) \qquad \text{(II A-7.23)}$$

where $c_1 = \sqrt{gd_1}$ and $c_2 = \sqrt{gd_2}$.

For two dimensional motions we have for continuity of mass flow across the plane $x = 0$

$$d_1 u_1 = d_2 u_2 \qquad \text{at } x = 0 \qquad \text{(II A-7.24)}$$

For continuity of the wave surface

$$\eta_1 = \eta_2 \qquad \text{at } x = 0 \qquad \text{(II A-7.25)}$$

These conditions reduce to

$$\frac{d_1}{c_1}\Big[F(t) - f(t)\Big] = \frac{d_2}{c_2}\,\phi(t),$$

$$F(t) + f(t) = \phi(t) \qquad \text{(II A-7.26)}$$

Solving for the transmission coefficient $T = \phi/F$ and the reflection coefficient $R = f/F$ we find

$$T = \frac{\phi}{F} = \frac{2\,c_1}{c_1 + c_2}$$

$$\text{(II A-7.27)}$$

$$R = \frac{f}{F} = \frac{c_1 - c_2}{c_1 \quad c_2}$$

Since $T^2 + R^2 = 1$ the energy in the incident wave is equal to the sum of energies in the reflected and transmitted waves. These quantities are plotted in Figure II A-29 and compared with results obtained in Section 3 by energy transmission methods. The basic assumption in the derivation of the shallow water theory is that the vertical acceleration of the water particles is negligible. This condition is clearly violated at the barrier itself but may hold for cross sections some distance from the barrier. The connection formulae may then be expected to relate the variables at two cross sections on opposite sides of the barrier at distances from it which are small compared to the wave length and moderate multiples of the depth.

The extension of this analysis to the case of a simple rectangular bar of width W, and height h is easily carried out (see for example, Jeffreys, 1944). The transmission coefficient is found to be

$$T = \frac{1}{\sqrt{1 + \frac{1}{4}\left(\frac{c_s}{c_b} - \frac{c_b}{c_s}\right)^2 \sin^2 \frac{2\pi W}{L_b}}} \qquad \text{(II A-7.28)}$$

the subscripts s and b referring to conditions seaward of the barrier and over the barrier respectively. Since the waves have constant period

$$L_b = \sqrt{\frac{d_b}{d_s}}\; L_s = \sqrt{1 - \frac{h}{d}}\; L_s$$

We see that

$$\frac{d - h}{L_b} = \sqrt{1 - \frac{h}{d}}\;\frac{d}{L_s}$$

and thus if the waves are initially of the shallow water type they always are. The numerical values of T are plotted in Figure II A-30.

Minimum transmission occurs for

$$\frac{2\pi W}{L_s\sqrt{1 - \frac{h}{d}}} = (n+\tfrac{1}{2})\pi \qquad , \quad n = 0, 1, 2, --- \quad \text{(II A-7.29)}$$

or

$$\frac{W}{L_s} = (n+\tfrac{1}{2})\,\tfrac{1}{2}\sqrt{1 - \frac{h}{d}}$$

and complete transmission occurs when

$$\frac{W}{L_s} = \frac{n}{2}\sqrt{1 - \frac{h}{d}} \qquad , \; n = 0, 1, 2, --- \qquad \text{(II A7.30)}$$

For $\frac{h}{d} = \frac{3}{4}$ one finds minimum transmission for $\frac{W}{L_s} = \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, ---$ and complete transmission for the intermediate values $\frac{W}{L_s} = 0, \frac{2}{8}, \frac{4}{8}, \frac{6}{8}, ---$. The transmission coefficient for minimum transmission is

$$T_{min} = \frac{2}{\frac{c_s}{c_b} + \frac{c_b}{c_s}} = \frac{2}{\alpha + \frac{1}{\alpha}} \qquad \text{(II A-7.31)}$$

where $\alpha = \sqrt{1 - \frac{h}{d}}$. The transmission is quite appreciable until the barrier approaches very closely to the surface. For example if $T_{min} = 0.1$ we find $d - h \simeq \frac{d}{400}$, so that if d = 40 ft. the bar would be 0.1 ft. below the surface. For barriers near the surface one has a type of resonance phenomenon the value of T rapidly oscillating between T_{min} and 1 as h increases. The extent of transmission appears to depend quite critically on the value of $\frac{h}{d}$ in this region. Actually the theory becomes invalid here.

The theory has been discussed for the case of a continuous shelf of the form

$$h\ (x) = \frac{d + (d - h)\ e^{nx}}{1 + e^{nx}} \qquad \text{(II A-7.32)}$$

by Yoshida (1948) (see Figure II A-31a). It is easily seen from II A-7.32 that as $n \to \infty$ the shelf approaches a simple discontinuity at x = 0 of height h, which was considered earlier. The quantity $2\ell = \frac{2\pi}{n}$ measures the "effective" width of the shelving region. Yoshida finds that the transmission as defined previously is given by

$$T^2 = \frac{B}{A} \frac{\sinh 2\pi A \sinh 2\pi B}{\left[\sinh \pi\ (A + B)\right]^2} \qquad \text{(II A-7.33)}$$

where $A = \frac{2\ell}{L_a}$, $B = \frac{2\ell}{L_b}$ and L_a, L_b are the wave lengths in depths d and d - h respectively. The reflection coefficient is

$$R = \frac{\sinh \pi (B - A)}{\sinh \pi (B + A)} \qquad \text{(II A-7.34)}$$

The simple discontinuity is approached for $n \to \infty$ and one finds

$$T \to \frac{2}{1 + \frac{L_b}{L_a}}$$

$$\text{(II A-7.35)}$$

$$R \to \frac{L_a - L_b}{L_a + L_b}$$

which are the same as those given earlier for this case. For the gradual transition, $A \gg 1$ and $B \gg 1$, we find

$$R^2 \cong e^{-2\pi A} - e^{-2\pi B} \approx 0$$

(II A-7.36)

$$T = \sqrt{\frac{B}{A}} = (1 - \frac{h}{d})^{-1/4}$$

the reflection nearly vanishing and the transmission following Green's law.

Yoshida (1948) treats the case of a bank and trench by choosing $y = e^{nx}$

$$h(y) = \left[c_1 - \frac{A_1 y}{1-y} - \frac{B_1 y}{(1-y)^2} \right]^{-1} \qquad \text{(II A-7.37)}$$

the profiles being shown in Figure II A-31b. In terms of the indicated variables

$$b = \frac{1}{c_1 + A_1} , \quad d = \frac{1}{c_1}$$

$$h_M = \frac{1}{c_1 + \frac{(A_1 + B_1)^2}{4B_1}}$$

The type of reflection and transmission depends on the quantity

$$\delta = i\sqrt{\frac{1}{4} + kB_1}$$

where $kB_1 = \frac{16\lambda^2}{L_M}(1 - \frac{h_M}{d})$. For the bank, $B_1 > 0$, δ is always imaginary and even for the trench, $B_1 < 0$, δ may be imaginary if the trench is shallow or narrow, or if the wave period is longer, so that $|k B_1| < \frac{1}{4}$. However δ is real if $|k B_1| > \frac{1}{4}$, which corresponds to the case of the deep or broad trench or the shorter period wave. Then if δ is imaginary

$$R^2 = \frac{\cosh 2\pi(A - B) + \cos (2\pi |\delta|)}{\cosh 2\pi(A + B) + \cos (2\pi |\delta|)} \qquad \text{(II A-7.38)}$$

If δ is real

$$R^2 = \frac{\cosh 2\pi(A - B) + \cosh 2\pi\delta}{\cosh 2\pi(A + B) + \cosh 2\pi\delta} \qquad \text{(II A-7.39)}$$

Yoshida (1950) has also considered the case of a finite depth change completed within an interval small compared with the wave length, while outside of this region the depth is constant.

Dean (1944) treated the motion of shallow water sinusoidal waves over a small bar of the form

$$h = d\left(1 - \frac{H}{d\left[1 + \left(\frac{x}{\ell}\right)^2\right]}\right) \qquad \text{(II A-7.40)}$$

as shown in Figure II A-31c. For small H a first approximation furnishes the reflection coefficient at a great distance from the bar of

$$R = \frac{\pi^2 H \ell}{dL} e^{-4\pi \frac{\ell}{L}} \qquad \text{(II A-7.41)}$$

The reflection process is seen to depend upon the barrier height relative to depth, H/d, and the effective barrier width relative to the incident wave length, $2\ell/L$. We see that $R \to 0$ as $\ell/L \to 0$ and as $\ell/L \to \infty$. In the former case the wave does not "feel the barrier" and in the latter case the change is so gradual that there is no reflection. The assumption implicit in the shallow water theory that the vertical acceleration is negligible is invalid, however, if ℓ/L is too small. R is a maximum, for given H/d, if $\ell = L/4\pi$ and $R_{max} = \pi H/4\, d\, e \simeq 0.29\, H/d$ which is small. The wave length decreases as the wave moves over the bar and one finds in the case of maximum reflection, where $\ell = L/4\pi$, that the change in wave length is approximately $0.15\, L\, \frac{H}{d}$. This makes it appear that the presence of a bar will be difficult to detect from the air by measuring the change of wave length.

The velocity of a wave crest is

$$c_{crest} = \frac{\sqrt{gd}\,(1 - r^2)}{1 + r^2 + 2r\cos(2\sigma t + \epsilon)} \qquad \text{(II A-7.42)}$$

the elevation due to an incident and reflected wave being

$$\eta = \cos(kx - \sigma t) + r\cos(kx + \sigma t + \epsilon), \quad 0 < r < 1 \qquad \text{(II A-7.43)}$$

For large negative values of x, $\epsilon = \pi/2$, $r = R$. The crest velocity lies in the range

$$0 < \frac{1 - r}{1 + r}\sqrt{gd} \leq c_{crest} \leq \frac{1 + r}{1 - r}\sqrt{gd} \qquad \text{(II A-7.44)}$$

so that over part of the wave period the velocity $\sqrt{gd}$ will be exceeded by a wide margin if r is nearly 1. If a mean velocity is taken over an interval several wave periods in length reflection will not play a large part. These results indicate that if r is large and the time interval is substantially less than the wave period the calculated mean velocities should vary considerably. As an extreme example, consider the case of a bar at Clatsop Spit for which d = 30 ft., H = 23 ft., ℓ = 170 ft. For maximum

reflection $L = 2140$ ft., a value outside the realm of wind waves. For $L = 300$ ft. we find $R = 0.0035$ indicating a negligible reflected wave far from the bar.

For oblique incidence on a simple shelf (see Figure II A-32) the preceding methods apply with little alteration (see Arthur,1950). The reflection and transmission coefficients are

$$R = \frac{c_1 \cos\theta_1 - c_2 \cos\theta_2}{c_1 \cos\theta_1 + c_2 \cos\theta_2}$$

$$T = \frac{2\, c_1 \cos\theta_1}{c_1 \cos\theta_1 + c_2 \cos\theta_2} \qquad \text{(II A-7.45)}$$

Together with Snell's law

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2} \qquad \text{(II A-7.46)}$$

these equations describe the process completely. For $\theta_1 = 0$, the waves are normally incident, and R and T reduce to the expressions previously found for this case.

e. Shallow Water Waves on a Beach of Constant Slope:

The motion of shallow water waves has been investigated for a rectangular canal of gradually varying cross-section by Green (1837) (see Lamb, p. 273). The approximation involved in this theory is that the transverse dimensions of the canal as well as their derivatives db/dx and dh/dx vary slowly over the region of one wave-length. Under these conditions, the surface profile is represented by

$$\eta = \frac{1}{b^{1/2}\, d^{1/4}} \left[F\,(\theta - t) + f(\theta + t) \right] \qquad \text{(II A-7.47)}$$

where θ is defined by $dx/d\theta = \sqrt{gd}$. The wave velocity is then $\sqrt{gd}$. The quantities $F(\theta - t)$ and $f(\theta + t)$ represent waves travelling in the positive and negative x-directions, respectively. The problem of transition over a section whose cross-section changes continuously has been studied by Rayleigh (1880). It was shown that if the transition was completed over a space which is a moderate multiple of one wave length there is practically no reflection; while when the cross section changes abruptly the reflection is considerable depending on the actual change in dimensions.

When the waves crests are parallel to a beach of constant slope the general solution is still possible (see Lamb, p. 274, Hirono (1946), Stoker (1947)). Suppose the depth uniformly increases from the end $x = 0$. Then putting $d = d_o x/a$, $K = \sigma^2 a/gd_o$, we obtain for a simple harmonic motion
$\eta = Z(x) \cos(\sigma t + \epsilon)$

$$\frac{d}{dx}\left(x \frac{dZ}{dx}\right) + KZ = 0 \qquad \text{(II A-7.47)}$$

The general solution of this equation is

$$Z(x) = A\, J_o\,(2\sqrt{Kx}\,) + B\, Y_o\,(2\sqrt{Kx}\,) \qquad \text{(II A-7.49)}$$

where J_o and Y_o are the regular and singular Bessel functions of zero order respectively; Y_o has a logarithmic singularity at $x = 0$. The asymptotic approximations to Y_o and J_o for large x are

$$J_o(2\sqrt{Kx}) \simeq \frac{\cos\ (2\sqrt{Kx} - \frac{\pi}{4})}{\sqrt{\pi\sqrt{Kx}}}$$

and (II A-7.50)

$$Y_o(2\sqrt{Kx}) \simeq \frac{\sin\ (2\sqrt{Kx} - \frac{\pi}{4})}{\sqrt{\pi\sqrt{Kx}}}$$

This would show that this approximate theory requires the waves to have zero amplitude at $x = \infty$, but one should realize that the shallow water theory would not apply for large x so that one must begin with given wave conditions in shallow water. The wave length L for large x is given by the relation

$$\frac{2\pi}{L}\, x = 2\sqrt{Kx}$$

or for constant period one has $L \propto \sqrt{x}$ so that the wave length increases indefinitely.

The surface profile of a wave progressing toward shore is

$$\eta = A\left[\cos\sigma t\ Y_o\ (2\sqrt{Kx}) + \sin\sigma t\ J_o\ (2\sqrt{Kx}\,)\right] \qquad \text{(II A-7.51)}$$

For waves a sufficient distance from shore so that the asymptotic form can be used

$$\eta \simeq \frac{A}{\sqrt{\pi\sqrt{Kx}}}\ \sin\ (-2\sqrt{Kx} + \sigma t + \frac{\pi}{4}) \qquad \text{(II A-7.52)}$$

In general we have

$$\eta = A\sqrt{J_o^2\ (2\sqrt{Kx}\,) + Y_o^2\ (2\sqrt{Kx})}\ \sin\ (\sigma t - \theta\ ) \qquad \text{(II A-7.53)}$$

where

$$\theta = \tan^{-1} \frac{Y_o\ (2\sqrt{Kx})}{J_o\ (2\sqrt{Kx})}$$

The wave velocity of these waves is

$$c = \sigma / \frac{\partial \theta}{\partial x} = \pi \sigma x (J_o^2 + Y_o^2) = \pi \sqrt{gd} \sqrt{kx} \, (J_o^2 + Y_o^2) \qquad \text{(II A-7.54)}$$

The wave crests and troughs are found from the solution of the equation

$$\frac{\partial \eta}{\partial x} = A \left[\cos \sigma t \; Y_o' + \sin \sigma t \; J_o' \right] = 0$$

If x_o is the coordinate of a wave crest then

$$\frac{dx_o}{dt} = \sqrt{gd} \, \frac{J_1^2 (2\sqrt{Kx}) + Y_1^2 (2\sqrt{Kx})}{J_o Y_1 - J_1 Y_o}$$

$$= \frac{-\pi k}{\sigma} gd \, (J_1^2 + Y_1^2) \qquad \text{(II A-7.55)}$$

The elevation of the top of the wave is

$$\eta(x_o) = \frac{A}{\pi \sqrt{k x_o}} \; \frac{1}{\sqrt{J_1^2 + Y_1^2}} \qquad \text{(II A-7.56)}$$

The error in velocity is approximately determined by expanding the Bessel functions about the origin. We find

$$\frac{\sqrt{gd} - c}{\sqrt{gd}} \approx 0.00435 \, q^2 / \frac{d}{L} \qquad \text{(II A-7.57)}$$

For all but the smallest values of d/L the error is quite negligible. For example, if the slope $q = 1/10$ the error is less than 1% if $d/L > 0.00435$.

8. Wave Refraction:

When the quantity d/L is small, generally if d/L is approximately 1/2 or smaller, the bottom effects modify the wave motion. As a wave train moves into shoaling water one finds in addition to a change in height and wave length, discussed previously, a refraction tending to align the wave crests with the beach contours. This effect can be explained by means of Huyghen's principle, well known in geometrical optics. Each small section of a wave crest can be regarded as the source of a secondary disturbance which propagates radially outwards with a velocity associated with the wave length and depth at this section. The envelope of the disturbances after a small time increment is the new wave front. Since the wave velocity decreases with depth the crest tends to swing around becoming parallel to the contours. If we assume that the refraction does not take place so rapidly that the crest does not have time to level out then the total energy transmitted between neighboring orthogonals must remain constant. The wave height changes with the orthogonal spacing according to Equation II A-3.61.

A particularly useful tool, widely used in the corresponding optical problem is Fermat's principle which states that a point must move with the wave velocity over the true orthogonal to the wave crests in such a wave that the time taken between two fixed points is less than that over any other path joining the same two points. This is equivalent to stating that over the true orthogonal having arc length s

$$\int_{s_o}^{s_1} \frac{ds}{c} \quad \text{is stationary} \qquad \text{(II A-8.1)}$$

as compared with neighboring curves joining the same points. The necessary condition for this is expressed in Cartesian coordinates by Euler's equation

$$\frac{d}{dx}\left(\frac{\partial m}{\partial y'}\right) - \frac{\partial m}{\partial y} = 0 \qquad \text{(II A-8.2)}$$

where

$$m = \frac{\sqrt{1+y'^2}}{c}$$

In our treatment c is a function of d and L_o alone. The exact solution of this equation is simple only for parallel depth contours, i.e., the depth varies only in one direction, say the y-direction. Then a first integration is possible with the result that

$$y' \frac{\partial m}{\partial y'} - m = \text{const.} \qquad \text{(II A-8.3)}$$

Substituting and rewriting this reduces to

$$\frac{\sin \alpha}{\sin \alpha_o} = \frac{c}{c_o} \qquad \text{(II A-8.4)}$$

where α is the angle the orthogonal makes with the y-axis and the subscript o refers to the initial values on the orthogonal in deep water. This formula is known as Snell's law. As long as the contours are parallel this law is applicable so that the angle α depends only on the period in deep water, the initial direction of approach and the depth where α is desired. If the waves did not break first one would always have $\alpha = 0$ at the beach. The results are given in Figure II A-33 for cylindrical waves. If the ratio of wave lengths is unaffected by the refraction one can easily modify the curves for cylindrical waves so as to apply to short-crested waves. This has been carried out and the results given in Figure II A-34 for L=L'.

The spacing of orthogonals is easily determined by considering refraction at a single depth discontinuity as in Figure II A-35. One finds

$$\frac{1}{K} = \frac{\cos\alpha_o}{\Delta l_o} = \frac{\cos\alpha}{\Delta l} \qquad \text{(II A-8.5)}$$

For a continuous variation of depth one has such a law holding for each infinitesimal variation and hence in general

$$\frac{\cos\alpha}{\cos\alpha_o} = \frac{\Delta l}{\Delta l_o} \qquad \text{(II A-8.6)}$$

If the contours are no longer parallel, the commonly used approximation, is the application of Snell's law, proceeding from one contour to another in a straight line and using an average depth. The practical use of these methods has been described by Johnson, O'Brien, and Isaacs (1948). Essentially the same problem encountered here occurs in the determination of electron trajectories through given electrostatic fields (see Liebmann, 1950). For the electron problem the graphical application of Snell's law was discussed by Jacob (1938), who considers that there may be an error in the final position of the trajectory of as much as 50%. Liebmann is of the opinion that of the possible graphical methods the application of Snell's law is the simplest and quickest, but on the whole the least accurate.

A similar investigation is possible for parallel circular contours. One easily finds a "Snell's law" which is

$$\frac{r}{r_o}\frac{\sin\alpha}{\sin\alpha_o} = \frac{c}{c_o} \qquad \text{(II A-8.7)}$$

where α is the angle between the orthogonal and the radial line and the subscript o refers to the initial conditions in deep water. The orthogonal itself is obtained on solution of this equation, as before. One sees that the wave tends to align itself with the contours. For a bottom of constant slope the angle approaches zero, for small r, like $r^{3/2}$. This is considerably faster than for the straight parallel contours, in which case α approaches zero, for small y, like $y^{1/2}$. It is evident that if both c and r can change by large fractions relative to their initial values that there will be considerable refraction so that about a circular island the wave heights on the "sheltered" side will be large. The actual behavior in any particular case can easily be obtained by graphical methods based on the generalized "Snell's law" or for controllable accuracy by a numerical step by step integration of the associated second order ordinary differential equation.

Snell's law can also be interpreted for waves generated in shallow water and propagated toward deep water. Such situations occur for waves reflected from the shore and for incoming waves passing over a submarine bar (see Isaacs, Williams, and Eckart, (1950)). If we let the subscript i refer to the initial conditions in shallow water then

$$\frac{\sin\alpha}{\sin\alpha_i} = \frac{c}{c_i} \qquad \text{(II A-8.8)}$$

for a beach with straight parallel contours. An orthogonal directed toward deep water will be refracted back to shore if

$$c = \frac{c_i}{\sin \alpha_i} \quad \text{at some point} \qquad \text{(II A-8.9)}$$

The point at which the orthogonal turns back is, in extreme cases, nearly in deep water so that the limiting condition reads

$$L_o = \frac{2\pi d}{\sin^2 \alpha_i} \qquad \text{(II A-8.10)}$$

d being the initial depth in shallow water.

The refraction of a wave group has been treated by Jeffreys (1950[2]) and Stoneley (1935) (see also an earlier paper of Stoneley's, 1935, Monthly Nat. Roy. Astron. Soc.) incorporating ideas of Jeffreys. Jeffreys treats the simple case of refraction at a depth discontinuity. An orthogonal originating at O is refracted at Q and observed at p

The time factor of the disturbance at p is

$$\exp i \; (\sigma t - k_1 s_1 - k_2 s_2) \qquad \text{(II A-8.11)}$$

where k_1, k_2 are the wave numbers associated with σ in the two media. We let s be the distance of Q from some fixed point on the boundary. The principle disturbance occurs for a stationary value of II A-8.11 with respect to σ and s. This gives

$$k_1 \frac{d s_1}{ds} + k_2 \frac{d s_2}{ds} = 0$$

$$t - s_1 \frac{\partial k_1}{\partial \sigma} - s_2 \frac{\partial k_2}{\partial \sigma} = 0 \qquad \text{(II A-8.12)}$$

These can be written in the form

$$\frac{1}{c_1} \frac{d s_1}{d s} + \frac{1}{c_2} \frac{d s_2}{d s} = 0 \qquad \text{(II A-8.13)}$$

$$\frac{s_1}{c_{g_1}} + \frac{s_2}{c_{g_2}} = t \qquad \text{(II A-8.14)}$$

The first of these is simply Snell's law for the refraction of an orthogonal which is governed by the wave velocity as for a regular wave train. The second equation corresponds to $\frac{x}{t} = c_g$ showing that the principle period is determined by the group velocities. In general the disturbance at p can be represented by a Fourier integral type solution. The usual method of treating such integrals by the method of stationary phase leads to the discussion given above of the exponential factor given in II A-8.11.

9. Waves in Running Water:

When the water has a constant horizontal velocity the particle motion is given to the first order in a/L by simply adding component velocities. The wave velocity remains unchanged. The velocity potential describing a flow with constant velocity U in the positive x-direction is

$$\phi = -U\,x + \frac{g\,a}{\sigma}\,\frac{\cosh k\,(d+y)}{\cosh k\,d}\cos\,(kx - \sigma t) \qquad \text{(II A-9.1)}$$

This is only an approximation since the actual flow will not be irrotational in general.

The more general situation of a current whose velocity varies with depth has been considered by Biesel (1950[3]) to a first approximation. In a coordinate system for which the wave motion is stationary and the mean current velocity is c(y) the stream function is taken in the form

$$\Psi = \Upsilon(y) - c_o\,a\,f(y)\,\sin kx \qquad \text{(II A-9.2)}$$

where

$$c = \frac{d\,\Upsilon}{dy}, \quad c_o = c(o)$$

Since the surface amplitude is a, we find f(o) = 1. For a rigid bottom f(-d) = 0. Substituting Ψ into the equation of motion

$$\frac{d\,(\Psi,\,\Delta\Psi)}{d\,(x,y)} = 0 \qquad \text{(II A-9.3)}$$

it is found that

$$\frac{f''}{f} = m^2 + \frac{c''}{c} \qquad \text{(II A-9.4)}$$

For constant pressure on the free surface Bernoulli's equation yields the condition

$$c_o\left[c_o\,f'\,(o) - c'\,(o)\right] = g \qquad \text{(II A-9.5)}$$

where terms of the order $(a/L)^2$ are neglected.

We see from II A-9.2 and II A-9.3 that to this approximation the velocities of propagation of waves in running water is independent of their amplitude.

For a linear velocity distribution

$$c = c_o (1 + Ky) \tag{II A-9.6}$$

one finds

$$f(y) = \frac{\sinh k (d + y)}{\sinh kd} \tag{II A-9.7}$$

and II A-9.4 becomes

$$c_o^2 \left[k \coth kd - K \right] = g \tag{II A-9.8}$$

This equation only has solutions for k, assuming $K \geq 0$, if

$$\sqrt{c_o\, c_1} < \sqrt{gd} \tag{II A-9.9}$$

where c_1 is the velocity at the bottom $= c_o (1 - Kd)$. In this case the wave length of possible stationary waves is smaller than in a uniform current of the same mean velocity.

The absolute velocity of waves running with or against the current is found to be

$$c_a = U_o - \frac{U_o - U_1}{2} \frac{\tanh k d}{kd} \pm \sqrt{\left[\frac{U_o - U_1}{2} \frac{\tanh kd}{kd}\right]^2 + g \frac{\tanh kd}{k}} \tag{II A-9.10}$$

For deep water one has $c_a = U_o \pm \sqrt{g/k}$, indicating a simple algebraic addition of the surface velocity and the velocity of propagation of this wave in still water. For shallow water

$$c_a = \frac{U_o + U_1}{2} \pm \sqrt{\left(\frac{U_o - U_1}{2}\right)^2 + gd} \tag{II A-9.11}$$

differing only slightly from the classical formula since $U_o - U_1$ is small in these circumstances.

Abdullah (1947) treats the deep water case in which the current velocity decreases exponentially with depth. The current is assumed to produce a perturbation in the original flow which will be so small that the square of the perturbation velocity can be neglected. The resulting analysis leads to an ordinary differential equation which is solved in

series form. The resulting equations are difficult to discuss so that the author gives only a few numerical values based on successive approximations to the solutions. For a current distribution of the form $U = V_o e^{\alpha y}$ he computes values for: $V_o = 15$ cm/sec, $\alpha = 10^{-3}$ c.g.s and $V_o = 100$ cm/sec, $\alpha = 10^{-4}$ c.g.s Plotting c vs. L one finds that in these cases the values of c as computed from the simple approximate formula

$$(c - V_o)^2 = \frac{g}{m} \qquad \text{(II A-9.12)}$$

are somewhat too high. The first approximation as given by Abdullah, namely

$$m = \frac{g - \alpha V_o (c - V_o)}{(c - V_o)^2} \qquad \text{(II A-9.13)}$$

seems to be sufficiently accurate for most purposes, the exact values of m being only slightly higher. The above situation applies when V_o/c is small. Some discussion is also given by Abdullah for the case when V_o/c is approximately unity.

The pertubation method has been employed by Thomson (1949) to discuss wave motion in weak currents. He finds that the physically possible values of phase speed lie in the range

$$U_{min} - \sqrt{gd} \leq c \leq U_{max} + \sqrt{gd} \qquad \text{(II A-9.14)}$$

Moreover

$$c \leq U_{max} + \sqrt{\frac{g}{k}} \qquad \text{(II A-9.15)}$$

The motion of waves on streams whose velocities vary with time is important in oceanography in connection with waves crossing tidal streams. The velocity changes considerably during the time taken for swell waves to cross the stream. The problem was treated as one of relative motion by Barber (1949) assuming the current did not vary with depth. The velocity difference between the first and last crests of a set of N waves of wave length L is

$$N\,L\,\frac{\partial (c + u)}{\partial x}$$

This must also be equal to the rate of change of N L with respect to time, i.e.,

$$N\left(\frac{\partial L}{\partial t} + (c + u)\frac{\partial L}{\partial x}\right) = N\,L\,\frac{\partial (c + u)}{\partial x} \qquad \text{(II A-9.16)}$$

Since

$$\frac{\partial c}{\partial x} = \frac{d\,c}{d\,L}\frac{\partial L}{\partial x}$$

we find

$$\frac{1}{L}\left[\frac{\partial L}{\partial t} + \left(u + c - L\frac{\partial c}{\partial L}\right)\frac{\partial L}{\partial x}\right] = \frac{\partial u}{\partial x} \qquad \text{(II A-9.17)}$$

The right side of this equation measures the rate of expansion of the water surface which is produced by the streaming motion while the left side is the fractional rate of increase of the wave length with time. This means that an observer following a given wave group through moving water, will find that the average wave length in the group expands or contracts at the same rate as the surface covered by the group. Knowing the latter quantity the wave length and thus velocity and period can be determined.

The change of wave length for a flow in the same or opposite direction to that of the wave motion is obtained by equating periods in a steady state. Letting L, T, c be variables describing the waves in a current U and those with subscript o those in the undisturbed region, then

$$\frac{L_o}{c_o} = \frac{L}{c + U} \qquad \text{(II A-9.18)}$$

where

$$c = \sqrt{\frac{gL}{2\pi}\tanh\frac{2\pi d}{L}}$$

This is a transcendental equation for L in terms of c_o, L_o, d, U. For deep water one can solve explicitly for L/L_o and one finds

$$\frac{c}{c_o} = \sqrt{\frac{L}{L_o}} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{U}{c_o}} \; . \qquad \text{(II A-9.19)}$$

Approximate values for the change in wave height can be obtained by calculating the amount of energy transmitted relative to a fixed coordinate system both inside and outside of the current. In deep water the latter is

$$\frac{E_o\, c_o}{2}$$

and the former

$$\frac{E\, c}{2} + E\, U \; .$$

Hence

$$\frac{E_o\, c_o}{2} = \frac{E\, c}{2} + E\, U \qquad \text{(II A-9.20)}$$

and since E is proportional to H^2

$$\left(\frac{H}{H_o}\right)^2 = \frac{1}{\frac{c}{c_o} + 2\frac{U}{c_o}} \qquad \text{(II A-9.21)}$$

which can be computed from II A-9.19.

When a wave meets a current discontinuity at an angle Johnson (1947) finds the appropriate connection formulas by requiring the wave to be continuous across the discontinuity as shown in Figure II A-36. One sees that

$$\frac{c_o}{\sin\alpha} = U + \frac{c}{\sin\beta} \qquad \frac{L_o}{\sin\alpha} = \frac{L}{\sin\beta} \tag{II A-9.22}$$

from which one can find β and L/L_o given the other variables c_o, α, U, d. The energy transmitted is now

$$\frac{c_o\, b_o}{2} H_o^2 = \frac{cb}{2} H^2 + U \sin\beta\, b\, H^2 \tag{II A-9.23}$$

Solving for H/L we find that for deep water

$$\left(\frac{H}{L}\right)^2 = \left(\frac{H_o}{L_o}\right)^2 \frac{\cos\alpha}{\cos\beta} \frac{\left(1 - \frac{V}{c_o} \sin\alpha\right)}{1 + \frac{V}{c_o} \sin\alpha} \tag{II A-9.24}$$

The refraction can be calculated more generally when currents and under-water topography both play a role by employing a method given by Zermelo (1931) for determining the best path of an airplane travelling between two given points in a region of variable winds (see also Arthur, (1950)) in the sense that the travel time is a minimum, with the notation indicated in Figure II A-37. Zermelo finds that for the minimal path and constant c

$$\frac{d\theta}{dt} = -\frac{\partial U_x}{\partial y} \cos^2\theta + \left(\frac{\partial U_x}{\partial x} - \frac{\partial U_y}{\partial y}\right) \sin\theta \cos\theta + \frac{\partial U_y}{\partial x} \sin^2\theta \tag{II A-9.25}$$

while if the velocity c is variable De Mira Fernandes (1932) finds that

$$\frac{d\theta}{dt} = -\frac{\partial (U_c + c)}{\partial y} \cos\theta + \frac{\partial (U_c + c)}{\partial x} \sin\theta \tag{II A-9.26}$$

where V_c is the component of U in the c-direction. Together with the velocity equations

$$\frac{dx}{dt} = U_x + c \cos\theta \qquad \frac{dy}{dt} = U_y + c \sin\theta \tag{II A-9.27}$$

we have a set of three first order ordinary differential equations which can be solved for x, y, θ as functions of t thus determining the orthogonals.

10. The Effect of Rigid Obstacles on Deep Water Waves:

a. Wave Diffraction:

In analogy with the use of the word diffraction in electromagnetic and sound theory we consider the (resulting) wave motion when an incident regular periodic wave train is perturbed by a vertical barrier extending from the bottom to a point well above the free surface. Since the dependence of Φ on the depth coordinate is given by $\Phi = \cosh k(y+d)$ $\phi(x,z)$ we find that ϕ must satisfy the equation

$$\phi_{xx} + \phi_{zz} + k^2 \phi = 0 \qquad \text{(II A-10.1)}$$

The analogy with sound waves arises when one notices that the wave equation

$$\phi_{xx} + \phi_{zz} = \frac{1}{c^2} \phi_{tt} \qquad \text{(II A-10.2)}$$

assumes the form of equation II A-10.1 for a harmonic time dependence of . As a result one can take over the solutions of water wave problems almost directly from corresponding solutions of the wave equation in the theories of electromagnetics and sound.

One of the earliest solutions of a diffraction problem was obtained by Sommerfeld (1895) for the diffraction of a plane wave of sound by a semi-infinite screen. The solution was elegantly expressed in a closed form by means of Fresnel integrals. The solutions of all other diffraction problems have so far required the use of some form of approximation, such as the use of infinite series. For normal wave incidence the solution of Sommerfeld's diffraction problem has been solved very neatly by Lamb (1945, p. 538), using parabolic coordinates. This coordinate system naturally suggests itself for this problem since it is such that one of the parabolic coordinate lines corresponds to the semi-infinite screen ($y = 0$, $x > 0$). Expressing equation II A-10.1 in parabolic coordinates ξ, η it is a relatively simple matter to show that it is possible to find solutions of the form $e^{iky} f(\xi+\eta)$ and $e^{-iky} g(\xi-\eta)$ which satisfy all the conditions of the problem: incident plane wave, zero normal velocity on the screen.

The first discussion of this solution in connection with water waves appears to be due to Larras (1942). It was also discovered by Penny and Price (1944). More recent discussions with experimental verifications are those of Putnam and Arthur (1948), Blue and Johnson (1949), Lacombe (1949), Carry and Chapus (1951). Since the solution has been discussed in detail in the above references and in another section of this Manual a repetition is unnecessary.

We consider in more detail the diffraction of a cylindrical wave train in water of constant depth by a vertical circular cylinder resting on the bottom and projecting from the water. The corresponding problem of sound wave diffraction is given in Morse's book (1948). It was first adapted by Havelock (1940) to the treatment of diffraction of deep water waves by a vertical cylinder intended to represent a ship. The results are readily modified to the case of water of an arbitrary finite depth. A special application is the determination of the pressure distribution on cylindrical piles.

The velocity potential of the motion has the form

$$\phi = \frac{g\,a}{\sigma} \frac{\cosh k\,(d+y)}{\cosh kd}\, e^{i\,(kx+\sigma t)} + \frac{\cosh k\,(d+y)}{\cosh kd}\, e^{i\sigma t}\, \phi'\,(x, z)$$

(II A-10.3)

the real part of the solution being used. The first term in this expression represents the incident cylindrical wave moving in the negative x-direction. The vertical cylinder is of radius, a, about the origin of the coordinate system and extends from the bottom y = -d to a point some distance above the waves. Cylindrical coordinates are naturally adapted to the problem. The plane wave e^{ikx} can be expanded in an infinite series of the Bessel functions $J_n\,(kr)$, $r = \sqrt{x^2 + y^2}$. The scattered waves can be represented by a more general type of series, terms of which are those indicated by the separation of variables, involving singular Bessel function with argument k r multiplied by cos n θ , i.e., $H_n^2\,(k\,r)\cos n\,\theta$. For details of the calculation we refer to Havelock's paper (1940).

Neglecting terms of the order of $(H/L)^2$ we find that the pressure distribution on the cylinder is given by

$$p(y) = \frac{2\,g\,\rho\,H}{k} \frac{\cosh k\,(d+y)}{\cosh kd}\, A\,(k\,a) \cos\,(\sigma t - \theta) \qquad \text{(II A-10.4)}$$

where A (k a) and θ, expressible in terms of Bessel functions, are plotted in Figures II A-38 and II A-39 as a function of a/L. The time history of the surface profile of the incident wave on a line parallel to the wave crests through the center of the cylinder is $\eta = \frac{H}{2}\sin\,(\sigma t)$. For small k a, θ is small and the maximum pressure and moment occur 90° out of phase with the wave profile where the acceleration is a maximum. For a pile cantilevered at the bottom the moment of the resulting force about the bottom is

$$m\,(d) = \frac{2\,g\,\rho\,H}{k^3}\, D\,(kd)\, A\,(ka) \cos\,(\sigma t - \theta\,) \qquad \text{(II A-10.5)}$$

where D (kd) is plotted in Figure II A-40 as a function of d/L. The effective leverarm of the total force measured from the bottom is therefore

$$\ell = k_o\, D\,(kd) \qquad \text{(II A-10.6)}$$

where $k_o = \frac{2\pi}{L_o}$ is the deep water wave number.

Simple approximate formulas are available for cylinders whose diameter is small compared to the wave length. One finds that

$$A\,(k\,a) \simeq \frac{\pi}{2}(k\,a)^2 = \frac{\pi^3}{2}\left(\frac{2a}{L}\right)^2 \qquad \text{(II A-10.7)}$$

For small cylinders, and only for these, we find that the pressure and moment are proportional to the cross-sectional area of the pile.

In very shallow water we find approximately

$$D\,(kd) \simeq \frac{1}{2}\,(kd)^2 = 2\pi^2 \left(\frac{d}{L}\right)^2 \qquad \text{(II A-10.8)}$$

In particular this implies that $\ell \simeq \frac{d}{2}$, as one would expect. Combining the two approximations one finds that for small diameter piles in very shallow water

$$m\,(d) \simeq \pi^2 \rho g \,\frac{H}{L}\, a^2\, d^2 \cos\sigma t \qquad \text{(II A-10.9)}$$

It is not our intention to compare these theoretical results with experimental data; this will be done in a forthcoming report. Suffice it to say that the agreement with experiments in both model and full scale is quite good. Other effects if they occur must be quite small for waves of moderate steepness. One is led to conclude that an empirical formula used by Morison (1950) which contains a multitude of experimentally determined coefficients and arbitrary assumptions must be used with extreme caution since its dependence on physical parameters is quite different from that deduced theoretically. The effect of skin friction in the periodic flows considered has been shown theoretically to have a negligible influence on the moment. Work is continuing on the theoretical determination of the wake drag in such flows.

In general a useful method of solving simple diffraction problems is to find a coordinate system in which the projected perimeter of the obstacle forms a coordinate line. For the cylinder one naturally uses polar coordinates the surface of the obstacle being $r = a$. For Sommerfeld's diffraction problem the natural coordinate system is parabolic, the semi-infinite line coinciding with the degenerate parabola forming one of the coordinate axes. The incident and reflected waves are represented in terms of infinite series obtained by separation of variables in equation II A-10.1 written in terms of the new variables. The boundary condition of zero normal velocity on the coordinate line leads to the determination of the coefficients of a generalized Fourier series which is a relatively simple matter in many cases. Such a method can be applied to the diffraction through a breakwater gap by use of confocal elliptic cylindrical coordinates. This was first employed by Morse and Rubenstein (1938) for the diffraction of electromagnetic and sound waves and adapted to the corresponding water wave problem by Carrand Stelzriede (1951).

b. Underwater Obstacles:

Dean (1945[1]) has treated the reflection of surface waves by a submerged circular cylinder with horizontal axis parallel to the wave crests. He shows quite generally that the reflection coefficient at great distances from the cylinder is zero, so that the only effect of the cylinder at great distances is to produce a phase shift between the incident and transmitted waves, their amplitudes being the same. It has been shown that the reflection coefficient for shallow water waves incident on a submarine

bar is non-zero though it is usually small. This theory applies to the case where the lower boundary of the water extends on each side of the bar an indefinite distance. The result of Dean's analysis in deep water is to suggest that the existence of water of moderate or small depth is a necessary condition for the existence of a reflected wave at large distances from a symmetrical submerged obstacle.

Ursell (1949) has used a different method for solving the same problem of a submerged cylinder which is also permitted to execute small harmonic oscillations. The reflection from a fixed cylinder of radius r whose axis is submerged to a depth f has been computed for

$$\frac{2\pi r}{L} = \frac{4}{3}, \qquad \frac{2\pi f}{L} = \frac{5}{3}$$

a case also treated by Dean. There is no reflected wave as was first shown by Dean. The velocity potential of the resultant wave motion is

$$\phi = \frac{g\,a}{\sigma}\, e^{+ky}\left[0.998 \sin(kx+\sigma t) + 0.061 \sin(kx+\sigma t)\right] \qquad \text{(II A-10.10)}$$

For a pulsating cylinder of radius $r = r_o\,(1+\epsilon \cos \sigma t)$ where ϵ is small the surface amplitude is given by

$$2\pi r_o\, \epsilon\, e^{-kf}\,(k\, r_o - A_1\, k^3\, r_o^{\,3}) + 0\,(k^5\, r_o^{\,6}) \qquad \text{(II A-10.11)}$$

where

$$A_1 = 2\, e^{-2kf} \int_{-\infty}^{2\,kf} e^{u}\, \frac{du}{u} - \frac{1}{2\,kf}$$

Dean (1945) treats the reflection of surface waves by a plane vertical barrier submerged to a depth f below the still water level. He finds if f/L is sufficiently small that

$$R = \frac{I_1}{\sqrt{I_1^{\,2} + I_3^{\,2}}}, \quad T = \frac{I_3}{\sqrt{I_1^{\,2} + I_3^{\,2}}} \qquad \text{(II A-10.12)}$$

where $I_1 \simeq \ln \frac{2}{kf} - 0.577$

$I_3 \simeq \pi$

For $\frac{2\pi f}{L} = \frac{1}{2}$ and $\frac{1}{4}$ numerical methods give values of T of 0.968 and 0.902 respectively while the approximate formula above gives corresponding values of 0.964 and 0.900. Selected values of R and T as computed from the approximate formulas are given in Figure II A-41 as taken from Dean's report.

Thus one would expect a height reduction of 10% for a wave passing over such a barrier submerged to a depth $f = T^2/5$, T being the wave period in seconds. It seems reasonable to suppose that for a given f other types of symmetrical barriers, such as two inclined planes meeting in a horizontal line, will give higher transmission coefficients and lower reflection coefficients since the transition is more gradual.

Ursell (1947) has treated wave motion past a vertical barrier extending from the surface to a depth h'. The transmission and reflection coefficients are given in Figure II A-42.

11. The Effect of Viscosity on Water Waves:

The first treatment of the effect of ordinary viscosity on the motion of surface waves in deep water was given by Stokes (1845). He computed the mean rate at which the surface forces necessary to preserve the sinusoidal motion with viscosity do work equating this to the energy dissipated in the free motion (see Lamb, p. 624). In the absence of surface forces, such as tangential drag due to wind

$$\frac{d}{dt}\left(\frac{1}{2}\rho k c^2 a^2\right) = -2\mu k^3 c^2 a^2 \tag{II A-11.1}$$

or

$$\frac{da}{dt} = -2\nu k^2 a$$

and therefore

$$a = a_o e^{-2\nu k^2 t} \tag{II A-11.2}$$

μ being the coefficient of viscosity and ν the kinematic coefficient of viscosity. This type of calculation is intended to apply to all but the smallest waves. The modulus of decay, which is the time necessary for the amplitude to drop to $\frac{1}{e} \cong 0.368$ of its initial value is in the case of water

$$\tau = 0.184 \, L^2 \text{ hours} \tag{II A-11.3}$$

if L is expressed in feet. For waves of the size one has in the ocean the damping is very small, for example, if L = 10 ft., τ = 18.4 hours, while if L = 100 ft., $\tau \cong$ 77 days.

A more complete investigation has been made by Basset (1888) and Lamb (1930, p. 625). They solve the Navier-Stokes equations for the motion of viscous fluids neglecting terms of order $(H/L)^2$. Except for very small wave lengths the motion is shown to be irrotational with a velocity potential of

$$\phi = A e^{-2\nu k^2 t + ky + i(kx \pm \sigma t)} \tag{II A-11.4}$$

leading to a surface amplitude as given by the approximate treatment of Stokes. An approximate value of the vorticity is

$$\omega = \mp 2\sigma k a e^{-2\nu k^2 t + \beta y} \cos\left\{kx \pm (\sigma t + \beta y)\right\} \tag{II A-11.5}$$

where $\beta = \sqrt{\frac{\sigma}{2\nu}}$. The variations due to viscosity diffuse rapidly being quite analogous to those of the temperature variations in thermal conduction,

or of density in the theory of diffusion. The oscillatory wave motion has the effect of reversing the sign of the vorticity, so that for depths comparable with $2\pi/\beta$ ($=0.0129\sqrt{T}$ ft. for water) the vorticity is practically zero.

The effect of finite depth on such calculations has been investigated by Basset (1888), Hough (1896) and Biesel (1949[2]). The latter author corrects some approximations of Basset's and deduces formulas different from any previous ones. Biesel finds that

$$a = a_o\, e^{-\left[\frac{1}{\sinh 2kd}\sqrt{\frac{k^2\nu}{2\sigma}} + \frac{2k^2\nu}{\sigma}\,\frac{\cosh 4kd + \cosh 2kd - 1}{\cosh 4kd - 1}\right]\sigma t} \qquad \text{(II A-11.6)}$$

The first term in the exponent will be the largest for moderate depth and fluids of small viscosity such as water. For larger depths one finds again the deep water formula (II A-11.1). When the amplitude is constant at a given point as a function of time but varies with position one has

$$a = a_o\, e^{-\frac{2}{1+\frac{2kd}{\sinh 2kd}}\left[\frac{1}{\sinh 2kd}\sqrt{\frac{k^2\nu}{2\sigma}} + \frac{2k^2\nu}{\sigma}\,\frac{\cosh 4kd + \cosh 2kd - 1}{\cosh 4kd - 1}\right] kx} \qquad \text{(II A-11.7)}$$

For the time dependent amplitude one finds that relative difference between the moduli of decay in general and in deep water is

$$\frac{\tau - \tau_{deep}}{\tau} = \frac{\cosh 2kd}{2\sinh^2 2kd} - \frac{1}{2k\sinh 2kd}\sqrt{\frac{\sigma}{\nu}} \qquad \text{(II A-11.8)}$$

For a wave 40 cm. long with $d/L = 0.5$ this is 0.626.

The damping of ocean waves by eddy viscosity has been recently considered by Bowden (1950). He proposed a coefficient of eddy viscosity $N = K\,c\,a$, where K = constant. This form is indicated on dimensional grounds and is shown to be in conformity with van Karman's similarity hypothesis for shearing flow. For waves of a single period we consider the quasi-stationary state of wave energy varying with distance but constant in time at any given point. The rate of energy dissipation due to eddy viscosity is

$$-\frac{c}{2}\frac{dE}{dx} = 2\rho K k^3 c^3 a^3 = W \qquad \text{(II A-11.9)}$$

Assuming that the energy is propagated with the appropriate group velocity – $E = \frac{1}{2}\rho g a^2$, $k c^2 = g$ and hence

$$\frac{1}{a} = \frac{1}{a_o} + \frac{64\pi^4 K}{g^2 T^4}\, x \qquad \text{(II A-11.10)}$$

The corresponding formula of Sverdrup and Munk (1947) who attribute the decay solely to air resistance is

$$a = a_o \exp\left[-\frac{4\pi^2 s\rho'}{\rho g T^2} x\right] \qquad \text{(II A-11.11)}$$

The shortest periods decay most rapidly, the discrimination being most rapid for dissipation by eddy viscosity. This causes a shift of the period associated with the maximum amplitude toward higher values.

In general T and c increase with x so that one has

$$\frac{1}{2}\left(c\frac{dE}{dx} + E\frac{dc}{dx}\right) = -W \qquad \text{(II A-11.12)}$$

and hence

$$\frac{da}{dx} = -\frac{1}{2}\frac{a}{c}\frac{dc}{dx} - \frac{4Kg^2a^2}{c^4} \qquad \text{(II A-11.13)}$$

Assuming $\frac{dc}{dx} = \sigma$ (a constant) one finds

$$\frac{1}{aT^{1/2}} = \frac{1}{a_oT_o^{1/2}} + \frac{128\pi^4 Kx}{7g^2(T-T_o)}\left(\frac{1}{T_o^{1/2}} - \frac{1}{T^{7/2}}\right) \qquad \text{(II A-11.14)}$$

The corresponding solution for dissipation by air resistance is

$$aT^{1/2} = a_o T_o^{1/2} \exp\left[-\frac{4\pi^2 s\rho'}{\rho g T_o T} x\right]$$

The scatter in values of s and K computed from ocean wave data as given by Sverdrup and Munk and laboratory channel data as given by Johnson and Rice (1951) is about the same and both are quite large so that one cannot decide whether the decay law for eddy viscosity or air resistance fits the observations better.

Keulegan (1948) has treated the gradual damping of solitary waves by bottom friction. He considers the rate of energy dissipation in the laminar boundary layer formed at the bottom as given by Boussinesq. For unit channel width this rate of energy dissipation

$$\frac{dE_1}{dt} = 4\pi^{-3/2}\left(\frac{4}{3}\right)^{\frac{1}{4}}\rho\,\nu^{\frac{1}{2}}\,g^{5/4}\,h_1^{7/4} \qquad \text{(II A-11.15)}$$

where h_1 is the elevation of the crest above the mean level, and ν is the kinematic coefficient of viscosity. Denoting the initial value of the crest height by h_{10} the value at a distance is given by

$$\left(\frac{h_1}{d}\right)^{-\frac{1}{4}} - \left(\frac{h_{10}}{d}\right)^{-\frac{1}{4}} = K\frac{s}{d} \qquad \text{(II A-11.16)}$$

where

$$K = \frac{1}{12}\left(1+\frac{2\,d}{B}\right)\sqrt{\frac{\nu}{g^{\frac{1}{2}}\;d^{3/2}}}$$

B is the channel width and s is the distance travelled.

The extinction of a limited wave takes place in a different way. For a rectangular wave of elevation h and length L Keulegan finds

$$h_1 = h_{10}\,e^{-\alpha\frac{s}{d}} \qquad \text{(II A-11.17)}$$

where

$$\alpha = \frac{1}{\sqrt{\pi}}\left(1+\frac{2\,d}{B}\right)\left(\frac{L}{d}\right)^{-\frac{1}{2}}\left(\frac{\nu}{g^{1/2}\;d^{3/2}}\right)^{\frac{1}{2}}$$

The relative damping is independent of the initial height in this case. For the solitary wave the effective wave length depends on the height and hence so does the logarithmic decrement of the wave height.

12. Generation of Waves by Wind:

Kelvin (1871) first treated the irrotational motion of air over a sinusoidal wave profile. Equating the difference in pressure between air and water to the pressure due to surface tension at the water's surface one finds the following expression for the wave velocity

$$c_w = \frac{\sigma V}{1+\sigma} \pm \sqrt{c_o^2 - \frac{\sigma V^2}{(1+\sigma)^2}} \qquad \text{(II A-12.1)}$$

σ being the ratio of the density of air to that of water, c_o the wave velocity in still air and V the horizontal wind velocity far from the water's surface. The wave velocity

$$c_o = \sqrt{\frac{1-\sigma}{1+\sigma}\left(\frac{g}{k} + T'\,k\right)} \qquad \text{(II A-12.2)}$$

has a minimum value, c_m given by $c_m^{\,2} = \frac{1-\sigma}{1+\sigma}\,2\sqrt{g\,T'}$, T' being $\frac{T}{\rho-\rho'}$ where T is the surface tension. The plane surface is then stable for propagation of all wave lengths if

$$V < \frac{1+\sigma}{\sqrt{\sigma}}\,c_m \qquad \text{(II A-12.3)}$$

but for larger wind velocities instability causes waves to form and grow. The factor $e^{i\sigma t}$ in the wave amplitude can be written in this case as $e^{\pm\alpha t + i\beta t}$, where

$$\frac{\alpha}{k} = \sqrt{\frac{\sigma}{(1+\sigma)^2}\,V^2 - c_o^{\,2}}\;, \quad \beta = \frac{\sigma}{1+\sigma}\,k\,V$$

indicating the possibility of an amplitude increasing exponentially with time for $\alpha > 0$. For air over water $\sigma = 0.00129$ and the maximum value of V for stable waves is roughly 12.5 nautical miles per hour. For larger wind velocities waves will be formed on an originally calm water surface with the smallest possible wave length $L_{cap} = 0.68$ in. being that of capillary waves. The rate of travel of the first waves formed is 0.31 in/sec.

This theory has been extended by Jeffreys (1925) to the case of short-crested waves. He finds that

$$c^2 = \frac{\sigma V}{1+\sigma} \pm \sqrt{c_1^2 - \frac{\sigma V^2}{(1+\sigma)^2}} \qquad \text{(II A-12.4)}$$

where

$$c_1 = \sqrt{\frac{1-\sigma}{1+\sigma}\left(\frac{g\,r}{k^2} + \frac{T'\,r^3}{k^2}\right)}$$

If the waves are long-crested $n = 0$ and c^2 is equal to the expression of Kelvin's. In order for a wave to develop and grow

$$\begin{aligned} V^2 &\geq \frac{1+\sigma}{\sqrt{\sigma}}\, c_1 \\ &\geq \frac{1+\sigma}{\sqrt{\sigma}}\, c_o \qquad \text{(II A-12.5)} \\ &\geq \frac{1+\sigma}{\sqrt{\sigma}}\, c_m \end{aligned}$$

Thus we see that a long-crested wave is the first to be generated with the lightest winds which are capable of generating any waves at all.

The stability of surface waves has been investigated by Abdullah (1949) for small perturbations from the undisturbed state in which the wind velocity U' is constant. The current distribution is assumed to decay exponentially with depth, i.e., $U = V_o\, e^{\alpha y}$. The stability is difficult to discuss in general since the square root portion of the phase velocity is a complicated function of the wave length. If $\alpha = 0$ the phase velocity reduces to that for two fluids flowing one above the other with constant velocities. In this simpler case the condition for stability was found to be

$$L_s \geq \frac{2\pi}{g}\,\frac{\rho\rho'(\Delta U)^2}{(\rho-\rho')(\rho+\rho')} \qquad \text{(II A-12.6)}$$

where $\Delta U = U' - V_o$ and ρ, ρ' are the densities of water and air respectively. For a first approximation Abdullah finds that the stability condition for a general $\alpha > 0$ is

$$L \geq L_s + \frac{\frac{\alpha}{m}\rho\rho' V_o(\Delta U) - \left(\frac{1}{2}\frac{\alpha}{m}\rho V_o\right)^2}{(\rho - \rho')(\rho + \rho')} \qquad \text{(II A-12.7)}$$

For $U' = 10$ m/sec., $\rho = 1$, $\rho' = 1.3 \times 10^{-3}$, $\alpha = 10^{-3}$ c.g.s units the critical wave lengths for $V_o = 15$ cm/sec and $V_o = 100$ cm/sec. are 8.124 cm and 9.683 cm. respectively while the simpler formula II A-12.1 gives corresponding values of 8.1 cm. and 6.25 cm.

Jeffreys (1925) pointed out that all the predictions of Kelvin's theory of potential flow are in disagreement with observations. Jeffreys found by observation that the minimum velocity for wave formation is about 3.61 ft/sec., the wave length of waves first generated is from 2 to 3 in., and the velocity of such waves is about 1 ft/sec. The wind necessary to raise a deep water wave of 10 sec. period must have a velocity of at least 1425 ft/sec., an impossible situation.

The errors were attributed by Jeffreys to irregular motion of the air. Since it appeared that the waves first formed were quite regular it was assumed that the discontinuities and turbulence in the water were unimportant. Kelvin's theory on the other hand assumes that the air flows smoothly over the wave profile while one knows from experience that separation generally takes place in air flow over an obstacle of surface. The main part of the airstream "jets" over the crest striking the next windward crest and being deflected upwards, thus produces high pressures at the turning point. The leeward side is sheltered to a certain extent although there may be an eddy producing a flow opposite in direction to the original flow. Since the pressure is acting in the direction of the particle velocity if the wind moves faster than the wave the wave tends to grow. This imposes a restriction on possible wave lengths for a given wind velocity. By analogy with the air force on an inclined plate the reaction is supposed normal to the wave profile and proportional to $\rho V^2 \frac{\partial \eta}{\partial x}$. The reaction being a complicated function of x can nevertheless be expressed as a Fourier series in the variable $kx - \sigma t$; and it is clear that the only component contributing to the net amount of work done has the same wave length as the surface wave. The force per unit area on a wave of unit width is then of the form

$$f = s \rho g (V - c)^2 \frac{\partial \eta}{\partial x} \qquad \text{(II A-12.8)}$$

s being a numerical constant (called the sheltering coefficient) ranging between 0 and 1, and probably less than 1/2. Now the rate of dissipation of energy in a wave per unit surface area is

$$R = \mu r \sigma^2 a^2 \coth rd \qquad \text{(II A-12.9)}$$

for small steepness ratios, a generalization of a previous result for infinitely deep water. The rate at which the air pressure does work on the surface is $f \frac{\partial \eta}{\partial t}$, the average value of which is

$$\frac{1}{c} s \rho' (V - c)^2 \sigma k a^2 \qquad \text{(II A-12.10)}$$

The energy equation becomes

$$\frac{d}{dt}\left(\frac{1}{4}\rho\frac{\sigma^2 a^2}{r} \coth rd\right) = \frac{1}{2} s \rho' (V - c)^2 \sigma k a^2 - \mu r \sigma^2 a^2 \coth rd \qquad \text{(II A-12.11)}$$

if one neglects tangential stress. A necessary condition for wave growth is therefore that the right hand side of this equation be positive, i.e.,

$$\frac{(V - c)^2}{c} > \frac{4\nu\rho}{s\rho'} \frac{r}{\tanh kd} \qquad \text{(II A-12.12)}$$

or neglecting surface tension

$$(V - c)^2 c \geq \frac{4\nu g\rho}{s\rho'} \frac{r^2}{k^2} \geq \frac{4\nu g\rho}{s\rho'} \qquad \text{(II A-12.13)}$$

For a given wind this is least for $c = \frac{1}{3}V$; the smallest wind which could maintain waves being

$$V_{min} = 3\left(\frac{\nu g\rho}{s\rho'}\right)^{1/3} \left(\frac{r^2}{k^2}\right)^{1/3} \qquad \text{(II A-12.14)}$$

This velocity is smallest for long-crested waves. For $\nu = 0.018$ cm^2/sec. $V_{min} = 73s^{-1/3}$ for long-crested waves and then

$$k^3 = 5\ s^2 \qquad \text{(II A-12.15)}$$

Measurements by Jeffreys of the critical wind velocity V_{min} of 3.4, 3.6 and 3.8 ft/sec. leads to values of s of 0.318, 0.269, 0.229 and wave-lengths of 8.0, 8.8 and 9.8 cm.

When a mixture of waves of different lengths are present the problem is complicated by their non-linear mode of combination as well as by the calculation of the resultant stress components which in general no longer have the wave length of the principal wave under consideration. Physically it is supposed that a short wave superimposed on a longer one would soon decay for it will in general be of smaller height and will thus be sheltered by the larger wave for much of its travel, while its trough will be filled with water from the splashing crest of the longer wave as it passes over the crest. Thus, the waves in a storm area will presumably consist of many large short-crested waves combined with a low amplitude long-crested swell which has not had sufficient time to build up to its maximum size.

Jeffreys has estimated the total thrust of the wind in a horizontal direction to be equal to the skin friction determined by other means. For a wave surface in the form of circular arcs with a crest angle of 120° he finds that for waves on the verge of formation the skin friction coefficient is $s' = 0.013$, the thrust per unit area being $s'\rho' V^2$ where V is the velocity of wind relative to water. Taking a value of $s' = 0.002$ from experiments for flow over solids it is found for this circular arc profile that the largest waves should travel with about 3/4 of the wind velocity. With this condition it is found for short-crested waves that

$$\nu \leq 6.7 \times 10^{-9}\ V^3 \quad \text{(c.g.s. units)}$$

Attributing the damping in mid-ocean mainly to eddy viscosity experimental values for ν are obtained of the order of $\nu = 8 \times 10^{-4}\ V^2$. This inconsistency is not satisfactorily accounted for.

Bondi (1942) has extended Jeffrey's results in shallow water by considering the boundary layer formed at the bottom. It is shown that Jeffreys' inequality must be replaced by

$$\frac{(V - c)^2}{c} \geq \frac{4\rho}{s\rho'}\left[\nu r \coth rd + \sqrt{\frac{\nu\sigma}{2}}\,\frac{1}{4 \sinh^2 rd}\right] \qquad \text{(II A-12.16)}$$

The first waves produced by a rising wind will again be long-crested. For deep water the result is identical with that of Jeffreys while for smaller depths the difference measured by the second term on the right hand side of II A-12.16 is quite large. Bondi using a wider range of depths than did Jeffreys, arrived at an approximate s value of 0.25.

Stanton (1937) made a harmonic analysis of the pressure distribution on a wooden model of a wave for which d/L = 0.66, a/L = 0.197 and found that the pressure was far from being representable in terms of a single harmonic term only.

Sverdrup and Munk (1947) modified Jeffreys' theory for deep water by keeping his value for the rate of normal transfer of energy but including also a term representing the tangential energy transfer, which is

$$R_T = \frac{1}{L}\int_0^L \tau \mu_o \, dx \qquad \text{(II A-12.17)}$$

where μ_o = horizontal surface velocity and τ is the shear distribution. They assume that $\tau = \gamma^2 \rho' U^2$ where $\gamma^2 = 2.6 \times 10^{-3}$ for wind velocities of the order of 16 ft/sec. or greater. For waves of infinitesimal steepness the integral R_T is zero. For waves of finite height R_T is calculated by including the mass transport term in μ which is inconsistent with the notion that the tangential stress should be parallel to the wave surface.

The inconsistency has been removed by Schaaf and Sauer (1950) who find that for

$$\tau = \frac{\rho}{2}\, C_D\, (U + c - V)^2$$

$$q_a = \pi U \frac{a}{L}\, \phi\left(\frac{x}{L}\right) \qquad \text{(II A-12.18)}$$

C_D being an undetermined coefficient of drag and $\phi(\frac{x}{L})$ a periodic function of $\frac{x}{L}$, that

$$R_T = \rho C_D \pi^2 \left(\frac{a}{L}\right)^2 \frac{c\,U}{2}\left[\frac{U}{2} - c + 2\eta(U - c)\right] \qquad \text{(II A-12.19)}$$

This shows that $R_T < 0$ for $c > 0$, and thus contrary to the conclusion of Munk and Sverdrup, waves cannot grow when the wave velocity exceeds the wind velocity. When calculations of this kind depend upon terms of the order of $(a/L)^2$ the results are objectionable for one must in general consider rotational motions. This is evident from the measurements of particle velocities for wind waves by Johnson and Rice (1951).

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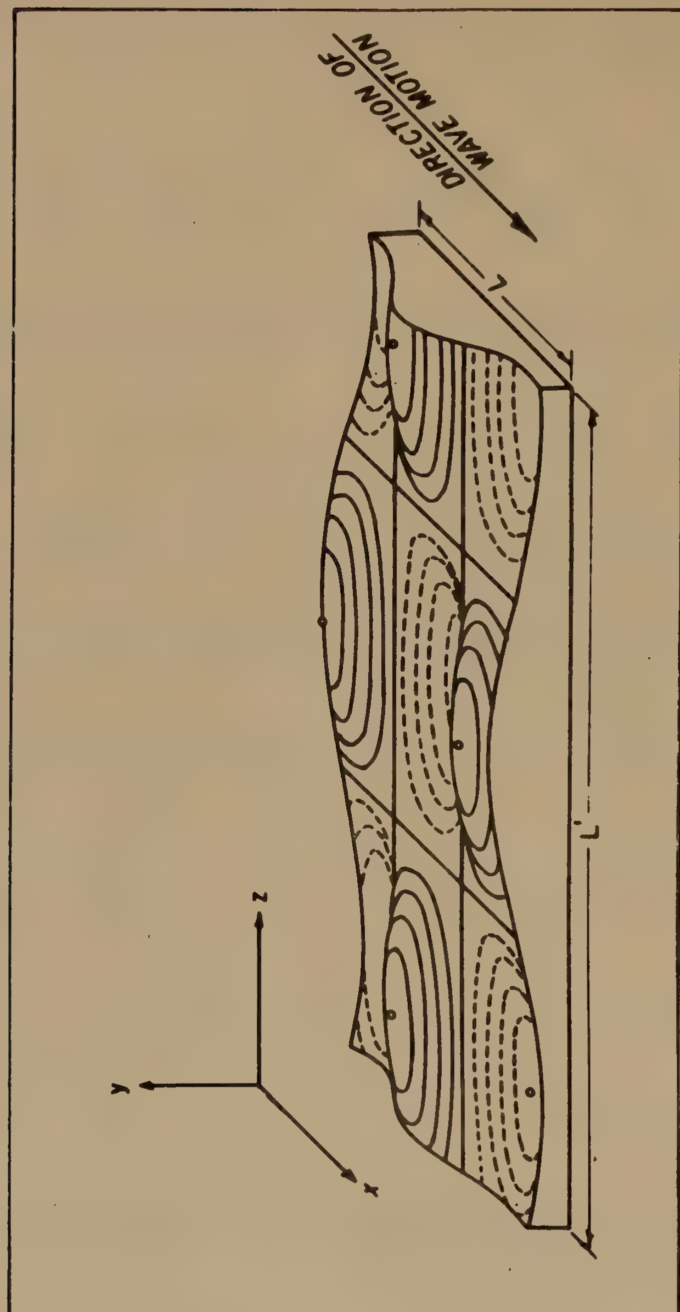


FIG. IIA-1 - PICTORIAL SKETCH OF SHORT-CRESTED WAVE CONTOURS
(L' = 7L)

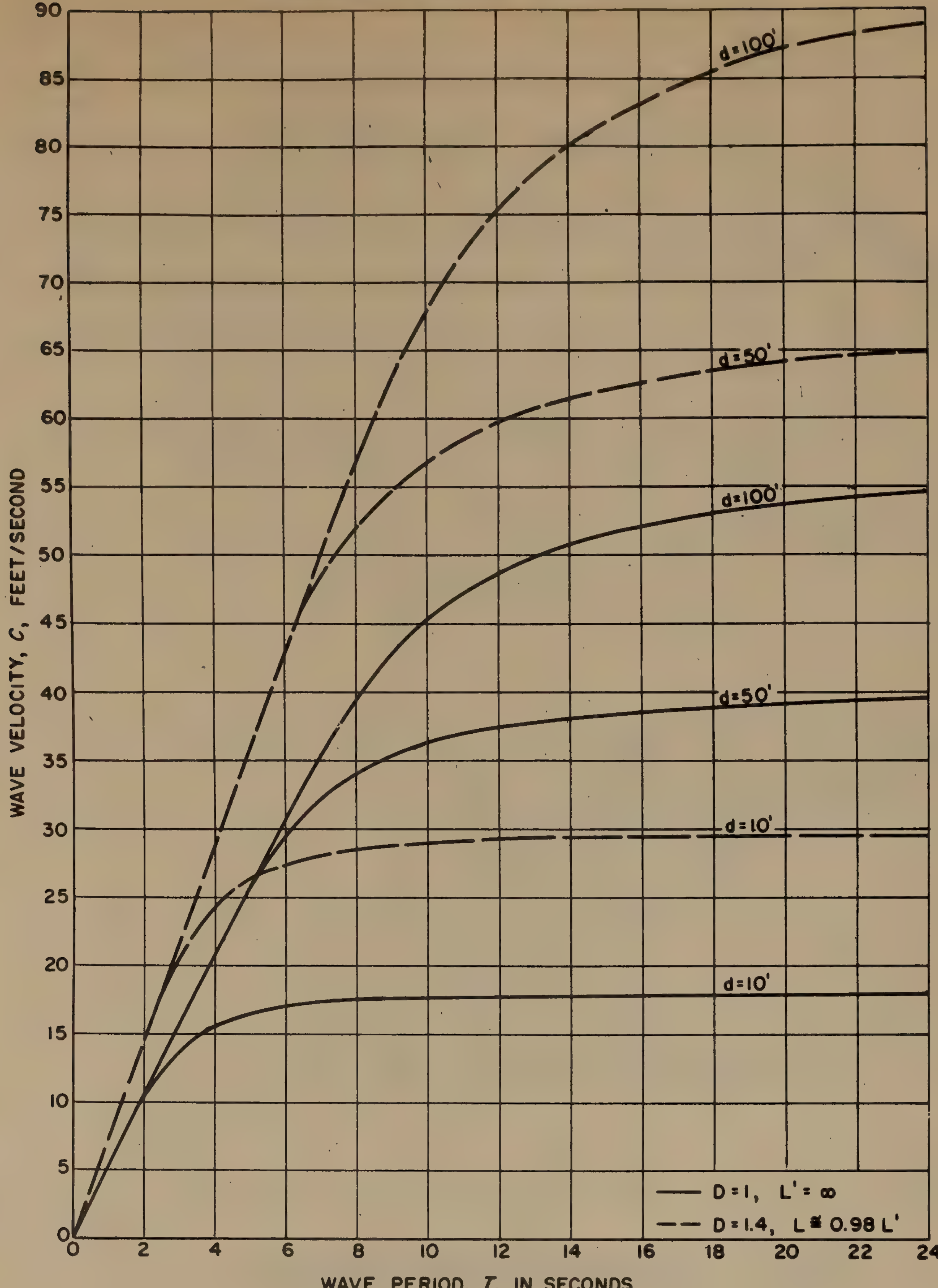


FIG. II A-2 - WAVE VELOCITY VERSUS WAVE PERIOD
FOR DIFFERENT VALUES OF d AND L/L'

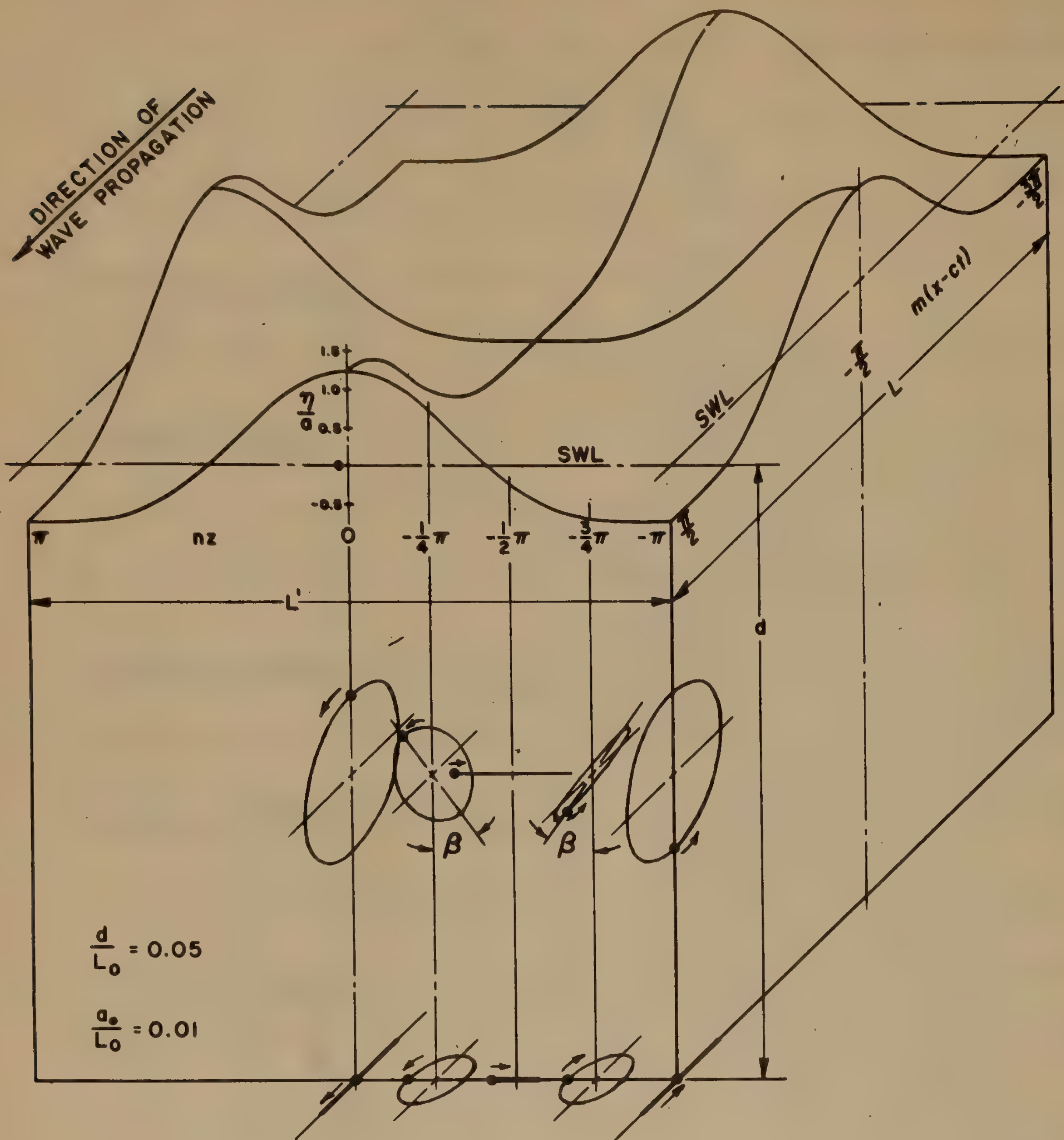


FIG. IIA-3 -- SECOND ORDER SURFACE PROFILES
FIRST ORDER ELLIPTICAL PARTICLE ORBITS

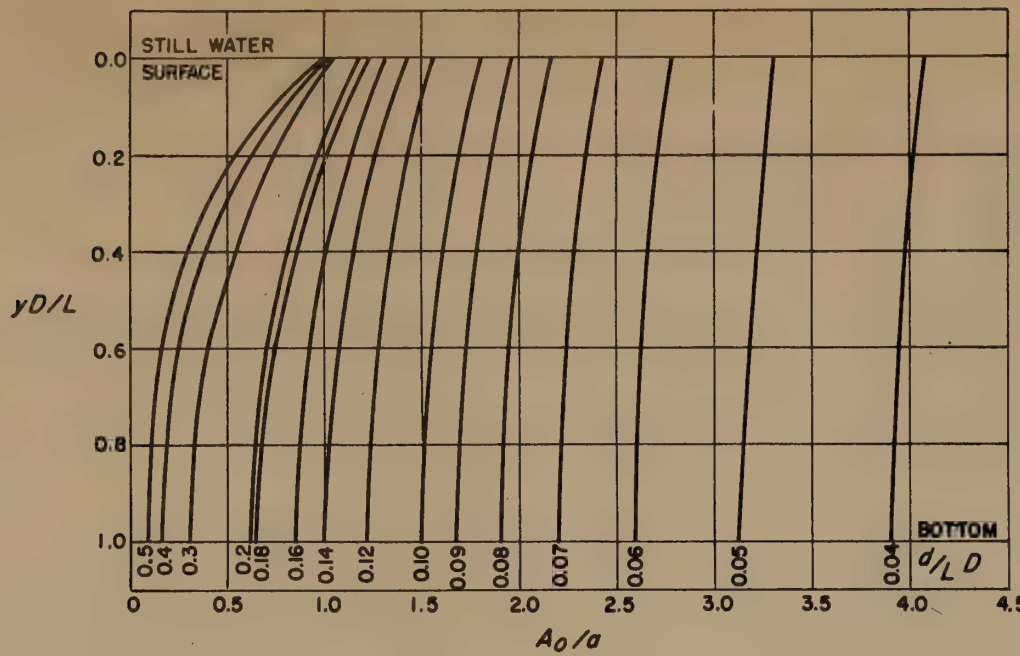


FIG. IIA-4 - DEPTH DEPENDENCE OF HORIZONTAL AMPLITUDE OF PARTICLE OSCILLATION

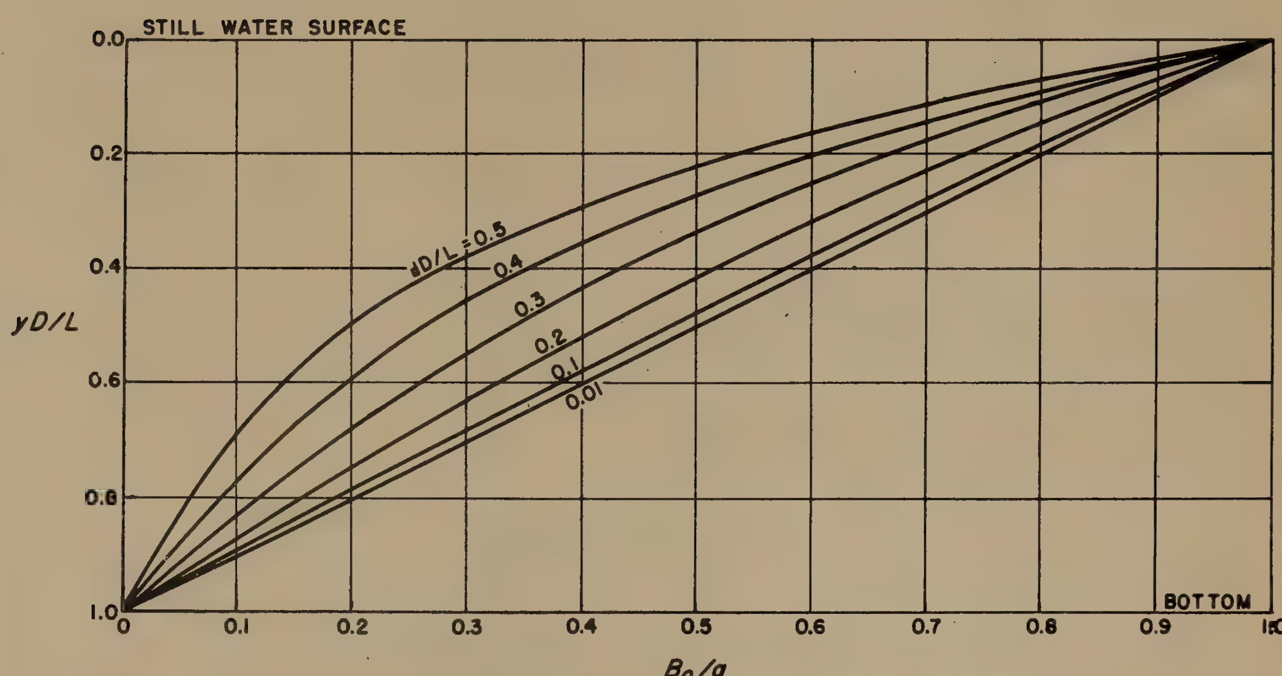


FIG. IIA-5 - DEPTH DEPENDENCE OF VERTICAL AMPLITUDE OF PARTICLE OSCILLATION

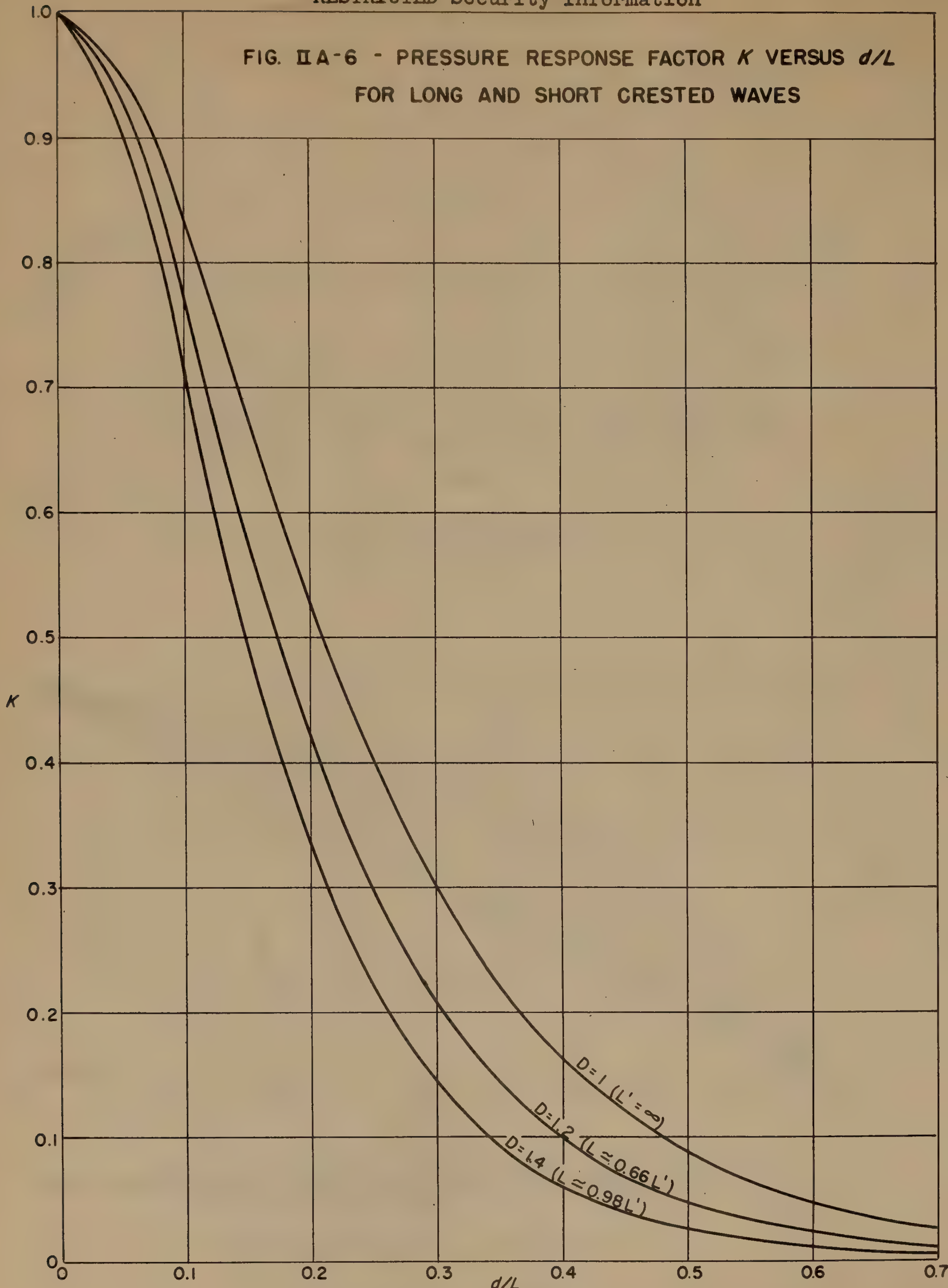


FIG. IIA-6 - PRESSURE RESPONSE FACTOR K VERSUS d/L
FOR LONG AND SHORT CRESTED WAVES
K
d/L
D=1 (L'=∞)
D=1.2 (L≃0.66L')
D=1.4 (L≃0.98L')
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

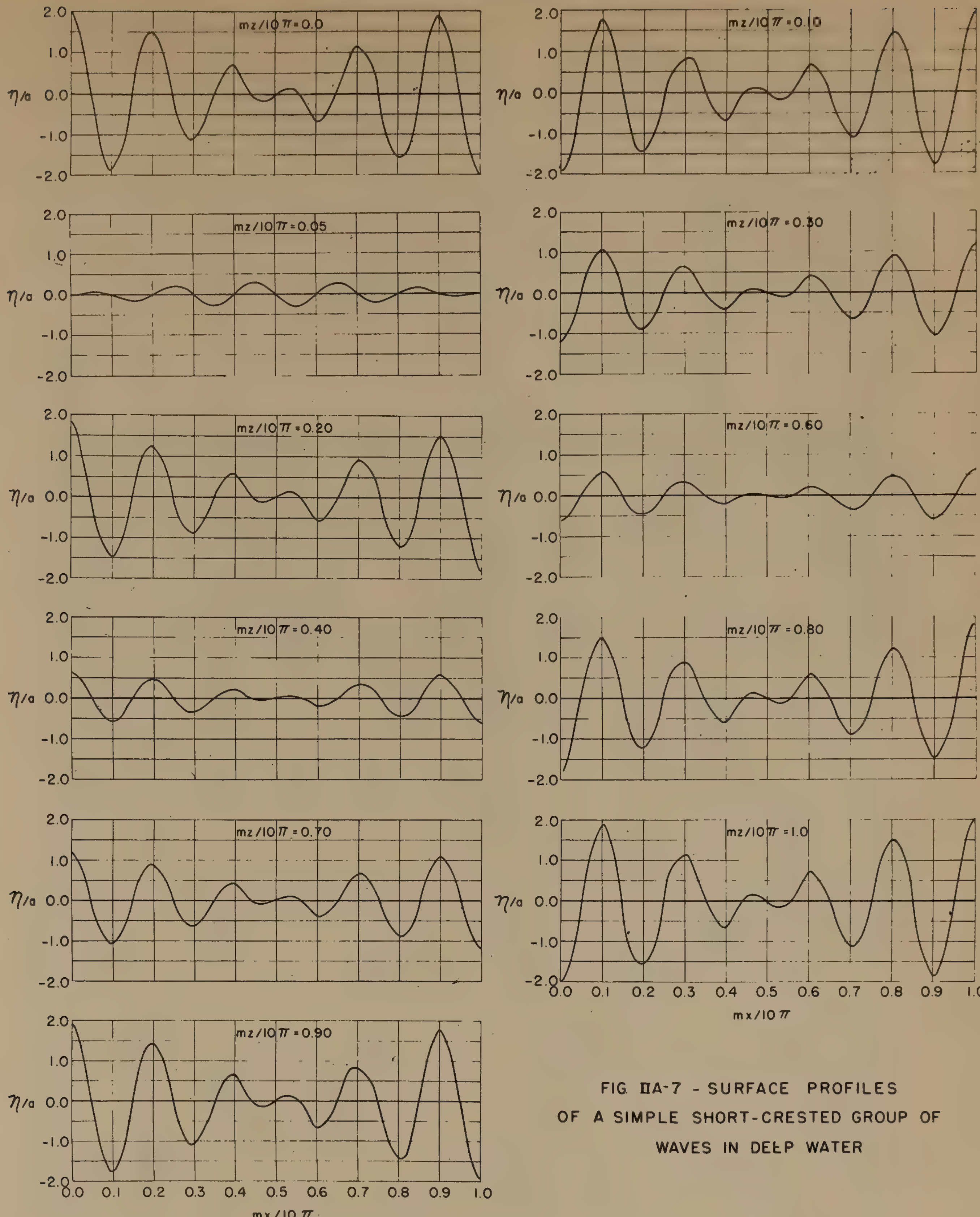


FIG. IIA-7 - SURFACE PROFILES OF A SIMPLE SHORT-CRESTED GROUP OF WAVES IN DEEP WATER

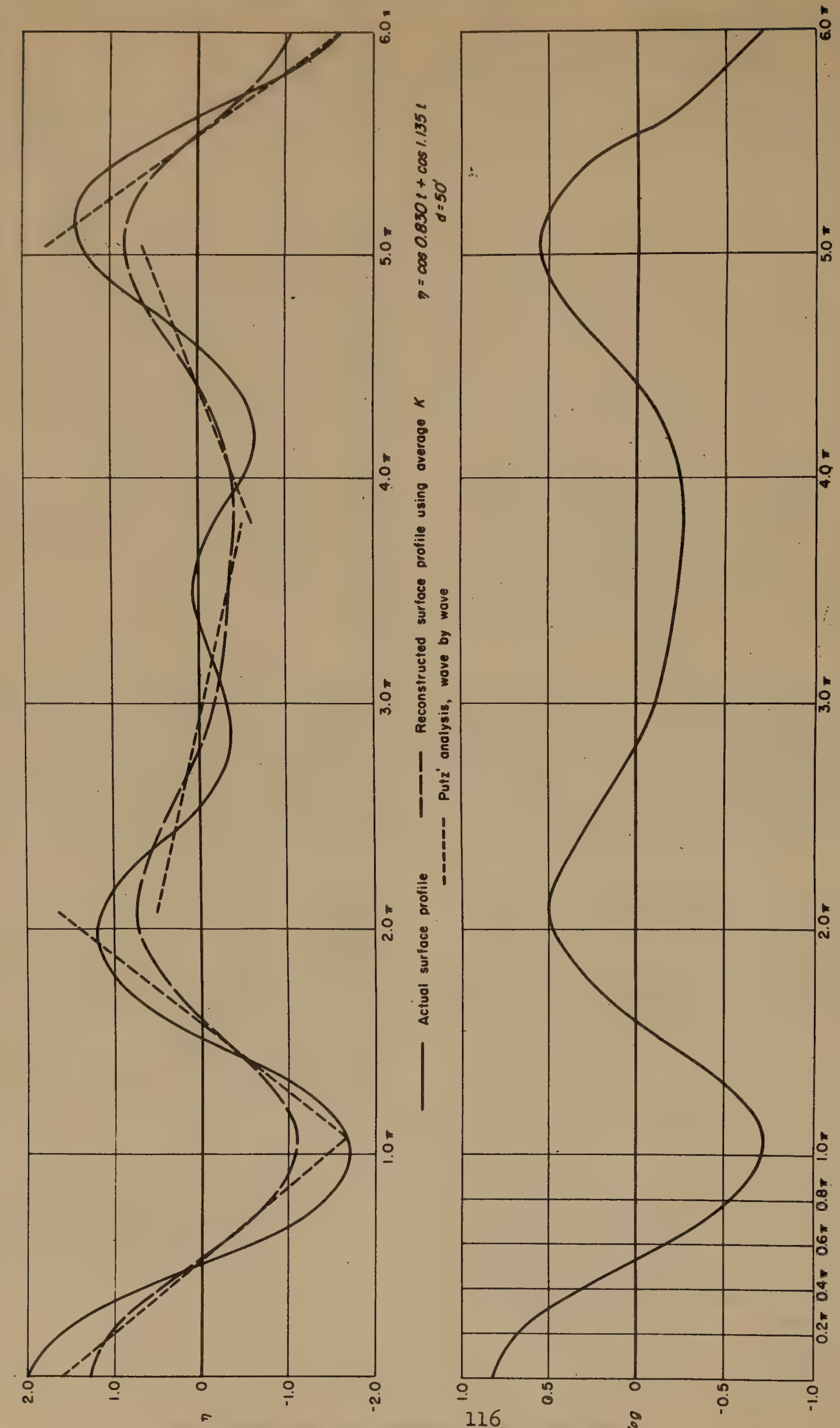


FIG. IIA-8 - SURFACE PROFILES FROM A HYPOTHETICAL UNDERWATER PRESSURE RECORD

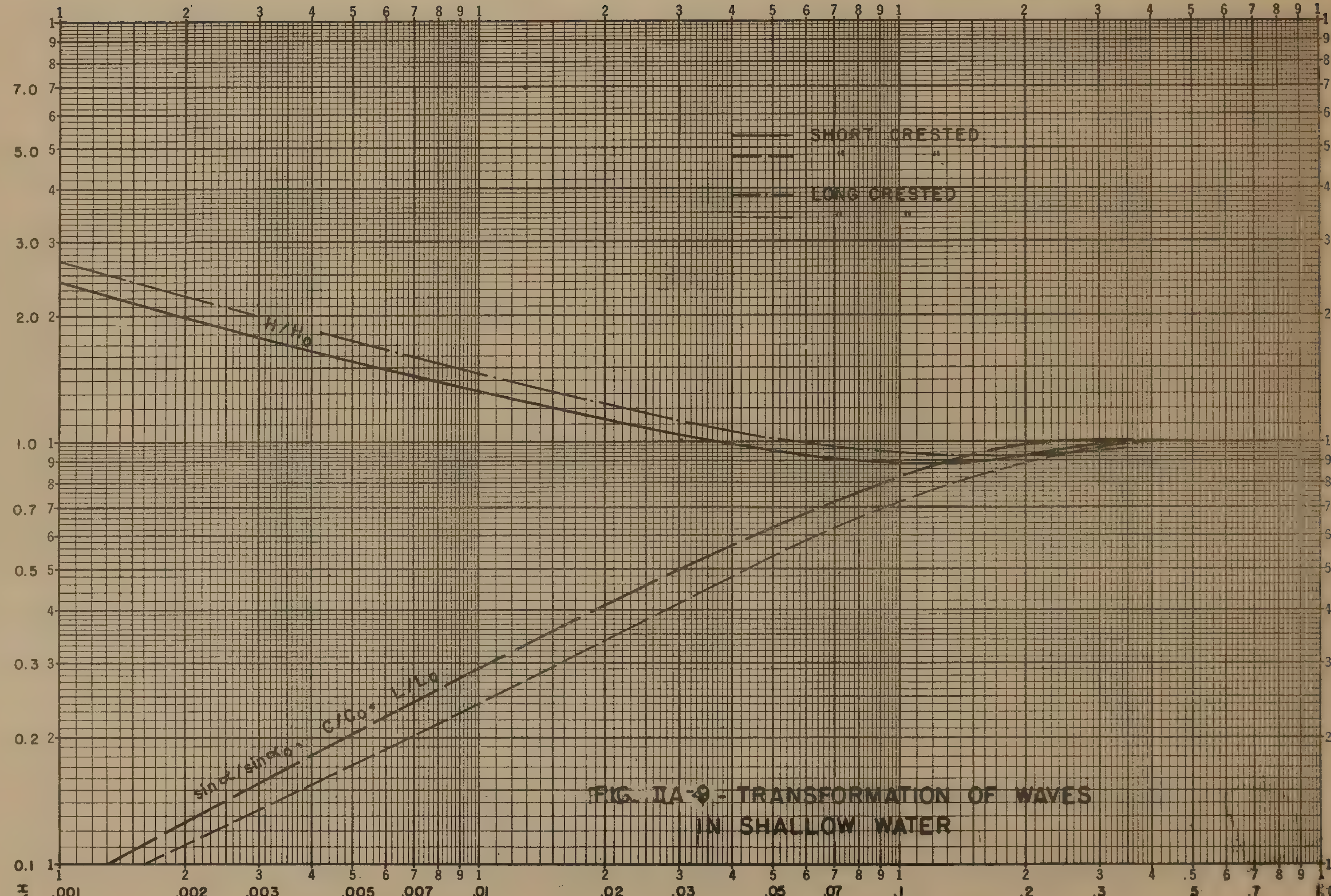


FIG. IIA-9 - TRANSFORMATION OF WAVES IN SHALLOW WATER

HYD-5422

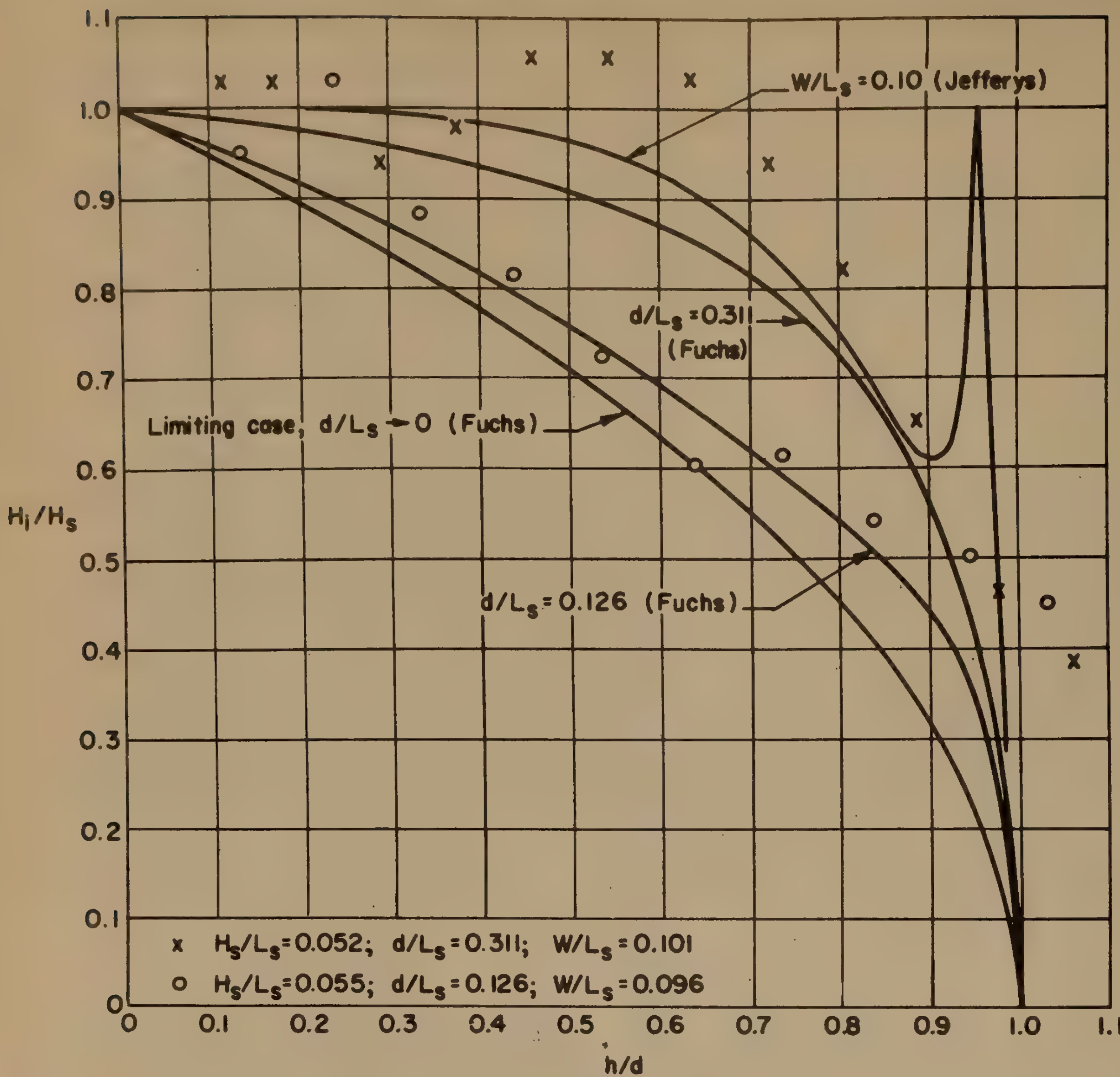


FIG. IIA-10 - EFFECT OF RELATIVE DEPTH ON WAVE ACTION OVER A RECTANGULAR UNDERWATER BARRIER

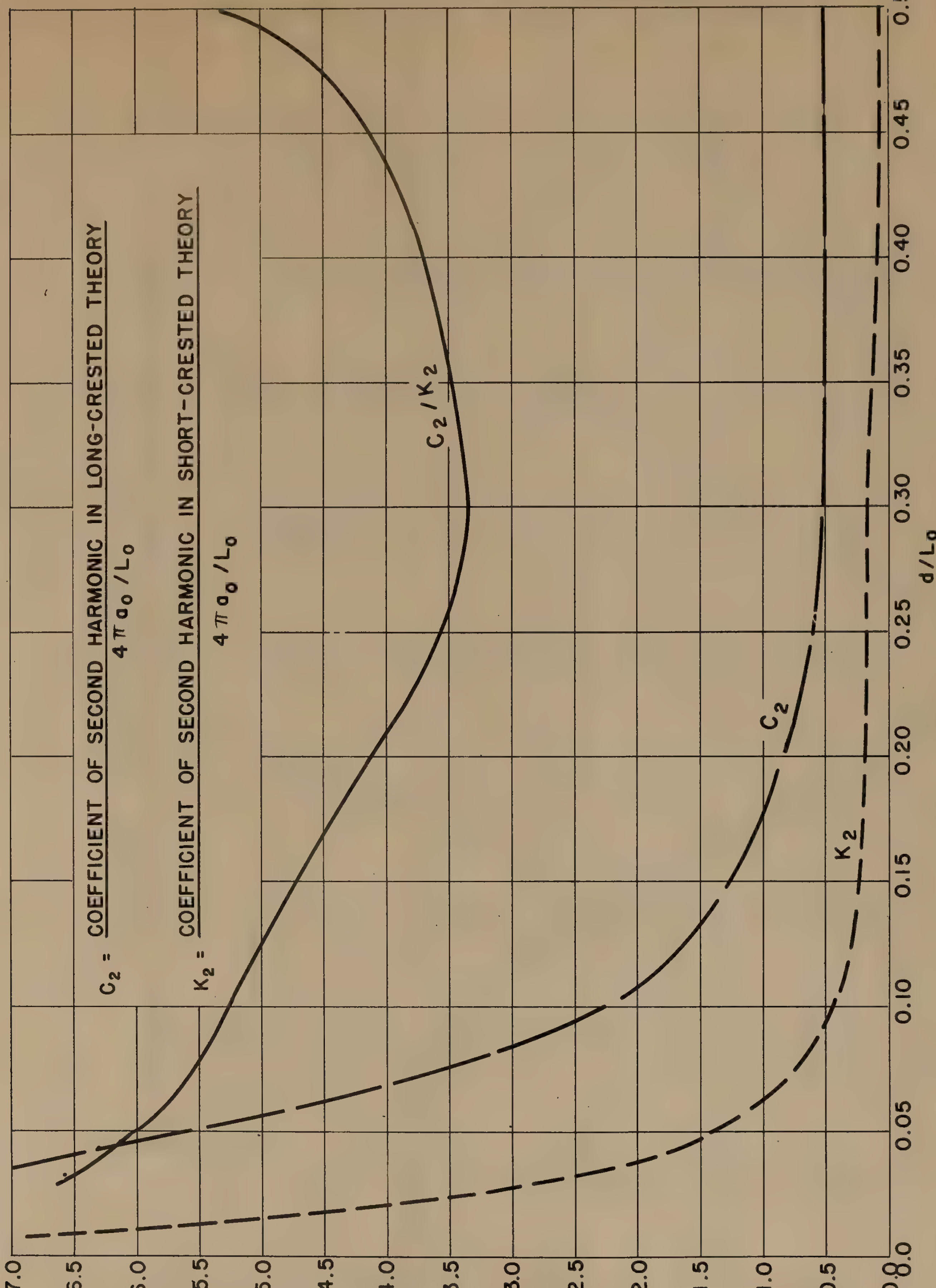


FIG. IIA-11 - COEFFICIENTS OF SECOND HARMONICS IN EQUATIONS FOR WAVE PROFILES

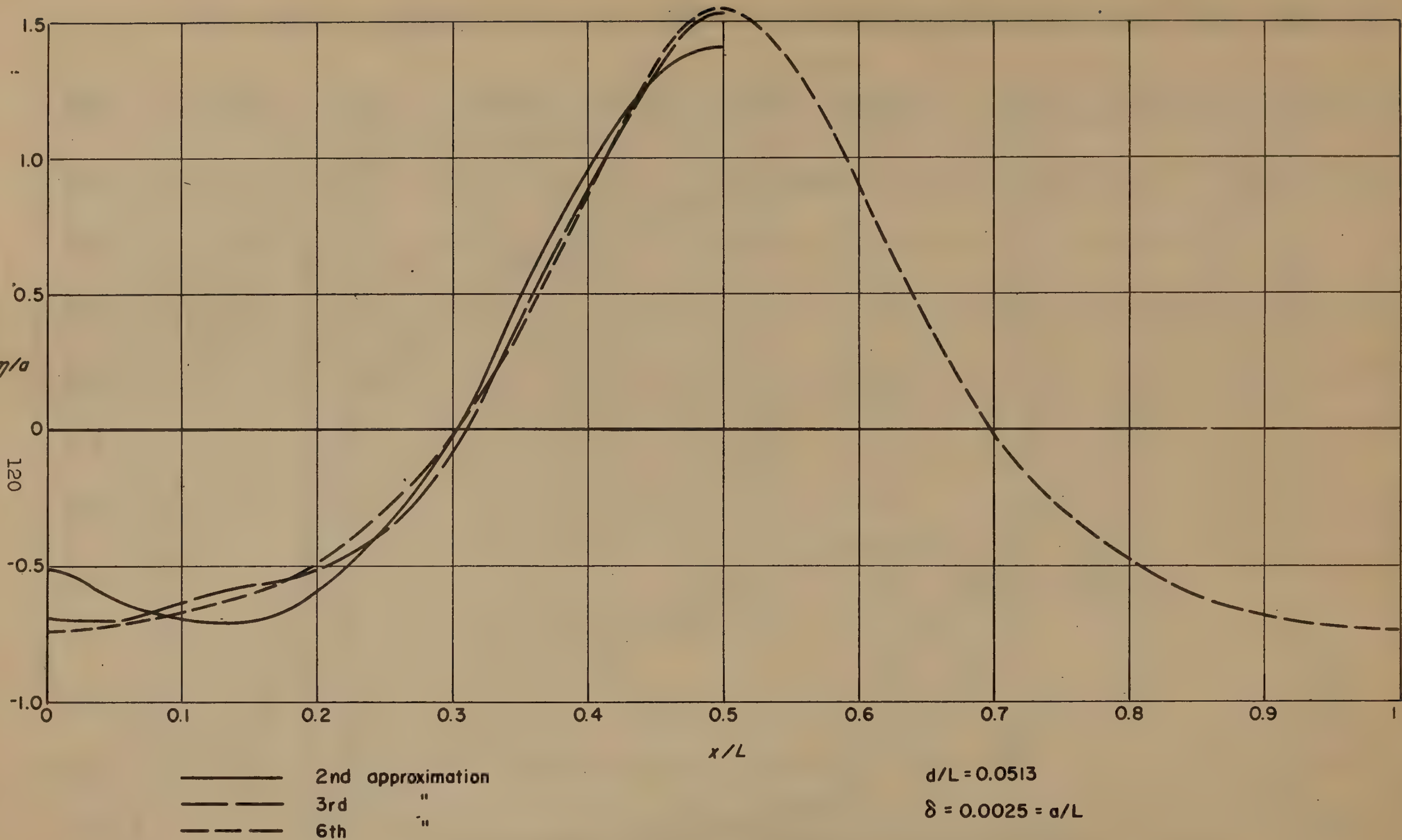


FIG. IIA-12 - COMPARISON OF DIFFERENT ORDERS OF APPROXIMATION TO A PERIODIC SURFACE PROFILE IN DEEP WATER

RESTRICTED Security Information

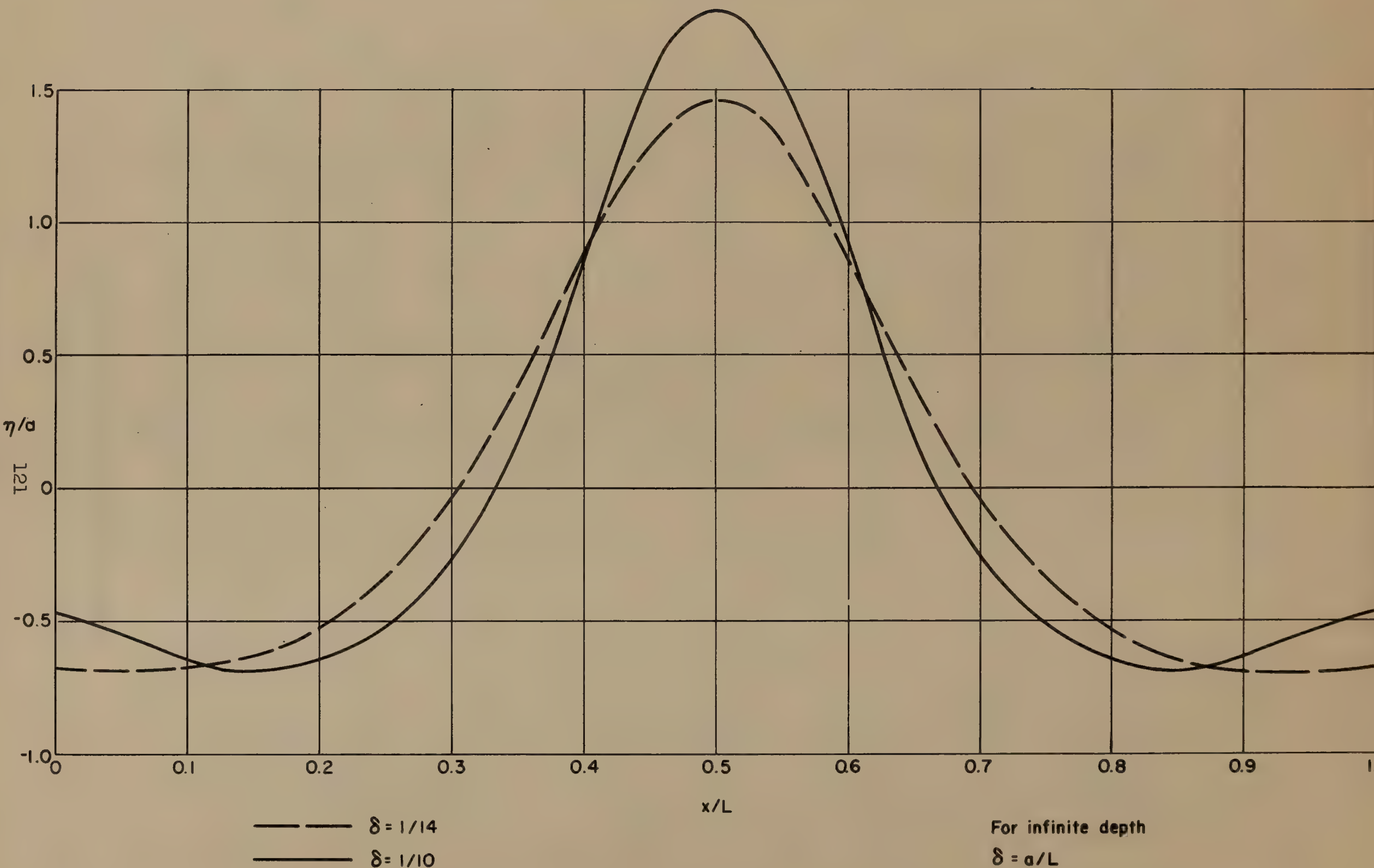


FIG. IIA-13 - PERIODIC SURFACE PROFILES OF VARIOUS STEEPNESSES IN DEEP WATER

RESTRICTED Security Information

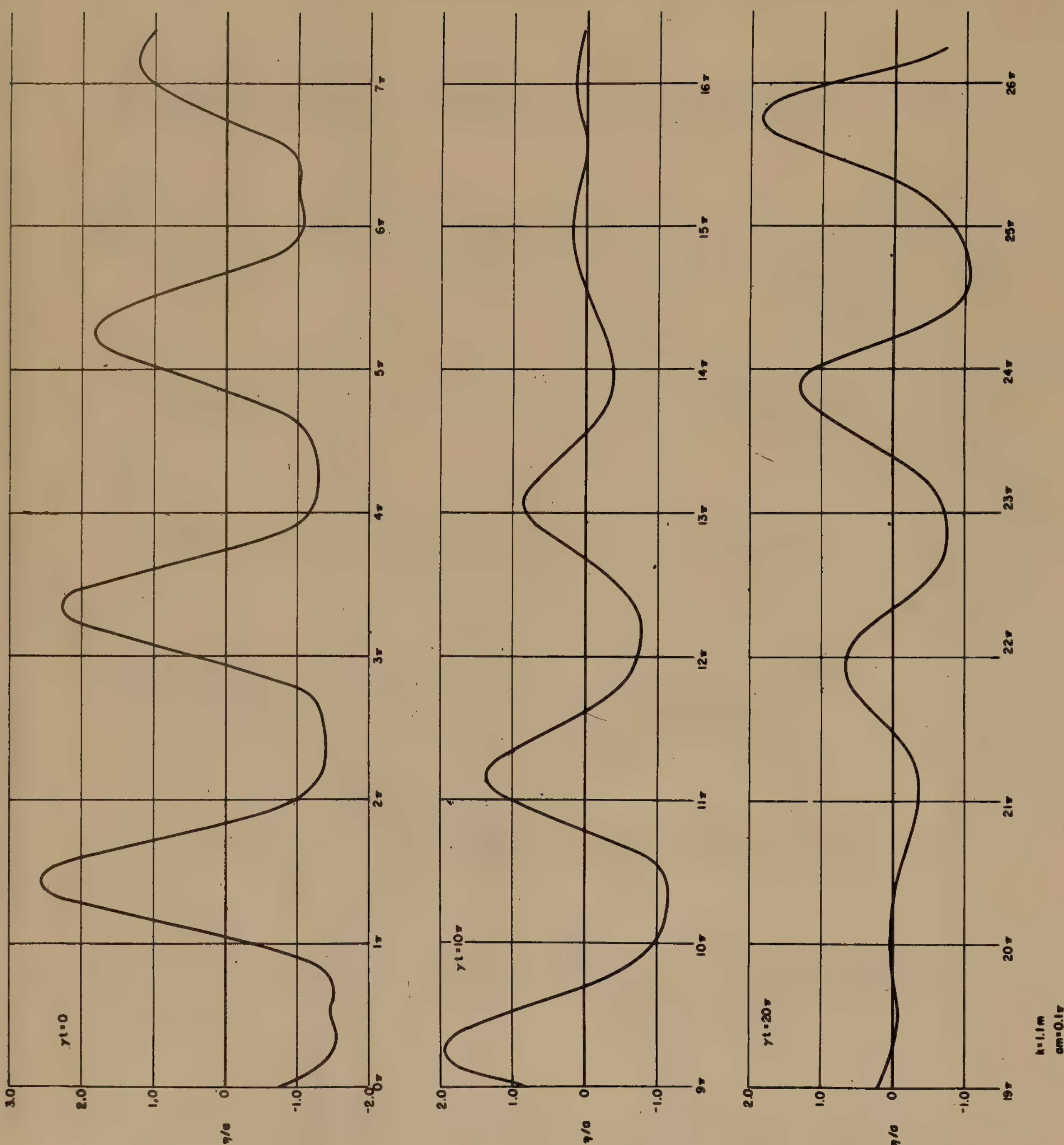


FIG. IIA-14 - SURFACE PROFILES RESULTING FROM NONLINEAR SUPERPOSITION OF TWO WAVES OF NEIGHBORING WAVE-LENGTHS

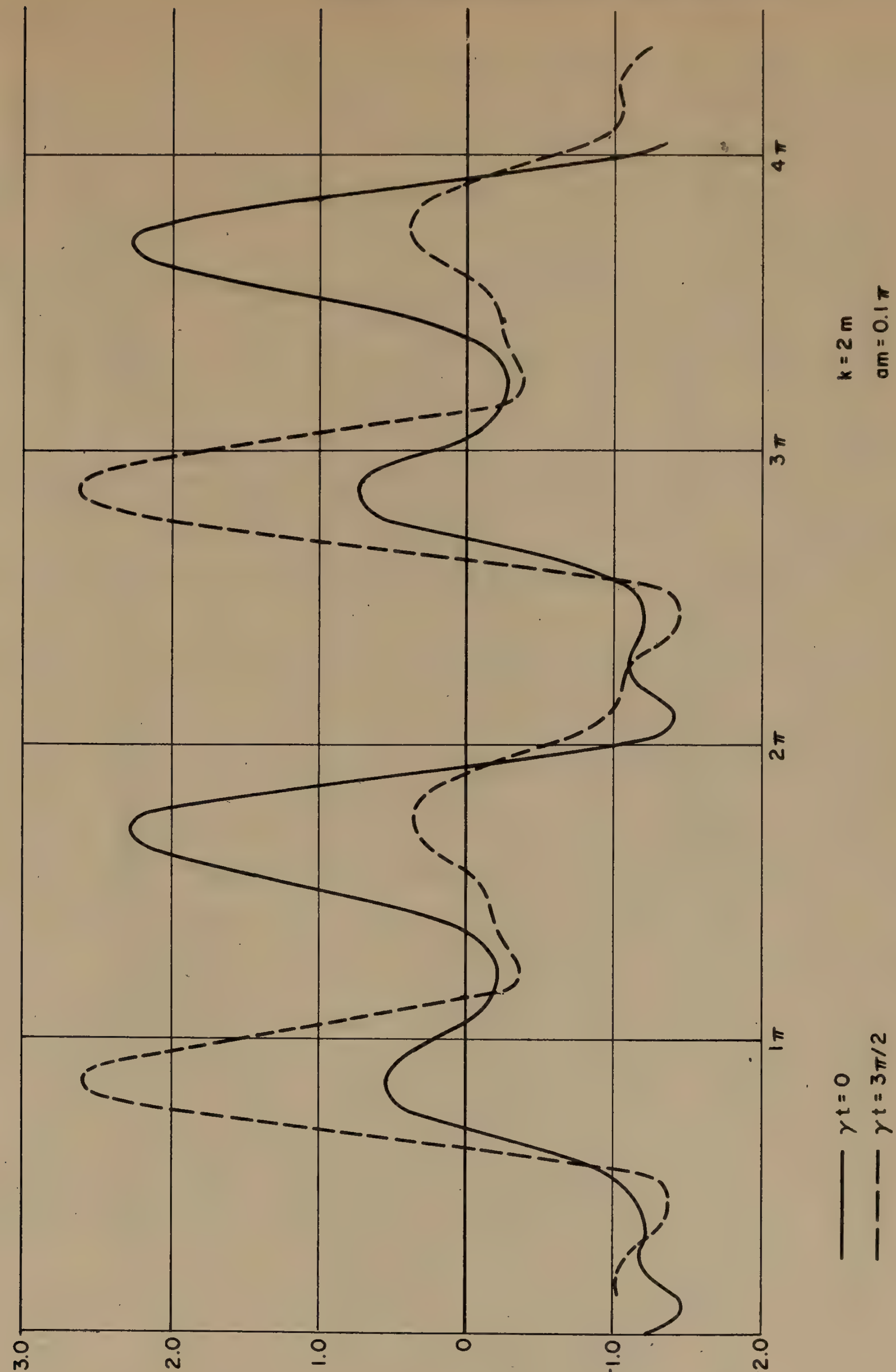


FIG. IIA-15 - SURFACE PROFILES RESULTING FROM NONLINEAR SUPERPOSITION OF TWO WAVES OF VERY DIFFERENT WAVE-LENGTHS

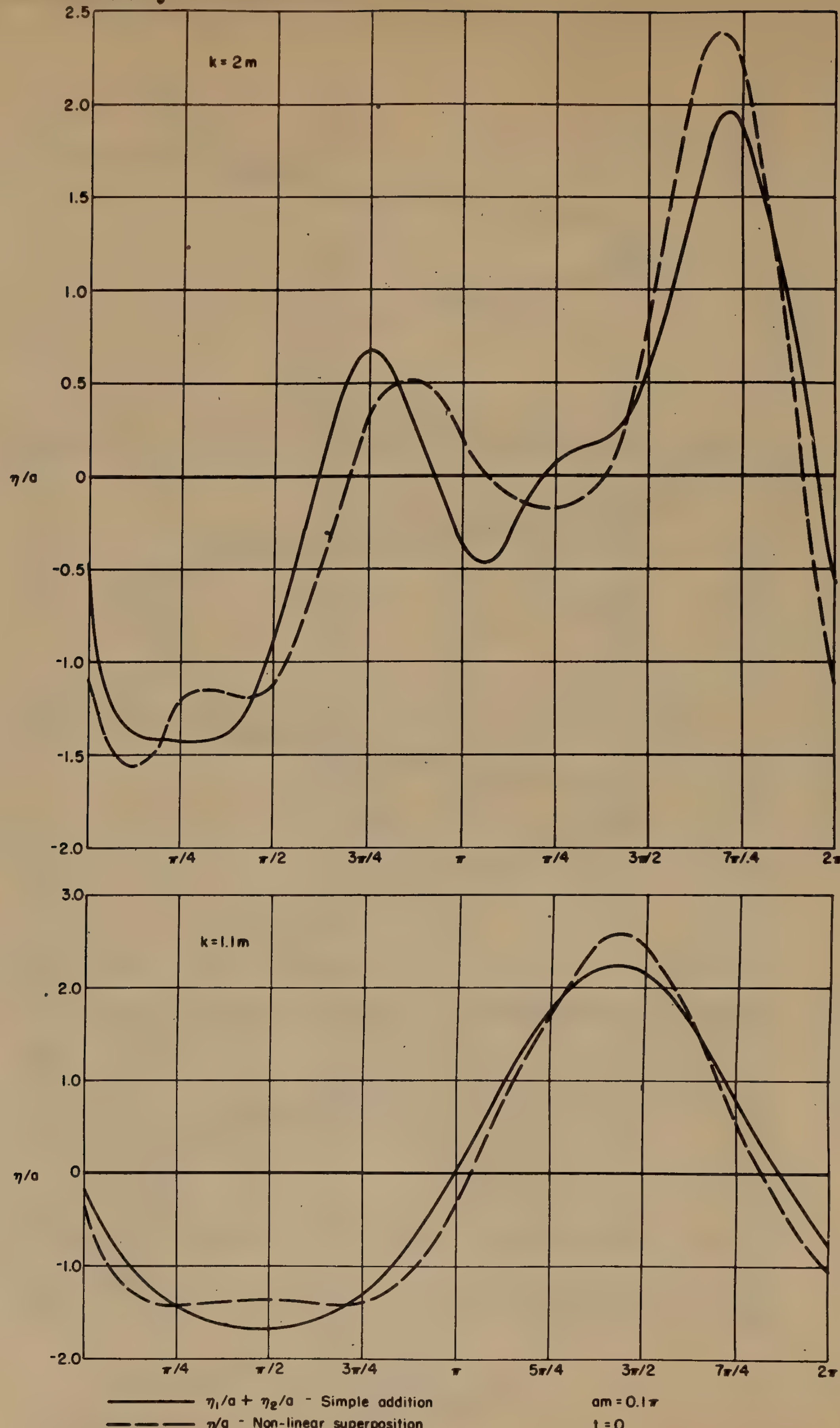


FIG. II A-16 - EFFECT OF NONLINEARITY ON SUPERPOSITION OF WAVE PROFILES

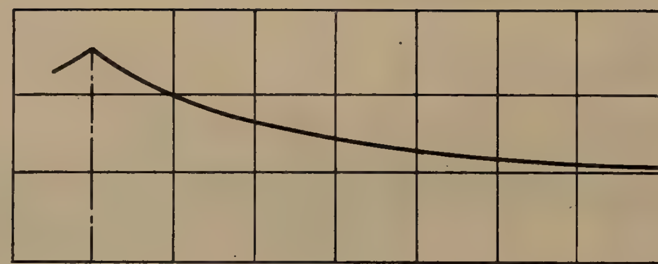


FIG. IIA-17 - SURFACE PROFILE OF AN EXTREME SOLITARY WAVE

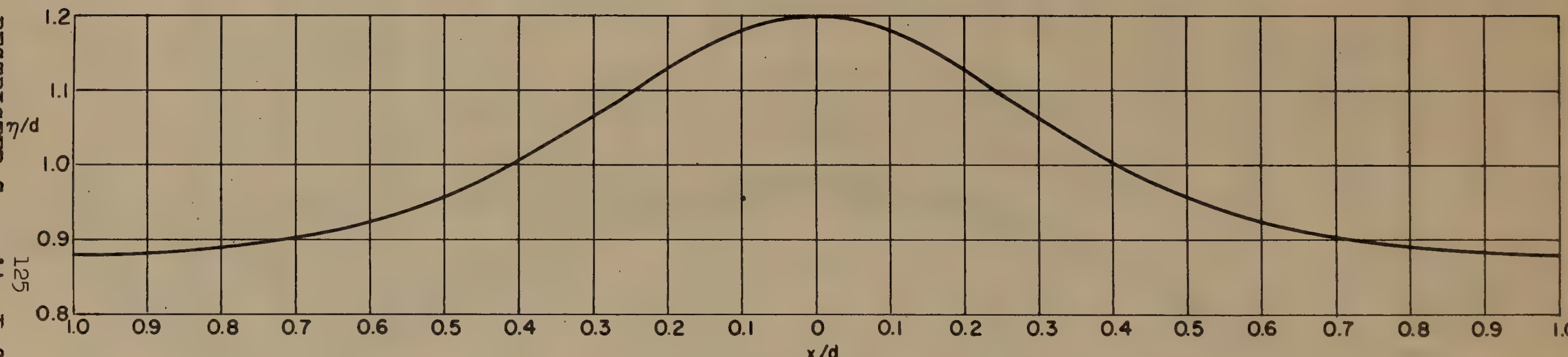


FIG. IIA-18 - SURFACE PROFILE OF A CNOIDAL WAVE

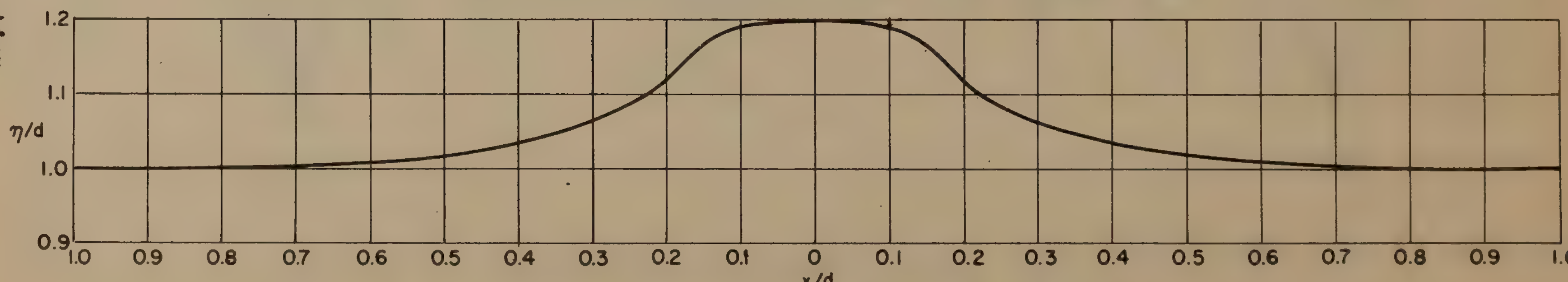


FIG. IIA-19 - SURFACE PROFILE OF A SOLITARY WAVE OF MODERATE HEIGHT

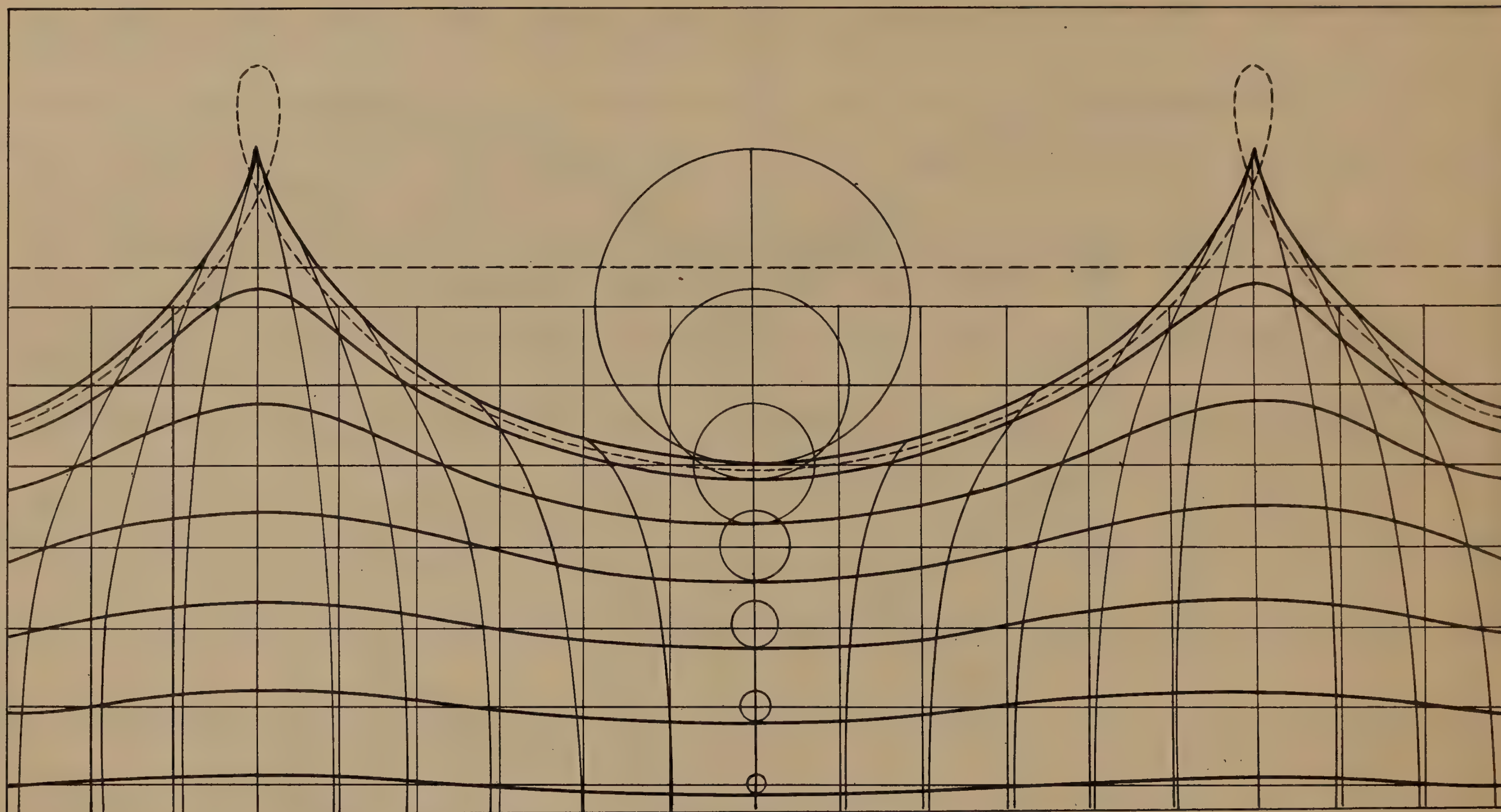


FIG. IIA-20 - PARTICLE POSITIONS AND STREAMLINES
IN A TROCHOIDAL WAVE

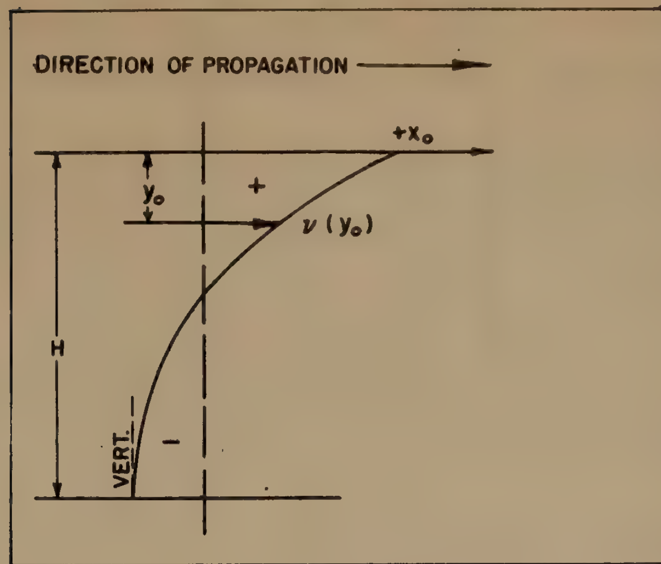


FIG. IIA-21 - MASS TRANSPORT VELOCITIES FOR MICHE'S ROTATIONAL WAVES

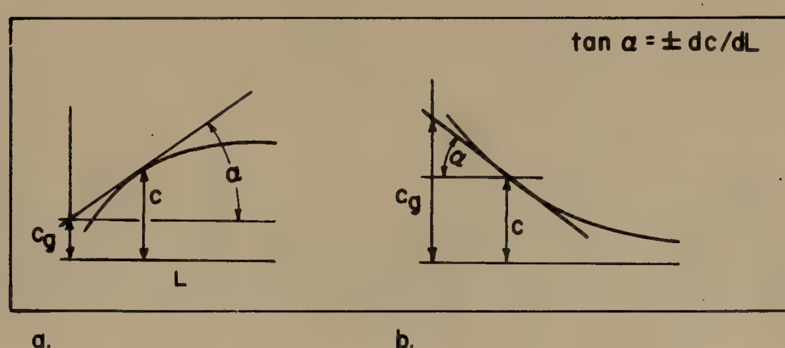


FIG. IIA-22 - CONSTRUCTION OF THE GROUP VELOCITY FOR A GIVEN PHASE VELOCITY FOR
(a) NORMAL DISPERSION (ex. GRAVITY WAVES) AND
(b) ANAMALOUS DISPERSION (ex. CAPILLARY WAVES)

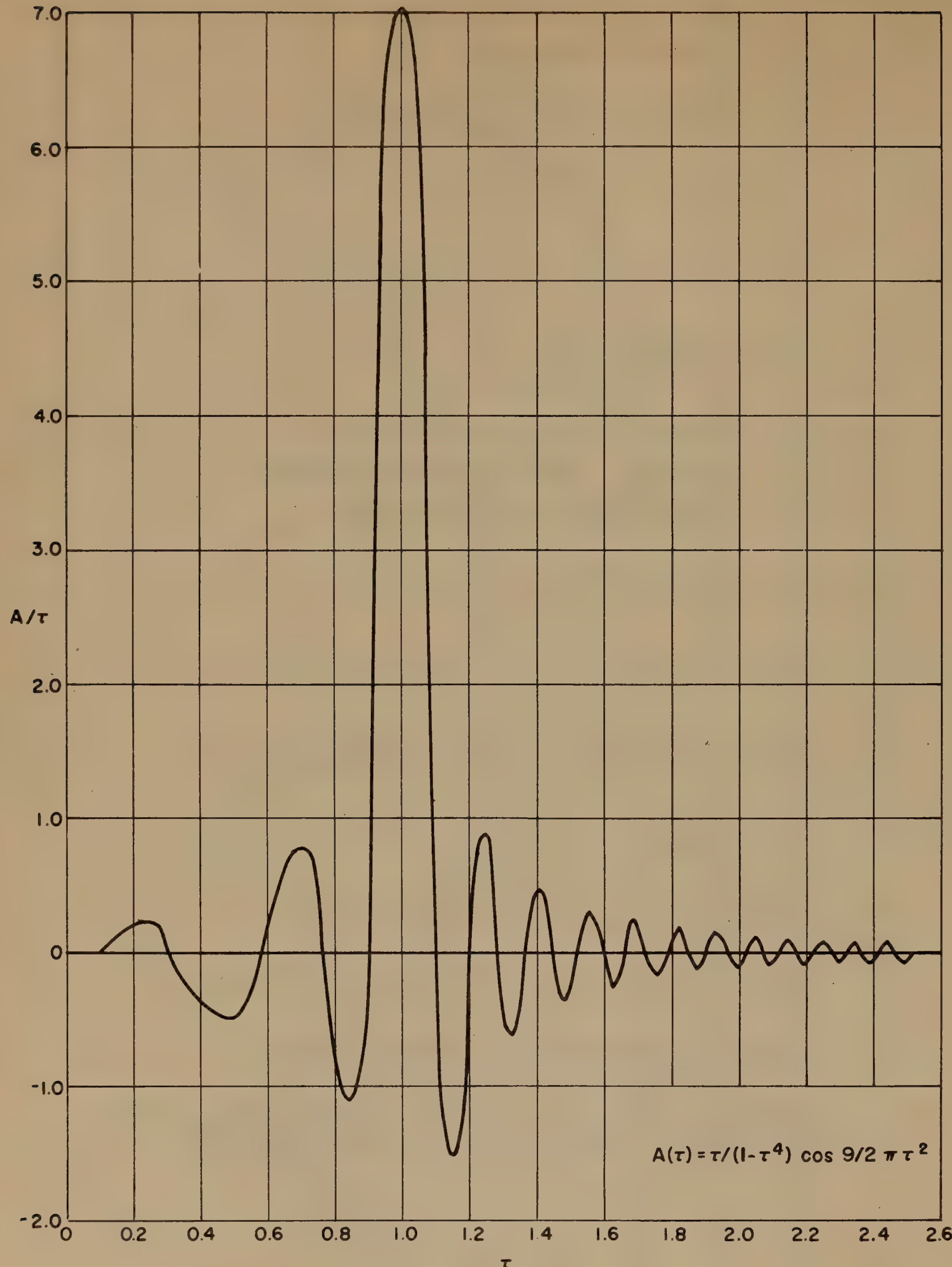


FIG. IIA-23 - TIME HISTORY OF WAVE ENVELOPE FOR GROUP RESULTING FROM AN INITIALLY FINITE WAVE-TRAIN

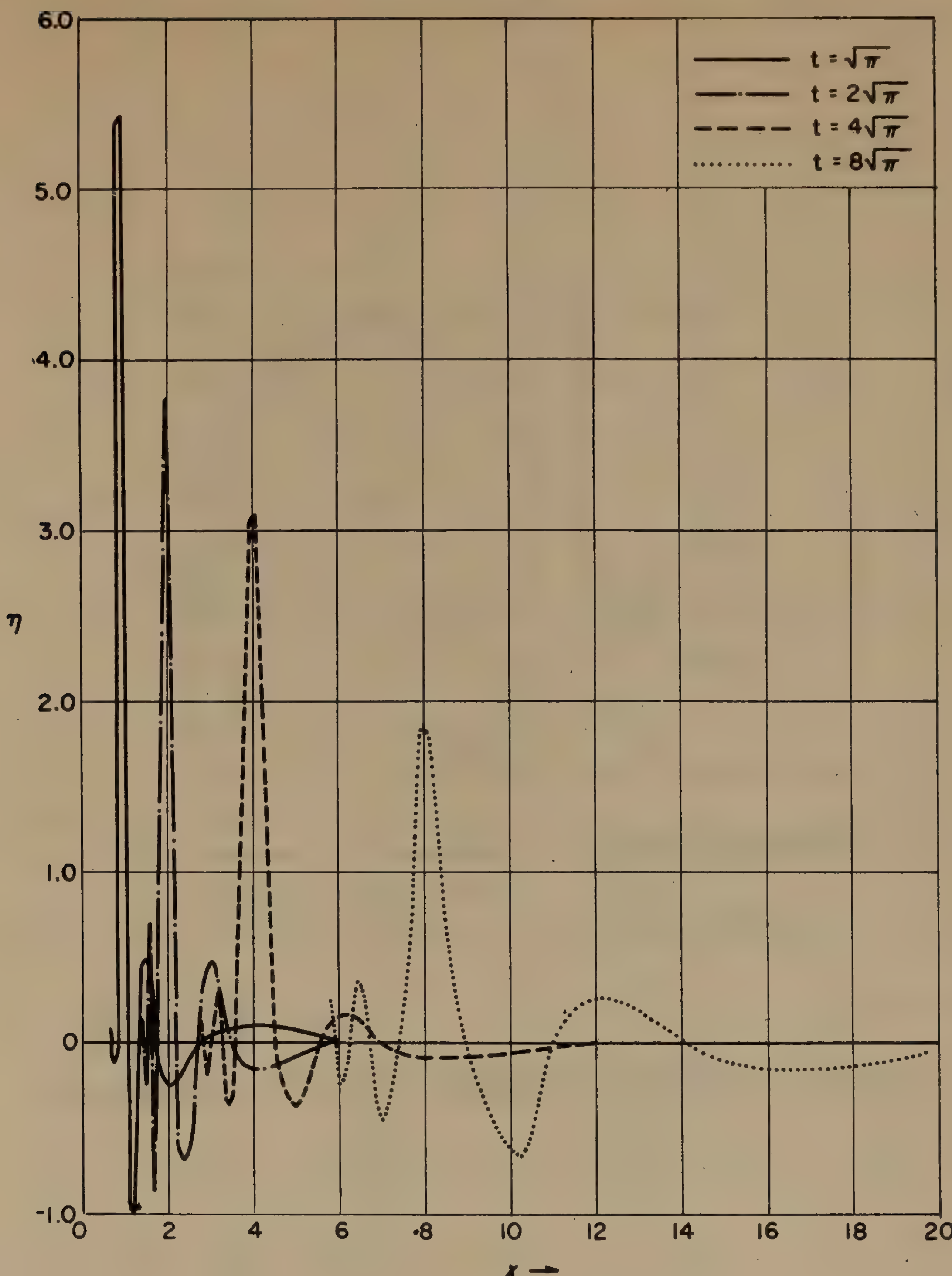


FIG. IIA-24 - SURFACE PROFILES OF WAVE ENVELOPE FOR GROUP RESULTING FROM AN INITIALLY FINITE WAVE-TRAIN

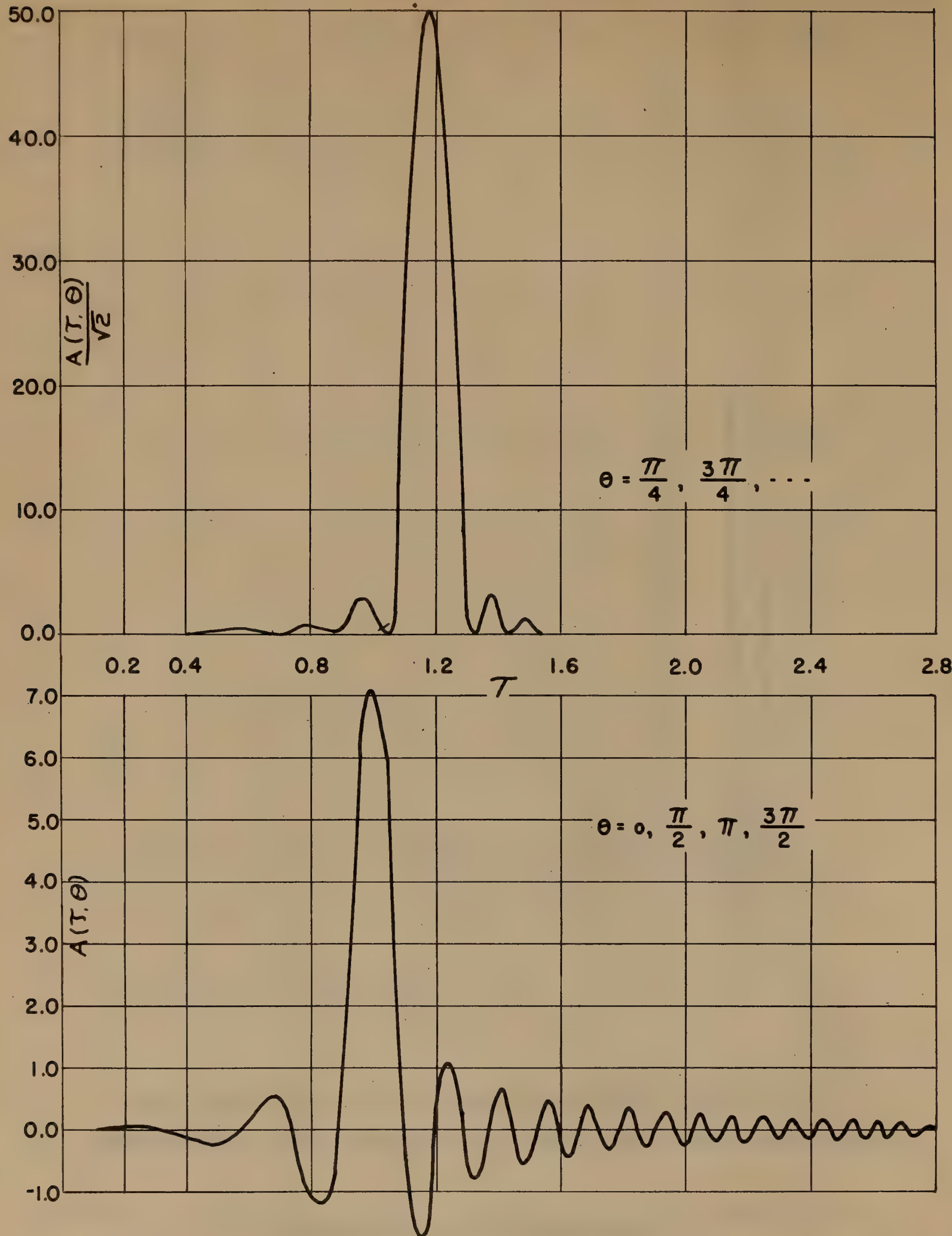


FIG. IIA-2B - AMPLITUDE. FACTOR $A(\tau, \theta)$ FOR LOCALIZED DISPLACEMENT
4 1/2 WAVE LENGTHS LONG

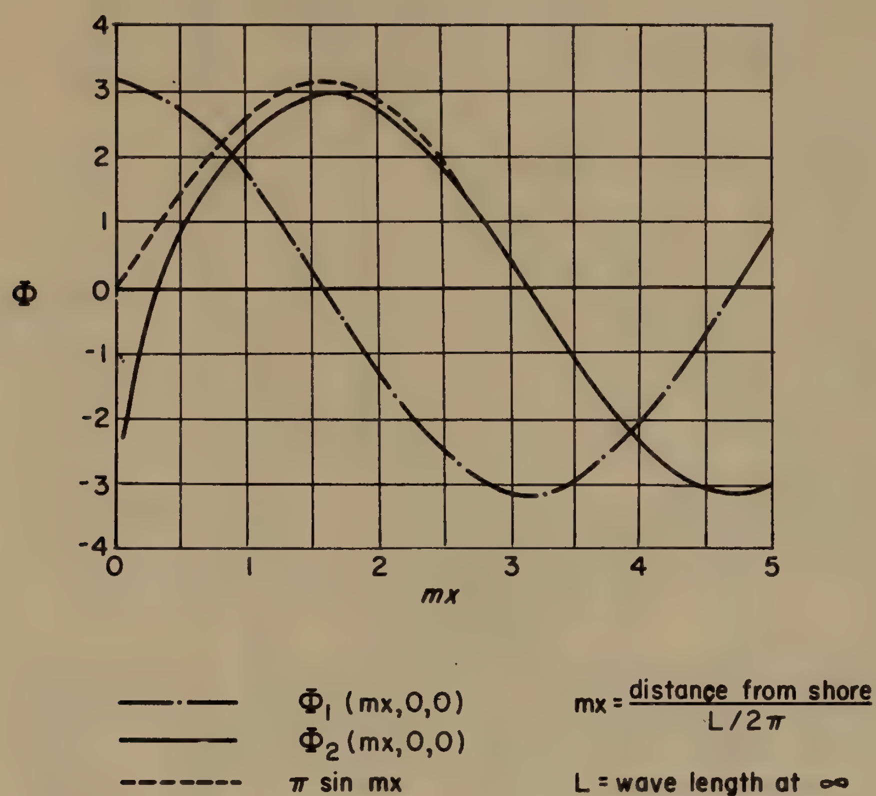


FIG. IIA-26 - STANDING WAVES FOR A VERTICAL CLIFF

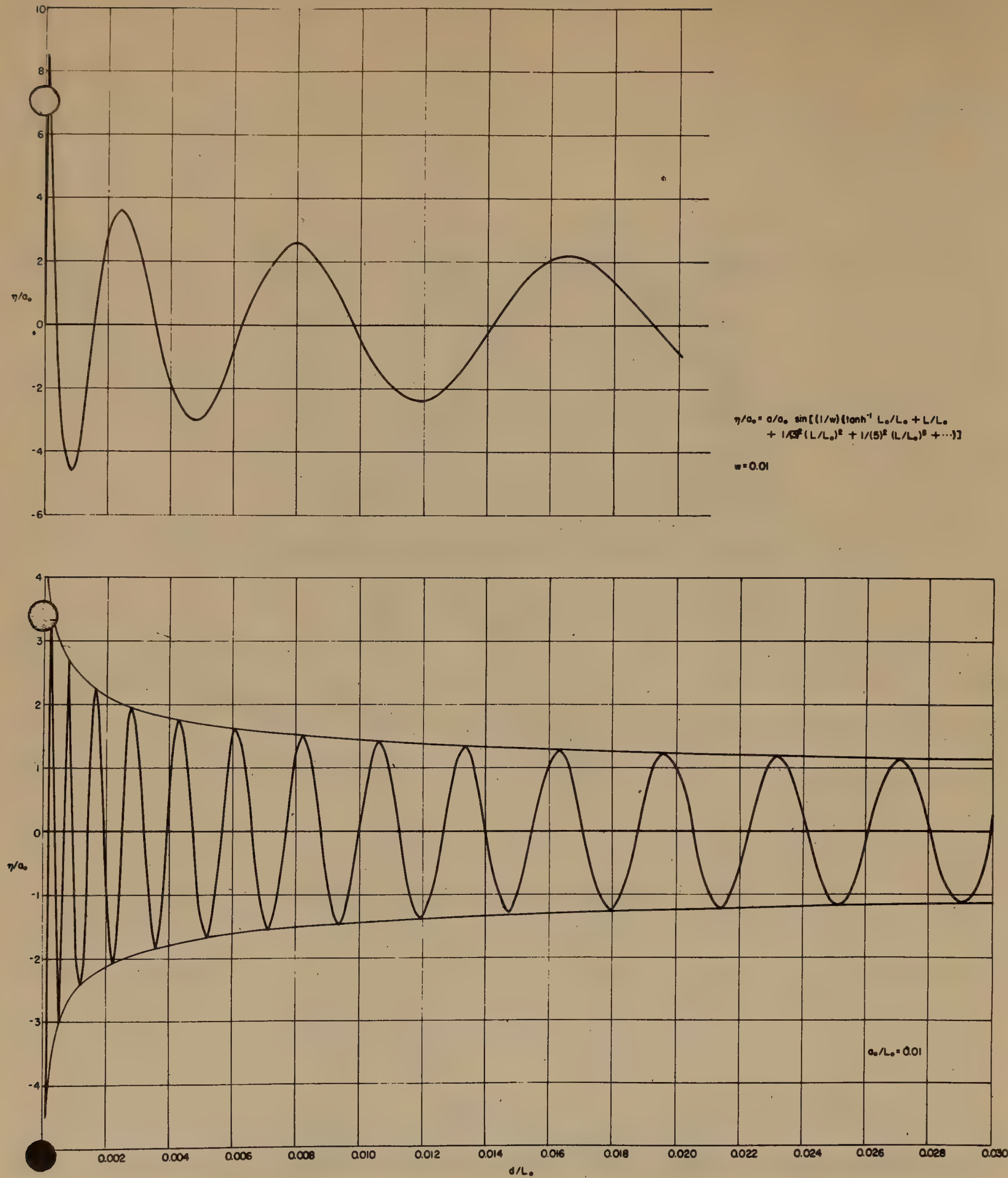


FIG. IIA-27 - PROGRESSIVE WAVE PROFILES ON A BEACH OF 1/100 SLOPE

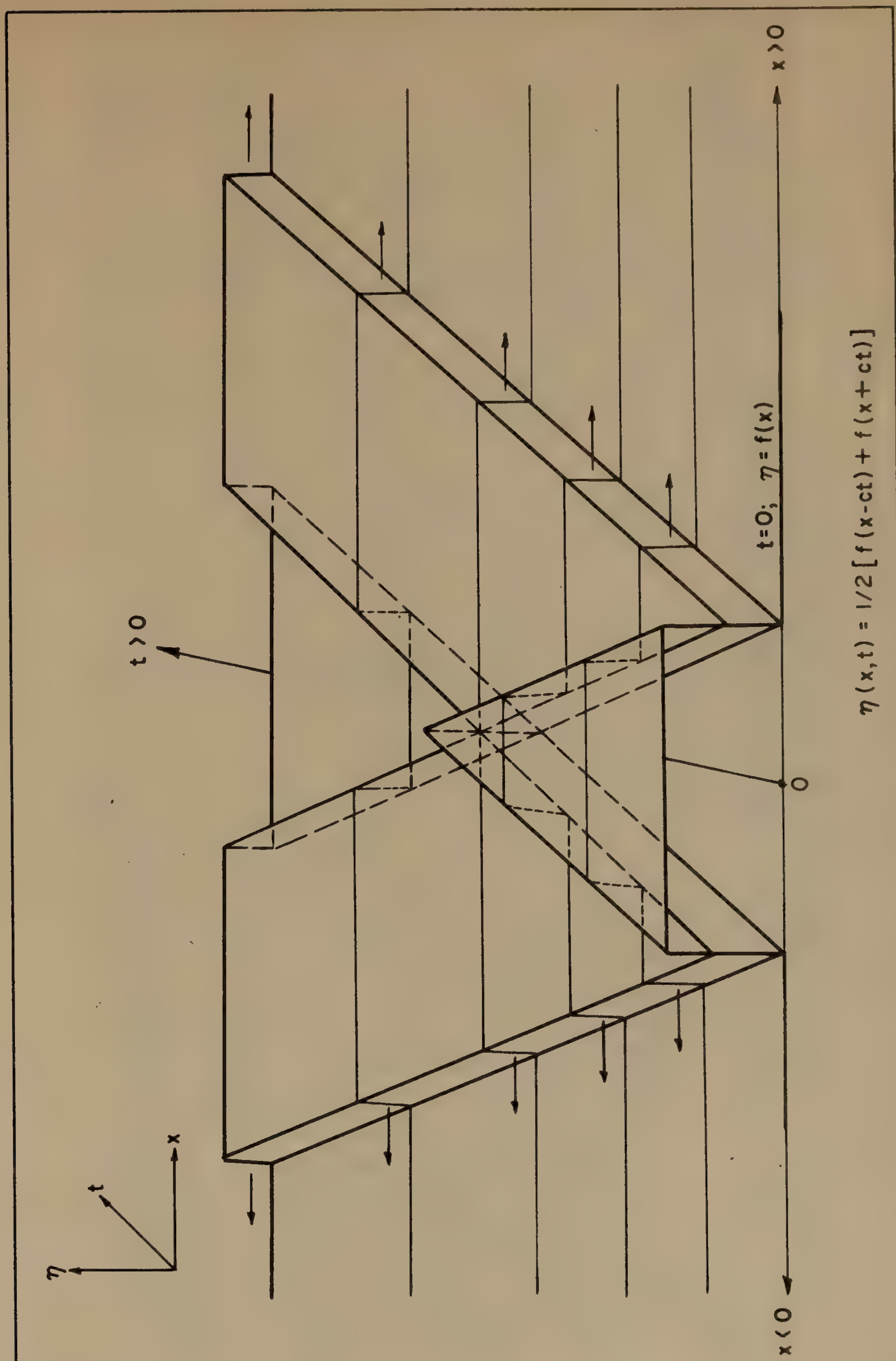


FIG. IIA-28 - ISOMETRIC DRAWING OF THE SURFACE BEHAVIOR DUE TO A LOCALIZED DISTURBANCE IN SHALLOW WATER

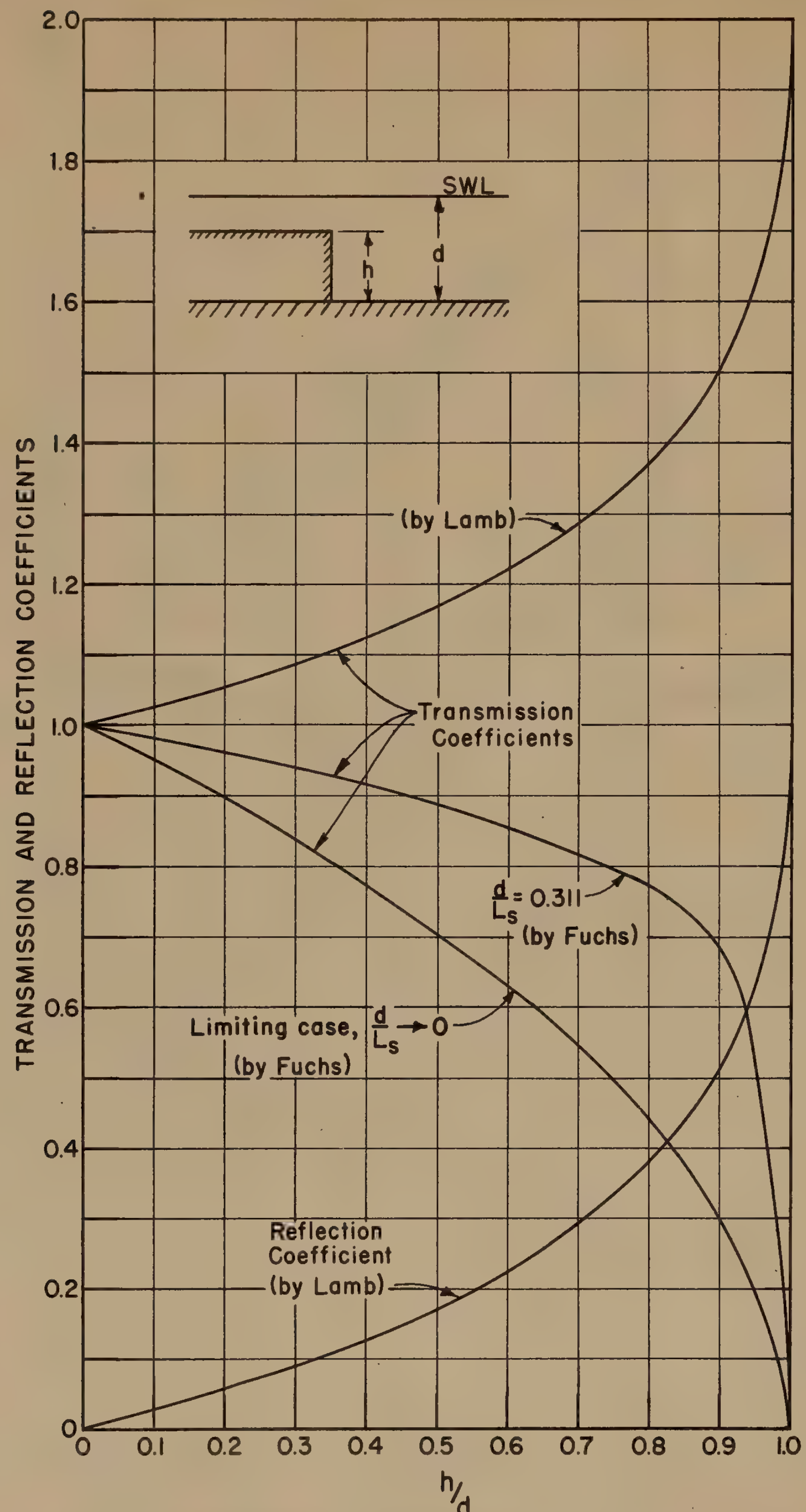


FIG. IIA-29 --THEORETICAL REFLECTION AND TRANSMISSION COEFFICIENTS FOR WAVE ACTION ON A VERTICAL REEF.

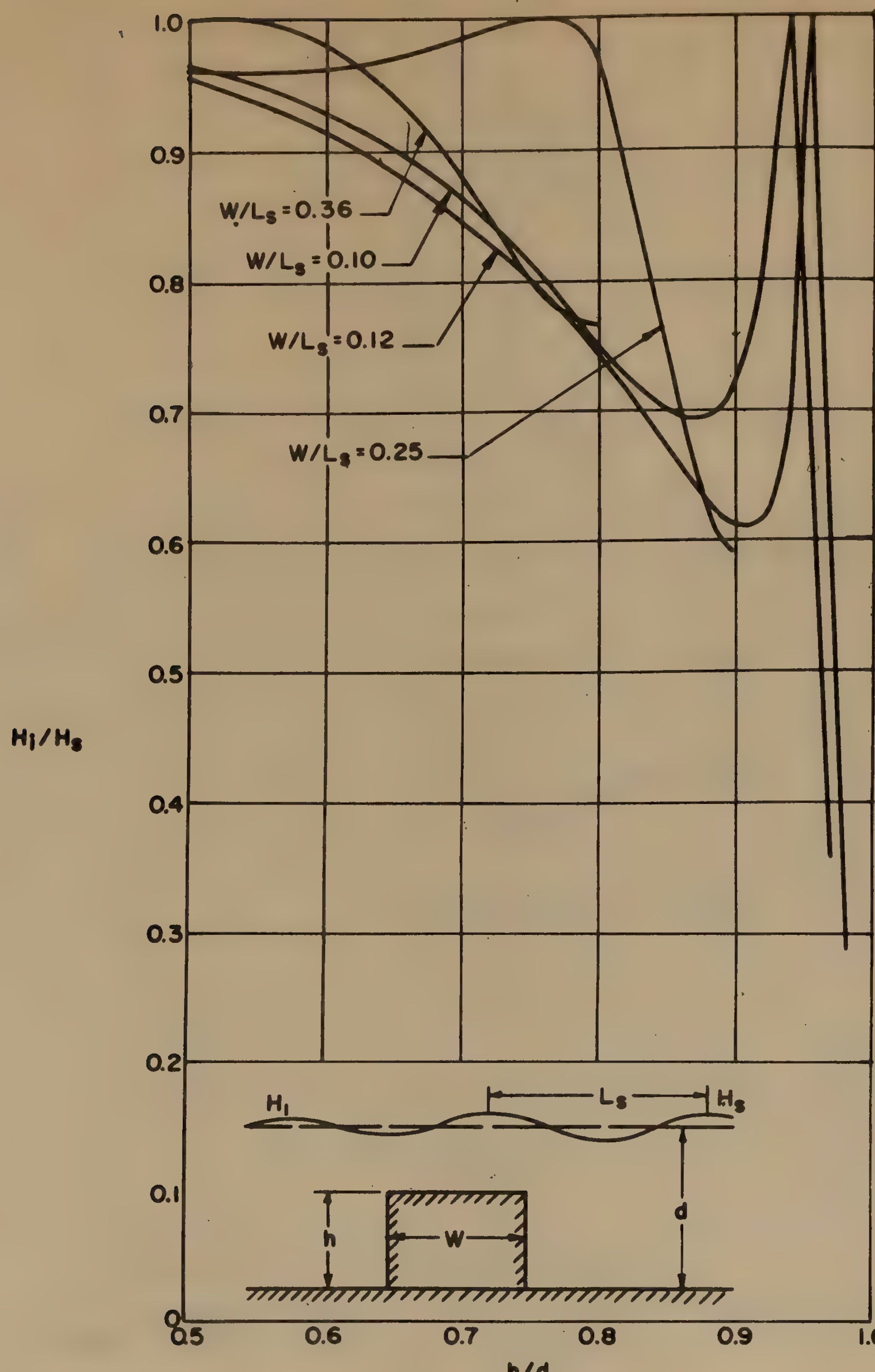
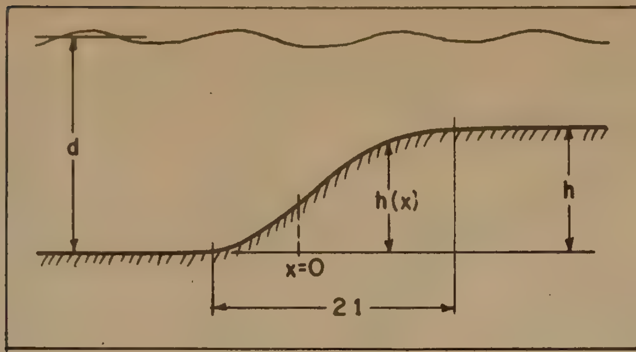
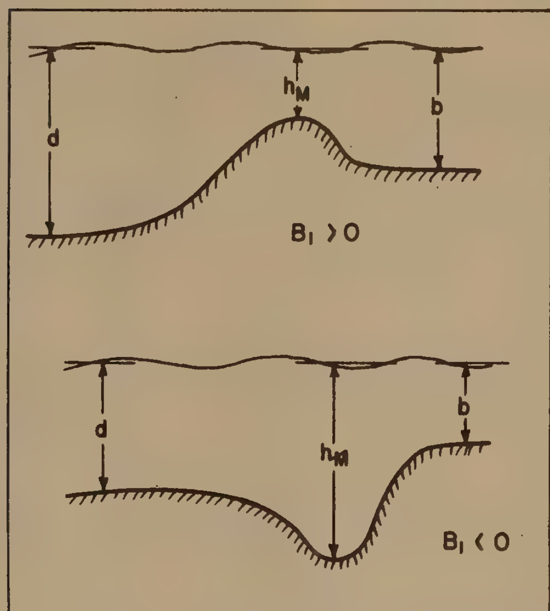


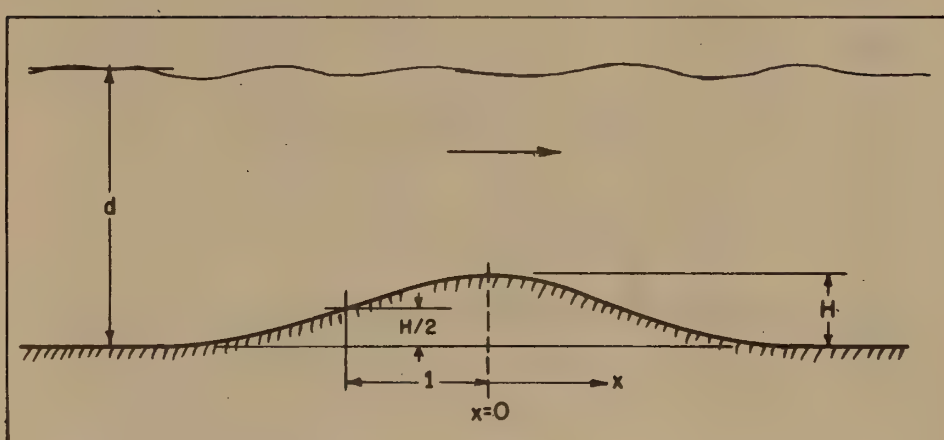
FIG. IIA-30 - THEORETICAL TRANSMISSION COEFFICIENTS OBTAINED BY JEFFERYS FOR WAVE ACTION OVER A RECTANGULAR BARRIER



a. Continuous shelf (Yoshida)



b. Submarine bank and trench (Yoshida)



c. Symmetrical bar (Dean)

FIG. IIA-31 - VARIOUS TYPES OF UNDERWATER BARRIERS IN SHALLOW WATER

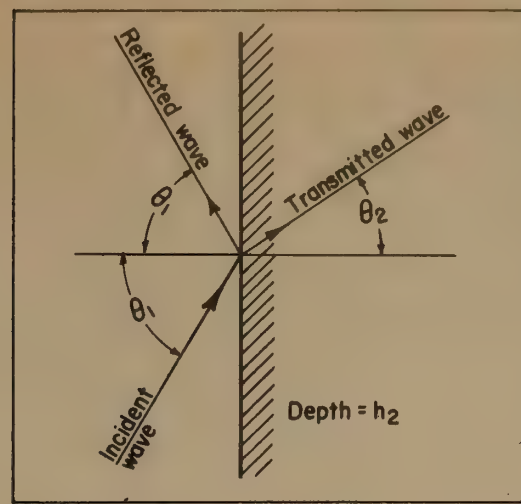


FIG. IIA-32 - OBLIQUE WAVE INCIDENCE ON A REEF IN SHALLOW WATER

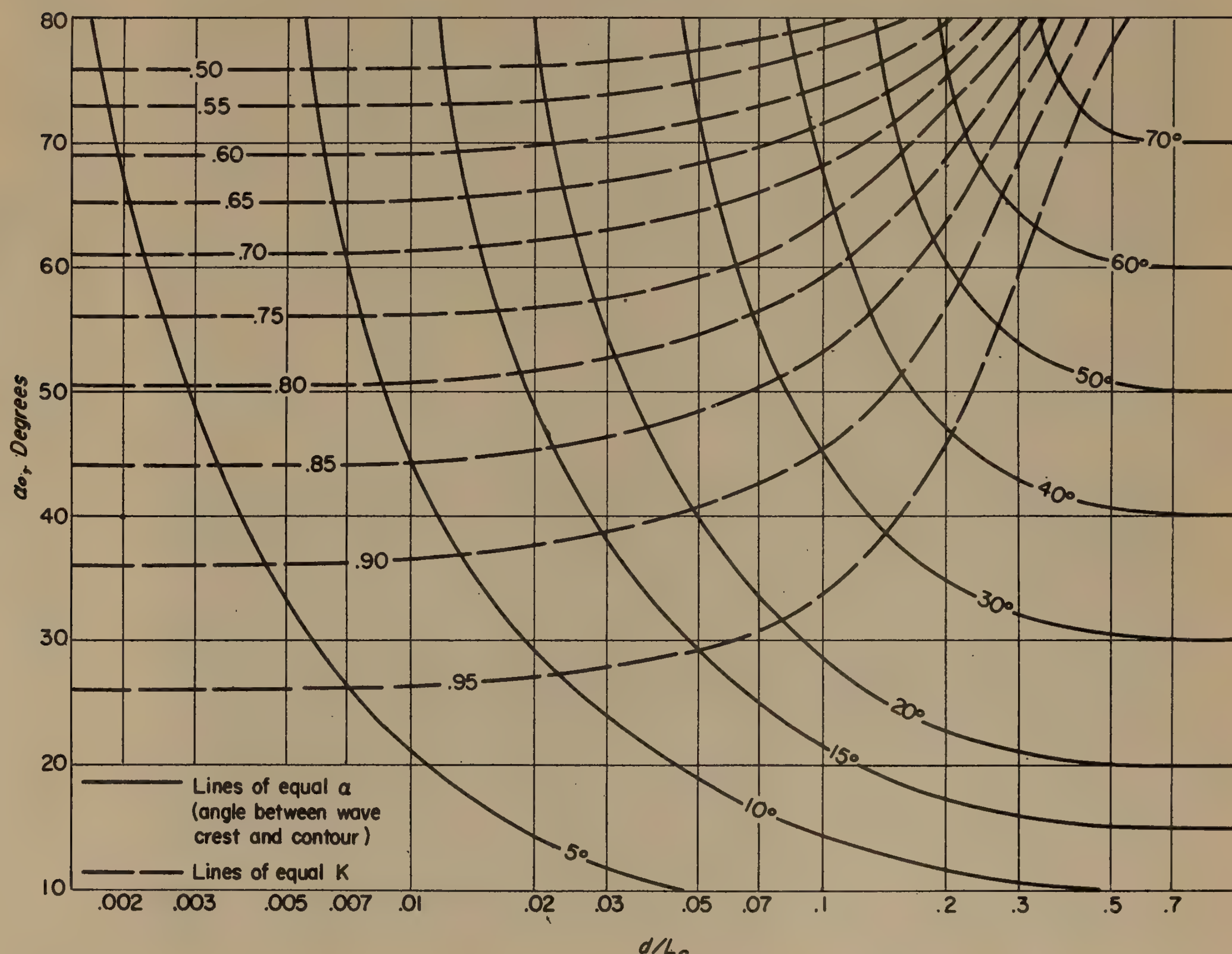


FIG. IIA-33 - CHANGE IN WAVE DIRECTION AND HEIGHT DUE TO REFRACTION ON BEACHES WITH STRAIGHT, PARALLEL DEPTH CONTOURS.
LONG-CRESTED WAVES ($L' = \infty$)

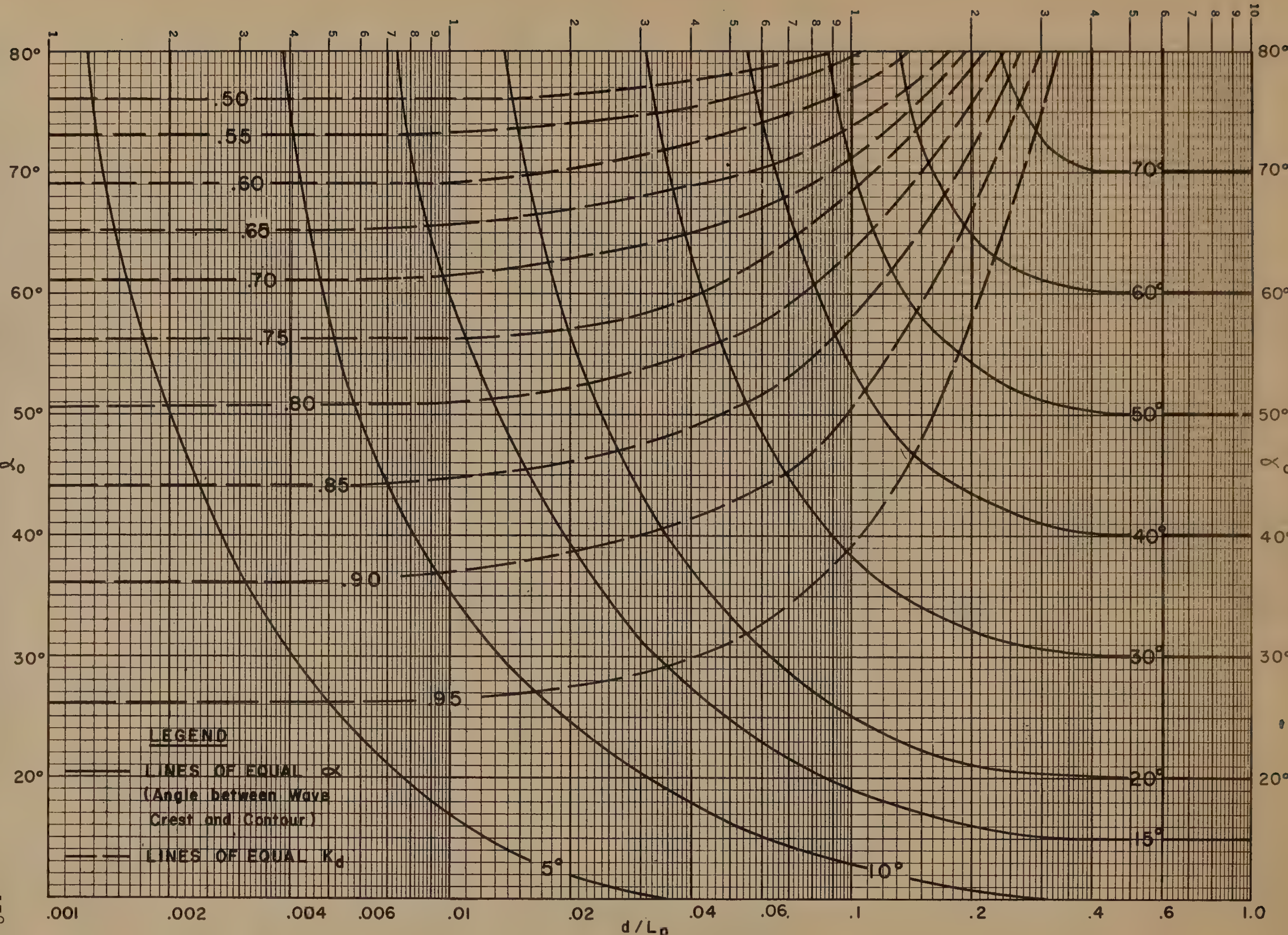


FIG. IIA-34 - CHANGE IN WAVE DIRECTION AND HEIGHT DUE TO REFRACTION ON BEACHES WITH STRAIGHT, PARALLEL DEPTH CONTOURS FOR SHORT-CRESTED (L = L')

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HYD-5429

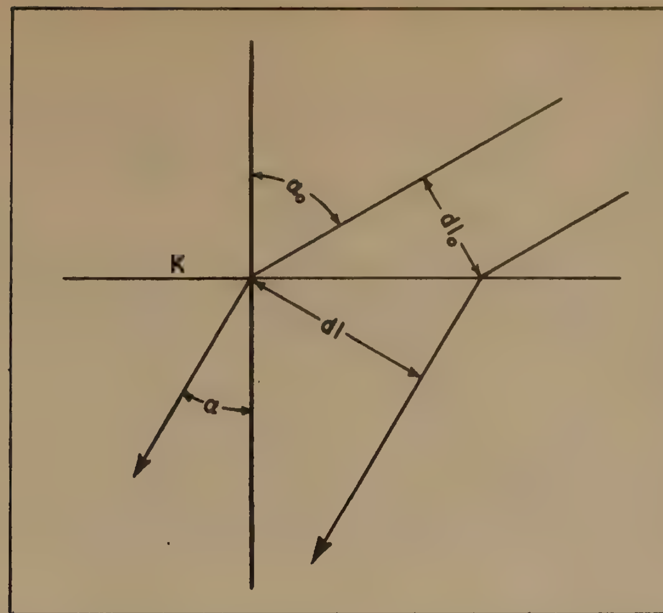

FIG. IIA-35 - REFRACTION AT A DEPTH DISCONTINUITY

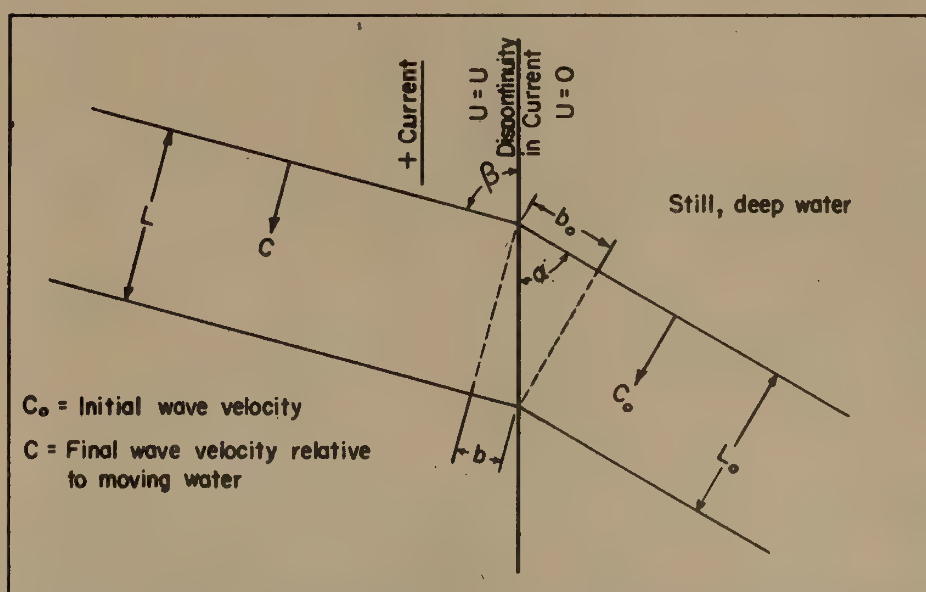

FIG. IIA-36 - REFRACTION AT A CURRENT DISCONTINUITY

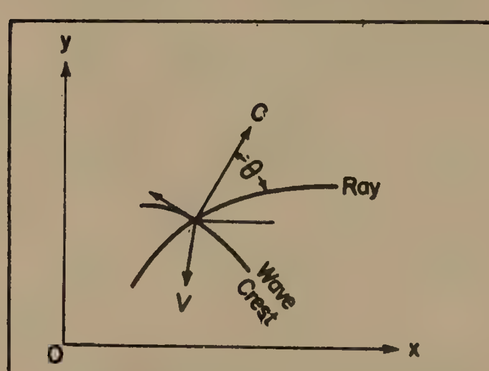

FIG. IIA-37 - REFRACTION DUE TO CURRENTS AND DEPTH CHANGES

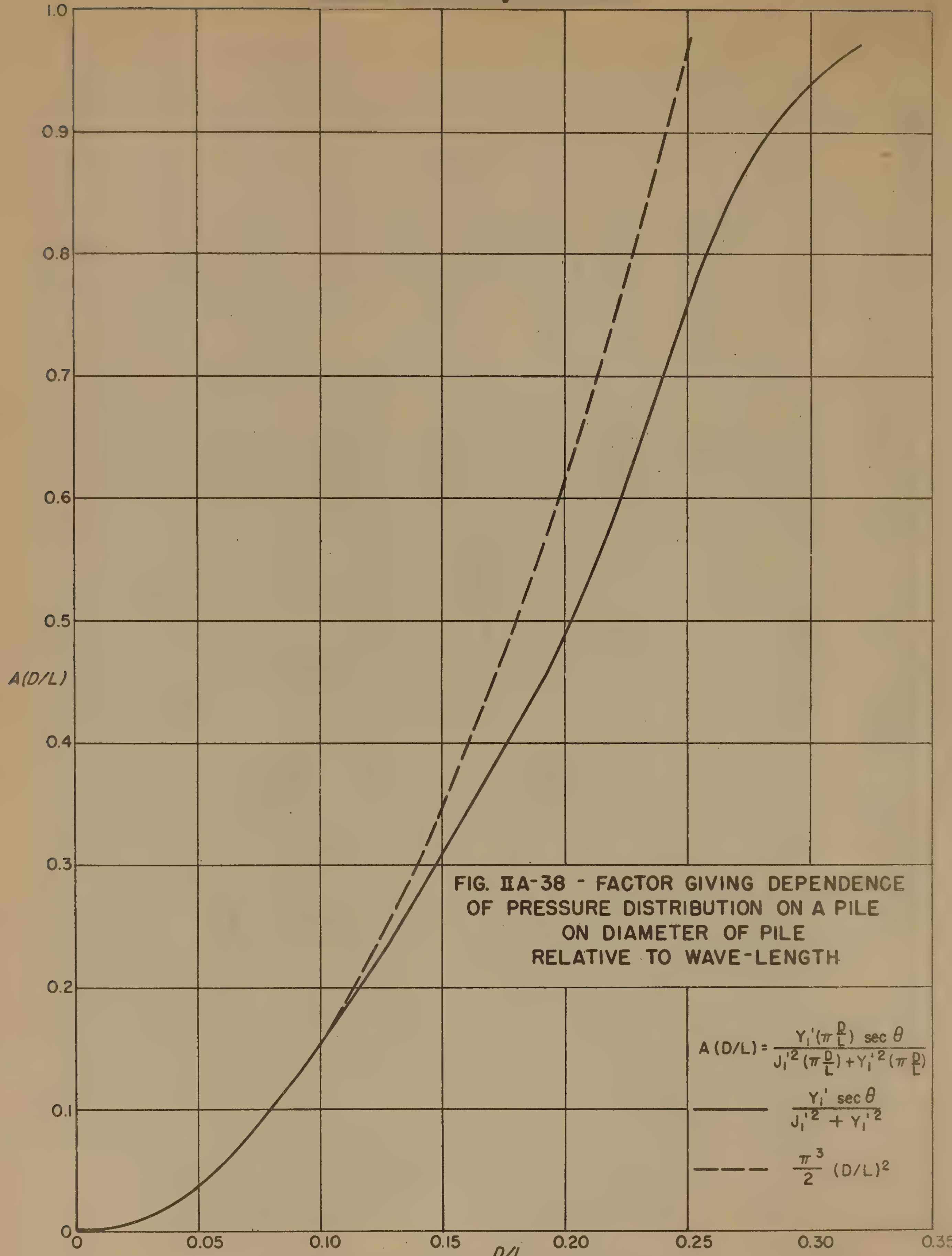

FIG. IIA-38 - FACTOR GIVING DEPENDENCE OF PRESSURE DISTRIBUTION ON A PILE ON DIAMETER OF PILE RELATIVE TO WAVE-LENGTH

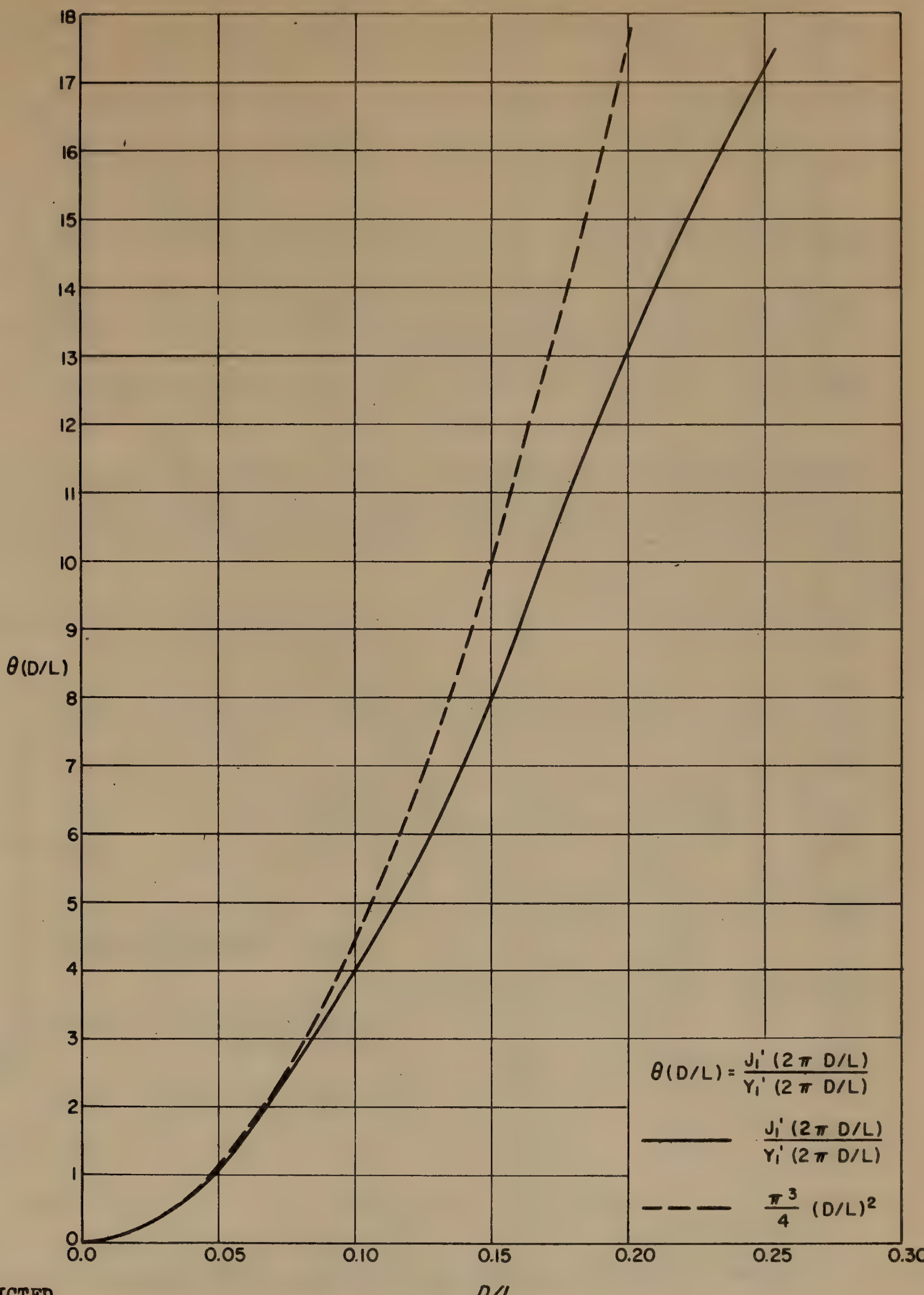

FIG. IIA-39 - PHASE ANGLE OF PRESSURE DISTRIBUTION ON A PILE RELATIVE TO PHASE ANGLE FOR STILL WATER LEVEL OF INCIDENT WAVE

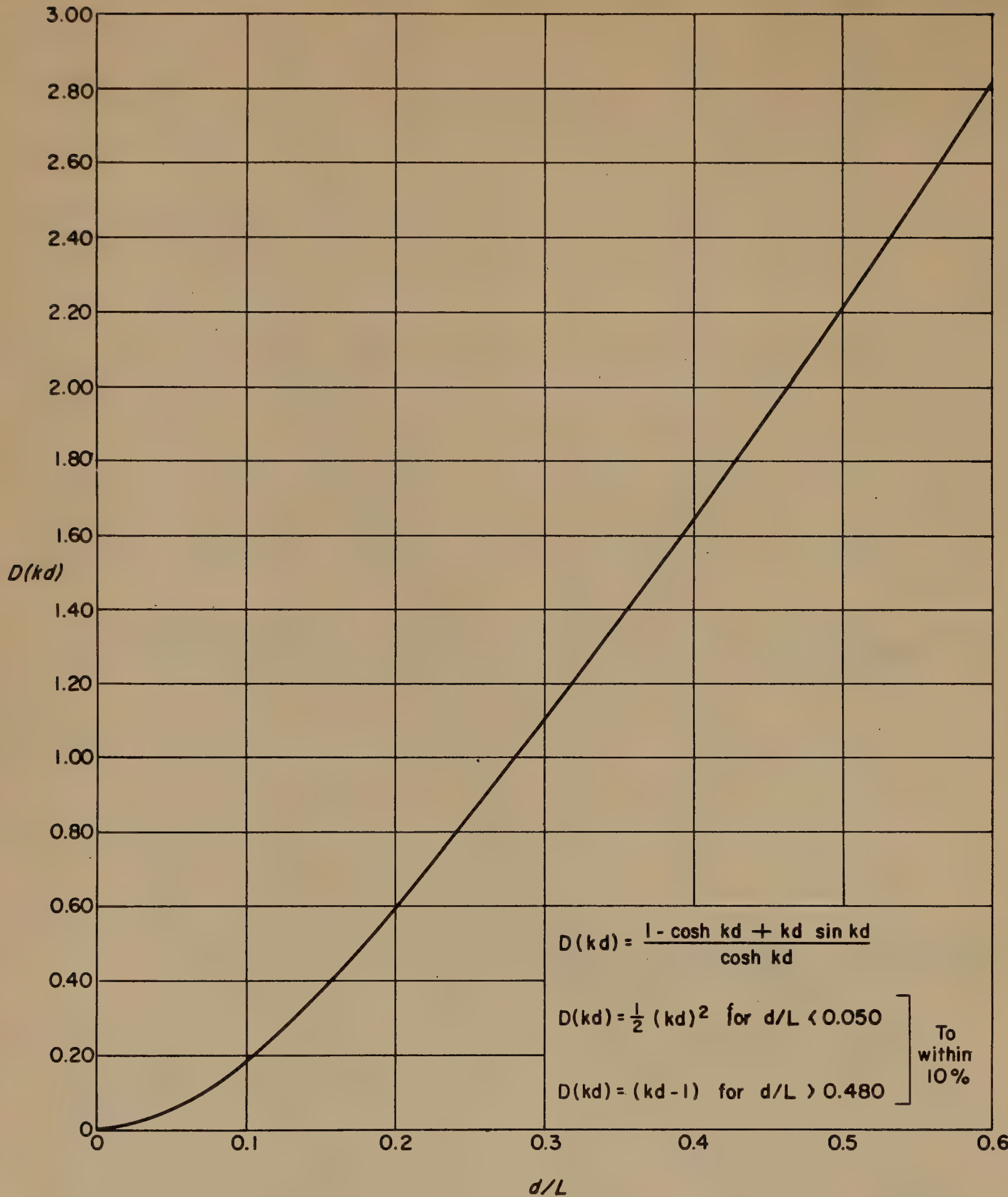

FIG. IIA-40 - FACTOR GIVING DEPENDENCE OF PRESSURE DISTRIBUTION ON A CYLINDRICAL PILE ON *d/L*

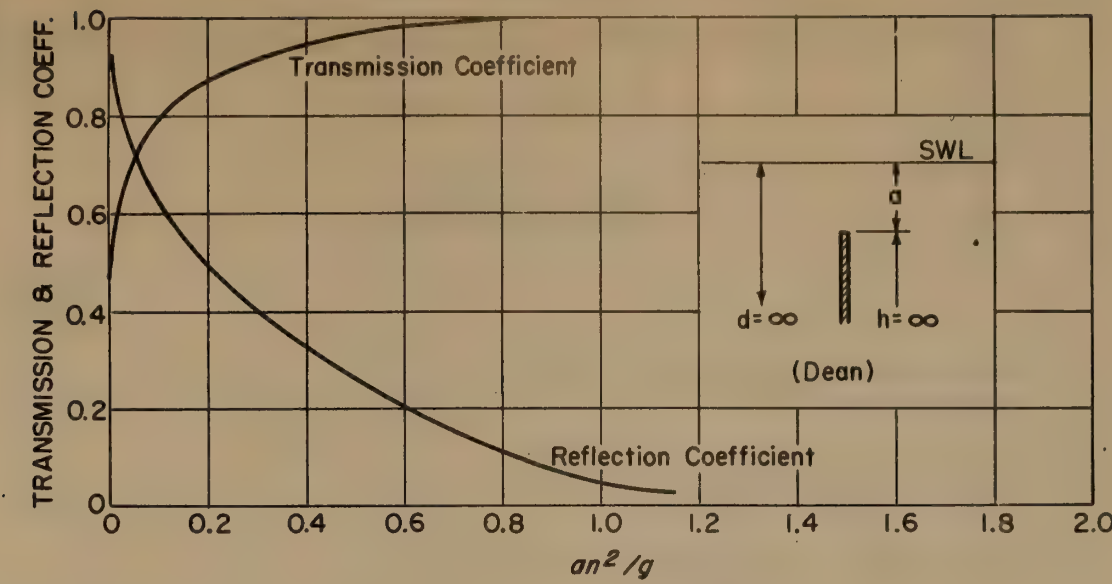

FIG. IIA-41 - REFLECTION AND TRANSMISSION COEFFICIENTS FOR WAVE ACTION OVER A SUBMERGED VERTICAL PLANE BARRIER

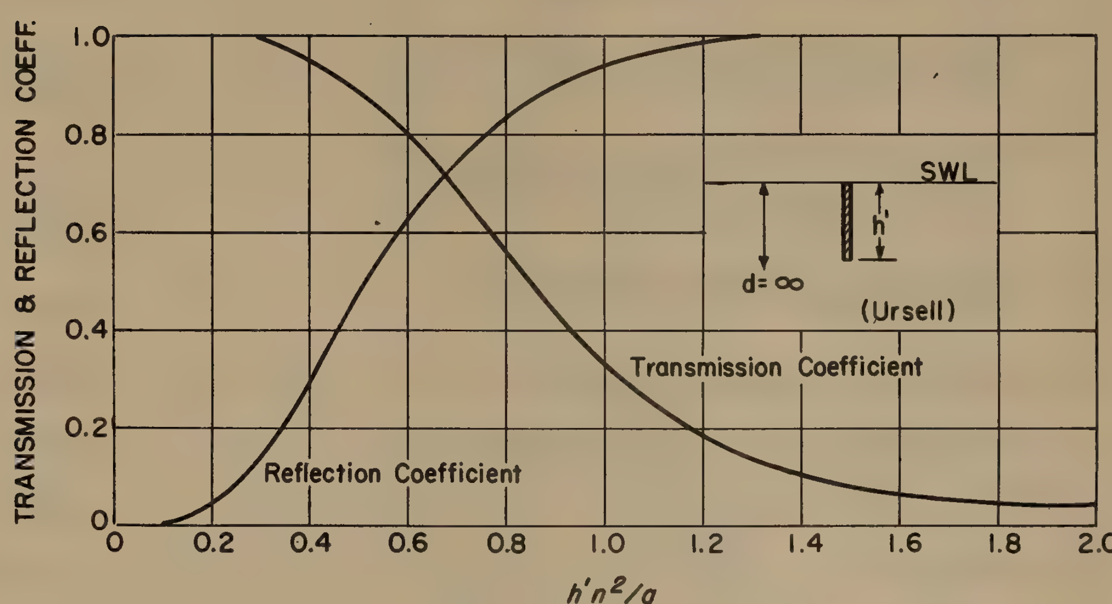

FIG. IIA-42 - REFLECTION AND TRANSMISSION COEFFICIENTS FOR WAVE ACTION OVER A VERTICAL PLANE BARRIER EXTENDING FROM THE SURFACE TO A DEPTH h'

B. USEFUL GRAPHS
BY
STAFF

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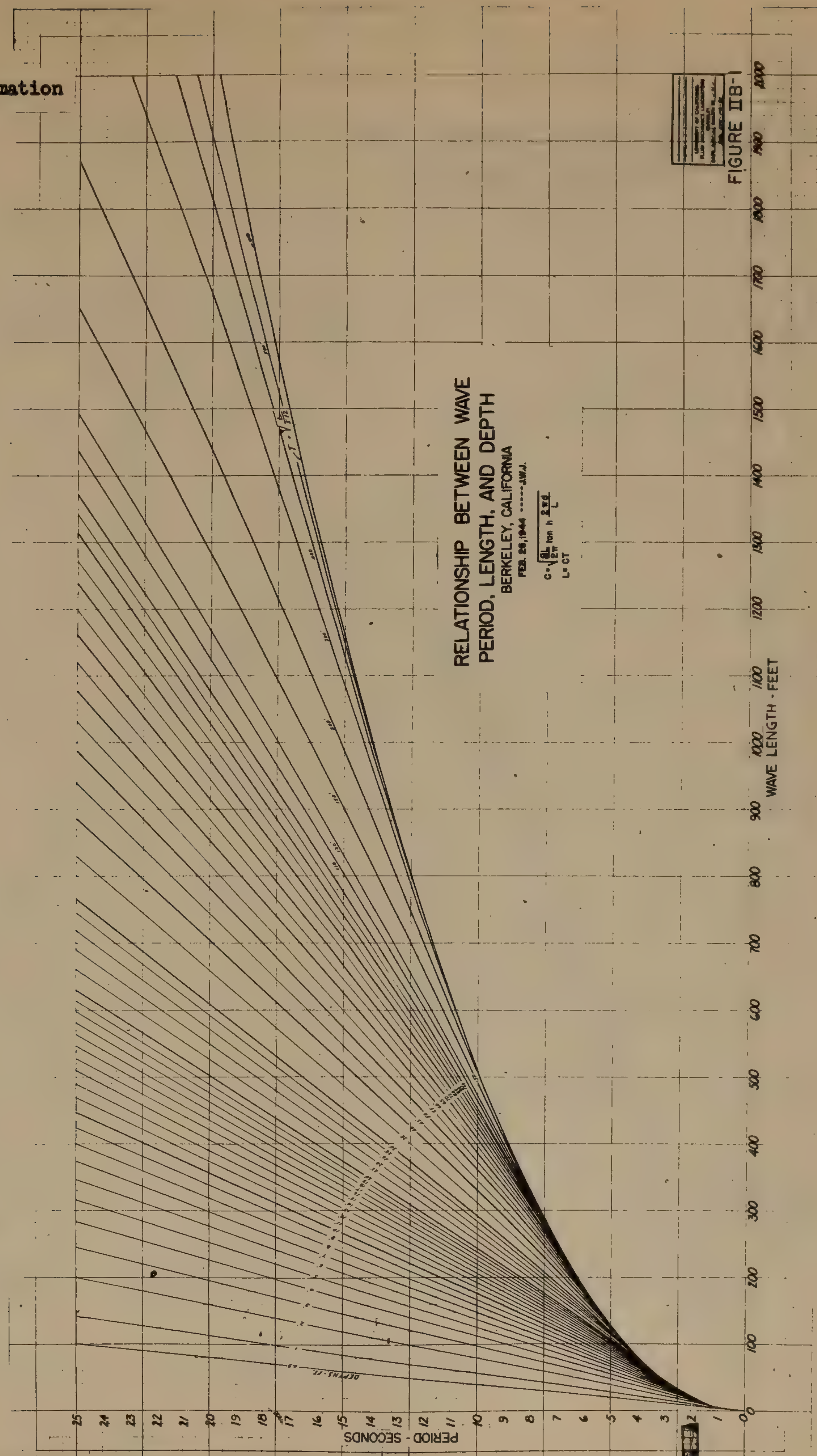


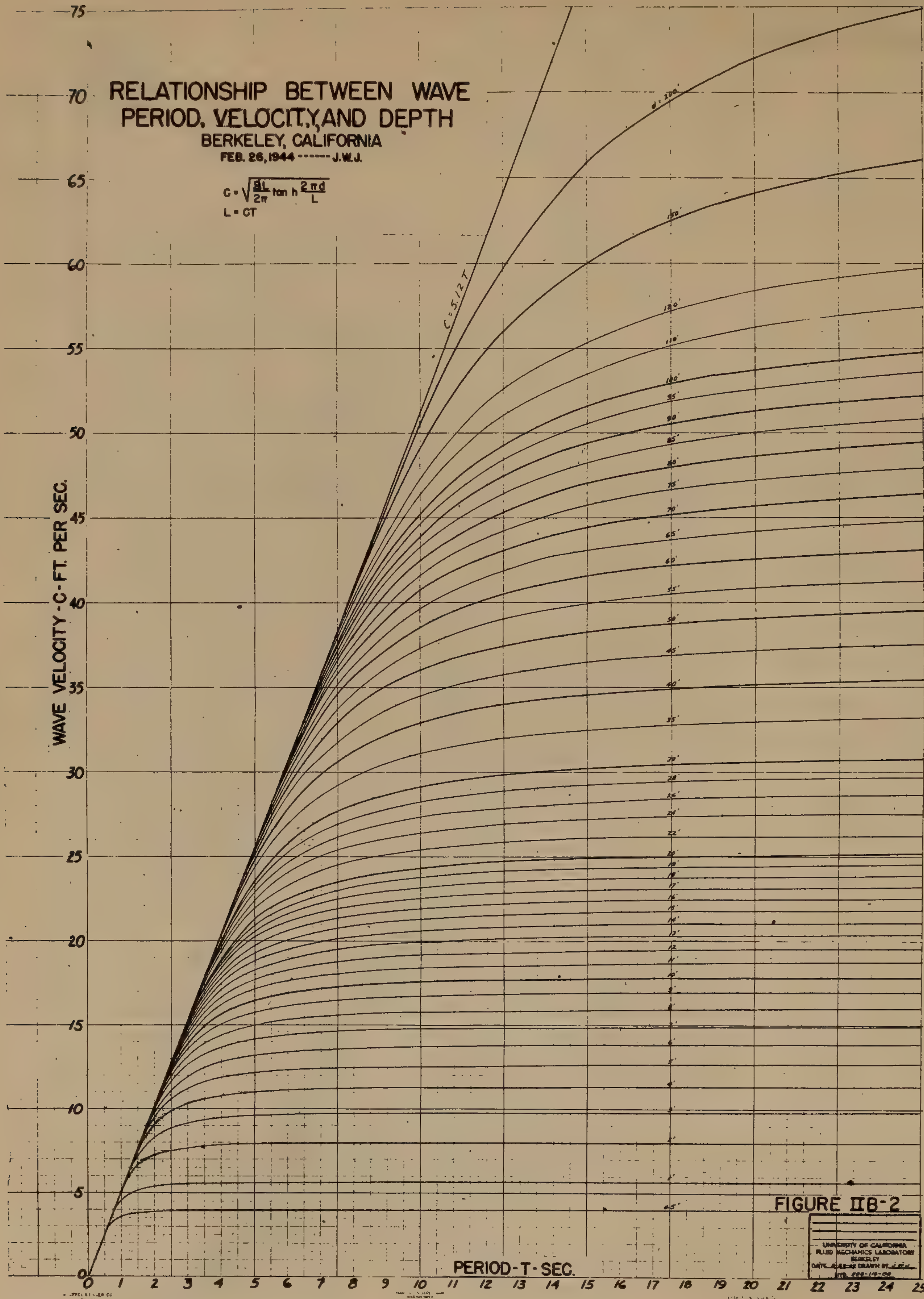

RELATIONSHIP BETWEEN WAVE
PERIOD, VELOCITY, AND DEPTH
BERKELEY, CALIFORNIA
FEB. 26, 1944 ------ J.W.J.
$C = \sqrt{\frac{gL}{2\pi}\tanh\frac{2\pi d}{L}}$
L = CT
C = 5.12 T
WAVE VELOCITY - C - FT. PER SEC.
PERIOD - T - SEC.
UNIVERSITY OF CALIFORNIA
FLUID MECHANICS LABORATORY
BERKELEY

FIGURE IIB-2

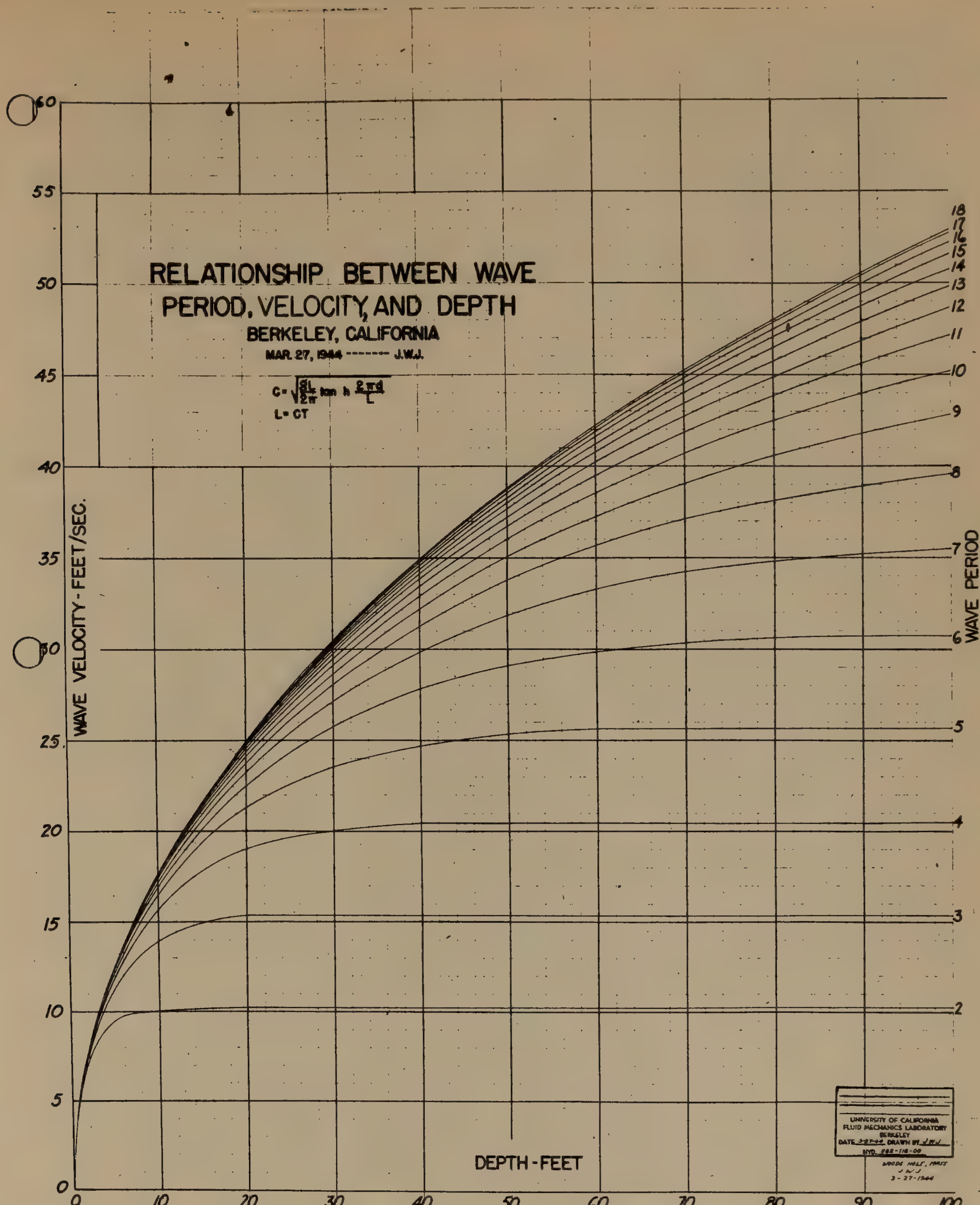

RELATIONSHIP BETWEEN WAVE
PERIOD, VELOCITY, AND DEPTH
BERKELEY, CALIFORNIA
MAR. 27, 1944 ------- J.W.J.
L= CT
WAVE VELOCITY - FEET/SEC.
WAVE PERIOD
DEPTH - FEET
UNIVERSITY OF CALIFORNIA
FLUID MECHANICS LABORATORY
BERKELEY

FIGURE IIB-3

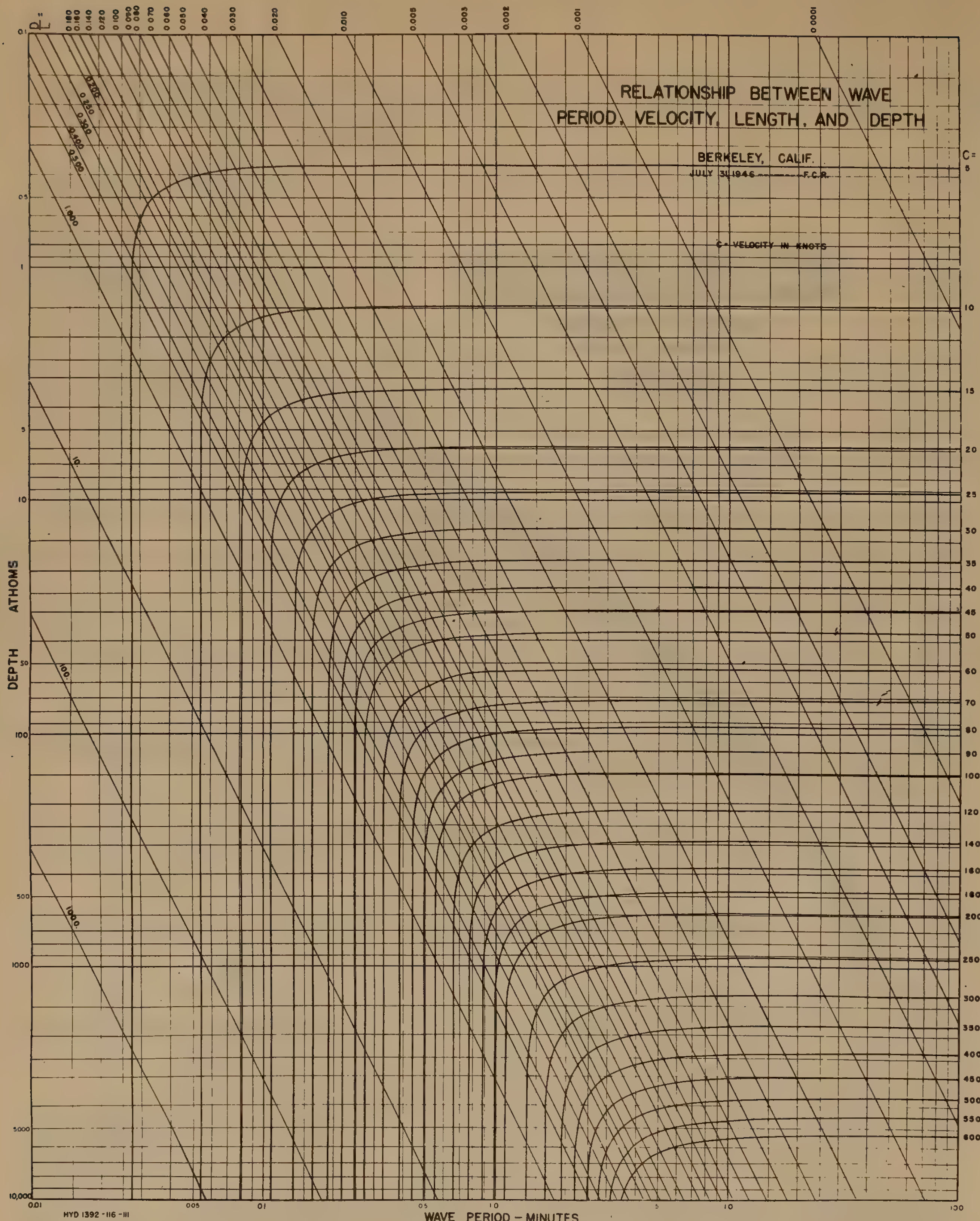

RELATIONSHIP BETWEEN WAVE PERIOD, VELOCITY, LENGTH, AND DEPTH
BERKELEY, CALIF.
JULY 31, 1946 F.C.R.
C = VELOCITY IN KNOTS
DEPTH FATHOMS
WAVE PERIOD - MINUTES
HYD 1392-116-111

FIGURE IIB-4

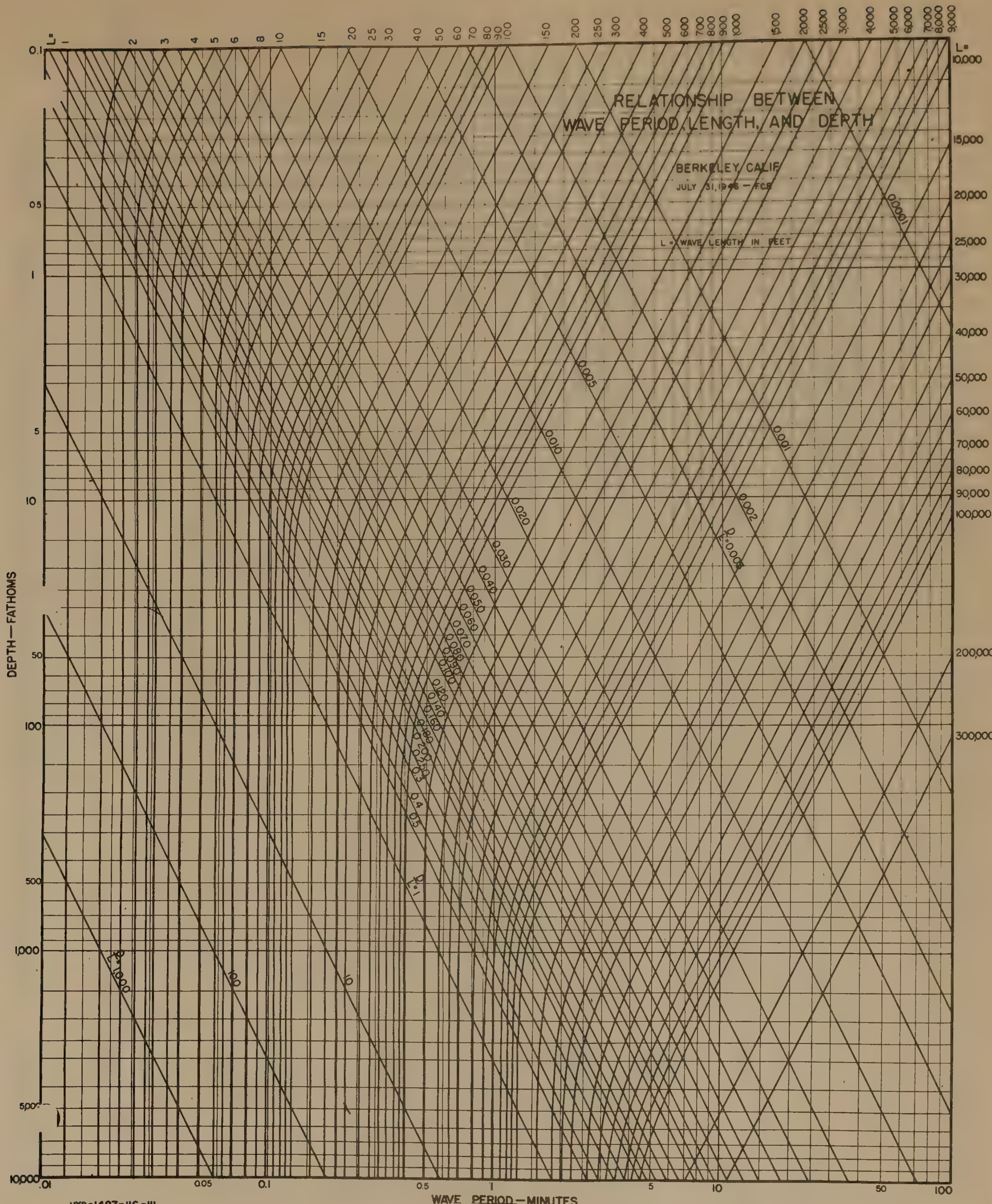
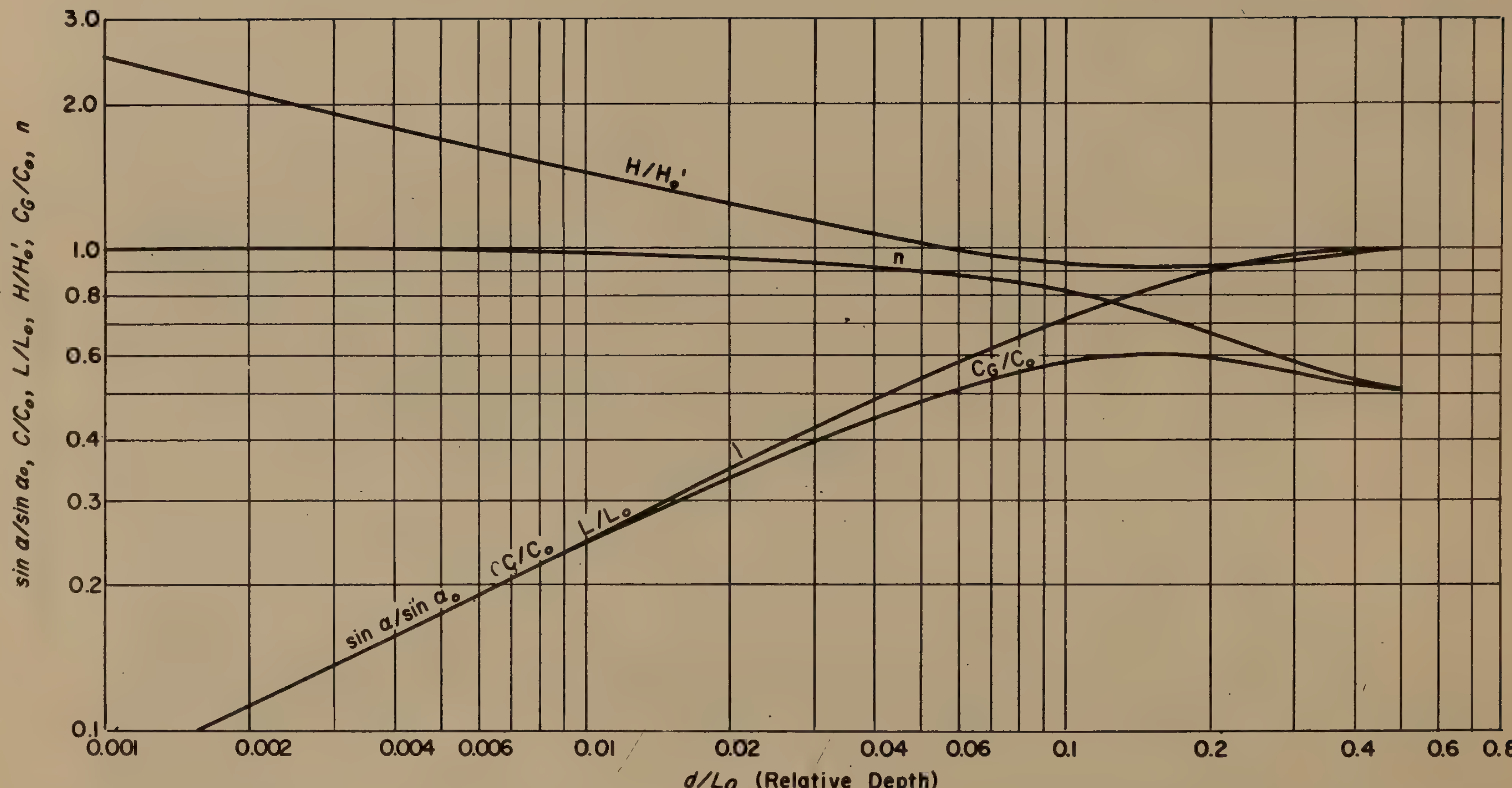

RELATIONSHIP BETWEEN
WAVE PERIOD, LENGTH, AND DEPTH
BERKELEY, CALIF.
JULY 31, 1946 — FCB
L = WAVE LENGTH IN FEET
DEPTH — FATHOMS
WAVE PERIOD — MINUTES
HYD-1487-116-111

FIGURE IIB-5



H = Wave height
L = Wave length
C = Wave velocity
α = Angle of wave crest with shoreline; for straight, parallel contours

d = Water depth
n = Fraction of wave energy advancing with wave velocity
$-'$ = Superscript refers to waves not affected by refraction
$-_o$ = Subscript refers to deep water conditions

FIG. IIB-6 - WAVE CHARACTERISTICS IN SHALLOW WATER RELATIVE TO DEEP WATER

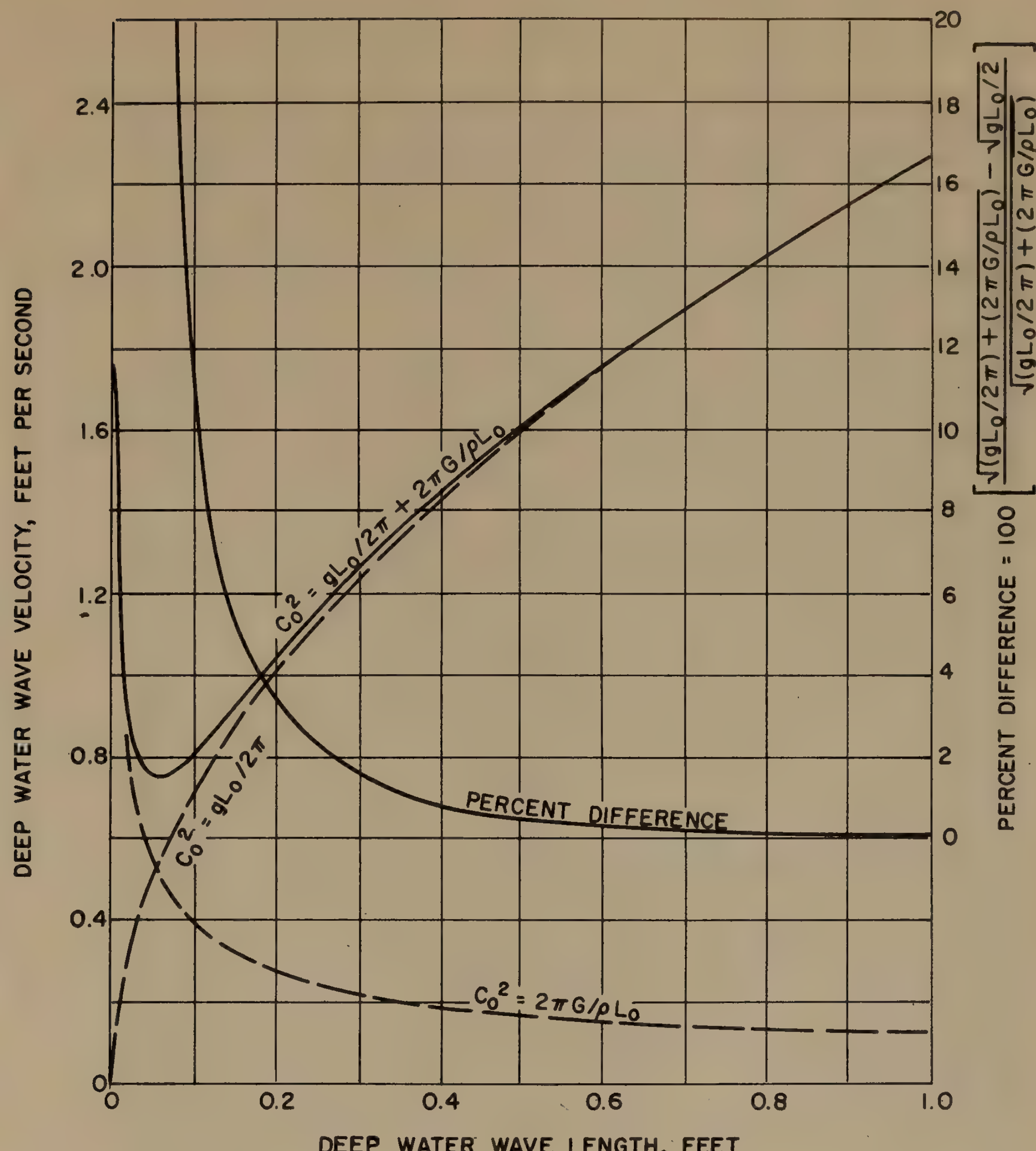


FIG. IIB-7 - EFFECT OF SURFACE TENSION ON DEEP WATER WAVE VELOCITY IN FRESH WATER AT 70°F

HYD. 3275-116-224

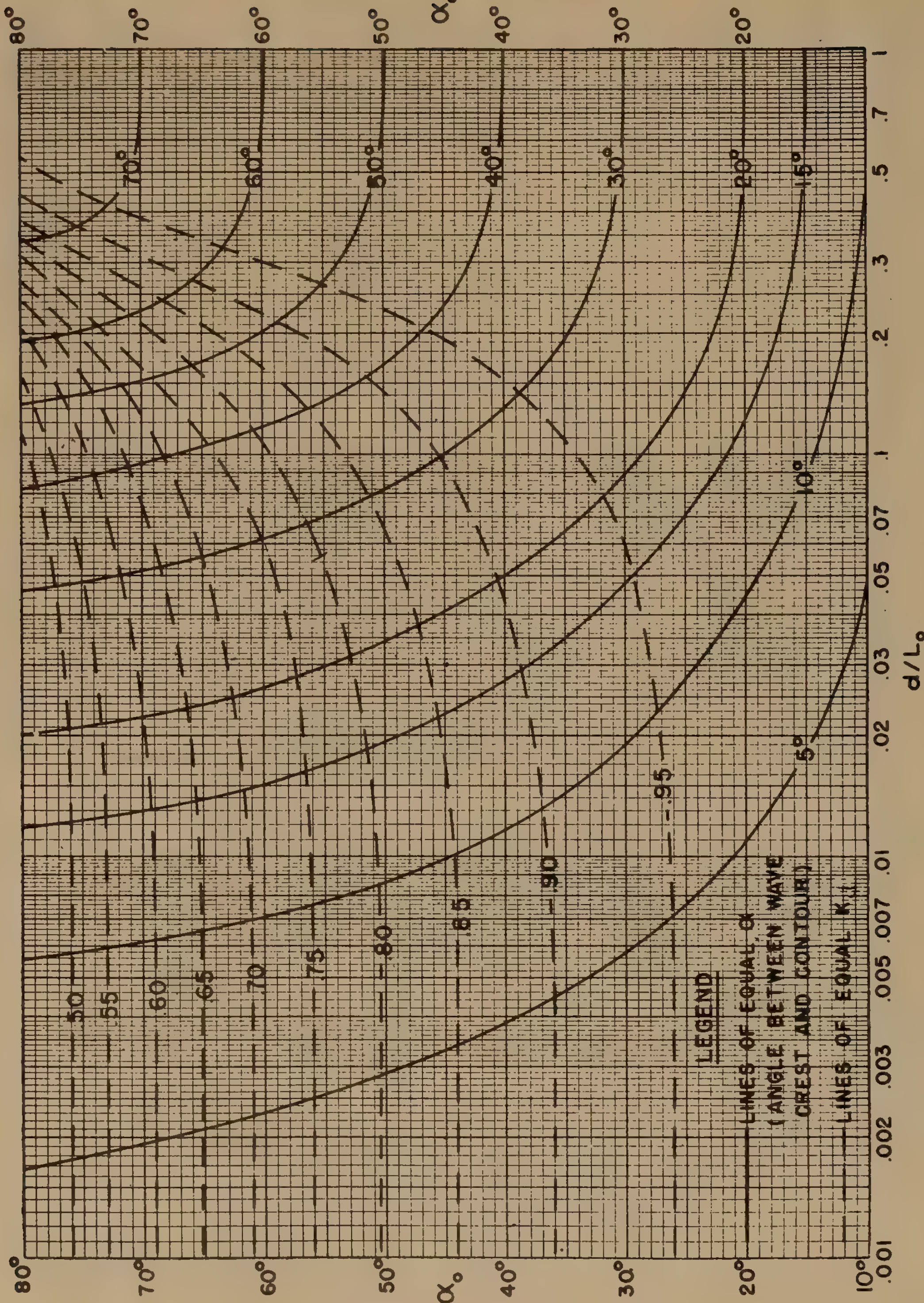


Fig. IIIB-8 - Change in Wave Direction and Height Due to Refraction on Beaches with Straight, Parallel Depth Contours

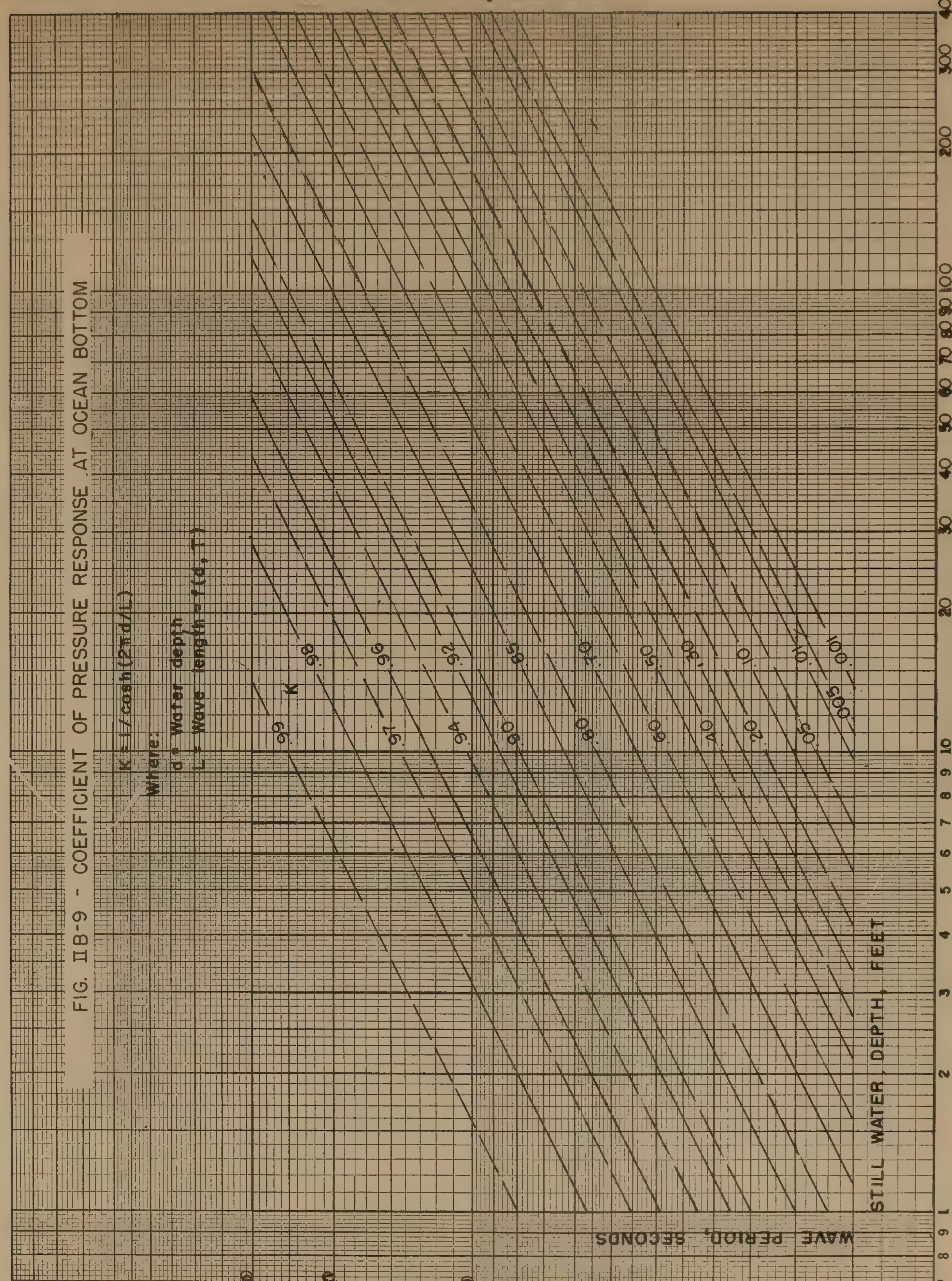


FIG. IIB-9 - COEFFICIENT OF PRESSURE RESPONSE AT OCEAN BOTTOM
K = 1/cosh(2πd/L)
Where:
d = Water depth
L = Wave length = f(d,T)
K
.99
.98
.97
.96
.94
.92
.90
.85
.80
.70
.60
.50
.40
.30
.20
.10
.05
.01
.005
.001
WAVE PERIOD, SECONDS
STILL WATER DEPTH, FEET

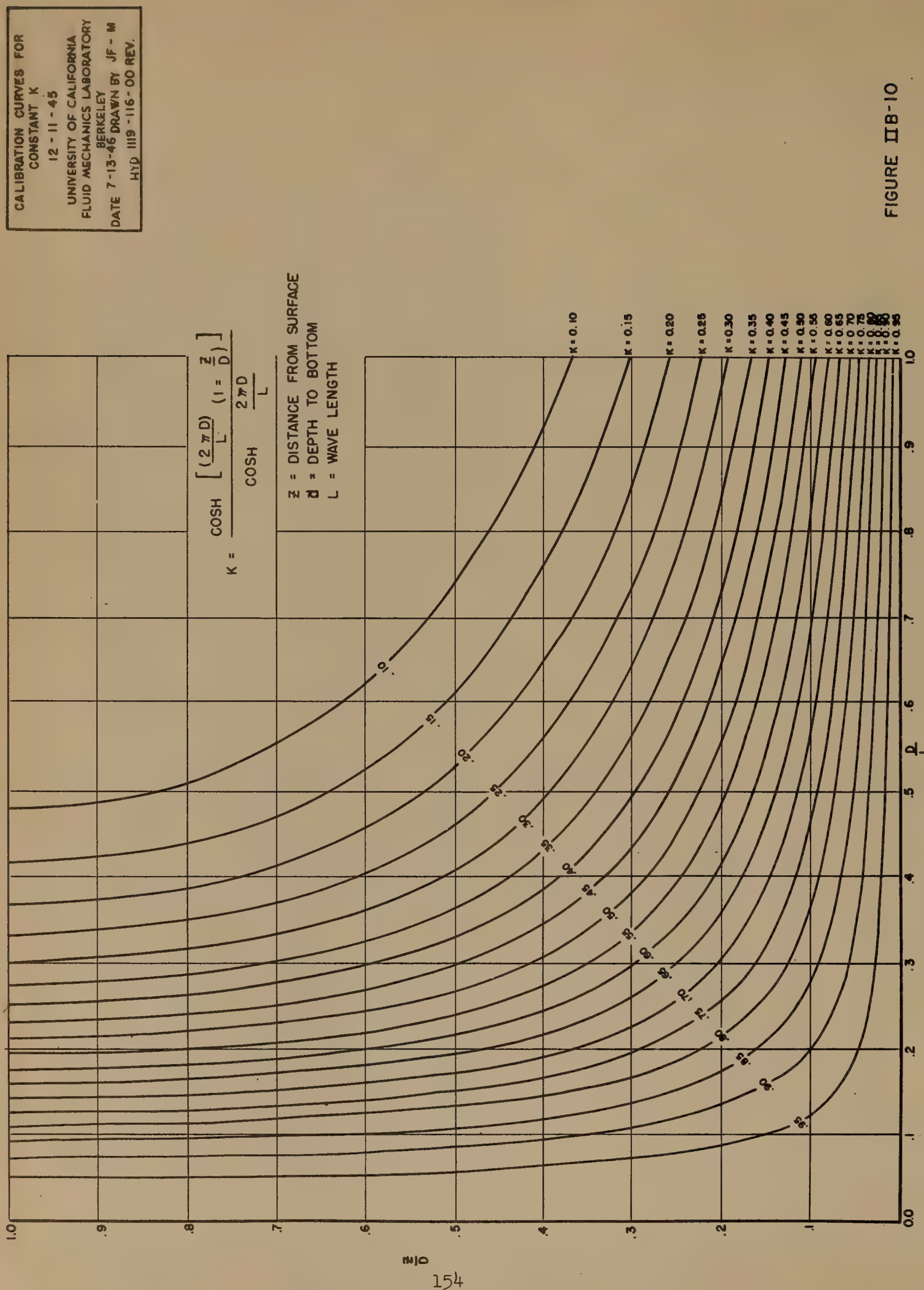


FIGURE IIB-10

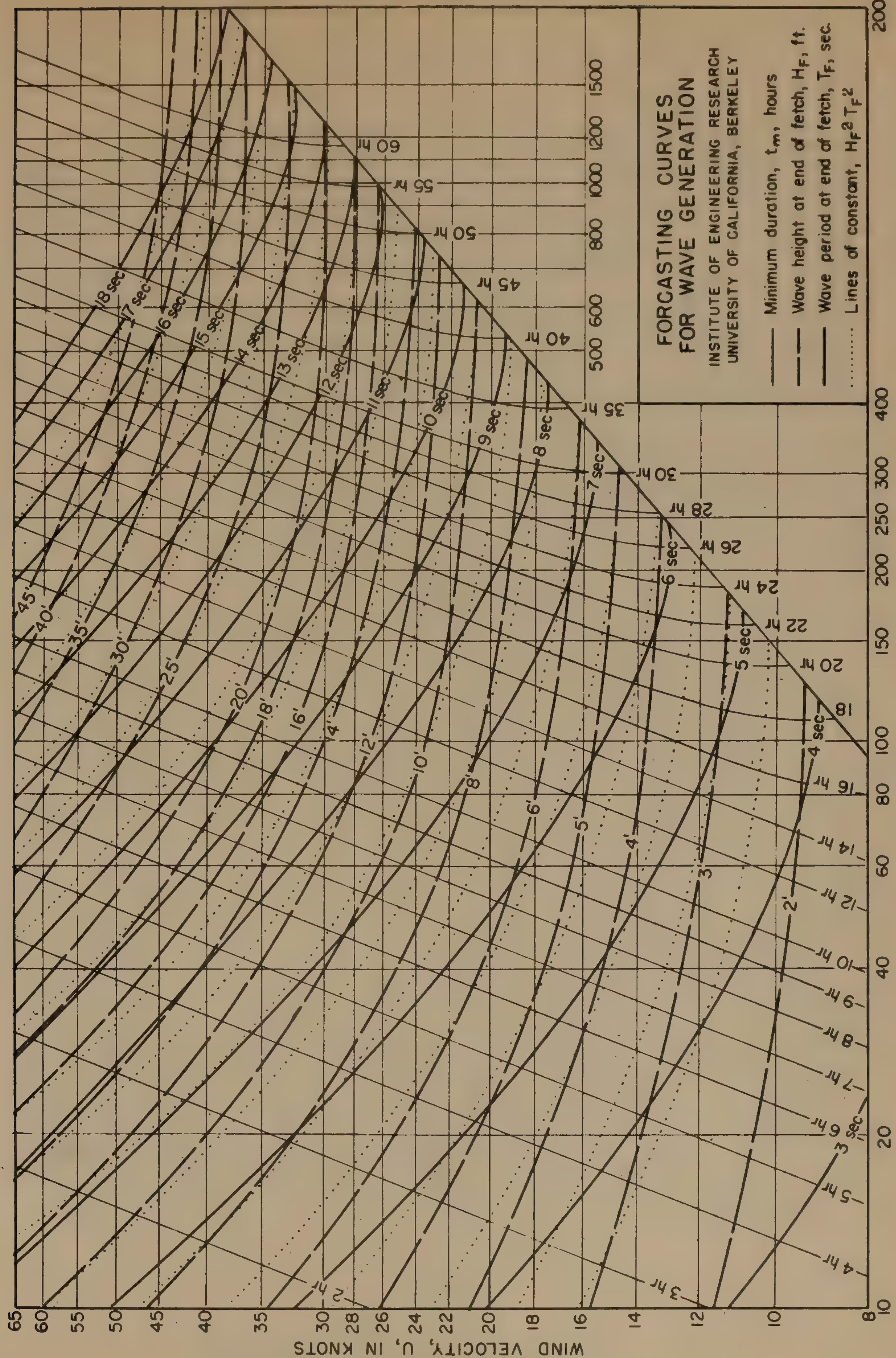


FIGURE II B-11

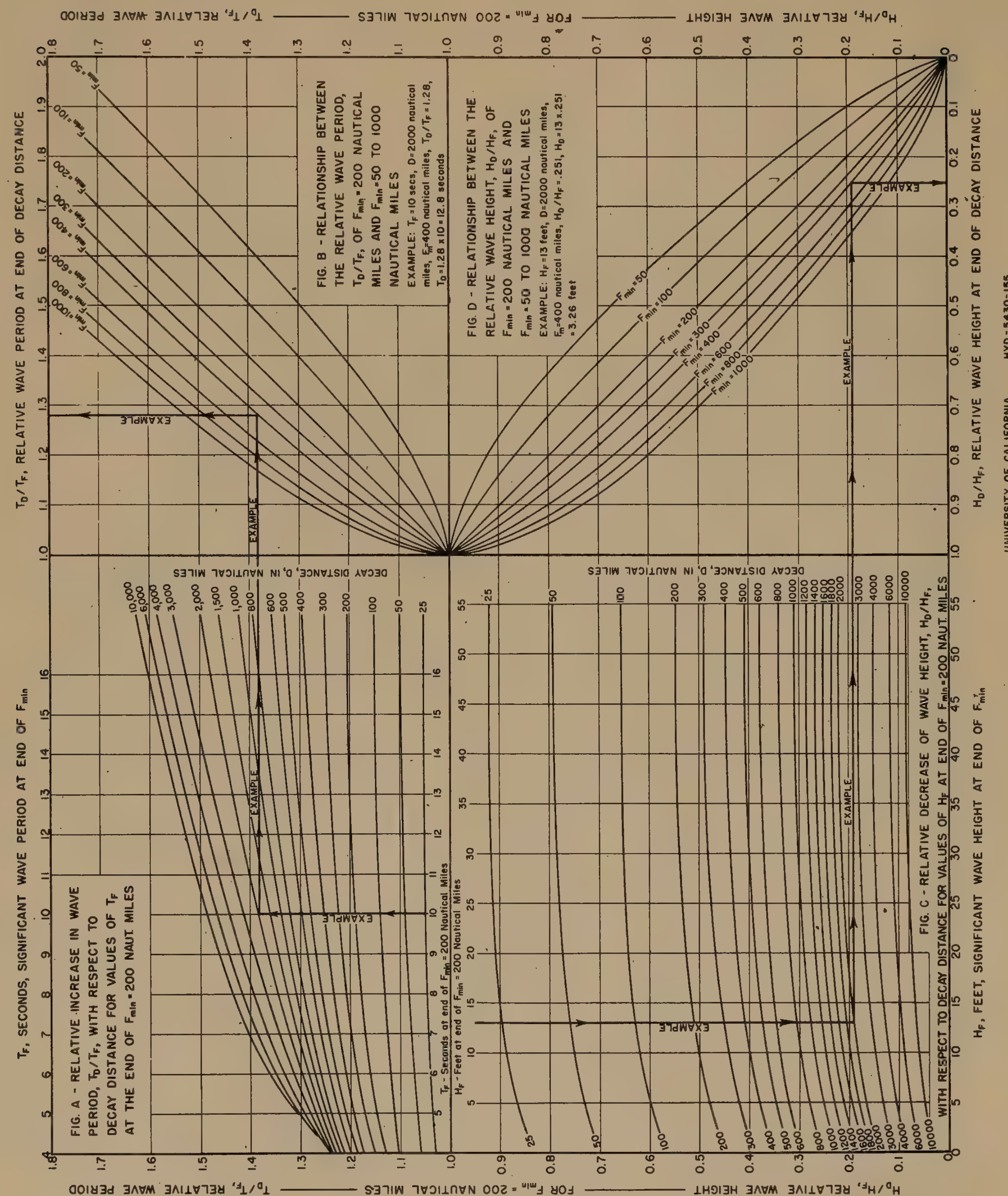
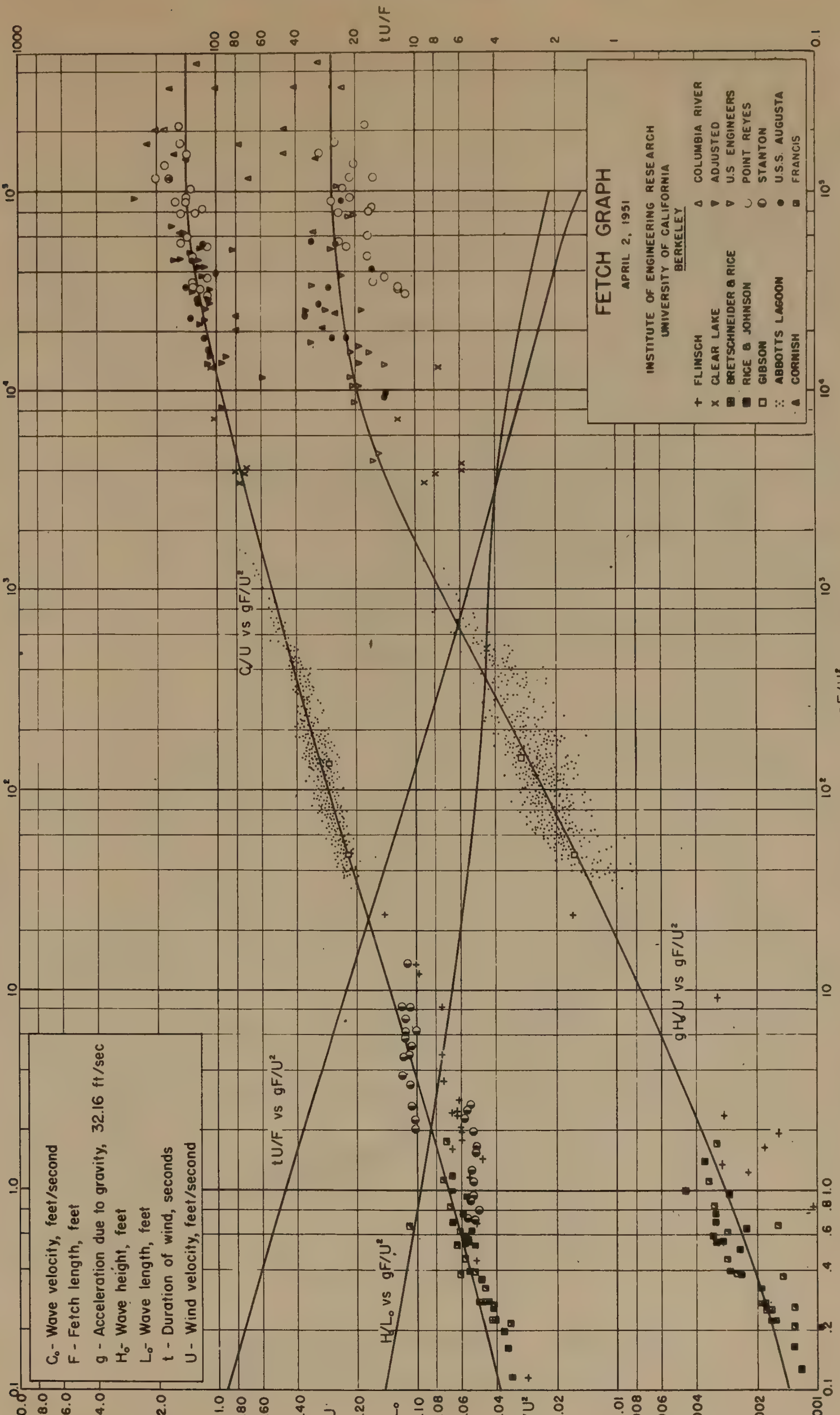


FIGURE IIB-12



HYD-5155-155

FIGURE II B-13

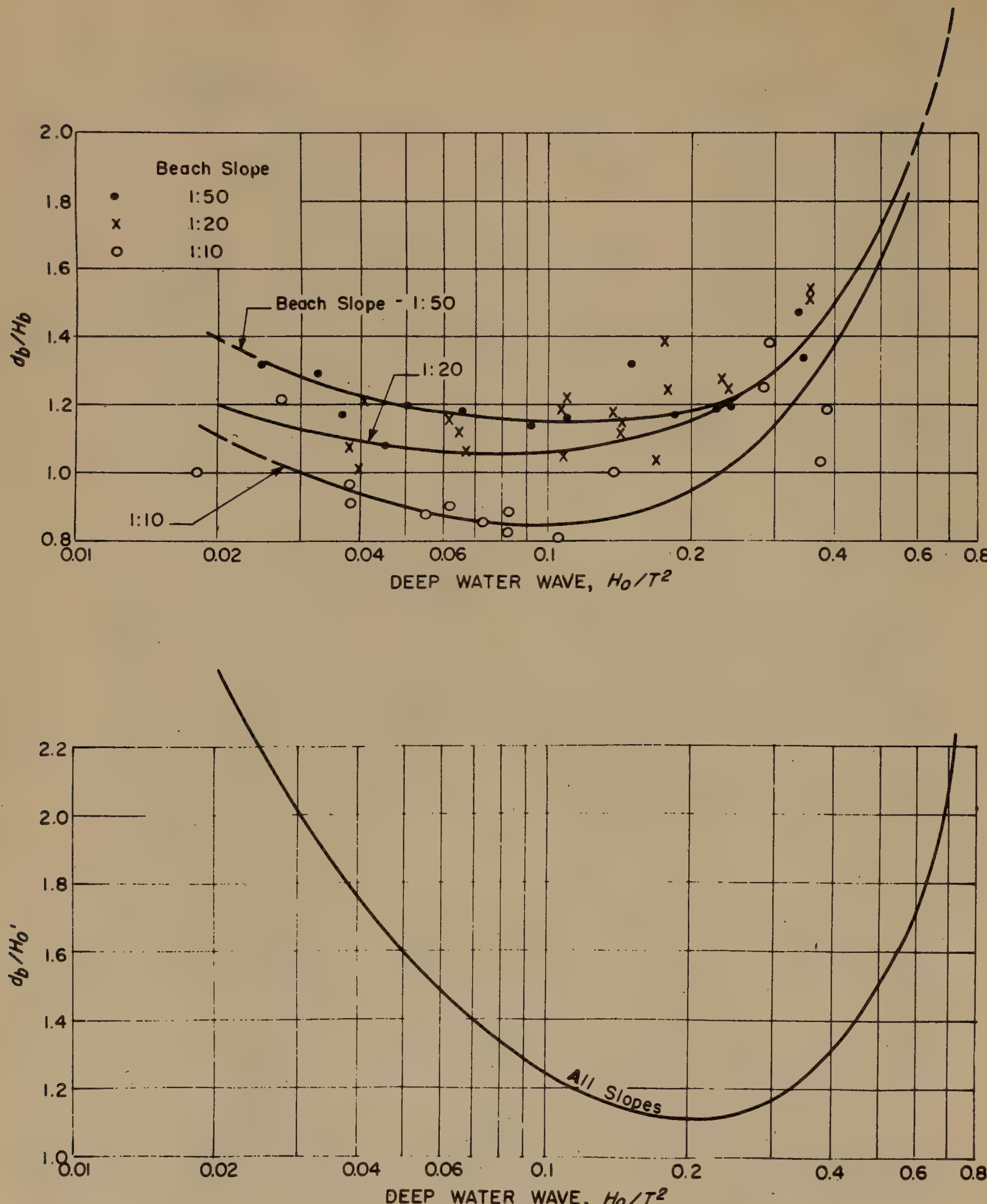


FIG. IIB-14 - BREAKER DEPTH INDEX

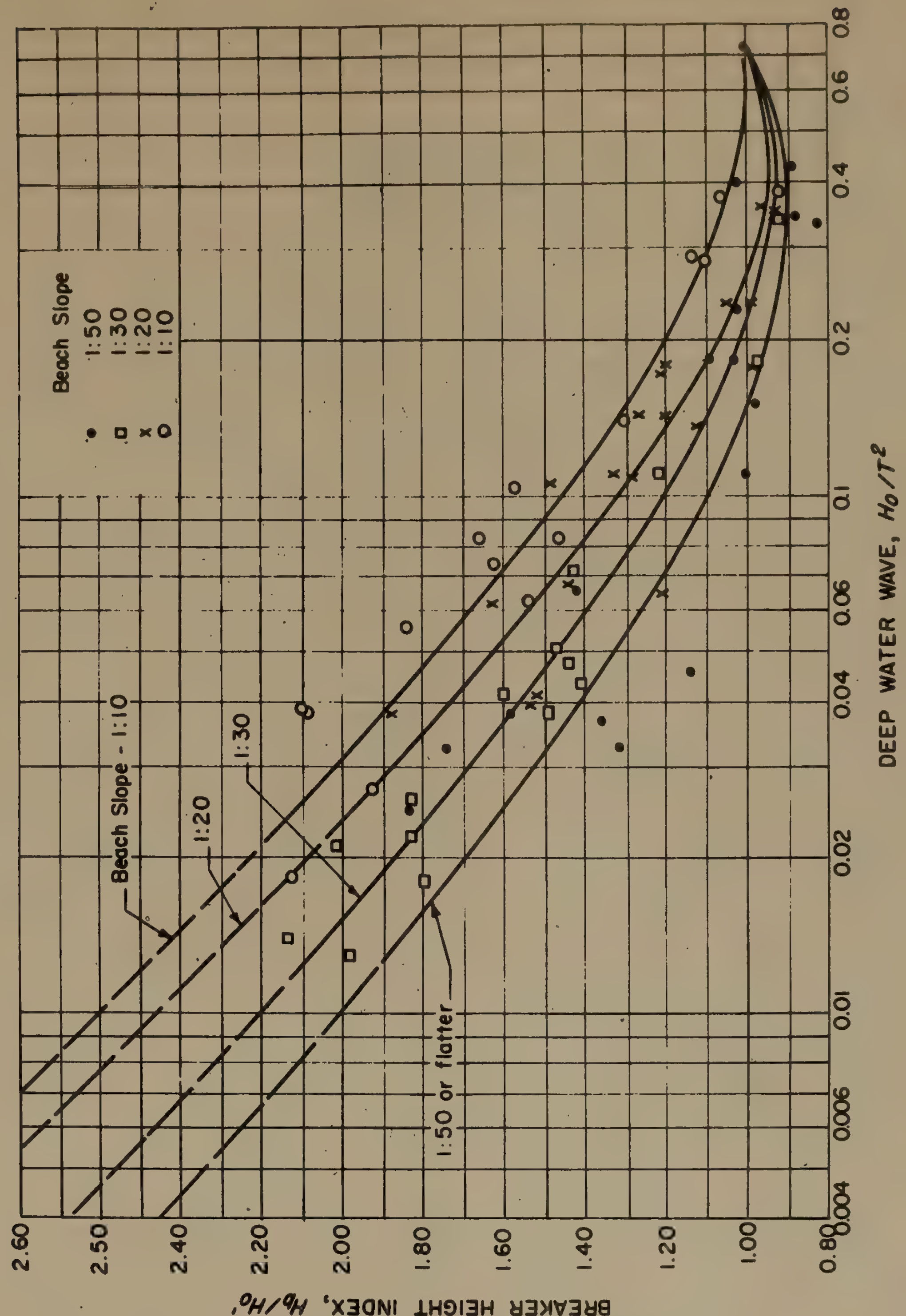


FIG. IIB-15 - BREAKER HEIGHT INDEX AS A FUNCTION OF DEEP WATER WAVE AND BEACH SLOPE

SECTION II. WAVE THEORY

C. TABLES OF FUNCTIONS
BY

R. L. WIEGEL

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C. TABLES OF FUNCTIONS

1. Introduction:

The purpose of this tabulation is to assemble, for easy accessibility, some of the functions that are used most frequently in investigations involving various surface wave phenomena.

Most of the relationships involve the independent variables d/L, where d is the depth of water and L is the wave length in that depth of water. As the most easily measured wave dimension is its period T, it is more convenient to use d/L_o as the independent relationship since $L_o = gT^2/2\pi$, where L_o is the "deep water" wave length. It was possible to develop the tables with d/L_o as the independent variable, in even increments, by successive approximations using the relationship

$$d/L \times \tanh 2\pi d/L = d/L_o \qquad \text{(II C-1.1)}$$

Values of d/L as a function of d/L_o have been tabulated (column 2).

2. Functions of d/L_o Tabulated in Table II C-1:

a. $2\pi d/L$, Sinh $2\pi d/L$ and Cosh $2\pi d/L$:

Values of $2\pi d/L$, sinh $2\pi d/L$ and cosh $2\pi d/L$ have been tabulated as functions of d/L_o (columns 3, 4 and 5).

b. L/L_o, C/C_o, and Tanh $2\pi d/L$:

The equation for wave velocity (of a small amplitude wave) is

$$C = \sqrt{(gL/2\pi) \tanh 2\pi d/L} \qquad \text{(II C-2.1)}$$

in any depth of water. In deep water $\tanh 2\pi d/L \rightarrow 1$ and hence

$$C_o = \sqrt{gL_o/2\pi} \qquad \text{(II C-2.2)}$$

So

$$(C/C_o) = \sqrt{\frac{(g/2\pi)L \tanh 2\pi d/L}{(g/2\pi)\, L_o}} = \sqrt{(L/L_o) \tanh 2\pi d/L}$$

as

$$L = C\,T$$

$$(C/C_o)(L/T)/(L_o/T) = (L/L_o) \tanh 2\pi d/L,$$

$$C/C_o = \tanh 2\pi d/L = L/L_o \qquad \text{(II C-2.3)}$$

Values of $\tanh 2\pi d/L$ (hence, C/C_o and L/L_o) have been tabulated as a function of d/L_o (column 3).

c. Pressure Response Factor:

In order to utilize underwater pressure instruments, it is necessary to know what height wave gives a particular pressure at some depth below the still water level. The simple relationship is given by

$$K = H_i/H = P/P_o = \frac{\cosh\left[2\pi d/L(1 - Z/d)\right]}{\cosh\,(2\pi d/L)} \qquad \text{(II C-2.4)}$$

where K is the sub-surface pressure response coefficient, H is the wave height, H_i is the equivalent "height" at the instrument at depth Z below the water surface. These values of K have not been tabulated.

For the case of an instrument placed on the bottom, this equation simplifies to

$$K = 1/\cosh 2\pi d/L \qquad \text{(II C-2.5)}$$

This is the column which has been tabulated (column 7).

d. $4\pi d/L$, Sinh $4\pi d/L$ and Cosh $4\pi d/L$:

Values of $4\pi d/L$, sinh $4\pi d/L$ and cosh $4\pi d/L$ have been tabulated as functions of d/L_o (columns 8, 9 and 10).

e. Fraction of Energy Advancing with the Wave:

According to the irrotational wave theory for waves of small amplitude only a fraction of the energy advances with the wave form (i.e., with the wave velocity C, rather than the group velocity C_g. The equation for this fraction, n, is

$$n = \tfrac{1}{2}\left[1 + (4\pi d/L)/\sinh 4\pi d/L\right] \qquad \text{(II C-2.6)}$$

Values of n have been tabulated as functions of d/L_o (column 11).

f. Ratio of Group Velocity to Deep Water Wave Velocity:

$$C_g/C_o = (C_g/C)(C/C_o) = n \tanh 2\pi d/L \qquad \text{(II C-2.7)}$$

Values of C_g/C_o have been tabulated as a function of d/L_o (column 12).

g. Ratio of Wave Height to Unrefracted Deep Water Wave Height:

A wave entering shoaling water changes in height because of the effect of shoaling and because of refraction. The effect due to shoaling alone for small amplitude waves is

$$D_d = H/H_o' = \sqrt{(1/2)(1/n)(C_o/C)} \qquad \text{(II C-2.8)}$$

where H_o' is the unrefracted deep water wave height. Values of H/H_o' have been tabulated as a function of d/L_o (column 13).

h. Energy Coefficient:

In the Gerstner theory of waves the energy coefficient M is an important factor. The equation for this is

$$M = \pi^2/2 \tanh^2 2\pi d/L \qquad \text{(II C-2.9)}$$

which is used in the equation

$$E = (wLH^2/8)(1 - MH^2/L^2)$$

Values of M have been tabulated as a function of d/L_o (column 14).

3. Table II C-2:

Values of C_o and L_o are tabulated for values of T (wave period) from 3.0 to 26.9 seconds. Values of C_o (the deep water wave velocity) are tabulated both in feet per second and knots and values of L_o (deep water wave length) are tabulated in feet.

4. Accuracy of Tables:

The values were computed using 5 place figures. The last figure was dropped and the tables prepared using 4 figures.

In practice it has been found that 3 figures are all that are usually needed to give the desired accuracy. However, when dealing with differences it is necessary to use all 4 figures.

d/L_o	d/L	$2\pi d/L$	TANH* $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
0	0	0	0	0	1	1	0	0	1	1	0		
.0001000	.003990	.02507	.02506	.02507	1.0003	.9997	.05014	.05016	1.001	.9998	.02506	4.467	7,855
.0002000	.005643	.03546	.03544	.03547	1.0006	.9994	.07091	.07097	1.003	.9996	.03543	3.757	3,928
.0003000	.006912	.04343	.04340	.04344	1.0009	.9991	.08686	.08697	1.004	.9994	.04339	3.395	2,620
.0004000	.007982	.05015	.05011	.05018	1.0013	.9987	.1003	.1005	1.005	.9992	.05007	3.160	1,965
.0005000	.008925	.05608	.05602	.05611	1.0016	.9984	.1122	.1124	1.006	.9990	.05596	2.989	1,572
.0006000	.009778	.06144	.06136	.06148	1.0019	.9981	.1229	.1232	1.008	.9988	.06128	2.856	1,311
.0007000	.01056	.06637	.06627	.06642	1.0022	.9978	.1327	.1331	1.009	.9985	.06617	2.749	1,124
.0008000	.01129	.07096	.07084	.07102	1.0025	.9975	.1419	.1424	1.010	.9983	.07072	2.659	983.5
.0009000	.01198	.07527	.07513	.07534	1.0028	.9972	.1505	.1511	1.011	.9981	.07499	2.582	874.3
.001000	.01263	.07935	.07918	.07943	1.0032	.9969	.1587	.1594	1.013	.9979	.07902	2.515	787.0
.001100	.01325	.08323	.08304	.08333	1.0035	.9966	.1665	.1672	1.014	.9977	.08285	2.456	715.6
.001200	.01384	.08694	.08672	.08705	1.0038	.9962	.1739	.1748	1.015	.9975	.08651	2.404	656.1
.001300	.01440	.09050	.09026	.09063	1.0041	.9959	.1810	.1820	1.016	.9973	.09001	2.357	605.8
.001400	.01495	.09393	.09365	.09407	1.0044	.9956	.1879	.1890	1.018	.9971	.09338	2.314	562.6
.001500	.01548	.09723	.09693	.09739	1.0047	.9953	.1945	.1957	1.019	.9969	.09663	2.275	525
.001600	.01598	.1004	.1001	.1006	1.0051	.9949	.2009	.2022	1.020	.9967	.09977	2.239	493
.001700	.01648	.1035	.1032	.1037	1.0054	.9946	.2071	.2086	1.022	.9965	.1028	2.205	463
.001800	.01696	.1066	.1062	.1068	1.0057	.9943	.2131	.2147	1.023	.9962	.1058	2.174	438
.001900	.01743	.1095	.1091	.1097	1.0060	.9940	.2190	.2207	1.024	.9960	.1087	2.145	415
.002000	.01788	.1123	.1119	.1125	1.0063	.9937	.2247	.2266	1.025	.9958	.1114	2.119	394
.002100	.01832	.1151	.1146	.1154	1.0066	.9934	.2303	.2323	1.027	.9956	.1141	2.094	376
.002200	.01876	.1178	.1173	.1181	1.0069	.9931	.2357	.2379	1.028	.9954	.1161	2.070	359
.002300	.01918	.1205	.1199	.1208	1.0073	.9928	.2410	.2433	1.029	.9952	.1193	2.047	343
.002400	.01959	.1231	.1225	.1234	1.0076	.9925	.2462	.2487	1.031	.9950	.1219	2.025	329
.002500	.02000	.1257	.1250	.1260	1.0079	.9922	.2513	.2540	1.032	.9948	.1243	2.005	316
.002600	.02040	.1282	.1275	.1285	1.0082	.9919	.2563	.2592	1.033	.9946	.1268	1.986	304
.002700	.02079	.1306	.1299	.1310	1.0085	.9916	.2612	.2642	1.034	.9944	.1292	1.967	292
.002800	.02117	.1330	.1323	.1334	1.0089	.9912	.2661	.2692	1.036	.9942	.1315	1.950	282
.002900	.02155	.1354	.1346	.1358	1.0092	.9909	.2708	.2741	1.037	.9939	.1338	1.933	272

*Also: b_s/a_s, C/C_o, L/L_o

d/L_o	d/L	$2\pi d/L$	[illegible] $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.003000	.02192	.1377	.1369	.1382	1.0095	.9906	.2755	.2796	1.038	.9937	.1360	1.917	263
.003100	.02228	.1400	.1391	.1405	1.0098	.9903	.2800	.2837	1.040	.9935	.1382	1.902	255
.003200	.02264	.1423	.1413	.1427	1.0101	.9900	.2845	.2884	1.041	.9933	.1404	1.887	247
.003300	.02300	.1445	.1435	.1449	1.0104	.9897	.2890	.2930	1.042	.9931	.1425	1.873	240
.003400	.02335	.1467	.1456	.1472	1.0108	.9893	.2934	.2976	1.043	.9929	.1446	1.860	233
.003500	.02369	.1488	.1477	.1494	1.0111	.9890	.2977	.3021	1.045	.9927	.1466	1.847	226
.003600	.02403	.1510	.1498	.1515	1.0114	.9887	.3020	.3065	1.046	.9925	.1487	1.834	220
.003700	.02436	.1531	.1519	.1537	1.0117	.9884	.3061	.3109	1.047	.9923	.1507	1.822	214
.003800	.02469	.1551	.1539	.1558	1.0121	.9881	.3103	.3153	1.049	.9921	.1527	1.810	208
.003900	.02502	.1572	.1559	.1579	1.0124	.9878	.3144	.3196	1.050	.9919	.1546	1.799	203
.004000	.02534	.1592	.1579	.1599	1.0127	.9875	.3184	.3238	1.051	.9917	.1565	1.788	198
.004100	.02566	.1612	.1598	.1619	1.0130	.9872	.3224	.3280	1.052	.9915	.1584	1.777	193
.004200	.02597	.1632	.1617	.1639	1.0133	.9869	.3263	.3322	1.054	.9912	.1602	1.767	189
.004300	.02628	.1651	.1636	.1659	1.0137	.9865	.3302	.3362	1.055	.9910	.1621	1.756	184
.004400	.02659	.1671	.1655	.1678	1.0140	.9862	.3341	.3403	1.056	.9908	.1640	1.746	180
.004500	.02689	.1690	.1674	.1698	1.0143	.9859	.3380	.3444	1.058	.9906	.1658	1.737	176
.004600	.02719	.1708	.1692	.1717	1.0146	.9856	.3417	.3483	1.059	.9904	.1676	1.727	172
.004700	.02749	.1727	.1710	.1736	1.0149	.9853	.3454	.3523	1.060	.9902	.1693	1.718	169
.004800	.02778	.1745	.1728	.1754	1.0153	.9849	.3491	.3562	1.062	.9900	.1711	1.709	165
.004900	.02807	.1764	.1746	.1773	1.0156	.9846	.3527	.3601	1.063	.9898	.1728	1.701	162
.005000	.02836	.1782	.1764	.1791	1.0159	.9843	.3564	.3640	1.064	.9896	.1746	1.692	159
.005100	.02864	.1800	.1781	.1809	1.0162	.9840	.3599	.3678	1.066	.9894	.1762	1.684	156
.005200	.02893	.1818	.1798	.1827	1.0166	.9837	.3635	.3715	1.067	.9892	.1779	1.676	153
.005300	.02921	.1835	.1815	.1845	1.0169	.9834	.3670	.3753	1.068	.9889	.1795	1.669	150
.005400	.02948	.1852	.1832	.1863	1.0172	.9831	.3705	.3790	1.069	.9887	.1811	1.662	147
.005500	.02976	.1870	.1848	.1880	1.0175	.9828	.3739	.3827	1.071	.9885	.1827	1.654	145
.005600	.03003	.1887	.1865	.1898	1.0178	.9825	.3774	.3864	1.072	.9883	.1843	1.647	142
.005700	.03030	.1904	.1881	.1915	1.0182	.9822	.3808	.3900	1.073	.9881	.1859	1.640	140
.005800	.03057	.1921	.1897	.1932	1.0185	.9818	.3841	.3937	1.075	.9879	.1874	1.633	137
.005900	.03083	.1937	.1913	.1949	1.0188	.9815	.3875	.3972	1.076	.9877	.1890	1.626	135

d/L_o	d/L	$2\pi\,d/L$	TANH $2\pi\,d/L$	SINH $2\pi\,d/L$	COSH $2\pi\,d/L$	K	$4\pi\,d/L$	SINH $4\pi\,d/L$	COSH $4\pi\,d/L$	n	C_G/C_o	H/H_o'	M
.006000	.03110	.1954	.1929	.1967	1.0192	.9812	.3908	.4008	1.077	.9875	.1905	1.620	133
.006100	.03136	.1970	.1945	.1983	1.0195	.9809	.3941	.4044	1.079	.9873	.1920	1.614	130
.006200	.03162	.1987	.1961	.2000	1.0198	.9806	.3973	.4079	1.080	.9871	.1935	1.607	128
.006300	.03188	.2003	.1976	.2016	1.0201	.9803	.4006	.4114	1.081	.9869	.1950	1.601	126
.006400	.03213	.2019	.1992	.2033	1.0205	.9799	.4038	.4148	1.083	.9867	.1965	1.595	124
.006500	.03238	.2035	.2007	.2049	1.0208	.9796	.4070	.4183	1.084	.9865	.1980	1.589	123
.006600	.03264	.2051	.2022	.2065	1.0211	.9793	.4101	.4217	1.085	.9863	.1994	1.583	121
.006700	.03289	.2066	.2037	.2081	1.0214	.9790	.4133	.4251	1.087	.9860	.2009	1.578	119
.006800	.03313	.2082	.2052	.2097	1.0217	.9787	.4164	.4285	1.088	.9858	.2023	1.572	117
.006900	.03338	.2097	.2067	.2113	1.0221	.9784	.4195	.4319	1.089	.9856	.2037	1.567	116
.007000	.03362	.2113	.2082	.2128	1.0224	.9781	.4225	.4352	1.091	.9854	.2051	1.561	114
.007100	.03387	.2128	.2096	.2144	1.0227	.9778	.4256	.4386	1.092	.9852	.2065	1.556	112
.007200	.03411	.2143	.2111	.2160	1.0231	.9774	.4286	.4419	1.093	.9850	.2079	1.551	111
.007300	.03435	.2158	.2125	.2175	1.0234	.9771	.4316	.4452	1.095	.9848	.2093	1.546	109
.007400	.03459	.2173	.2139	.2190	1.0237	.9768	.4346	.4484	1.096	.9846	.2106	1.541	108
.007500	.03482	.2188	.2154	.2205	1.0240	.9765	.4376	.4517	1.097	.9844	.2120	1.536	106
.007600	.03506	.2203	.2168	.2221	1.0244	.9762	.4406	.4549	1.099	.9842	.2134	1.531	105
.007700	.03529	.2218	.2182	.2236	1.0247	.9759	.4435	.4582	1.100	.9840	.2147	1.526	104
.007800	.03552	.2232	.2196	.2251	1.0250	.9756	.4464	.4614	1.101	.9838	.2160	1.521	102
.007900	.03576	.2247	.2209	.2265	1.0253	.9753	.4493	.4646	1.103	.9836	.2173	1.517	101
.008000	.03598	.2261	.2223	.2280	1.0257	.9750	.4522	.4678	1.104	.9834	.2186	1.512	100
.008100	.03621	.2275	.2237	.2295	1.0260	.9747	.4551	.4709	1.105	.9832	.2199	1.508	98.6
.008200	.03644	.2290	.2250	.2310	1.0263	.9744	.4579	.4741	1.107	.9830	.2212	1.503	97.5
.008300	.03666	.2304	.2264	.2324	1.0266	.9741	.4607	.4772	1.108	.9827	.2225	1.499	96.3
.008400	.03689	.2318	.2277	.2338	1.0270	.9737	.4636	.4803	1.109	.9825	.2237	1.495	95.2
.008500	.03711	.2332	.2290	.2353	1.0273	.9734	.4664	.4834	1.111	.9823	.2250	1.491	94.1
.008600	.03733	.2346	.2303	.2367	1.0276	.9731	.4691	.4865	1.112	.9821	.2262	1.487	93.0
.008700	.03755	.2360	.2317	.2381	1.0280	.9728	.4719	.4896	1.113	.9819	.2275	1.482	91.9
.008800	.03777	.2373	.2330	.2396	1.0283	.9725	.4747	.4927	1.115	.9817	.2287	1.478	90.9
.008900	.03799	.2387	.2343	.2410	1.0286	.9722	.4774	.4957	1.116	.9815	.2300	1.474	89.9

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.009000	.03821	.2401	.2356	.2424	1.0290	.9718	.4801	.4988	1.118	.9813	.2312	1.471	88.9
.009100	.03842	.2414	.2368	.2438	1.0293	.9715	.4828	.5018	1.119	.9811	.2324	1.467	88.0
.009200	.03864	.2428	.2381	.2452	1.0296	.9712	.4855	.5049	1.120	.9809	.2336	1.463	87.1
.009300	.03885	.2441	.2394	.2465	1.0299	.9709	.4882	.5079	1.122	.9807	.2348	1.459	86.1
.009400	.03906	.2455	.2407	.2479	1.0303	.9706	.4909	.5109	1.123	.9805	.2360	1.456	85.2
.009500	.03928	.2468	.2419	.2493	1.0306	.9703	.4936	.5138	1.124	.9803	.2371	1.452	84.3
.009600	.03949	.2481	.2431	.2507	1.0309	.9700	.4962	.5168	1.126	.9801	.2383	1.448	83.5
.009700	.03970	.2494	.2443	.2520	1.0313	.9697	.4988	.5198	1.127	.9799	.2394	1.445	82.7
.009800	.03990	.2507	.2456	.2534	1.0316	.9694	.5014	.5227	1.128	.9797	.2406	1.442	81.8
.009900	.04011	.2520	.2468	.2547	1.0319	.9691	.5040	.5257	1.130	.9794	.2417	1.438	81.0
.01000	.04032	.2533	.2480	.2560	1.0322	.9688	.5066	.5286	1.131	.9792	.2429	1.435	80.2
.01100	.04233	.2660	.2598	.2691	1.0356	.9656	.5319	.5574	1.145	.9772	.2539	1.403	73.1
.01200	.04426	.2781	.2711	.2817	1.0389	.9625	.5562	.5853	1.159	.9751	.2643	1.375	67.1
.01300	.04612	.2898	.2820	.2938	1.0423	.9594	.5795	.6125	1.173	.9731	.2743	1.350	62.1
.01400	.04791	.3010	.2924	.3056	1.0456	.9564	.6020	.6391	1.187	.9710	.2838	1.327	57.8
.01500	.04964	.3119	.3022	.3170	1.0490	.9533	.6238	.6651	1.201	.9690	.2928	1.307	54.0
.01600	.05132	.3225	.3117	.3281	1.0524	.9502	.6450	.6906	1.215	.9670	.3014	1.288	50.8
.01700	.05296	.3328	.3209	.3389	1.0559	.9471	.6655	.7158	1.230	.9649	.3096	1.271	47.9
.01800	.05455	.3428	.3298	.3495	1.0593	.9440	.6856	.7405	1.244	.9629	.3176	1.255	45.3
.01900	.05611	.3525	.3386	.3599	1.0628	.9409	.7051	.7650	1.259	.9609	.3253	1.240	43.0
.02000	.05763	.3621	.3470	.3701	1.0663	.9378	.7242	.7891	1.274	.9588	.3327	1.226	41.0
.02100	.05912	.3714	.3552	.3800	1.0698	.9348	.7429	.8131	1.289	.9568	.3399	1.213	39.1
.02200	.06057	.3806	.3632	.3898	1.0733	.9317	.7612	.8368	1.304	.9548	.3468	1.201	37.4
.02300	.06200	.3896	.3710	.3995	1.0768	.9287	.7791	.8603	1.319	.9528	.3535	1.189	35.9
.02400	.06340	.3984	.3786	.4090	1.0804	.9256	.7967	.8837	1.335	.9508	.3600	1.178	34.4
.02500	.06478	.4070	.3860	.4184	1.0840	.9225	.8140	.9069	1.350	.9488	.3662	1.168	33.1
.02600	.06613	.4155	.3932	.4276	1.0876	.9195	.8310	.9310	1.366	.9468	.3722	1.159	31.9
.02700	.06747	.4239	.4002	.4367	1.0912	.9164	.8478	.9530	1.381	.9448	.3781	1.150	30.8
.02800	.06878	.4322	.4071	.4457	1.0949	.9133	.8643	.9760	1.397	.9428	.3838	1.141	29.8
.02900	.07007	.4403	.4138	.4546	1.0985	.9103	.8805	.9988	1.413	.9408	.3893	1.133	28.8

d/L_o	d/L	$2\pi\, d/L$	TANH $2\pi\, d/L$	SINH $2\pi\, d/L$	COSH $2\pi\, d/L$	K	$4\pi\, d/L$	SINH $4\pi\, d/L$	COSH $4\pi\, d/L$	n	C_G/C_o	H/H'_o	M
.03000	.07135	.4483	.4205	.4634	1.1021	.9073	.8966	1.022	1.430	.9388	.3947	1.125	27.9
.03100	.07260	.4562	.4269	.4721	1.1059	.9042	.9124	1.044	1.446	.9369	.4000	1.118	27.1
.03200	.07385	.4640	.4333	.4808	1.1096	.9012	.9280	1.067	1.462	.9349	.4051	1.111	26.3
.03300	.07507	.4717	.4395	.4894	1.1133	.8982	.9434	1.090	1.479	.9329	.4100	1.104	25.6
.03400	.07630	.4794	.4457	.4980	1.1171	.8952	.9588	1.113	1.496	.9309	.4149	1.098	24.8
.03500	.07743	.4868	.4517	.5064	1.1209	.8921	.9737	1.135	1.513	.9289	.4196	1.092	24.19
.03600	.07867	.4943	.4577	.5147	1.1247	.8891	.9886	1.158	1.530	.9270	.4242	1.086	23.56
.03700	.07984	.5017	.4635	.5230	1.1285	.8861	1.0033	1.180	1.547	.9250	.4287	1.080	22.97
.03800	.08100	.5090	.4691	.5312	1.1324	.8831	1.018	1.203	1.564	.9230	.4330	1.075	22.42
.03900	.08215	.5162	.4747	.5394	1.1362	.8801	1.032	1.226	1.582	.9211	.4372	1.069	21.90
.04000	.08329	.5233	.4802	.5475	1.1401	.8771	1.047	1.248	1.600	.9192	.4414	1.064	21.40
.04100	.08442	.5304	.4857	.5556	1.1440	.8741	1.061	1.271	1.617	.9172	.4455	1.059	20.92
.04200	.08553	.5374	.4911	.5637	1.1479	.8711	1.075	1.294	1.636	.9153	.4495	1.055	20.46
.04300	.08664	.5444	.4964	.5717	1.1518	.8688	1.089	1.317	1.654	.9133	.4534	1.050	20.03
.04400	.08774	.5513	.5015	.5796	1.1558	.8652	1.103	1.340	1.672	.9114	.4571	1.046	19.62
.04500	.08883	.5581	.5066	.5876	1.1599	.8621	1.116	1.363	1.691	.9095	.4607	1.042	19.23
.04600	.08991	.5649	.5116	.5954	1.1639	.8592	1.130	1.386	1.709	.9076	.4643	1.038	18.85
.04700	.09098	.5717	.5166	.6033	1.1679	.8562	1.143	1.409	1.728	.9057	.4679	1.034	18.49
.04800	.09205	.5784	.5215	.6111	1.1720	.8532	1.157	1.433	1.747	.9037	.4713	1.030	18.15
.04900	.09311	.5850	.5263	.6189	1.1760	.8503	1.170	1.456	1.766	.9018	.4746	1.026	17.82
.05000	.09416	.5916	.5310	.6257	1.1802	.8473	1.183	1.479	1.786	.8999	.4779	1.023	17.50
.05100	.09520	.5981	.5357	.6344	1.1843	.8444	1.196	1.503	1.805	.8980	.4811	1.019	17.19
.05200	.09623	.6046	.5403	.6421	1.1884	.8415	1.209	1.526	1.825	.8961	.4842	1.016	16.90
.05300	.09726	.6111	.5449	.6499	1.1926	.8385	1.222	1.550	1.845	.8943	.4873	1.013	16.62
.05400	.09829	.6176	.5494	.6575	1.1968	.8356	1.235	1.574	1.865	.8924	.4903	1.010	16.35
.05500	.09930	.6239	.5538	.6652	1.2011	.8326	1.248	1.598	1.885	.8905	.4932	1.007	16.09
.05600	.1003	.6303	.5582	.6729	1.2053	.8297	1.261	1.622	1.906	.8886	.4960	1.004	15.84
.05700	.1013	.6366	.5626	.6805	1.2096	.8267	1.273	1.646	1.926	.8867	.4988	1.001	15.60
.05800	.1023	.6428	.5668	.6880	1.2138	.8239	1.286	1.670	1.947	.8849	.5015	.9985	15.36
.05900	.1033	.6491	.5711	.6956	1.2181	.8209	1.298	1.695	1.968	.8830	.5042	.9958	15.13

d/L_o	d/L	$2\pi\, d/L$	TANH $2\pi\, d/L$	SINH $2\pi\, d/L$	COSH $2\pi\, d/L$	K	$4\pi\, d/L$	SINH $4\pi\, d/L$	COSH $4\pi\, d/L$	n	C_G/C_o	H/H'_o	M
.06000	.1043	.6553	.5753	.7033	1.2225	.8180	1.311	1.719	1.989	.8811	.5068	.9932	14.91
.06100	.1053	.6616	.5794	.7110	1.2270	.8150	1.3231	1.744	2.011	.8792	.5094	.9907	14.70
.06200	.1063	.6678	.5834	.7187	1.2315	.8121	1.336	1.770	2.033	.8773	.5119	.9883	14.50
.06300	.1073	.6739	.5874	.7256	1.2355	.8093	1.348	1.795	2.055	.8755	.5143	.9860	14.30
.06400	.1082	.6799	.5914	.7335	1.2402	.8063	1.360	1.819	2.076	.8737	.5167	.9837	14.11
.06500	.1092	.6860	.5954	.7411	1.2447	.8035	1.372	1.845	2.098	.8719	.5191	.9815	13.92
.06600	.1101	.6920	.5993	.7486	1.2492	.8005	1.384	1.870	2.121	.8700	.5214	.9793	13.74
.06700	.1111	.6981	.6031	.7561	1.2537	.7977	1.396	1.896	2.144	.8682	.5236	.9772	13.57
.06800	.1120	.7037	.6069	.7633	1.2580	.7948	1.408	1.921	2.166	.8664	.5258	.9752	13.40
.06900	.1130	.7099	.6106	.7711	1.2628	.7919	1.420	1.948	2.189	.8646	.5279	.9732	13.24
.07000	.1139	.7157	.6144	.7783	1.2672	.7890	1.432	1.974	2.213	.8627	.5300	.9713	13.08
.07100	.1149	.7219	.6181	.7863	1.2721	.7861	1.444	2.000	2.236	.8609	.5321	.9694	12.92
.07200	.1158	.7277	.6217	.7937	1.2767	.7833	1.455	2.026	2.260	.8591	.5341	.9676	12.77
.07300	.1168	.7336	.6252	.8011	1.2813	.7804	1.467	2.053	2.284	.8572	.5360	.9658	12.62
.07400	.1177	.7395	.6289	.8088	1.2861	.7775	1.479	2.080	2.308	.8554	.5380	.9641	12.48
.07500	.1186	.7453	.6324	.8162	1.2908	.7747	1.490	2.107	2.332	.8537	.5399	.9624	12.34
.07600	.1195	.7511	.6359	.8237	1.2956	.7719	1.502	2.135	2.357	.8519	.5417	.9607	12.21
.07700	.1205	.7569	.6392	.8312	1.3004	.7690	1.514	2.162	2.382	.8501	.5435	.9591	12.08
.07800	.1214	.7625	.6427	.8386	1.3051	.7662	1.525	2.189	2.407	.8483	.5452	.9576	11.95
.07900	.1223	.7683	.6460	.8462	1.3100	.7634	1.537	2.217	2.432	.8465	.5469	.9562	11.83
.08000	.1232	.7741	.6493	.8538	1.3149	.7605	1.548	2.245	2.458	.8448	.5485	.9548	11.71
.08100	.1241	.7799	.6526	.8614	1.3198	.7577	1.560	2.274	2.484	.8430	.5501	.9534	11.59
.08200	.1251	.7854	.6558	.8687	1.3246	.7549	1.571	2.303	2.511	.8413	.5517	.9520	11.47
.08300	.1259	.7911	.6590	.8762	1.3295	.7522	1.583	2.331	2.537	.8395	.5533	.9506	11.36
.08400	.1268	.7967	.6622	.8837	1.3345	.7494	1.594	2.360	2.563	.8378	.5548	.9493	11.25
.08500	.1277	.8026	.6655	.8915	1.3397	.7464	1.605	2.389	2.590	.8360	.5563	.9481	11.14
.08600	.1286	.8080	.6685	.8989	1.3446	.7437	1.616	2.418	2.617	.8342	.5577	.9469	11.04
.08700	.1295	.8137	.6716	.9064	1.3497	.7409	1.628	2.448	2.644	.8325	.5591	.9457	10.94
.08800	.1304	.8193	.6747	.9141	1.3548	.7381	1.639	2.478	2.672	.8308	.5605	.9445	10.84
.08900	.1313	.8250	.6778	.9218	1.3600	.7353	1.650	2.508	2.700	.8290	.5619	.9433	10.74

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.09000	.1322	.8306	.6808	.9295	1.3653	.7324	1.661	2.538	2.728	.8273	.5632	.9422	10.65
.09100	.1331	.8363	.6838	.9372	1.3706	.7296	1.672	2.568	2.756	.8255	.5645	.9411	10.55
.09200	.1340	.8420	.6868	.9450	1.3759	.7268	1.684	2.599	2.785	.8238	.5658	.9401	10.46
.09300	.1349	.8474	.6897	.9525	1.3810	.7241	1.695	2.630	2.814	.8221	.5670	.9391	10.37
.09400	.1357	.8528	.6925	.9600	1.3862	.7214	1.706	2.662	2.843	.8204	.5682	.9381	10.29
.09500	.1366	.8583	.6953	.9677	1.3917	.7186	1.717	2.693	2.873	.8187	.5693	.9371	10.21
.09600	.1375	.8639	.6982	.9755	1.3970	.7158	1.728	2.726	2.903	.8170	.5704	.9362	10.12
.09700	.1384	.8694	.7011	.9832	1.4023	.7131	1.739	2.757	2.933	.8153	.5716	.9353	10.04
.09800	.1392	.8749	.7039	.9908	1.4077	.7104	1.750	2.790	2.963	.8136	.5727	.9344	9.962
.09900	.1401	.8803	.7066	.9985	1.4131	.7076	1.761	2.822	2.994	.8120	.5737	.9335	9.884
.1000	.1410	.8858	.7093	1.006	1.4187	.7049	1.772	2.855	3.025	.8103	.5747	.9327	9.808
.1010	.1419	.8913	.7120	1.014	1.4242	.7022	1.783	2.888	3.057	.8086	.5757	.9319	9.734
.1020	.1427	.8967	.7147	1.022	1.4297	.6994	1.793	2.922	3.088	.8069	.5766	.9311	9.661
.1030	.1436	.9023	.7173	1.030	1.4354	.6967	1.805	2.956	3.121	.8052	.5776	.9304	9.590
.1040	.1445	.9076	.7200	1.037	1.4410	.6940	1.815	2.990	3.153	.8036	.5785	.9297	9.519
.1050	.1453	.9130	.7226	1.045	1.4465	.6913	1.826	3.024	3.185	.8019	.5794	.9290	9.451
.1060	.1462	.9184	.7252	1.053	1.4523	.6886	1.837	3.059	3.218	.8003	.5803	.9282	9.384
.1070	.1470	.9239	.7277	1.061	1.4580	.6859	1.848	3.094	3.251	.7986	.5812	.9276	9.318
.1080	.1479	.9293	.7303	1.069	1.4638	.6833	1.858	3.128	3.284	.7970	.5820	.9269	9.254
.1090	.1488	.9343	.7327	1.076	1.4692	.6806	1.869	3.164	3.319	.7954	.5828	.9263	9.191
.1100	.1496	.9400	.7352	1.085	1.4752	.6779	1.880	3.201	3.353	.7937	.5836	.9257	9.129
.1110	.1505	.9456	.7377	1.093	1.4814	.6752	1.891	3.237	3.388	.7920	.5843	.9251	9.068
.1120	.1513	.9508	.7402	1.101	1.4871	.6725	1.902	3.274	3.423	.7904	.5850	.9245	9.009
.1130	.1522	.9563	.7426	1.109	1.4932	.6697	1.913	3.312	3.459	.7888	.5857	.9239	8.950
.1140	.1530	.9616	.7450	1.117	1.4990	.6671	1.923	3.348	3.494	.7872	.5864	.9234	8.891
.1150	.1539	.9670	.7474	1.125	1.5051	.6645	1.934	3.385	3.530	.7856	.5871	.9228	8.835
.1160	.1547	.9720	.7497	1.133	1.5108	.6619	1.944	3.423	3.566	.7840	.5878	.9223	8.780
.1170	.1556	.9775	.7520	1.141	1.5171	.6592	1.955	3.462	3.603	.7824	.5884	.9218	8.726
.1180	.1564	.9827	.7543	1.149	1.5230	.6566	1.966	3.501	3.641	.7808	.5890	.9214	8.673
.1190	.1573	.9882	.7566	1.157	1.5293	.6539	1.977	3.540	3.678	.7792	.5896	.9209	8.621

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.1200	.1581	.9936	.7589	1.165	1.5356	.6512	1.987	3.579	3.716	.7776	.5902	.9204	8.569
.1210	.1590	.9989	.7612	1.174	1.5418	.6486	1.998	3.620	3.755	.7760	.5907	.9200	8.518
.1220	.1598	1.004	.7634	1.182	1.5479	.6460	2.008	3.659	3.793	.7745	.5913	.9196	8.468
.1230	.1607	1.010	.7656	1.190	1.5546	.6433	2.019	3.699	3.832	.7729	.5918	.9192	8.419
.1240	.1615	1.015	.7678	1.198	1.5605	.6407	2.030	3.740	3.871	.7713	.5922	.9189	8.371
.1250	.1624	1.020	.7700	1.207	1.5674	.6381	2.041	3.782	3.912	.7698	.5926	.9186	8.324
.1260	.1632	1.025	.7721	1.215	1.5734	.6356	2.051	3.824	3.952	.7682	.5931	.9182	8.278
.1270	.1640	1.030	.7742	1.223	1.5795	.6331	2.061	3.865	3.992	.7667	.5936	.9178	8.233
.1280	.1649	1.036	.7763	1.231	1.5862	.6305	2.072	3.907	4.033	.7652	.5940	.9175	8.189
.1290	.1657	1.041	.7783	1.240	1.5927	.6279	2.082	3.950	4.074	.7637	.5944	.9172	8.146
.1300	.1665	1.046	.7804	1.248	1.5990	.6254	2.093	3.992	4.115	.7621	.5948	.9169	8.103
.1310	.1674	1.052	.7824	1.257	1.6060	.6228	2.104	4.036	4.158	.7606	.5951	.9166	8.061
.1320	.1682	1.057	.7844	1.265	1.6124	.6202	2.114	4.080	4.201	.7591	.5954	.9164	8.020
.1330	.1691	1.062	.7865	1.273	1.6191	.6176	2.125	4.125	4.245	.7575	.5958	.9161	7.978
.1340	.1699	1.068	.7885	1.282	1.6260	.6150	2.135	4.169	4.288	.7560	.5961	.9158	7.937
.1350	.1708	1.073	.7905	1.291	1.633	.6123	2.146	4.217	4.334	.7545	.5964	.9156	7.897
.1360	.1716	1.078	.7925	1.300	1.640	.6098	2.156	4.262	4.378	.7530	.5967	.9154	7.857
.1370	.1724	1.084	.7945	1.308	1.647	.6073	2.167	4.309	4.423	.7515	.5969	.9152	7.819
.1380	.1733	1.089	.7964	1.317	1.654	.6047	2.177	4.355	4.468	.7500	.5972	.9150	7.781
.1390	.1741	1.094	.7983	1.326	1.660	.6022	2.188	4.402	4.514	.7485	.5975	.9148	7.744
.1400	.1749	1.099	.8002	1.334	1.667	.5998	2.198	4.450	4.561	.7471	.5978	.9146	7.707
.1410	.1758	1.105	.8021	1.343	1.675	.5972	2.209	4.498	4.607	.7456	.5980	.9144	7.671
.1420	.1766	1.110	.8039	1.352	1.681	.5947	2.219	4.546	4.654	.7441	.5982	.9142	7.636
.1430	.1774	1.115	.8057	1.360	1.688	.5923	2.230	4.595	4.663	.7426	.5984	.9141	7.602
.1440	.1783	1.120	.8076	1.369	1.696	.5898	2.240	4.644	4.751	.7412	.5986	.9140	7.567
.1450	.1791	1.125	.8094	1.378	1.703	.5873	2.251	4.695	4.800	.7397	.5987	.9139	7.533
.1460	.1800	1.131	.8112	1.388	1.710	.5847	2.261	4.746	4.850	.7382	.5989	.9137	7.499
.1470	.1808	1.136	.8131	1.397	1.718	.5822	2.272	4.798	4.901	.7368	.5990	.9136	7.465
.1480	.1816	1.141	.8149	1.405	1.725	.5798	2.282	4.847	4.951	.7354	.5992	.9135	7.432
.1490	.1825	1.146	.8166	1.415	1.732	.5773	2.293	4.901	5.001	.7339	.5993	.9134	7.400

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.1500	.1833	1.152	.8183	1.424	1.740	.5748	2.303	4.954	5.054	.7325	.5994	.9133	7.369
.1510	.1841	1.157	.8200	1.433	1.747	.5723	2.314	5.007	5.106	.7311	.5994	.9133	7.339
.1520	.1850	1.162	.8217	1.442	1.755	.5699	2.324	5.061	5.159	.7296	.5995	.9132	7.309
.1530	.1858	1.167	.8234	1.451	1.762	.5675	2.335	5.115	5.212	.7282	.5996	.9132	7.279
.1540	.1866	1.173	.8250	1.460	1.770	.5651	2.345	5.169	5.265	.7268	.5996	.9132	7.250
.1550	.1875	1.178	.8267	1.469	1.777	.5627	2.356	5.225	5.320	.7254	.5997	.9131	7.221
.1560	.1883	1.183	.8284	1.479	1.785	.5602	2.366	5.283	5.376	.7240	.5998	.9130	7.191
.1570	.1891	1.188	.8301	1.488	1.793	.5577	2.377	5.339	5.432	.7226	.5999	.9129	7.162
.1580	.1900	1.194	.8317	1.498	1.801	.5552	2.387	5.398	5.490	.7212	.5996	.9130	7.134
.1590	.1908	1.199	.8333	1.507	1.809	.5528	2.398	5.454	5.544	.7198	.5998	.9130	7.107
.1600	.1917	1.204	.8349	1.517	1.817	.5504	2.408	5.513	5.603	.7184	.5998	.9130	7.079
.1610	.1925	1.209	.8365	1.527	1.825	.5480	2.419	5.571	5.660	.7171	.5998	.9130	7.052
.1620	.1933	1.215	.8381	1.536	1.833	.5456	2.429	5.630	5.718	.7157	.5998	.9130	7.026
.1630	.1941	1.220	.8396	1.546	1.841	.5432	2.440	5.690	5.777	.7144	.5998	.9130	7.000
.1640	.1950	1.225	.8411	1.555	1.849	.5409	2.450	5.751	5.837	.7130	.5998	.9130	6.975
.1650	.1958	1.230	.8427	1.565	1.857	.5385	2.461	5.813	5.898	.7117	.5997	.9131	6.949
.1660	.1966	1.235	.8442	1.574	1.865	.5362	2.471	5.874	5.959	.7103	.5996	.9132	6.924
.1670	.1975	1.240	.8457	1.584	1.873	.5339	2.482	5.938	6.021	.7090	.5996	.9132	6.900
.1680	.1983	1.246	.8472	1.594	1.882	.5315	2.492	6.003	6.085	.7076	.5995	.9133	6.876
.1690	.1992	1.251	.8486	1.604	1.890	.5291	2.503	6.066	6.148	.7063	.5994	.9133	6.853
.1700	.2000	1.257	.8501	1.614	1.899	.5267	2.513	6.130	6.212	.7050	.5993	.9134	6.830
.1710	.2008	1.262	.8515	1.624	1.907	.5243	2.523	6.197	6.275	.7036	.5992	.9135	6.807
.1720	.2017	1.267	.8529	1.634	1.915	.5220	2.534	6.262	6.342	.7023	.5991	.9136	6.784
.1730	.2025	1.272	.8544	1.644	1.924	.5197	2.544	6.329	6.407	.7010	.5989	.9137	6.761
.1740	.2033	1.277	.8558	1.654	1.933	.5174	2.555	6.395	6.473	.6997	.5988	.9138	6.738
.1750	.2042	1.282	.8572	1.664	1.941	.5151	2.565	6.465	6.541	.6984	.5987	.9139	6.716
.1760	.2050	1.288	.8586	1.675	1.951	.5127	2.576	6.534	6.610	.6971	.5985	.9140	6.694
.1770	.2058	1.293	.8600	1.685	1.959	.5104	2.586	6.603	6.679	.6958	.5984	.9141	6.672
.1780	.2066	1.298	.8614	1.695	1.968	.5081	2.597	6.672	6.747	.6946	.5982	.9142	6.651
.1790	.2075	1.304	.8627	1.706	1.977	.5058	2.607	6.744	6.818	.6933	.5980	.9144	6.631

d/L_o	d/L	$2\pi\ d/L$	TANH $2\pi\ d/L$	SINH $2\pi\ d/L$	COSH $2\pi\ d/L$	K	$4\pi\ d/L$	SINH $4\pi\ d/L$	COSH $4\pi\ d/L$	n	C_G/C_o	H/H_o'	M
.1800	.2083	1.309	.8640	1.716	1.986	.5036	2.618	6.818	6.891	.6920	.5979	.9145	6.611
.1810	.2092	1.314	.8653	1.727	1.995	.5013	2.629	6.890	6.963	.6907	.5977	.9146	6.591
.1820	.2100	1.320	.8666	1.737	2.004	.4990	2.639	6.963	7.035	.6895	.5975	.9148	6.571
.1830	.2108	1.325	.8680	1.748	2.013	.4967	2.650	7.038	7.109	.6882	.5974	.9149	6.550
.1840	.2117	1.330	.8693	1.758	2.022	.4945	2.660	7.113	7.183	.6870	.5972	.9150	6.530
.1850	.2125	1.335	.8706	1.769	2.032	.4922	2.671	7.191	7.260	.6857	.5969	.9152	6.511
.1860	.2134	1.341	.8718	1.780	2.041	.4899	2.681	7.267	7.336	.6845	.5967	.9154	6.492
.1870	.2142	1.346	.8731	1.791	2.051	.4876	2.692	7.345	7.412	.6832	.5965	.9155	6.474
.1880	.2150	1.351	.8743	1.801	2.060	.4854	2.702	7.421	7.488	.6820	.5963	.9157	6.456
.1890	.2159	1.356	.8755	1.812	2.070	.4832	2.712	7.500	7.566	.6808	.5961	.9159	6.438
.1900	.2167	1.362	.8767	1.823	2.079	.4809	2.723	7.581	7.647	.6796	.5958	.9161	6.421
.1910	.2176	1.367	.8779	1.834	2.089	.4787	2.734	7.663	7.728	.6784	.5955	.9163	6.403
.1920	.2184	1.372	.8791	1.845	2.099	.4765	2.744	7.746	7.810	.6772	.5952	.9165	6.385
.1930	.2192	1.377	.8803	1.856	2.108	.4743	2.755	7.827	7.891	.6760	.5950	.9167	6.368
.1940	.2201	1.383	.8815	1.867	2.118	.4721	2.765	7.911	7.974	.6748	.5948	.9169	6.351
.1950	.2209	1.388	.8827	1.879	2.128	.4699	2.776	7.996	8.059	.6736	.5946	.9170	6.334
.1960	.2218	1.393	.8839	1.890	2.138	.4677	2.787	8.083	8.145	.6724	.5944	.9172	6.317
.1970	.2226	1.399	.8850	1.901	2.148	.4655	2.797	8.167	8.228	.6712	.5941	.9174	6.300
.1980	.2234	1.404	.8862	1.913	2.158	.4633	2.808	8.256	8.316	.6700	.5938	.9176	6.284
.1990	.2243	1.409	.8873	1.924	2.169	.4611	2.819	8.346	8.406	.6689	.5935	.9179	6.268
.2000	.2251	1.414	.8884	1.935	2.178	.4590	2.829	8.436	8.495	.6677	.5932	.9181	6.253
.2010	.2260	1.420	.8895	1.947	2.189	.4569	2.840	8.524	8.583	.6666	.5929	.9183	6.237
.2020	.2268	1.425	.8906	1.959	2.199	.4547	2.850	8.616	8.674	.6654	.5926	.9186	6.222
.2030	.2277	1.430	.8917	1.970	2.210	.4526	2.861	8.708	8.766	.6642	.5923	.9188	6.206
.2040	.2285	1.436	.8928	1.982	2.220	.4504	2.872	8.803	8.860	.6631	.5920	.9190	6.191
.2050	.2293	1.441	.8939	1.994	2.231	.4483	2.882	8.897	8.953	.6620	.5917	.9193	6.176
.2060	.2302	1.446	.8950	2.006	2.242	.4462	2.893	8.994	9.050	.6608	.5914	.9195	6.161
.2070	.2310	1.451	.8960	2.017	2.252	.4441	2.903	9.090	9.144	.6597	.5911	.9197	6.147
.2080	.2319	1.457	.8971	2.030	2.263	.4419	2.914	9.187	9.240	.6586	.5908	.9200	6.133
.2090	.2328	1.462	.8981	2.042	2.274	.4398	2.925	9.288	9.342	.6574	.5905	.9202	6.119

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.2100	.2336	1.468	.8991	2.055	2.285	.4377	2.936	9.389	9.442	.6563	.5901	.9205	6.105
.2110	.2344	1.473	.9001	2.066	2.295	.4357	2.946	9.490	9.542	.6552	.5898	.9207	6.091
.2120	.2353	1.479	.9011	2.079	2.307	.4336	2.957	9.590	9.642	.6541	.5894	.9210	6.077
.2130	.2361	1.484	.9021	2.091	2.318	.4315	2.967	9.693	9.744	.6531	.5891	.9213	6.064
.2140	.2370	1.489	.9031	2.103	2.329	.4294	2.978	9.796	9.847	.6520	.5838	.9215	6.051
.2150	.2378	1.494	.9041	2.115	2.340	.4274	2.989	9.902	9.952	.6509	.5884	.9218	6.037
.2160	.2387	1.500	.9051	2.128	2.351	.4253	2.999	10.01	10.06	.6498	.5881	.9221	6.024
.2170	.2395	1.506	.9061	2.142	2.364	.4232	3.010	10.12	10.17	.6488	.5878	.9223	6.011
.2180	.2404	1.511	.9070	2.154	2.375	.4211	3.021	10.23	10.28	.6477	.5874	.9226	5.999
.2190	.2412	1.516	.9079	2.166	2.386	.4191	3.031	10.34	10.38	.6467	.5871	.9228	5.987
.2200	.2421	1.521	.9088	2.178	2.397	.4171	3.042	10.45	10.50	.6456	.5868	.9231	5.975
.2210	.2429	1.526	.9097	2.192	2.409	.4151	3.052	10.56	10.61	.6446	.5864	.9234	5.963
.2220	.2438	1.532	.9107	2.204	2.421	.4131	3.063	10.68	10.72	.6436	.5861	.9236	5.951
.2230	.2446	1.537	.9116	2.218	2.433	.4111	3.074	10.79	10.84	.6425	.5857	.9239	5.939
.2240	.2455	1.542	.9125	2.230	2.444	.4091	3.085	10.91	10.95	.6414	.5854	.9242	5.927
.2250	.2463	1.548	.9134	2.244	2.457	.4071	3.095	11.02	11.07	.6404	.5850	.9245	5.915
.2260	.2472	1.553	.9143	2.257	2.469	.4051	3.106	11.15	11.19	.6394	.5846	.9248	5.903
.2270	.2481	1.559	.9152	2.271	2.481	.4031	3.117	11.27	11.31	.6383	.5842	.9251	5.891
.2280	.2489	1.564	.9161	2.284	2.493	.4011	3.128	11.39	11.44	.6373	.5838	.9254	5.880
.2290	.2498	1.569	.9170	2.297	2.506	.3991	3.138	11.51	11.56	.6363	.5834	.9258	5.869
.2300	.2506	1.575	.9178	2.311	2.518	.3971	3.149	11.64	11.68	.6353	.5830	.9261	5.858
.2310	.2515	1.580	.9186	2.325	2.531	.3952	3.160	11.77	11.81	.6343	.5826	.9264	5.848
.2320	.2523	1.585	.9194	2.338	2.543	.3932	3.171	11.90	11.93	.6333	.5823	.9267	5.838
.2330	.2532	1.591	.9203	2.352	2.556	.3912	3.182	12.03	12.07	.6323	.5819	.9270	5.827
.2340	.2540	1.596	.9211	2.366	2.569	.3893	3.192	12.15	12.19	.6313	.5815	.9273	5.816
.2350	.2549	1.602	.9219	2.380	2.581	.3874	3.203	12.29	12.33	.6304	.5811	.9276	5.806
.2360	.2558	1.607	.9227	2.393	2.594	.3855	3.214	12.43	12.47	.6294	.5807	.9279	5.796
.2370	.2566	1.612	.9235	2.408	2.607	.3836	3.225	12.55	12.59	.6284	.5804	.9282	5.786
.2380	.2575	1.618	.9243	2.422	2.620	.3816	3.236	12.69	12.73	.6275	.5800	.9285	5.776
.2390	.2584	1.623	.9251	2.436	2.634	.3797	3.247	12.83	12.87	.6265	.5796	.9288	5.766

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.2400	.2592	1.629	.9259	2.450	2.647	.3779	3.257	12.97	13.01	.6256	.5792	.9291	5.756
.2410	.2601	1.634	.9267	2.464	2.660	.3760	3.268	13.11	13.15	.6246	.5788	.9294	5.746
.2420	.2610	1.640	.9275	2.480	2.674	.3741	3.279	13.26	13.30	.6237	.5784	.9298	5.736
.2430	.2618	1.645	.9282	2.494	2.687	.3722	3.290	13.40	13.44	.6228	.5780	.9301	5.727
.2440	.2627	1.650	.9289	2.508	2.700	.3704	3.301	13.55	13.59	.6218	.5776	.9304	5.718
.2450	.2635	1.656	.9296	2.523	2.714	.3685	3.312	13.70	13.73	.6209	.5272	.9307	5.710
.2460	.2644	1.661	.9304	2.538	2.728	.3666	3.323	13.85	13.88	.6200	.5768	.9310	5.701
.2470	.2653	1.667	.9311	2.553	2.742	.3648	3.334	14.00	14.04	.6191	.5764	.9314	5.692
.2480	.2661	1.672	.9318	2.568	2.755	.3629	3.344	14.15	14.19	.6182	.5760	.9317	5.684
.2490	.2670	1.678	.9325	2.583	2.770	.3610	3.355	14.31	14.35	.6173	.5756	.9320	5.675
.2500	.2679	1.683	.9332	2.599	2.784	.3592	3.367	14.47	14.51	.6164	.5752	.9323	5.667
.2510	.2687	1.689	.9339	2.614	2.798	.3574	3.377	14.62	14.66	.6155	.5748	.9327	5.658
.2520	.2696	1.694	.9346	2.629	2.813	.3556	3.388	14.79	14.82	.6146	.5744	.9330	5.650
.2530	.2705	1.700	.9353	2.645	2.828	.3537	3.399	14.95	14.99	.6137	.5740	.9333	5.641
.2540	.2714	1.705	.9360	2.660	2.842	.3519	3.410	15.12	15.15	.6128	.5736	.9336	5.633
.2550	.2722	1.711	.9367	2.676	2.856	.3501	3.421	15.29	15.32	.6120	.5732	.9340	5.624
.2560	.2731	1.716	.9374	2.691	2.871	.3483	3.432	15.45	15.49	.6111	.5728	.9343	5.616
.2570	.2740	1.722	.9381	2.707	2.886	.3465	3.443	15.63	15.66	.6102	.5724	.9346	5.608
.2580	.2749	1.727	.9388	2.723	2.901	.3447	3.454	15.80	15.83	.6093	.5720	.9349	5.600
.2590	.2757	1.732	.9394	2.739	2.916	.3430	3.465	15.97	16.00	.6085	.5716	.9353	5.592
.2600	.2766	1.738	.9400	2.755	2.931	.3412	3.476	16.15	16.18	.6076	.5712	.9356	5.585
.2610	.2775	1.744	.9406	2.772	2.946	.3394	3.487	16.33	16.36	.6068	.5707	.9360	5.578
.2620	.2784	1.749	.9412	2.788	2.962	.3376	3.498	16.51	16.54	.6060	.5703	.9363	5.571
.2630	.2792	1.755	.9418	2.804	2.977	.3359	3.509	16.69	16.73	.6052	.5699	.9367	5.563
.2640	.2801	1.760	.9425	2.820	2.992	.3342	3.520	16.88	16.91	.6043	.5695	.9370	5.556
.2650	.2810	1.766	.9431	2.837	3.008	.3325	3.531	17.07	17.10	.6035	.5691	.9373	5.548
.2660	.2819	1.771	.9437	2.853	3.023	.3308	3.542	17.26	17.28	.6027	.5687	.9377	5.541
.2670	.2827	1.776	.9443	2.870	3.039	.3291	3.553	17.45	17.45	.6018	.5683	.9380	5.534
.2680	.2836	1.782	.9449	2.886	3.055	.3274	3.564	17.64	17.67	.6010	.5679	.9383	5.527
.2690	.2845	1.788	.9455	2.904	3.071	.3256	3.575	17.84	17.87	.6002	.5675	.9386	5.520

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.2700	.2854	1.793	.9461	2.921	3.088	.3239	3.587	18.04	18.07	.5994	.5671	.9390	5.513
.2710	.2863	1.799	.9467	2.938	3.104	.3222	3.598	18.24	18.27	.5986	.5667	.9393	5.506
.2720	.2872	1.804	.9473	2.956	3.120	.3205	3.610	18.46	18.49	.5978	.5663	.9396	5.499
.2730	.2880	1.810	.9478	2.973	3.136	.3189	3.620	18.65	18.67	.5971	.5659	.9400	5.493
.2740	.2889	1.815	.9484	2.990	3.153	.3172	3.631	18.86	18.89	.5963	.5655	.9403	5.486
.2750	.2898	1.821	.9490	3.008	3.170	.3155	3.642	19.07	19.10	.5955	.5651	.9406	5.480
.2760	.2907	1.826	.9495	3.025	3.186	.3139	3.653	19.28	19.30	.5947	.5647	.9410	5.474
.2770	.2916	1.832	.9500	3.043	3.203	.3122	3.664	19.49	19.51	.5940	.5643	.9413	5.468
.2780	.2924	1.837	.9505	3.061	3.220	.3106	3.675	19.71	19.74	.5932	.5639	.9416	5.462
.2790	.2933	1.843	.9511	3.079	3.237	.3089	3.686	19.93	19.96	.5925	.5635	.9420	5.456
.2800	.2942	1.849	.9516	3.097	3.254	.3073	3.697	20.16	20.18	.5917	.5631	.9423	5.450
.2810	.2951	1.854	.9521	3.115	3.272	.3057	3.709	20.39	20.41	.5910	.5627	.9426	5.444
.2820	.2960	1.860	.9526	3.133	3.289	.3040	3.720	20.62	20.64	.5902	.5623	.9430	5.438
.2830	.2969	1.866	.9532	3.152	3.307	.3024	3.731	20.85	20.87	.5895	.5619	.9433	5.432
.2840	.2978	1.871	.9537	3.171	3.325	.3008	3.742	21.09	21.11	.5887	.5615	.9436	5.426
.2850	.2987	1.877	.9542	3.190	3.343	.2992	3.754	21.33	21.35	.5880	.5611	.9440	5.420
.2860	.2996	1.882	.9547	3.209	3.361	.2976	3.765	21.57	21.59	.5873	.5607	.9443	5.414
.2870	.3005	1.888	.9552	3.228	3.379	.2959	3.776	21.82	21.84	.5866	.5603	.9446	5.409
.2880	.3014	1.893	.9557	3.246	3.396	.2944	3.787	22.05	22.07	.5859	.5600	.9449	5.403
.2890	.3022	1.899	.9562	3.264	3.414	.2929	3.798	22.30	22.32	.5852	.5596	.9452	5.397
.2900	.3031	1.905	.9567	3.284	3.433	.2913	3.809	22.54	22.57	.5845	.5592	.9456	5.392
.2910	.3040	1.910	.9572	3.303	3.451	.2898	3.821	22.81	22.83	.5838	.5588	.9459	5.386
.2920	.3049	1.916	.9577	3.323	3.471	.2882	3.832	23.07	23.09	.5831	.5584	.9463	5.380
.2930	.3058	1.922	.9581	3.343	3.490	.2866	3.843	23.33	23.35	.5824	.5580	.9466	5.375
.2940	.3067	1.927	.9585	3.362	3.508	.2851	3.855	23.60	23.62	.5617	.5576	.9469	5.371
.2950	.3076	1.933	.9590	3.382	3.527	.2835	3.866	23.86	23.88	.5810	.5572	.9473	5.366
.2960	.3085	1.938	.9594	3.402	3.546	.2820	3.877	24.12	24.15	.5804	.5568	.9476	5.361
.2970	.3094	1.944	.9599	3.422	3.565	.2805	3.888	24.40	24.42	.5797	.5564	.9480	5.356
.2980	.3103	1.950	.9603	3.442	3.585	.2790	3.900	24.68	24.70	.5790	.5560	.9483	5.351
.2990	.3112	1.955	.9607	3.462	3.604	.2775	3.911	24.96	24.98	.5784	.5556	.9486	5.347

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.3000	.3121	1.961	.9611	3.483	3.624	.2760	3.922	25.24	25.26	.5777	.5552	.9490	5.342
.3010	.3130	1.967	.9616	3.503	3.643	.2745	3.933	25.53	25.55	.5771	.5549	.9493	5.337
.3020	.3139	1.972	.9620	3.524	3.663	.2730	3.945	25.82	25.83	.5764	.5545	.9496	5.332
.3030	.3148	1.978	.9624	3.545	3.683	.2715	3.956	26.12	26.14	.5758	.5541	.9499	5.328
.3040	.3157	1.984	.9629	3.566	3.703	.2700	3.968	26.42	26.44	.5751	.5538	.9502	5.323
.3050	.3166	1.989	.9633	3.587	3.724	.2685	3.979	26.72	26.74	.5745	.5534	.9505	5.318
.3060	.3175	1.995	.9637	3.609	3.745	.2670	3.990	27.02	27.04	.5739	.5530	.9509	5.314
.3070	.3184	2.001	.9641	3.630	3.765	.2656	4.002	27.33	27.35	.5732	.5527	.9512	5.309
.3080	.3193	2.007	.9645	3.651	3.786	.2641	4.013	27.65	27.66	.5726	.5523	.9515	5.305
.3090	.3202	2.012	.9649	3.673	3.806	.2627	4.024	27.96	27.98	.5720	.5519	.9518	5.300
.3100	.3211	2.018	.9653	3.694	3.827	.2613	4.036	28.28	28.30	.5714	.5515	.9522	5.296
.3110	.3220	2.023	.9656	3.716	3.848	.2599	4.047	28.60	28.62	.5708	.5511	.9525	5.292
.3120	.3230	2.029	.9660	3.738	3.870	.2584	4.058	28.93	28.95	.5701	.5507	.9528	5.288
.3130	.3239	2.035	.9664	3.760	3.891	.2570	4.070	29.27	29.28	.5695	.5504	.9531	5.284
.3140	.3248	2.041	.9668	3.782	3.912	.2556	4.081	29.60	29.62	.5689	.5500	.9535	5.280
.3150	.3257	2.046	.9672	3.805	3.934	.2542	4.093	29.94	29.96	.5683	.5497	.9538	5.276
.3160	.3266	2.052	.9676	3.828	3.956	.2528	4.104	30.29	30.31	.5678	.5494	.9541	5.272
.3170	.3275	2.058	.9679	3.851	3.978	.2514	4.116	30.64	30.65	.5672	.5490	.9544	5.268
.3180	.3284	2.063	.9682	3.873	4.000	.2500	4.127	30.99	31.00	.5666	.5486	.9547	5.264
.3190	.3294	2.069	.9686	3.896	4.022	.2486	4.139	31.35	31.37	.5660	.5483	.9550	5.260
.3200	.3302	2.075	.9690	3.919	4.045	.2472	4.150	31.71	31.72	.5655	.5479	.9553	5.256
.3210	.3311	2.081	.9693	3.943	4.068	.2459	4.161	32.07	32.08	.5649	.5476	.9556	5.252
.3220	.3321	2.086	.9696	3.966	4.090	.2445	4.173	32.44	32.46	.5643	.5472	.9559	5.249
.3230	.3330	2.092	.9700	3.990	4.114	.2431	4.185	32.83	32.84	.5637	.5468	.9562	5.245
.3240	.3339	2.098	.9703	4.014	4.136	.2418	4.196	33.20	33.22	.5632	.5465	.9565	5.241
.3250	.3349	2.104	.9707	4.038	4.160	.2404	4.208	33.60	33.61	.5627	.5462	.9568	5.237
.3260	.3357	2.110	.9710	4.061	4.183	.2391	4.219	33.97	33.99	.5621	.5458	.9571	5.234
.3270	.3367	2.115	.9713	4.085	4.206	.2378	4.231	34.37	34.38	.5616	.5455	.9574	5.231
.3280	.3376	2.121	.9717	4.110	4.230	.2364	4.242	34.77	34.79	.5610	.5451	.9577	5.227
.3290	.3385	2.127	.9720	4.135	4.254	.2351	4.254	35.18	35.19	.5605	.5448	.9580	5.223

d/L_o	d/L	$2\pi\, d/L$	TANH $2\pi\, d/L$	SINH $2\pi\, d/L$	COSH $2\pi\, d/L$	K	$4\pi\, d/L$	SINH $4\pi\, d/L$	COSH $4\pi\, d/L$	n	C_G/C_o	H/H_o'	M
.3300	.3394	2.133	.9723	4.159	4.277	.2338	4.265	35.58	35.59	.5599	.5444	.9583	5.220
.3310	.3403	2.138	.9726	4.184	4.301	.2325	4.277	35.99	36.00	.5594	.5441	.9586	5.217
.3320	.3413	2.144	.9729	4.209	4.326	.2312	4.288	36.42	36.43	.5589	.5438	.9589	5.214
.3330	.3422	2.150	.9732	4.234	4.350	.2299	4.300	36.84	36.85	.5584	.5434	.9592	5.210
.3340	.3431	2.156	.9735	4.259	4.375	.2286	4.311	37.25	37.27	.5578	.5431	.9595	5.207
.3350	.3440	2.161	.9738	4.284	4.399	.2273	4.323	37.70	37.72	.5573	.5427	.9598	5.204
.3360	.3449	2.167	.9741	4.310	4.424	.2260	4.335	38.14	38.15	.5568	.5424	.9601	5.201
.3370	.3459	2.173	.9744	4.336	4.450	.2247	4.346	38.59	38.60	.5563	.5421	.9604	5.198
.3380	.3468	2.179	.9747	4.361	4.474	.2235	4.358	39.02	39.04	.5558	.5417	.9607	5.194
.3390	.3477	2.185	.9750	4.388	4.500	.2222	4.369	39.48	39.49	.5553	.5414	.9610	5.191
.3400	.3468	2.190	.9753	4.413	4.525	.2210	4.381	39.95	39.96	.5548	.5411	.9613	5.188
.3410	.3495	2.196	.9756	4.439	4.550	.2198	4.392	40.40	40.41	.5544	.5408	.9615	5.185
.3420	.3504	2.202	.9758	4.466	4.576	.2185	4.404	40.87	40.89	.5539	.5405	.9618	5.182
.3430	.3514	2.208	.9761	4.492	4.602	.2173	4.416	41.36	41.37	.5534	.5402	.9621	5.179
.3440	.3523	2.214	.9764	4.521	4.630	.2160	4.427	41.85	41.84	.5529	.5399	.9623	5.176
.3450	.3532	2.220	.9767	4.547	4.656	.2148	4.439	42.33	42.34	.5524	.5396	.9626	5.173
.3460	.3542	2.225	.9769	4.575	4.682	.2136	4.451	42.83	42.84	.5519	.5392	.9629	5.171
.3470	.3551	2.231	.9772	4.602	4.709	.2124	4.462	43.34	43.35	.5515	.5389	.9632	5.168
.3480	.3560	2.237	.9775	4.629	4.736	.2111	4.474	43.85	43.86	.5510	.5386	.9635	5.165
.3490	.3570	2.243	.9777	4.657	4.763	.2099	4.486	44.37	44.40	.5505	.5383	.9638	5.162
.3500	.3579	2.249	.9780	4.685	4.791	.2087	4.498	44.89	44.80	.5501	.5380	.9640	5.159
.3510	.3588	2.255	.9782	4.713	4.818	.2076	4.509	45.42	45.43	.5496	.5377	.9643	5.157
.3520	.3598	2.260	.9785	4.741	4.845	.2064	4.521	45.95	45.96	.5492	.5374	.9646	5.154
.3530	.3607	2.266	.9787	4.770	4.873	.2052	4.533	46.50	46.51	.5487	.5371	.9648	5.152
.3540	.3616	2.272	.9790	4.798	4.901	.2040	4.544	47.03	47.04	.5483	.5368	.9651	5.149
.3550	.3625	2.278	.9792	4.827	4.929	.2029	4.556	47.59	47.60	.5479	.5365	.9654	5.147
.3560	.3635	2.284	.9795	4.856	4.957	.2017	4.568	48.15	48.16	.5474	.5362	.9657	5.144
.3570	.3644	2.290	.9797	4.885	4.987	.2005	4.579	48.72	48.73	.5470	.5359	.9659	5.141
.3580	.3653	2.296	.9799	4.914	5.015	.1994	4.591	49.29	49.30	.5466	.5356	.9662	5.139
.3590	.3663	2.301	.9801	4.944	5.044	.1983	4.603	49.88	49.89	.5461	.5353	.9665	5.137

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.3600	.3672	2.307	.9804	4.974	5.072	.1972	4.615	50.47	50.48	.5457	.5350	.9667	5.134
.3610	.3682	2.313	.9806	5.004	5.103	.1960	4.627	51.08	51.09	.5453	.5347	.9670	5.132
.3620	.3691	2.319	.9808	5.034	5.132	.1949	4.638	51.67	51.67	.5449	.5344	.9673	5.130
.3630	.3700	2.325	.9811	5.063	5.161	.1938	4.650	52.27	52.28	.5445	.5342	.9675	5.127
.3640	.3709	2.331	.9813	5.094	5.191	.1926	4.661	52.89	52.90	.5441	.5339	.9677	5.125
.3650	.3719	2.337	.9815	5.124	5.221	.1915	4.673	53.52	53.53	.5437	.5336	.9680	5.123
.3660	.3728	2.342	.9817	5.155	5.251	.1904	4.685	54.15	54.16	.5433	.5333	.9683	5.121
.3670	.3737	2.348	.9819	5.186	5.281	.1894	4.697	54.78	54.79	.5429	.5330	.9686	5.118
.3680	.3747	2.354	.9821	5.217	5.312	.1883	4.708	55.42	55.43	.5425	.5327	.9688	5.116
.3690	.3756	2.360	.9823	5.248	5.343	.1872	4.720	56.09	56.10	.5421	.5325	.9690	5.114
.3700	.3766	2.366	.9825	5.280	5.374	.1861	4.732	56.76	56.77	.5417	.5322	.9693	5.112
.3710	.3775	2.372	.9827	5.312	5.406	.1850	4.744	57.43	57.44	.5413	.5319	.9696	5.110
.3720	.3785	2.378	.9830	5.345	5.438	.1839	4.756	58.13	58.14	.5409	.5317	.9698	5.107
.3730	.3794	2.384	.9832	5.377	5.469	.1828	4.768	58.82	58.83	.5405	.5314	.9700	5.105
.3740	.3804	2.390	.9834	5.410	5.502	.1818	4.780	59.52	59.53	.5402	.5312	.9702	5.103
.3750	.3813	2.396	.9835	5.443	5.534	.1807	4.792	60.24	60.25	.5398	.5309	.9705	5.101
.3760	.3822	2.402	.9837	5.475	5.566	.1797	4.803	60.95	60.95	.5394	.5306	.9707	5.099
.3770	.3832	2.408	.9839	5.508	5.598	.1786	4.815	61.68	61.68	.5390	.5304	.9709	5.097
.3780	.3841	2.413	.9841	5.541	5.631	.1776	4.827	62.41	62.42	.5387	.5301	.9712	5.095
.3790	.3850	2.419	.9843	5.572	5.661	.1766	4.838	63.13	63.14	.5383	.5299	.9714	5.093
.3800	.3860	2.425	.9845	5.609	5.697	.1756	4.851	63.91	63.91	.5380	.5296	.9717	5.091
.3810	.3869	2.431	.9847	5.643	5.731	.1745	4.862	64.67	64.67	.5376	.5294	.9719	5.090
.3820	.3879	2.437	.9848	5.677	5.765	.1735	4.875	65.45	65.46	.5372	.5291	.9721	5.088
.3830	.3888	2.443	.9850	5.712	5.798	.1725	4.885	66.16	66.17	.5369	.5288	.9724	5.086
.3840	.3898	2.449	.9852	5.746	5.833	.1715	4.898	67.02	67.03	.5365	.5286	.9726	5.084
.3850	.3907	2.455	.9854	5.780	5.866	.1705	4.910	67.80	67.81	.5362	.5284	.9728	5.082
.3860	.3917	2.461	.9855	5.814	5.900	.1695	4.922	68.61	68.62	.5359	.5281	.9730	5.081
.3870	.3926	2.467	.9857	5.850	5.935	.1685	4.934	69.45	69.46	.5355	.5279	.9732	5.079
.3880	.3936	2.473	.9859	5.886	5.970	.1675	4.946	70.28	70.29	.5352	.5276	.9735	5.077
.3890	.3945	2.479	.9860	5.921	6.005	.1665	4.958	71.12	71.13	.5349	.5274	.9737	5.076

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.3900	.3955	2.485	.9862	5.957	6.040	.1656	4.970	71.97	71.98	.5345	.5271	.9739	5.074
.3910	.3964	2.491	.9864	5.993	6.076	.1646	4.982	72.85	72.86	.5342	.5269	.9741	5.072
.3920	.3974	2.497	.9865	6.029	6.112	.1636	4.993	73.72	73.72	.5339	.5267	.9743	5.071
.3930	.3983	2.503	.9867	6.066	6.148	.1627	5.005	74.58	74.59	.5336	.5265	.9745	5.069
.3940	.3993	2.509	.9869	6.103	6.185	.1617	5.017	75.48	75.49	.5332	.5262	.9748	5.067
.3950	.4002	2.515	.9870	6.140	6.221	.1608	5.029	76.40	76.40	.5329	.5260	.9750	5.066
.3960	.4012	2.521	.9872	6.177	6.258	.1598	5.041	77.31	77.32	.5326	.5258	.9752	5.064
.3970	.4021	2.527	.9873	6.215	6.295	.1589	5.053	78.24	78.24	.5323	.5255	.9754	5.063
.3980	.4031	2.532	.9874	6.252	6.332	.1579	5.065	79.19	79.19	.5320	.5253	.9756	5.062
.3990	.4040	2.538	.9876	6.290	6.369	.1570	5.077	80.13	80.13	.5317	.5251	.9758	5.060
.4000	.4050	2.544	.9877	6.329	6.407	.1561	5.089	81.12	81.12	.5314	.5248	.9761	5.058
.4010	.4059	2.550	.9879	6.367	6.445	.1552	5.101	82.07	82.08	.5311	.5246	.9763	5.056
.4020	.4069	2.556	.9880	6.406	6.483	.1542	5.113	83.06	83.06	.5308	.5244	.9765	5.055
.4030	.4078	2.562	.9882	6.444	6.521	.1533	5.125	84.07	84.07	.5305	.5242	.9766	5.053
.4040	.4088	2.568	.9883	6.484	6.561	.1524	5.137	85.11	85.12	.5302	.5240	.9768	5.052
.4050	.4098	2.575	.9885	6.525	6.601	.1515	5.149	86.14	86.14	.5299	.5238	.9770	5.050
.4060	.4107	2.581	.9886	6.564	6.640	.1506	5.161	87.17	87.17	.5296	.5236	.9772	5.049
.4070	.4116	2.586	.9887	6.603	6.679	.1497	5.173	88.19	88.20	.5293	.5234	.9774	5.048
.4080	.4126	2.592	.9889	6.644	6.718	.1488	5.185	89.28	89.28	.5290	.5232	.9776	5.046
.4090	.4136	2.598	.9890	6.684	6.758	.1480	5.197	90.38	90.39	.5287	.5229	.9778	5.045
.4100	.4145	2.604	.9891	6.725	6.799	.1471	5.209	91.44	91.44	.5285	.5227	.9780	5.044
.4110	.4155	2.610	.9892	6.766	6.839	.1462	5.221	92.54	92.55	.5282	.5225	.9782	5.043
.4120	.4164	2.616	.9894	6.806	6.879	.1454	5.233	93.67	93.67	.5279	.5223	.9784	5.041
.4130	.4174	2.623	.9895	6.849	6.921	.1445	5.245	94.83	94.83	.5277	.5221	.9786	5.040
.4140	.4183	2.629	.9896	6.890	6.963	.1436	5.257	95.95	95.96	.5274	.5219	.9788	5.039
.4150	.4193	2.635	.9898	6.932	7.004	.1428	5.269	97.13	97.13	.5271	.5217	.9790	5.037
.4160	.4203	2.641	.9899	6.974	7.046	.1419	5.281	98.29	98.30	.5269	.5215	.9792	5.036
.4170	.4212	2.647	.9900	7.018	7.088	.1411	5.294	99.52	99.52	.5266	.5213	.9794	5.035
.4180	.4222	2.653	.9901	7.060	7.130	.1403	5.305	100.7	100.7	.5263	.5211	.9795	5.034
.4190	.4231	2.659	.9902	7.102	7.173	.1394	5.317	101.9	101.9	.5261	.5209	.9797	5.033

d/L_o	d/L	2π d/L	TANH 2π d/L	SINH 2π d/L	COSH 2π d/L	K	4π d/L	SINH 4π d/L	COSH 4π d/L	n	C_G/C_o	H/H'_o	M
.4200	.4241	2.665	.9904	7.146	7.215	.1386	5.329	103.1	103.1	.5258	.5208	.9798	5.031
.4210	.4251	2.671	.9905	7.190	7.259	.1378	5.341	104.4	104.4	.5256	.5206	.9800	5.030
.4220	.4260	2.677	.9906	7.234	7.303	.1369	5.353	105.7	105.7	.5253	.5204	.9802	5.029
.4230	.4270	2.683	.9907	7.279	7.349	.1361	5.366	107.0	107.0	.5251	.5202	.9804	5.028
.4240	.4280	2.689	.9908	7.325	7.392	.1353	5.378	108.3	108.3	.5248	.5200	.9806	5.027
.4250	.4289	2.695	.9909	7.371	7.438	.1345	5.390	109.7	109.7	.5246	.5198	.9808	5.026
.4260	.4298	2.701	.9910	7.412	7.479	.1337	5.402	110.9	110.9	.5244	.5196	.9810	5.025
.4270	.4308	2.707	.9911	7.457	7.524	.1329	5.414	112.2	112.2	.5241	.5195	.9811	5.024
.4280	.4318	2.713	.9912	7.503	7.570	.1321	5.426	113.6	113.6	.5239	.5193	.9812	5.023
.4290	.4328	2.719	.9913	7.550	7.616	.1313	5.438	115.0	115.0	.5237	.5191	.9814	5.022
.4300	.4337	2.725	.9914	7.595	7.661	.1305	5.450	116.4	116.4	.5234	.5189	.9816	5.021
.4310	.4347	2.731	.9915	7.642	7.707	.1298	5.462	117.8	117.8	.5232	.5187	.9818	5.020
.4320	.4356	2.737	.9916	7.688	7.753	.1290	5.474	119.2	119.3	.5230	.5186	.9819	5.019
.4330	.4366	2.743	.9917	7.735	7.800	.1282	5.486	120.7	120.7	.5227	.5184	.9821	5.018
.4340	.4376	2.749	.9918	7.783	7.847	.1274	5.499	122.2	122.2	.5225	.5182	.9823	5.017
.4350	.4385	2.755	.9919	7.831	7.895	.1267	5.511	123.7	123.7	.5223	.5181	.9824	5.016
.4360	.4395	2.762	.9920	7.880	7.943	.1259	5.523	125.2	125.2	.5221	.5179	.9826	5.015
.4370	.4405	2.768	.9921	7.922	7.991	.1251	5.535	126.7	126.7	.5218	.5177	.9828	5.014
.4380	.4414	2.774	.9922	7.975	8.035	.1244	5.547	128.3	128.3	.5216	.5176	.9829	5.013
.4390	.4424	2.780	.9923	8.026	8.088	.1236	5.560	129.9	129.9	.5214	.5174	.9830	5.012
.4400	.4434	2.786	.9924	8.075	8.136	.1229	5.572	131.4	131.4	.5212	.5172	.9832	5.011
.4410	.4443	2.792	.9925	8.124	8.185	.1222	5.584	133.0	133.0	.5210	.5171	.9833	5.010
.4420	.4453	2.798	.9926	8.175	8.236	.1214	5.596	134.7	134.7	.5208	.5169	.9835	5.009
.4430	.4463	2.804	.9927	8.228	8.285	.1207	5.608	136.3	136.3	.5206	.5168	.9836	5.008
.4440	.4472	2.810	.9928	8.274	8.334	.1200	5.620	137.9	137.9	.5204	.5166	.9838	5.007
.4450	.4482	2.816	.9929	8.326	8.387	.1192	5.632	139.6	139.7	.5202	.5165	.9839	5.006
.4460	.4492	2.822	.9930	8.379	8.438	.1185	5.644	141.4	141.4	.5200	.5163	.9841	5.005
.4470	.4501	2.828	.9930	8.427	8.486	.1178	5.657	143.1	143.1	.5198	.5161	.9843	5.005
.4480	.4511	2.834	.9931	8.481	8.540	.1171	5.669	144.8	144.8	.5196	.5160	.9844	5.004
.4490	.4521	2.840	.9932	8.532	8.590	.1164	5.681	146.6	146.6	.5194	.5158	.9846	5.003

d/L_o	d/L	$2\pi\ d/L$	TANH $2\pi\ d/L$	SINH $2\pi\ d/L$	COSH $2\ d/L$	K	$4\pi\ d/L$	SINH $4\pi\ d/L$	COSH $4\pi\ d/L$	n	C_G/C_o	H/H_o'	M
.4500	.4531	2.847	.9933	8.585	8.643	.1157	5.693	148.4	148.4	.5192	.5157	.9847	5.002
.4510	.4540	2.853	.9934	8.638	8.695	.1150	5.705	150.2	150.2	.5190	.5156	.9848	5.001
.4520	.4550	2.859	.9935	8.693	8.750	.1143	5.717	152.1	152.1	.5188	.5154	.9849	5.000
.4530	.4560	2.865	.9935	8.747	8.804	.1136	5.730	154.0	154.0	.5186	.5152	.9851	5.000
.4540	.4569	2.871	.9936	8.797	8.854	.1129	5.742	155.9	155.9	.5184	.5151	.9852	4.999
.4550	.4579	2.877	.9937	8.853	8.910	.1122	5.754	157.7	157.7	.5182	.5150	.9853	4.998
.4560	.4589	2.883	.9938	8.910	8.965	.1115	5.766	159.7	159.7	.5181	.5148	.9855	4.997
.4570	.4599	2.890	.9938	8.965	9.021	.1109	5.779	161.7	161.7	.5179	.5146	.9857	4.997
.4580	.4608	2.896	.9939	9.016	9.072	.1102	5.791	163.6	163.6	.5177	.5145	.9858	4.996
.4590	.4618	2.902	.9940	9.074	9.129	.1095	5.803	165.6	165.6	.5175	.5144	.9859	4.995
.4600	.4628	2.908	.9941	9.132	9.186	.1089	5.815	167.7	167.7	.5173	.5143	.9860	4.994
.4610	.4637	2.914	.9941	9.183	9.238	.1083	5.827	169.7	169.7	.5172	.5141	.9862	4.994
.4620	.4647	2.920	.9942	9.242	9.296	.1076	5.840	171.8	171.8	.5170	.5140	.9863	4.993
.4630	.4657	2.926	.9943	9.301	9.354	.1069	5.852	173.9	173.9	.5168	.5139	.9864	4.992
.4640	.4666	2.932	.9944	9.353	9.406	.1063	5.864	176.0	176.0	.5167	.5138	.9865	4.991
.4650	.4676	2.938	.9944	9.413	9.466	.1056	5.876	178.2	178.2	.5165	.5136	.9867	4.991
.4660	.4686	2.944	.9945	9.472	9.525	.1050	5.888	180.4	180.4	.5163	.5135	.9868	4.990
.4670	.4695	2.951	.9946	9.533	9.585	.1043	5.900	182.6	182.6	.5162	.5134	.9869	4.989
.4680	.4705	2.957	.9946	9.586	9.638	.1037	5.912	184.8	184.8	.5160	.5132	.9871	4.989
.4690	.4715	2.963	.9947	9.647	9.699	.1031	5.925	187.2	187.2	.5158	.5131	.9872	4.988
.4700	.4725	2.969	.9947	9.709	9.760	.1025	5.937	189.5	189.5	.5157	.5129	.9873	4.988
.4710	.4735	2.975	.9948	9.770	9.821	.1018	5.949	191.8	191.8	.5155	.5128	.9874	4.987
.4720	.4744	2.981	.9949	9.826	9.877	.1012	5.962	194.2	194.2	.5154	.5127	.9875	4.986
.4730	.4754	2.987	.9949	9.888	9.938	.1006	5.974	196.5	196.5	.5152	.5126	.9876	4.986
.4740	.4764	2.993	.9950	9.951	10.00	.1000	5.986	199.0	199.0	.5150	.5125	.9877	4.985
.4750	.4774	2.999	.9951	10.01	10.07	.09942	5.999	201.4	201.4	.5149	.5124	.9878	4.984
.4760	.4783	3.005	.9951	10.07	10.12	.09882	6.011	203.9	203.9	.5147	.5122	.9880	4.984
.4770	.4793	3.012	.9952	10.13	10.18	.09820	6.023	206.5	206.5	.5146	.5121	.9881	4.983
.4780	.4803	3.018	.9952	10.20	10.25	.09759	6.036	209.0	209.0	.5144	.5120	.9882	4.983
.4790	.4813	3.024	.9953	10.26	10.31	.09698	6.048	211.7	211.7	.5143	.5119	.9883	4.982

d/L_o	d/L	$2\pi\ d/L$	TANH $2\pi\ d/L$	SINH $2\pi\ d/L$	COSH $2\pi\ d/L$	K	$4\pi\ d/L$	SINH $4\pi\ d/L$	COSH $4\pi\ d/L$	n	C_G/C_o	H/H'_o	M
.4800	.4822	3.030	.9953	10.32	10.37	.09641	6.060	214.2	214.2	.5142	.5117	.9885	4.982
.4810	.4832	3.036	.9954	10.39	10.43	.09583	6.072	216.8	216.8	.5140	.5116	.9886	4.981
.4820	.4842	3.042	.9955	10.45	10.50	.09523	6.085	219.5	219.5	.5139	.5115	.9887	4.980
.4830	.4852	3.049	.9955	10.52	10.57	.09464	6.097	222.2	222.2	.5137	.5114	.9888	4.980
.4840	.4862	3.055	.9956	10.59	10.63	.09405	6.109	225.0	225.0	.5136	.5113	.9889	4.979
.4850	.4871	3.061	.9956	10.65	10.69	.09352	6.121	228.3	228.3	.5134	.5112	.9890	4.979
.4860	.4881	3.067	.9957	10.71	10.76	.09294	6.134	230.6	230.6	.5133	.5111	.9891	4.978
.4870	.4891	3.073	.9957	10.78	10.83	.09236	6.146	233.5	233.5	.5132	.5110	.9892	4.978
.4880	.4901	3.079	.9958	10.85	10.90	.09178	6.159	236.4	236.4	.5130	.5109	.9893	4.977
.4890	.4911	3.086	.9958	10.92	10.96	.09121	6.171	239.6	239.6	.5129	.5107	.9895	4.977
.4900	.4920	3.092	.9959	10.99	11.03	.09064	6.183	242.3	242.3	.5128	.5106	.9896	4.976
.4910	.4930	3.098	.9959	11.05	11.09	.09010	6.195	245.2	245.2	.5126	.5105	.9897	4.976
.4920	.4940	3.104	.9960	11.12	11.16	.08956	6.208	248.3	248.3	.5125	.5104	.9898	4.975
.4930	.4950	3.110	.9960	11.19	11.24	.08901	6.220	251.3	251.3	.5124	.5103	.9899	4.975
.4940	.4960	3.117	.9961	11.26	11.31	.08845	6.232	254.5	254.5	.5122	.5102	.9899	4.974
.4950	.4969	3.122	.9961	11.32	11.37	.08793	6.245	257.6	257.6	.5121	.5101	.9900	4.974
.4960	.4979	3.128	.9962	11.40	11.44	.08741	6.257	260.8	260.8	.5120	.5100	.9901	4.973
.4970	.4989	3.135	.9962	11.47	11.51	.08691	6.269	264.0	264.0	.5119	.5099	.9902	4.973
.4980	.4999	3.141	.9963	11.54	11.59	.08637	6.282	267.3	267.3	.5118	.5098	.9903	4.972
.4990	.5009	3.147	.9963	11.61	11.65	.08584	6.294	270.6	270.6	.5116	.5097	.9904	4.972
.5000	.5018	3.153	.9964	11.68	11.72	.08530	6.306	274.0	274.0	.5115	.5096	.9905	4.971
.5010	.5028	3.159	.9964	11.75	11.80	.08477	6.319	277.5	277.5	.5114	.5095	.9906	4.971
.5020	.5038	3.166	.9964	11.83	11.87	.08424	6.331	280.8	280.8	.5113	.5094	.9907	4.971
.5030	.5048	3.172	.9965	11.91	11.95	.08371	6.343	284.3	284.3	.5112	.5093	.9908	4.970
.5040	.5058	3.178	.9965	11.98	12.02	.08320	6.356	287.9	287.9	.5110	.5092	.9909	4.970
.5050	.5067	3.184	.9966	12.05	12.09	.08270	6.368	291.4	291.4	.5109	.5092	.9909	4.969
.5060	.5077	3.190	.9966	12.12	12.16	.08220	6.380	295.0	295.0	.5108	.5091	.9910	4.969
.5070	.5087	3.196	.9967	12.20	12.24	.08169	6.393	298.7	298.7	.5107	.5090	.9911	4.968
.5080	.5097	3.203	.9967	12.28	12.32	.08119	6.405	302.4	302.4	.5106	.5089	.9912	4.968
.5090	.5107	3.209	.9968	12.35	12.39	.08068	6.417	306.2	306.2	.5105	.5088	.9913	4.967

d/L_c	d/L	$2\pi\, d/L$	TANH $2\pi\, d/L$	SINH $2\pi\, d/L$	COSH $2\pi\, d/L$	K	$4\pi\, d/L$	SINH $4\pi\, d/L$	COSH $4\pi\, d/L$	n	C_G/C_o	H/H_o'	M
.5100	.5117	3.215	.9968	12.43	12.47	.08022	6.430	310.0	310.0	.5104	.5087	.9914	4.967
.5110	.5126	3.221	.9968	12.50	12.54	.07972	6.442	313.8	313.8	.5103	.5086	.9915	4.967
.5120	.5136	3.227	.9969	12.58	12.62	.07922	6.454	317.7	317.7	.5102	.5086	.9915	4.966
.5130	.5146	3.233	.9969	12.66	12.70	.07873	6.467	321.7	321.7	.5101	.5085	.9916	4.966
.5140	.5156	3.240	.9970	12.74	12.78	.07824	6.479	325.7	325.7	.5100	.5084	.9917	4.965
.5150	.5166	3.246	.9970	12.82	12.86	.07776	6.491	329.7	329.7	.5098	.5083	.9918	4.965
.5160	.5176	3.252	.9970	12.90	12.94	.07729	6.504	333.8	333.8	.5097	.5082	.9919	4.965
.5170	.5185	3.258	.9971	12.98	13.02	.07682	6.516	337.9	337.9	.5096	.5082	.9919	4.964
.5180	.5195	3.264	.9971	13.06	13.10	.07634	6.529	342.2	342.2	.5095	.5081	.9920	4.964
.5190	.5205	3.270	.9971	13.14	13.18	.07587	6.541	346.4	346.4	.5094	.5080	.9921	4.964
.5200	.5215	3.277	.9972	13.22	13.26	.07540	6.553	350.7	350.7	.5093	.5079	.9922	4.963
.5210	.5225	3.283	.9972	13.31	13.35	.07494	6.566	355.1	355.1	.5092	.5078	.9923	4.963
.5220	.5235	3.289	.9972	13.39	13.43	.07449	6.578	359.6	359.6	.5092	.5077	.9924	4.963
.5230	.5244	3.295	.9973	13.47	13.51	.07404	6.590	364.0	364.0	.5091	.5077	.9924	4.962
.5240	.5254	3.301	.9973	13.55	13.59	.07358	6.603	368.5	368.5	.5090	.5076	.9925	4.962
.5250	.5264	3.308	.9973	13.64	13.68	.07312	6.615	373.1	373.1	.5089	.5075	.9926	4.962
.5260	.5274	3.314	.9974	13.73	13.76	.07266	6.628	377.8	377.8	.5088	.5074	.9927	4.961
.5270	.5284	3.320	.9974	13.81	13.85	.07221	6.640	382.5	382.5	.5087	.5074	.9927	4.961
.5280	.5294	3.326	.9974	13.90	13.94	.07177	6.652	387.3	387.3	.5086	.5073	.9928	4.961
.5290	.5304	3.333	.9975	13.99	14.02	.07134	6.665	392.2	392.2	.5085	.5072	.9929	4.960
.5300	.5314	3.339	.9975	14.07	14.10	.07091	6.677	397.0	397.0	.5084	.5071	.9930	4.960
.5310	.5323	3.345	.9975	14.16	14.19	.07047	6.690	402.0	402.0	.5083	.5070	.9931	4.960
.5320	.5333	3.351	.9976	14.25	14.28	.07003	6.702	406.9	406.9	.5082	.5070	.9931	4.959
.5330	.5343	3.357	.9976	14.34	14.37	.06959	6.714	412.0	412.0	.5082	.5069	.9932	4.959
.5340	.5353	3.363	.9976	14.43	14.46	.06915	6.727	417.2	417.2	.5081	.5068	.9933	4.959
.5350	.5363	3.370	.9976	14.52	14.55	.06872	6.739	422.4	422.4	.5080	.5068	.9933	4.959
.5360	.5373	3.376	.9977	14.61	14.64	.06829	6.752	427.7	427.7	.5079	.5067	.9934	4.958
.5370	.5383	3.382	.9977	14.70	14.73	.06787	6.764	433.1	433.1	.5078	.5066	.9935	4.958
.5380	.5393	3.388	.9977	14.79	14.82	.06746	6.776	438.5	438.5	.5077	.5066	.9935	4.958
.5390	.5402	3.394	.9977	14.88	14.91	.06705	6.789	444.0	444.0	.5077	.5065	.9936	4.958

d/L_o	d/L	$2\pi\, d/L$	TANH $2\pi\, d/L$	SINH $2\pi\, d/L$	COSH $2\pi\, d/L$	K	$4\pi\, d/L$	SINH $4\pi\, d/L$	COSH $4\pi\, d/L$	n	C_G/C_o	H/H'_o	M
.5400	.5412	3.401	.9978	14.97	15.01	.06664	6.801	449.5	449.5	.5076	.5065	.9936	4.957
.5410	.5422	3.407	.9978	15.07	15.10	.06623	6.814	455.1	455.1	.5075	.5064	.9937	4.957
.5420	.5432	3.413	.9978	15.16	15.19	.06582	6.826	460.7	460.7	.5074	.5063	.9938	4.957
.5430	.5442	3.419	.9979	15.25	15.29	.06542	6.838	466.4	466.4	.5073	.5063	.9938	4.956
.5440	.5452	3.426	.9979	15.35	15.38	.06501	6.851	472.2	472.2	.5073	.5062	.9939	4.956
.5450	.5461	3.432	.9979	15.45	15.48	.06461	6.863	478.1	478.1	.5072	.5061	.9940	4.956
.5460	.5471	3.438	.9979	15.54	15.58	.06420	6.876	484.3	484.3	.5071	.5060	.9941	4.956
.5470	.5481	3.444	.9980	15.64	15.67	.06380	6.888	490.3	490.3	.5070	.5060	.9941	4.955
.5480	.5491	3.450	.9980	15.74	15.77	.06341	6.901	496.4	496.4	.5070	.5059	.9942	4.955
.5490	.5501	3.456	.9980	15.84	15.87	.06302	6.913	502.5	502.5	.5069	.5059	.9942	4.955
.5500	.5511	3.463	.9980	15.94	15.97	.06263	6.925	508.7	508.7	.5068	.5058	.9942	4.955
.5510	.5521	3.469	.9981	16.04	16.07	.06224	6.937	515.0	515.0	.5067	.5058	.9942	4.954
.5520	.5531	3.475	.9981	16.14	16.17	.06186	6.950	521.6	521.6	.5067	.5057	.9943	4.954
.5530	.5541	3.481	.9981	16.24	16.27	.06148	6.962	528.1	528.1	.5066	.5056	.9944	4.954
.5540	.5551	3.488	.9981	16.34	16.37	.06110	6.975	534.8	534.8	.5065	.5056	.9944	4.954
.5550	.5560	3.494	.9982	16.44	16.47	.06073	6.987	541.4	541.4	.5065	.5056	.9945	4.953
.5560	.5570	3.500	.9982	16.54	16.57	.06035	7.000	548.1	548.1	.5064	.5055	.9945	4.953
.5570	.5580	3.506	.9982	16.65	16.68	.05997	7.012	554.9	554.9	.5063	.5054	.9946	4.953
.5580	.5590	3.512	.9982	16.75	16.78	.05960	7.025	562.0	562.0	.5063	.5053	.9947	4.953
.5590	.5600	3.519	.9982	16.85	16.88	.05923	7.037	569.1	569.1	.5062	.5053	.9947	4.953
.5600	.5610	3.525	.9983	16.96	16.99	.05887	7.050	576.1	576.1	.5061	.5053	.9947	4.952
.5610	.5620	3.531	.9983	17.06	17.09	.05850	7.062	583.3	583.3	.5061	.5052	.9948	4.952
.5620	.5630	3.537	.9983	17.17	17.20	.05814	7.074	590.7	590.7	.5060	.5051	.9949	4.952
.5630	.5640	3.543	.9983	17.28	17.31	.05778	7.087	598.0	598.0	.5059	.5051	.9949	4.952
.5640	.5649	3.550	.9984	17.38	17.41	.05743	7.099	605.0	605.0	.5059	.5050	.9950	4.951
.5650	.5659	3.556	.9984	17.49	17.52	.05707	7.112	613.2	613.2	.5058	.5050	.9950	4.951
.5660	.5669	3.562	.9984	17.60	17.63	.05672	7.124	620.8	620.8	.5057	.5049	.9951	4.951
.5670	.5679	3.568	.9984	17.71	17.74	.05637	7.136	628.5	628.5	.5057	.5049	.9951	4.951
.5680	.5689	3.575	.9984	17.82	17.85	.05602	7.149	636.4	636.4	.5056	.5048	.9952	4.951
.5690	.5699	3.581	.9985	17.94	17.97	.05567	7.161	644.3	644.3	.5056	.5048	.9952	4.950

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.5700	.5709	3.587	.9985	18.05	18.08	.05532	7.174	652.4	652.4	.5055	.5047	.9953	4.950
.5710	.5719	3.593	.9985	18.16	18.19	.05497	7.186	660.5	660.5	.5054	.5047	.9953	4.950
.5720	.5729	3.600	.9985	18.28	18.31	.05463	7.199	668.8	668.8	.5054	.5046	.9954	4.950
.5730	.5738	3.606	.9985	18.39	18.42	.05430	7.211	677.2	677.2	.5053	.5046	.9954	4.950
.5740	.5748	3.612	.9985	18.50	18.53	.05396	7.224	685.6	685.6	.5053	.5045	.9955	4.950
.5750	.5758	3.618	.9986	18.62	18.64	.05363	7.236	694.3	694.3	.5052	.5045	.9955	4.949
.5760	.5768	3.624	.9986	18.73	18.76	.05330	7.249	703.2	703.2	.5052	.5044	.9956	4.949
.5770	.5778	3.630	.9986	18.85	18.88	.05297	7.261	711.9	711.9	.5051	.5044	.9956	4.949
.5780	.5788	3.637	.9986	18.97	19.00	.05264	7.274	720.8	720.8	.5051	.5043	.9957	4.949
.5790	.5798	3.643	.9986	19.09	19.12	.05231	7.286	729.9	729.9	.5050	.5043	.9957	4.949
.5800	.5808	3.649	.9987	19.21	19.24	.05198	7.298	739.0	739.0	.5049	.5043	.9957	4.948
.5810	.5818	3.656	.9987	19.33	19.36	.05166	7.311	748.1	748.1	.5049	.5042	.9958	4.948
.5820	.5828	3.662	.9987	19.45	19.48	.05134	7.323	757.5	757.5	.5048	.5042	.9958	4.948
.5830	.5838	3.668	.9987	19.58	19.60	.05102	7.336	767.0	767.0	.5048	.5041	.9959	4.948
.5840	.5848	3.674	.9987	19.70	19.73	.05070	7.348	776.7	776.7	.5047	.5041	.9959	4.948
.5850	.5858	3.680	.9987	19.81	19.84	.05040	7.361	786.5	786.5	.5047	.5040	.9960	4.948
.5860	.5867	3.686	.9987	19.94	19.96	.05009	7.373	796.4	796.4	.5046	.5040	.9960	4.948
.5870	.5877	3.693	.9988	20.06	20.09	.04978	7.386	806.5	806.5	.5046	.5040	.9960	4.947
.5880	.5887	3.699	.9988	20.19	20.21	.04947	7.398	816.5	816.5	.5045	.5039	.9961	4.947
.5890	.5897	3.705	.9988	20.32	20.34	.04916	7.411	826.7	826.7	.5045	.5039	.9961	4.947
.5900	.5907	3.712	.9988	20.45	20.47	.04885	7.423	837.1	837.1	.5044	.5038	.9962	4.947
.5910	.5917	3.718	.9988	20.57	20.60	.04855	7.436	847.6	847.6	.5044	.5038	.9962	4.947
.5920	.5927	3.724	.9988	20.70	20.73	.04824	7.448	858.2	858.2	.5043	.5037	.9963	4.947
.5930	.5937	3.730	.9989	20.83	20.86	.04794	7.460	868.9	868.9	.5043	.5037	.9963	4.946
.5940	.5947	3.737	.9989	20.97	20.99	.04764	7.473	879.8	879.8	.5043	.5037	.9963	4.946
.5950	.5957	3.743	.9989	21.10	21.12	.04735	7.485	890.8	890.8	.5042	.5036	.9964	4.946
.5960	.5967	3.749	.9989	21.23	21.25	.04706	7.498	901.9	901.9	.5042	.5036	.9964	4.946
.5970	.5977	3.755	.9989	21.35	21.37	.04677	7.510	913.4	913.4	.5041	.5036	.9964	4.946
.5980	.5987	3.761	.9989	21.49	21.51	.04648	7.523	925.0	925.0	.5041	.5035	.9965	4.946
.5990	.5996	3.767	.9989	21.62	21.64	.04619	7.535	936.5	936.5	.5040	.5035	.9965	4.946

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.6000	.6006	3.774	.9990	21.76	21.78	.04591	7.548	948.1	948.1	.5040	.5035	.9965	4.945
.6100	.6106	3.836	.9991	23.17	23.19	.04313	7.673	1,074	1,074	.5036	.5031	.9969	4.944
.6200	.6205	3.899	.9992	24.66	24.68	.04052	7.798	1,217	1,217	.5032	.5028	.9972	4.943
.6300	.6305	3.961	.9993	26.25	26.27	.03806	7.923	1,379	1,379	.5029	.5025	.9975	4.942
.6400	.6404	4.024	.9994	27.95	27.97	.03576	8.048	1,527	1,527	.5026	.5023	.9977	4.941
.6500	.6504	4.086	.9994	29.75	29.77	.03359	8.173	1,771	1,771	.5023	.5020	.9980	4.940
.6600	.6603	4.149	.9995	31.68	31.69	.03155	8.298	2,008	2,008	.5021	.5018	.9982	4.940
.6700	.6703	4.212	.9996	33.73	33.74	.02964	8.423	2,275	2,275	.5019	.5017	.9983	4.939
.6800	.6803	4.274	.9996	35.90	35.92	.02784	8.548	2,579	2,579	.5017	.5015	.9985	4.939
.6900	.6902	4.337	.9997	38.23	38.24	.02615	8.674	2,923	2,923	.5015	.5013	.9987	4.938
.7000	.7002	4.400	.9997	40.71	40.72	.02456	8.799	3,314	3,314	.5013	.5012	.9988	4.938
.7100	.7102	4.462	.9997	43.34	43.35	.02307	8.925	3,757	3,757	.5012	.5011	.9989	4.937
.7200	.7202	4.525	.9998	46.14	46.15	.02167	9.050	4,258	4,258	.5011	.5010	.9990	4.937
.7300	.7302	4.588	.9998	49.13	49.14	.02035	9.175	4,828	4.828	.5010	.5009	.9991	4.937
.7400	.7401	4.650	.9998	52.31	52.32	.01911	9.301	5,473	5,473	.5009	.5008	.9992	4.937
,7500	.7501	4.713	.9998	55.70	55.71	.01795	9.426	6,204	6,204	.5008	.5007	.9993	4.936
.7600	.7601	4.776	.9999	59.31	59.31	.01686	9.552	7,034	7,034	.5007	.5006	.9994	4.936
.7700	.7701	4.839	.9999	63.15	63.16	.01583	9.677	7,976	7,976	.5006	.5005	.9995	4.936
.7800	.7801	4.902	.9999	67.24	67.25	.01487	9.803	9,042	9,042	.5005	.5004	.9996	4.936
.7900	.7901	4.964	.9999	71.60	71.60	.01397	9.929	10,250	10,250	.5005	.5004	.9996	4.936
.8000	.8001	5.027	.9999	76.24	76.24	.01312	10.05	11,620	11,620	.5004	.5004	.9996	4.936
.8100	.8101	5.090	.9999	81.18	81.19	.01232	10.18	13,180	13,180	.5004	.5004	.9996	4.936
.8200	.8201	5.153	.9999	86.44	86.44	.01157	10.31	14,940	14,940	.5003	.5003	.9997	4.935
.8300	.8301	5.215	.9999	92.04	92.05	.01086	10.43	17,340	17,340	.5003	.5003	.9997	4.935
.8400	.8400	5.278	1.000	98.00	98.01	.01020	10.56	19,210	19,210	.5003	.5003	.9997	4.935
.8500	.8500	5.341	1.000	104.4	104.4	.009582	10.68	21,780	21,780	.5002	.5002	.9998	4.935
.8600	.8600	5.404	1.000	111.1	111.1	.009000	10.81	24,690	24,690	.5002	.5002	.9998	4.935
.8700	.8700	5.467	1.000	118.3	118.3	.008451	10.93	28,000	28,000	.5002	.5002	.9998	4.935
.8800	.8800	5.529	1.000	126.0	126.0	.007934	11.06	31,750	31,750	.5002	.5002	.9998	4.935
.8900	.8900	5.592	1.000	134.2	134.2	.007454	11.18	36,000	36,000	.5002	.5002	.9998	4.935

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.9000	.9000	5.655	1.000	142.9	142.9	.007000	11.31	40,810	40,810	.5001	.5001	.9999	4.935
.9100	.9100	5.718	1.000	152.1	152.1	.006574	11.44	46,280	46,280	.5001	.5001	.9999	4.935
.9200	.9200	5.781	1.000	162.0	162.0	.006173	11.56	52,470	52,470	.5001	.5001	.9999	4.935
.9300	.9300	5.844	1.000	172.5	172.5	.005797	11.69	59,500	59,500	.5001	.5001	.9999	4.935
.9400	.9400	5.906	1.000	183.7	183.7	.005445	11.81	67,470	67,470	.5001	.5001	.9999	4.935
.9500	.9500	5.969	1.000	195.6	195.6	.005113	11.94	76,490	76,490	.5001	.5001	.9999	4.935
.9600	.9600	6.032	1.000	203.5	203.5	.004914	12.06	86,740	86,740	.5001	.5001	.9999	4.935
.9700	.9700	6.095	1.000	222.8	222.8	.004489	12.19	98,350	98,350	.5001	.5001	.9999	4.935
.9800	.9800	6.158	1.000	236.1	236.1	.004235	12.32	111,500	111,500	.5001	.5001	.9999	4.935
.9900	.9900	6.220	1.000	251.4	251.4	.003977	12.44	126,500	126,500	.5000	.5000	1.000	4.935
1.000	1.000	6.283	1.000	267.7	267.7	.003735	12.57	143,400	143,400	.5000	.5000	1.000	4.935

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TABLE IIC – 2

TABLE OF THE DEEP WATER WAVE LENGTH (L_o) AND VELOCITY (C_o) AS A FUNCTION OF THE WAVE PERIOD (T) AS OBTAINED FROM

$L_o = \frac{g}{2\pi}T^2 = 5.118T^2$ AND $C_o = \frac{g}{2\pi}T = 5.118T$

WHERE T IS IN SECONDS, L_o IS IN FEET AND C_o IS IN FEET. THE WAVE VELOCITY IS ALSO GIVEN IN KNOTS, AS OBTAINED FROM $C_o = \frac{g}{2\pi}T = 3.030T$ WHERE T IS IN SECONDS

T Sec.	C_o Ft/Sec	C_o Knots	L_o Ft.	T Sec.	C_o Ft/Sec	C_o Knots	L_o Ft.	T Sec.	C_o Ft/Sec	C_o Knots	L_o Ft.
3.0	15.4	9.1	46.1	7.0	35.8	21.2	251	11.0	56.3	33.3	620
3.1	15.9	9.4	49.2	7.1	36.3	21.5	258	11.1	56.8	33.6	631
3.2	16.4	9.7	52.4	7.2	36.8	21.8	265	11.2	57.3	33.9	642
3.3	16.9	10.0	55.8	7.3	37.4	22.1	273	11.3	57.8	34.2	654
3.4	17.4	10.3	59.2	7.4	37.9	22.4	280	11.4	58.3	34.5	665
3.5	17.9	10.6	62.7	7.5	38.4	22.7	288	11.5	58.9	34.8	677
3.6	18.4	10.9	66.4	7.6	38.9	23.0	296	11.6	59.4	35.1	689
3.7	18.9	11.2	70.1	7.7	39.4	23.3	304	11.7	59.9	35.4	701
3.8	19.4	11.5	73.9	7.8	39.9	23.6	312	11.8	60.4	35.8	713
3.9	20.0	11.8	77.9	7.9	40.4	23.9	320	11.9	60.9	36.1	725
4.0	20.5	12.1	81.9	8.0	40.9	24.2	328	12.0	61.4	36.4	737
4.1	21.0	12.4	86.1	8.1	41.4	24.5	336	12.1	61.9	36.7	750
4.2	21.5	12.7	90.3	8.2	42.0	24.8	344	12.2	62.4	37.0	762
4.3	22.0	13.0	94.7	8.3	42.5	25.1	353	12.3	63.0	37.3	775
4.4	22.5	13.3	99.1	8.4	43.0	25.4	361	12.4	63.5	37.6	787
4.5	23.0	13.6	104	8.5	43.5	25.7	370	12.5	64.0	37.9	800
4.6	23.5	13.9	108	8.6	44.0	26.1	379	12.6	64.5	38.2	813
4.7	24.0	14.2	113	8.7	44.5	26.4	388	12.7	65.0	38.5	826
4.8	24.6	14.5	118	8.8	45.0	26.7	397	12.8	65.5	38.8	839
4.9	25.1	14.8	123	8.9	45.6	27.0	406	12.9	66.0	39.1	852
5.0	25.6	15.2	128	9.0	46.1	27.3	415	13.0	66.5	39.4	865
5.1	26.1	15.5	133	9.1	46.6	27.6	424	13.1	67.0	39.7	879
5.2	26.6	15.8	138	9.2	47.1	27.9	433	13.2	67.6	40.0	892
5.3	27.1	16.1	144	9.3	47.6	28.2	442	13.3	68.1	40.3	906
5.4	27.6	16.4	149	9.4	48.1	28.5	452	13.4	68.6	40.6	919
5.5	28.1	16.7	155	9.5	48.6	28.8	461	13.5	69.1	40.9	933
5.6	28.7	17.0	161	9.6	49.1	29.1	471	13.6	69.6	41.2	947
5.7	29.2	17.3	166	9.7	49.6	29.4	481	13.7	70.1	41.5	961
5.8	29.7	17.6	172	9.8	50.2	29.7	491	13.8	70.6	41.8	975
5.9	30.2	17.9	178	9.9	50.7	30.0	502	13.9	71.1	42.1	989
6.0	30.7	18.2	184	10.0	51.2	30.3	512	14.0	71.6	42.4	1004
6.1	31.2	18.5	191	10.1	51.7	30.6	522	14.1	72.2	42.7	1018
6.2	31.7	18.8	197	10.2	52.2	30.9	533	14.2	72.7	43.0	1032
6.3	32.2	19.1	203	10.3	52.7	31.2	543	14.3	73.2	43.3	1047
6.4	32.8	19.4	210	10.4	53.2	31.5	554	14.4	73.7	43.6	1062
6.5	33.3	19.7	216	10.5	53.7	31.8	564	14.5	74.2	43.9	1076
6.6	33.8	20.0	223	10.6	54.2	32.1	575	14.6	74.7	44.2	1091
6.7	34.3	20.3	230	10.7	54.8	32.4	586	14.7	75.2	44.5	1106
6.8	34.8	20.6	237	10.8	55.3	32.7	597	14.8	75.7	44.8	1121
6.9	35.3	20.9	244	10.9	55.8	33.0	608	14.9	76.2	45.1	1137

T Sec.	C_o Ft/Sec	C_o Knots	L_o Ft.	T Sec.	C_o Ft/Sec	C_o Knots	L_o Ft.	T Sec.	C_o Ft/Sec	C_o Knots	L_o Ft.
15.0	76.8	45.4	1152	19.0	97.2	57.6	1847	23.0	117.7	69.7	2707
15.1	77.3	45.8	1167	19.1	97.8	57.9	1867	23.1	118.2	70.0	2731
15.2	77.8	46.1	1183	19.2	98.3	58.2	1886	23.2	118.7	70.3	2755
15.3	78.3	46.4	1199	19.3	98.8	58.5	1906	23.3	119.2	70.6	2779
15.4	78.8	46.7	1214	19.4	99.3	58.8	1926	23.4	119.8	70.9	2803
15.5	79.3	47.0	1230	19.5	99.8	59.1	1946	23.5	120.3	71.2	2827
15.6	79.8	47.3	1246	19.6	100.3	59.4	1966	23.6	120.8	71.5	2851
15.7	80.4	47.6	1262	19.7	100.8	59.7	1986	23.7	121.3	71.8	2875
15.8	80.9	47.9	1277	19.8	101.3	60.0	2006	23.8	121.8	72.1	2899
15.9	81.4	48.2	1293	19.9	101.8	60.3	2027	23.9	122.3	72.4	2924
16.0	81.9	48.5	1310	20.0	102.4	60.6	2047	24.0	122.8	72.7	2948
16.1	82.4	48.8	1326	20.1	102.9	60.9	2068	24.1	123.3	73.0	2973
16.2	82.9	49.1	1343	20.2	103.4	61.2	2088	24.2	123.8	73.3	2997
16.3	83.4	49.4	1359	20.3	103.9	61.5	2109	24.3	124.4	73.6	3022
16.4	83.9	49.7	1376	20.4	104.4	61.8	2130	24.4	124.9	73.9	3047
16.5	84.4	50.0	1393	20.5	104.9	62.1	2151	24.5	125.4	74.2	3072
16.6	85.0	50.3	1410	20.6	105.4	62.4	2172	24.6	125.9	74.5	3097
16.7	85.5	50.6	1427	20.7	105.9	62.7	2193	24.7	126.4	74.8	3123
16.8	86.0	50.9	1444	20.8	106.4	63.0	2214	24.8	126.9	75.1	3148
16.9	86.5	51.2	1461	20.9	107.0	63.3	2235	24.9	127.4	75.4	3173
17.0	87.0	51.5	1479	21.0	107.5	63.6	2257	25.0	128.0	75.7	3199
17.1	87.5	51.8	1496	21.1	108.0	63.9	2278	25.1	128.5	76.0	3225
17.2	88.0	52.1	1514	21.2	108.5	64.2	2300	25.2	129.0	76.4	3250
17.3	88.5	52.4	1531	21.3	109.0	64.5	2322	25.3	129.5	76.7	3276
17.4	89.0	52.7	1549	21.4	109.5	64.8	2344	25.4	130.0	77.0	3302
17.5	89.6	53.0	1567	21.5	110	65.1	2366	25.5	130.5	77.3	3328
17.6	90.1	53.3	1585	21.6	110.5	65.4	2388	25.6	131.0	77.6	3354
17.7	90.6	53.6	1603	21.7	111.1	65.8	2410	25.7	131.5	77.9	3380
17.8	91.1	53.9	1621	21.8	111.6	66.0	2432	25.8	132.0	78.2	3407
17.9	91.6	54.2	1639	21.9	112.1	66.4	2455	25.9	132.6	78.5	3433
18.0	92.1	54.5	1658	22.0	112.6	66.7	2477	26.0	133.1	78.8	3460
18.1	92.6	54.8	1677	22.1	113.1	67.0	2500	26.1	133.6	79.1	3486
18.2	93.1	55.1	1695	22.2	113.6	67.3	2522	26.2	134.1	79.4	3513
18.3	93.6	55.4	1714	22.3	114.1	67.6	2545	26.3	134.6	79.7	3540
18.4	94.2	55.8	1732	22.4	114.6	67.9	2568	26.4	135.1	80.0	3567
18.5	94.7	56.1	1751	22.5	115.2	68.2	2591	26.5	135.6	80.3	3594
18.6	95.2	56.4	1770	22.6	115.7	68.5	2614	26.6	136.1	80.6	3621
18.7	95.7	56.7	1789	22.7	116.2	68.8	2637	26.7	136.6	80.9	3649
18.8	96.2	57.0	1809	22.8	116.7	69.1	2661	26.8	137.2	81.2	3676
18.9	96.7	57.3	1828	22.9	117.2	69.4	2684	26.9	137.7	81.5	3703

MANUAL

of

AMPHIBIOUS OCEANOGRAPHY

SECTION III

WAVE FORECASTING

by

C. L. Bretschneider

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MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION III: WAVE FORECASTING

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MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION III: WAVE FORECASTING

A. INTRODUCTION

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A. INTRODUCTION*

1. Introduction:

As a result of wartime research on ocean surface waves a method has been available since 1943 for the prediction of wave characteristics. The initial stimulus for the development came during the planning phases of the Invasion of North Africa, and the methods subsequently devised were later used in a number of amphibious operations. The same techniques have found useful peacetime application in amphibious training exercises and in problems connected with coastal engineering. Much of the application to date has consisted in applying wave prediction techniques to historical rather than current meteorological data, hence the term "wave hindcasting" is in wide use.

The wind-generated waves in the ocean are conveniently divided into the three categories sea, swell, and surf. The term sea refers to waves under the direct influence of generating winds, and swell to waves which have left the generating area and are travelling through regions of weak winds or calms. Upon reaching shore the waves peak up and break, the water mixed with air rushing forward in a swirling mass up the beach face. This area, composed of the breakers and foam lines, is known as the surf zone.

Sverdrup and Munk (Reference III A-4) have obtained relationships for the growth and decay of waves by considering the energy transfer from wind to waves during growth and the reverse transfer from waves to air during decay. Empirical data have been utilized in evaluating certain coefficients and constants of integration. Recently, Bretschneider (References III A-2 and 3) developed new forecasting curves, making use of new data and concepts. The growth of waves depends upon wind velocity, the duration time of the wind, and the distance over which the wind blows (called the fetch). Observations have shown that as the wind blows over a limited fetch, the wave height and period over the up-wind part of the fetch reaches a steady state after a limited amount of time. As time passes the steady state region expands over the whole fetch. The wave height and period at the end of the fetch are altered by air resistance as the waves decay. The changes in height and period and the travel time required depend upon the period and the decay distance.

The basic data required for sea and swell forecasts are wind velocity and duration, fetch, and decay distance. These data are obtained from synoptic weather maps or from weather forecasts made from such maps. The wave forecasting theory assumes that the wind velocity is constant over the fetch for the duration time. In practice, this condition is never attained, and skill and judgment are required in selecting fetch, duration time, and average values of wind velocity such that accurate forecasts are obtained. For this reason, persons with experience in meteorology have usually been selected for training in wave forecasting. During World War II

* Taken largely from Reference IIIA-1

a number of meteorologists in the armed services were trained to use the Sverdrup-Munk methods.

The waves in the sea are extremely complex in the sense that wave height, period, and length vary widely with respect to space and time in an apparently irregular fashion. This complexity and irregularity have made it essential to introduce statistical measures in order that wave records and observations may be characterized in a meaningful manner and the significance of the forecast be established. In many applications, only the heights and periods of the higher waves in a wave train are of practical significance. For this reason, the average height and period of the highest one-third of the waves are useful statistical measures. These averages have been called significant wave height and significant wave period.

2. References:

III A-1. Arthur, R.S. - "Wave Forecasting and Hindcasting" - Institute of Coastal Engineering, Long Beach, California, October 1950, University of California.

III A-2. Bretschneider, C.L. - "The Generation and Decay of Wind Waves in Deep Water" - Series 29, Issue 46 - Institute of Engineering Research, University of California, Berkeley, California, August 1951, (unpublished). RESTRICTED.

III A-3. Bretschneider, C.L. - "Revised Wave Forecasting Curves and Procedures" - Series 29, Issue 47. Institute of Engineering Research, University of California, Berkeley, California, September 1950 (unpublished). RESTRICTED.

III A-4. Sverdrup, H. U. and Munk, W. H. - "Wind, Sea and Swell: Theory of Relations for Forecasting" - U.S. Navy Hydrographic Office, Publication No. 601, Washington D.C. 1947.

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION III: WAVE FORECASTING

B. SEA AND SWELL FORECASTING

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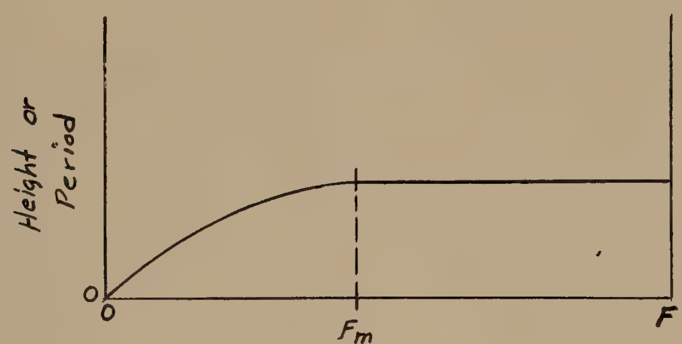
B. SEA AND SWELL FORECASTING*

1. The Growth and Decay of Wind Waves - A Short Summary:

The vertical distance between a trough of a wave to the following crest is called the wave height (H). The time taken for two successive crests (or troughs) to pass a point is known as the period (T), and the distance between successive crests is called the wave length (L). A group of waves, all of the same period, will travel with a velocity known as the group velocity (C_g = 1.52T knots).

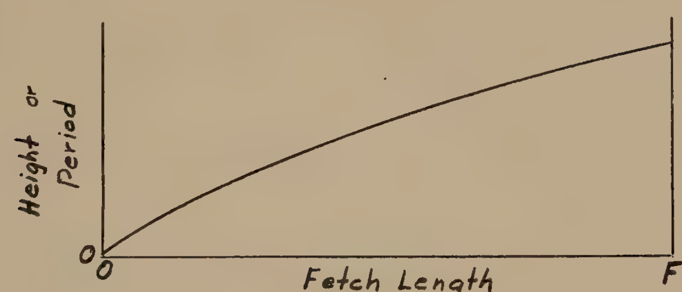
A distance of sea over which wind has been blowing is known as a fetch. The growth of waves within a fetch is governed by three things, the speed of the wind (U), the length of time this wind has been blowing, known as the duration (t_d), and the length of the fetch (f). In a non-growing fetch, a constant wind speed will develop waves limited in height and period at first by the duration of the wind, then by the length of the fetch.

At an early stage of development of the waves, their heights and periods at various points within the fetch will be distributed as in Sketch a.



Sketch a

At a later stage of development, the distribution of wave heights and periods will appear as in Sketch b.



Sketch b

* Taken entirely from References IIIB-3a and 4.

This second will be a steady state condition. That is, at any later time, if the wind speed remains constant, and the fetch length doesn't change, the same distribution of wave heights and periods will be found. A length of fetch for which a given wind speed will develop this steady-state condition regardless of how long the wind has been blowing is known as a minimum fetch (F_{min}). Similarly the length of time it takes a given wind blowing over a definite fetch length to cause a steady-state condition is known as the minimum duration (t_{min}).

In Sketch a that portion of the fetch from 0 to F_m has reached a steady state condition. If the wind has been blowing for t hours, the minimum duration for that portion of the fetch from 0 to F_m is t hours. Conversely the minimum fetch for a minimum duration of t hours would be F long. Note, in this case, the measured fetch F is longer than the minimum fetch (F_m) corresponding to the minimum duration t.

In Sketch b, the least time taken to develop the distribution shown would be the minimum duration corresponding to a minimum fetch of length F. Note that the measured duration of wind may be longer than this minimum duration.

In actuality the distributions shown in Sketches a and b are simplified distributions, since a spectrum of wave heights and periods are generated in a fetch. These simplified distributions really refer to what is known as significant waves, a statistical term which is used to describe the average of the heights and periods of the highest one-third of the waves in a group.

After leaving the fetch these waves travel to a point some distance away--a coast for example--with speeds proportional to their periods, (1.52 T knots, where T is in seconds). In this decay distance D (nautical miles), the longer period waves will move faster than those with shorter periods, and will arrive at the end of the decay distance before them. An observer at this point would see a group of waves whose significant period, called T_D, is longer than T_F. The period of a decaying wave, then, seems to increase.

The heights of waves do decrease after leaving a fetch, and at the end of the decay distance, the observed significant wave height, H_D, will be smaller than H_F.

The time for this group of waves to travel over the decay distance is approximately the ratio of D to the group velocity of waves with a period T_D, and is known as the travel time

$$t_D = \frac{D}{1.52\ T_D}$$

It is likely, in practice, that the maximum waves of a fetch will not be at the head of the measured fetch. That is, the distance of these maximum waves to a point P, for which forecasts are being made is not equal to D, the distance between P and the front of the fetch. We will call this new "decay distance" (to the maximum waves) D_m.

In this section we are concerned with the changes waves undergo in a time Z_t between two weather charts. In this time, a fetch of the first map in which waves have been generated may change in form and position, and the wind may change speed. In order to forecast, it is necessary to categorize these changes of fetch and wind characteristics. We do this by measuring:

(i) the change in the positions of the front and rear of the fetch,

(ii) The change in fetch length,

(iii) the change in wind speed,

(iv) and the advance of the energy front of the maximum wave in the time Z_t between charts.

Let us call the point for which forecasts are being made, P. If we then use the subscript $_1$ for the parameters to be measured on the first chart, they are

D_1 = the distance between the front of the fetch and P.

F_1 = the measured length of fetch.

$R_1 = D_1 + F_1$ = the distance between the rear of the fetch and P.

D_{m1} = the distance between the maximum waves of the fetch and P.

C_{g1} = the group velocity of the maximum waves

$C_{g1} Z_t$ = the advance of these waves between charts

F_{m1} = the minimum fetch needed for these maximum waves, with

U_1 = the wind speed in the fetch.

Using subscript $_2$ for the parameters of the second chart, it will be necessary to measure, D_2, F_2, R_2, and U_2. The techniques for forecasting presented in this Manual permit calculation of D_{m2}, C_{g2}, and F_{m2}.

For additional forecasting with a third chart, these last measured and calculated parameters take on the subscript $_1$ and a whole new set of parameters with subscript $_2$ are determined from the third chart.

2. Growth of Wind Waves

The growth of wind waves depends upon the net energy transfer from wind to waves. This energy transfer depends upon the wind velocity, the duration of the wind, and the distance over which the wind is blowing. The generation or growth of wind waves in deep

water can be represented conveniently by dimensionless relationships which take into consideration the variables promoting wave growth. In Figure III B-1 (the Fetch Graph) the relationship between C/U versus gF/U^2, gH/U^2 versus gF/U^2, H/L versus gF/U^2, and tU/F versus gF/U^2 are presented.

Because the growth of the waves is not apparent from Figure III B-1, Figure III B-2 was prepared as a special case to illustrate this effect. It is assumed that a 30-knot wind of constant speed and direction developed over an undisturbed water surface of unlimited extent blowing from a boundary surface, such as a coastline. Figure III B-2 shows the height and the period of the significant waves as functions of distance from the coast for the 5th, 10th, 20th, 30th and 40th hour after the wind started. It can be seen that the wave heights and periods increase with distance as the waves receive energy from the wind. As an example, when the wind has blown for 5 hours the wave height has increased from 0 at the coast line to 9.5 feet at a distance of 36 or more nautical miles from the coast, and the period has increased from 0 at the coast line to 6.6 seconds at a distance of 36 or more nautical miles. After the wind has blown for 20 hours the "significant wave" is 17.9 feet high and has a 10.7 second period at a distance of 245 nautical miles from the coast. It should be noted that for a fetch of 36 nautical miles the significant wave height and period do not change even though the wind continues more than 5 hours. It takes 20 hours to reach a steady state for a fetch of 245 nautical miles, whereas it takes only 5 hours to reach a steady state for a 36 nautical mile fetch.

The fetch distance in Figure III B-2 can be limited by either the presence of a coast line or by the shape of the wind systems over the open ocean. It can be seen that for a given wind velocity, the time needed to establish a steady state depends only upon the length of the fetch. For a given fetch this time depends upon the wind velocity and is longer for weak winds than for strong winds. This time is called the <u>minimum duration</u>, t_m, and is measured in hours. It can also be seen that, for a given wind velocity, the fetch length behind which a steady state is reached depends only upon the wind duration. For a given duration of wind this steady state fetch depends on the wind velocity and is shorter for weak winds than for strong winds. This fetch length may be called the <u>minimum fetch</u> F_m, and is measured in nautical miles. For a given wind velocity and significant wave there is only one minimum duration, t_m, and only one corresponding minimum fetch, F_m.

The minimum duration, t_m, as related to the minimum fetch, F_m, describes the limiting values of the generation of significant waves. <u>For any wind velocity the time necessary for the energy front of the significant waves to advance with group velocity over the steady state distance from the start to the end of the minimum fetch is equal to the minimum duration.</u> This must be true, since for any shorter duration the waves in the generating area have not reached a steady state over the entire area; they will have a lower group velocity, and the time for the waves to advance with lower group velocity would be greater than the minimum duration.

Figure III B-2 is merely an example and does not hold true for any wind velocity except 30 knots. (Similar figures for other wind velocities can be constructed). It is more convenient to have one graph representing all the wind velocities generating waves over different fetch lengths and with different wind durations. Plate I, Forecasting Curves for Wave Generation, has been prepared from Figure III B-1 for use in forecasting the growth of waves under the action of any wind velocity between 8 and 65 knots.

Similar to Figure III B-2, Figure III B-3 was constructed to show the increase in significant wave energy $(HT)^2$. This figure will be discussed later under the heading "Wind Duration and Minimum Duration".

3. Decay of Waves

When waves leave a generating area (fetch) and pass through a calm, or regions of lighter winds, they undergo a change. The significant period of the train of waves increases and the significant height decreases. The change in the significant wave depends upon the significant wave at the end of the fetch and the decay distance. In order that the data on the decay of waves could be presented in a logical manner, a concept, based on the following observations, has been introduced:

(i) Individual waves do not maintain their identity in deep water.

(ii) A spectrum of lengths and heights is present in both the fetch and decay areas.

(iii) At any particular decay distance the significant period decreases with time.

(iv) The significant period increases with decay distance in a manner different than that assumed by Sverdrup and Munk for their decay relationship.

(v) The travel time depends upon the group velocity associated with the period as measured at the end of the decay distance.

For any particular wave period there are a number of different significant wave heights which can be present, depending on the wind velocity in the fetch and the minimum fetch (or the minimum duration). This is shown as a special case in Figure III B-4 for a significant wave period of 10 seconds, (for which there are a number of different values for the significant wave height). It can be seen that a 10-second significant wave generated by a 60-knot wind blowing for 4 hours over a 45 nautical mile fetch will be 22 feet high; also a 10-second wave generated by a 25-knot wind blowing over 320 nautical miles for 26 hours will be 14 feet high. Although these two different significant wave heights have the same significant period, the increase of the significant wave period over equal decay distances, will be different.

Figure III B-5 presents another special case. The relationship between wind velocity and minimum fetch is shown for a 20-foot significant wave height for several values of significant wave period between 9.5 and 12.6 seconds. For a significant wave height of 20 feet with a significant wave period of 9.5 seconds a 60-knot wind would have to blow 3.9 hours over an undisturbed water surface of 40 nautical miles, and for a significant wave height of 20 feet with a significant wave period of 12.6 seconds a 30-knot wind would have to blow for 34 hours over an undisturbed water surface of 600 nautical miles. Although the significant wave heights are the same, the significant wave periods are different, and the decrease in significant wave height over equal decay distances will be different.

It has been determined (Reference III B-3) that the decay of waves can be represented by a family of dimensionless relationships; one relationship for each different fetch length, significant wave at the end of the fetch, and decay distance. Increase of significant wave period is expressed in terms of the dimensionless variables D/T_F^2 versus D/T_D^2 with D/F as the parameter. For the decrease of height, the relationship is D/H_F versus D/H_D with D/F as the parameter. (The relationship for period increase becomes dimensionless by multiplying D/T_F^2 and D/T_D^2 by 1/g, where g is the acceleration of gravity. In order to simplify computation g was taken as unity, rather than 32.2 ft/sec^2. D is in nautical miles.) Figures III B-6 and III B-7 show these relationships.

Figure III B-8 (a special solution of Figure III B-6) shows the increase of period with decay distance for a 10-second significant wave at the end of four different fetches. Each case has a different significant wave height, H_F, at the end of the fetch. It can be seen that the period at any decay distance for each of the four cases is different. Similarly Figure III B-9 (a special solution of Figure III B-7) shows the decrease in height of a 20-foot significant wave generated at the end of four different fetches. Each case has a different significant period, T_F, at the end of the fetch. It can be seen that the height at any decay distance for each of the four cases is different.

Plate II, a set of forecasting curves for the decay of waves, has been prepared from Figures III B-6 and III B-7. Standard decay curves were prepared for the decay of significant waves generated at the end of a minimum fetch of 200 nautical miles. These curves are shown in Plate II - Figure A (the relative increase of significant wave period) and Figure C (the relative decrease in significant wave height). Figures B and D in Plate II, are correction curves to be used for the relative wave period T_D/T_F and the relative wave height, H_D/H_F respectively, for any value of minimum fetch between 50 and 1000 nautical miles. That is, Figure B of Plate II represents the relationship between the relative wave period, T_D/T_F, for F_m of 200 nautical miles and the relative wave period, T_D/T_F, for F_m between 50 and 1000 nautical miles for the same corresponding values of T_F and D; the same is true of Figure D for the relative wave height.

Example of the use of Plate I and Plate II:

Given

U = 23.5 knots

F = 600 nautical miles

t_d = 33 hours

D = 2000 nautical miles

from Plate I

H_F = 13 feet

T_F = 10 seconds

F_m = 400 nautical miles

t_m = 33 hours

from Plate II

enter Figure A at T_F equal to 10 seconds, and proceed to D equals 2000 nautical miles (this gives T_D/T_F at D = 2000 nautical miles for a significant wave period, T_F, of 10-seconds at the end of minimum fetch of 200 nautical miles). Proceed horizontally to Figure B to F_m equals 410 nautical miles and read $T_D/T_F = 1.28$; $T_D = 10 \times 1.28 = 12.8$ seconds.

To determine the wave height, H_D, at the end of the decay enter Figure C at H_F = 13 feet and proceed to D = 2000 nautical miles. Proceed to Figure D until F_m = 400 nautical miles is reached and read $H_D/H_F = .25$, $H_D = .25 \times 13 = 3.3$ feet.

Knowing the significant wave height, it is possible to determine the maximum wave height from Figure III B-12. This figure is based on a statistical study (Reference III B-6) of wave distribution versus significant wave height.

The travel time of the swell is based on the average group velocity of the significant waves at the end of decay distance. That is,

$$t_D = D/C_g$$

and if

T_D = seconds

D = nautical miles

t_D = hours

then

$$t_D = 0.66\ D/T_D$$

The solution of this equation is found in Figure III B-13. In the preceding example t_D = 103 hours.

4. Wind Direction and Wind Velocity:

In making sea and swell forecasts it is important to determine the average surface wind velocity over a generating area. The geostrophic wind velocity is that velocity which results from a balance between the Coriolis force (deflecting force caused by the rotation of the earth) and the pressure gradient force (force established by horizontal distribution of mass in the atmosphere). The geostrophic wind occurs only with straight parallel isobars above the layer of air having a frictional drag on the earth, and is given by the equation

$$2 V_g \Omega \sin \phi - 1/\rho \left| \Delta P/\Delta n \right| = 0$$

where

V_g = geostrophic wind speed

Ω = angular velocity of the earth = $.729 \times 10^{-4}$ rad/sec

ϕ = latitude, in degrees

$\left|\Delta P/\Delta n\right|$ = absolute value of the pressure gradient

The direction of the flow is determined from Buy Ballot's law: "With your back to the wind the low pressure is to your left and the high pressure to your right in the Northern Hemisphere; the low pressure to the right and the high pressure to the left in the Southern Hemisphere". Figure III B-10 gives the solution of the geostrophic wind equation for 3-millibar and 5-millibar isobar spacing. For 1-millibar isobar spacing use 1/5 of the values given by the 5-millibar isobar spacing.

Since the geostrophic wind equation is for straight parallel isobars, which seldom occur, one has to take into consideration the curvature of the air flow, which is not necessarily the same as the curvature of the isobars. For stationary or slowly moving pressure systems, the curvature of the flow is about the same as the curvature of the isobars. For rapidly moving systems the curvature is greatest in the forward part of the storm, least on the right side of the path of low centers and left side of high centers. Other sections of high and low systems have a curvature of flow between these extreme values.

In addition to the forces involved in geostrophic flow, a centrifugal force due to the curvature in the air-flow pattern must be considered. For high pressure centers the centrifugal force acts in the same direction as the pressure gradient force, and in low pressure centers the centrifugal force acts against the pressure gradient. For equilibrium above the frictional air layer, the wind velocity is such that a balance exists among all the forces.

The surface wind does not blow parallel to the isobars and is reduced in speed from that at the geostrophic level. Because of

friction in the lower levels (i.e., below about 1500 feet) the wind blows at an angle to the isobars (toward lower pressure). A theoretical relationship between the wind at geostrophic level and the surface is discussed in most textbooks **(Ref. III B-5) on meteorology.** It has been found that surface winds over the ocean make an angle between 15° and 30° to the left of the isobars in the Northern Hemisphere so that the geostrophic wind is reduced by a factor of from about 0.8 to 0.5. An average value commonly used is 2/3.

Such a reduction in wind speed from the geostrophic wind level to the surface depends upon the stability of the air, or the lapse rate. If the air temperature T_aT_a is subtracted from the sea surface temperature T_sT_s, the resulting difference, called the sea surface temperature difference, is an indication of the stability of the air in the lower layers. A relationship between the wind ratio (ratio of approximate surface wind U' to the geostrophic wind V_g) and the sea air temperature difference was obtained from a statistical study of ocean weather reports and analyses. The results are summarized in Table III B-1, Stability Factors. Table III B-1 is based on moderate to straight isobars, and Table IIIB-2, Curvature Correction, was prepared to allow for curvature. Figure III B-11, Surface Wind Scale, was prepared from Reference III B-1.

TABLE III B-1

STABILITY FACTORS

$T_sT_s - T_aT_a$ (degrees F)	U'/V_g
< -7	0.55
-7 to 0	0.60
1 to 4	0.65
5 to 10	0.70
11 to 15	0.75
> 15	0.80

TABLE III B-2

CURVATURE CORRECTION

Curvature of Isobars	Great Cyclonic	Moderate or Straight	Great anti-cyclonic
Stable air	U = .85 U'	U = U'	U = 1.05 U'
Indifferent	U = .90 U'	U = U'	U = 1.1 U'
Unstable	U = .95 U'	U = U'	U = 1.15 U'

The following is suggested for curvature identification:

If the radius of cyclonic curvature is R^o = 5 degrees of latitude it is great cyclonic and R^o = 10 degrees is mild cyclonic; if the radius of anticyclonic curvature is R^o = 10 degrees of latitude it is great anticyclonic and R^o = 20 degrees is mild anticyclonic.

Example:

Given: Latitude = 35 degrees

Δ n = 1.0 degrees

Δ P = 3 millibar (isobar spacing)

Curvature = great cyclonic (R^o = 5° Latitude)

$T_sT_s - T_aT_a$ = + 5 degrees

Solution:

from Figure III B-10

V_g = 48 knots

from Figure IIIB-11

U/V_g = .62

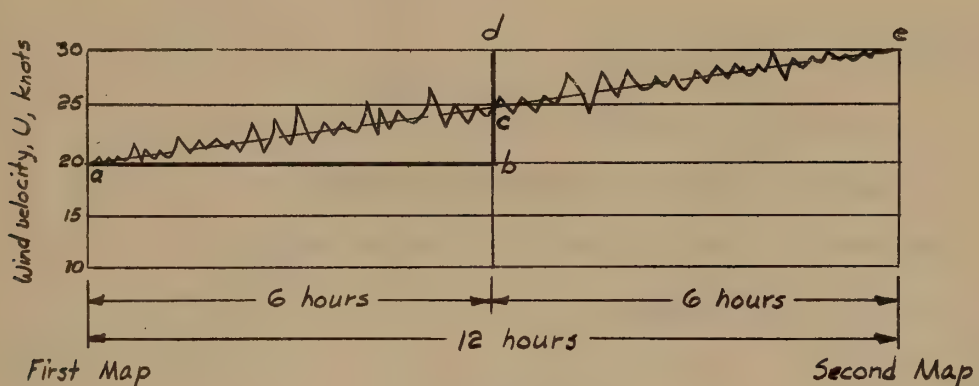
$\therefore$ U_s = 30 knots

5. Wind Duration and Minimum Duration:

The total energy per significant wave per foot of crest in deep water can be represented by $E = \rho gH^2L/8$ which reduces to $E = kH^2 T^2$, where k is constant, or E is proportional to $(HT)^2$, defined as the significant wave energy. For any minimum duration and minimum fetch there is a certain

value of $(HT)^2$ produced by the wind blowing on the waves. Figure IIIB-3 shows the significant wave energy $(HT)^2$ as a function of distance from the coast at intervals from 5 hours to 40 hours after a wind of 30 knots and 20 knots, respectively, started to blow over an undisturbed water surface. This figure shows that for a wind velocity of 30 knots the value of $(HT)^2$ is equal to 60^2 after t_{min} is equal to 5 hours and F_{min} equals 38 nautical miles. In order that $(HT)^2 = 60^2$ with a 20-knot wind blowing over an undisturbed water surface, it would take 16 hours, and the minimum fetch necessary would be 140 nautical miles. A 20-knot wind blowing for 40 hours over 500 nautical miles of undisturbed water surface would produce the same amount of significant wave energy $(HT)^2$ that a 30-knot wind would when blowing for 6.5 hours over 55 nautical miles of undisturbed water surface. If a 20-knot wind blows for 40 hours over 500 nautical miles and then increases immediately to 30 knots in the same direction and lasts for, say 12 hours, the total significant wave energy would be equivalent to that imparted by a 30-knot wind blowing for $12 + 6.5 = 18.5$ hours (from the preceding statement).

Winds do not usually increase immediately over a generating area, but instead increase gradually or irregularly throughout between 12-hourly weather maps. The following sketch (Sketch c) illustrates how a wind may vary between two 12-hourly weather maps.



Sketch c

The irregular line from a to e might represent the actual wind change between the 12-hour period (20 knots determined from the first weather map and 30 knots from the second weather map). The dashed line a-c-e would probably represent the best average wind change between maps. For all practical purposes it will be sufficiently accurate to assume that the 20-knot wind continued for 6 hours or half the time between weather maps and the 30-knot wind began and remained constant during the last 6 hours of the weather map. The duration of the 30-knot wind then will be 6 hours (half the map time) plus the time that it takes the 30-knot wind to generate the same amount of significant wave energy that a 20-knot wind would during its original duration plus 6 additional hours.

Example:

Given:

$F = 800$ nautical miles

$U = 20$ knots

$t_d = 34$ hours

next 12-hourly map shows

$F = 500$ nautical miles (same location as before)

$U = 30$ knots (in the same direction as before)

from the above sketch is seen that for

$U = 20$ knots

duration $= 34 + 6 = 40$ hours

from Figure III B-3 it is seen that this gives

$(HT)^2 = 85^2$

which would take a 30-knot wind only 6.5 hours to develop. Consequently, the duration of the 30-knot wind would be $6.5 + 6 = 12.5$ hours.

The lines of constant $(HT)^2$ on Plate I may be used to convert from one wind velocity and duration to another. In this particular example, enter Plate I at $U = 20$ knots and proceed until $t_d = 34 + 6 = 40$ hours, or $F = 800$ nautical miles, is first reached (since the fetch may limit the amount of significant energy), follow the dotted line back to $U = 30$ knots and read, $t_d = 6.5$ hours. The duration of the 30-knot wind will then be $6.5 + 6 = 12.5$ hours. In case the wind changes in direction use the component of the wind velocity on the previous map which is parallel to the existing wind velocity on the current map.

Plate I can be used for decreasing winds as well as increasing winds. However, if the decreasing winds are so great, that when following a line of constant $(HT)^2$ it is not possible to reach the new wind velocity, then the following is suggested:

(i) use $t_d = 60$ hours; this will give reliable heights, but not necessarily reliable periods and

(ii) analyze the previous map as diminutive swell (Reference III B-2) discussed later. This will give more reliable periods than (i).

6. Fetch:

The fetch is the horizontal length of the generating area in the direction of the wind, that is, the distance between the rear and the

front boundaries of the generating area. In general the fetch boundaries are determined by the following limitations:

(i) the coast line

(ii) meteorological fronts

(iii) curvature of the isobars

(iv) spreading of the isobars

The following steps are proposed as a guide in selecting a generating area of fetch:

(i) The fetch front can be located approximately by rotating a straight edge hinged at the forecasting location, and where the straight edge cuts the isobars at the front of the storm by approximately 10° to 15° connect a line. This locates the front of the fetch. Since the wind blows between 15° and 30° to the left of the isobars in the northern hemisphere, the winds generating the waves will be within 25° to 45° of a direct line to the forecasting location. Swing the straight edge back cutting the isobars at approximately 45° to 60°. This locates the rear of the fetch. The winds cutting the isobars will now be between (45° to 60°) minus (15° to 30°) on the opposite side of the direct line to the forecasting location. On most weather maps a straight line if not too far extended represents nearly a great circle. In case a weather map, such as the Northern or Southern Hemisphere polar sterographic maps, are used in making wave forecasts, in which a straight line deviates greatly from a great circle, a curved edge representing a great circle should be used.

The above procedure will not necessarily give the fetch of maximum intensity that will generate waves arriving later as swell at the forecasting location. One must also consider the area of maximum average wind velocity behind the fetch front. Usually the shortest distance to the fetch front will determine the decay distance and the extended line across the fetch will determine the fetch length.

(ii) When the decay distance is 500 or more nautical miles, that portion of the generating area with winds of Beaufort Force 4 or less can be neglected, and when the decay distance is 1000 nautical miles or more, that portion of the generating area with winds of Beaufort Force 5 or less can be neglected. This practice is allowable because waves raised under the above conditions will be reduced to small values when travelling through the respective decay distances. Only that portion of the fetch with stronger winds need be considered. If the winds are greatly different in different parts of the generating area, two fetches should be selected.

(iii) When cyclonic storms of great intensity are found, waves will be generated in different directions. In this case the isobars in the generating area will have larger curvature, and two fetches or more should be selected.

7. Changing Fetch and Wind Speed

Various forecasting situations may be classified by recording the possible changes the measured fetch of the first chart F_1, can undergo in the time Z_t between charts. These changes may be determined by noting:

whether the rear of F_2 is behind, even with, or ahead of energy front; and if behind,

whether the rear of F_2 is behind, even with or ahead of the position of the front of F_1; and in either case,

whether the front of F_2 is behind, even with or ahead of the energy front.

In certain cases we must also determine whether the front of F_2 is ahead of or behind the rear of F_1, and whether the measured fetch F_1 is less than, equal to, or greater than the minimum fetch F_{m1}.

On the following pages, the eight possible forecasting situations will be analyzed. In sections a through e these situations are dealt with by considering; that the wind speed does not change in the time Z_t between weather charts; and that the maximum significant waves are at the head of the fetch F_1. A summary of the criteria of fetch changes for each situation precedes each analysis. These derivations are extended to the case in which these maximum waves are within or at the rear of F_1 in section f, and in section g they are further generalized to the case in which the wind speed changes between weather maps. Section h deals with the decay of these waves.

a. Forecasting Situation (1):

I. The rear of F_2 is behind the energy front . . $R_2 > D_{m1} - C_{g1}Z_t$

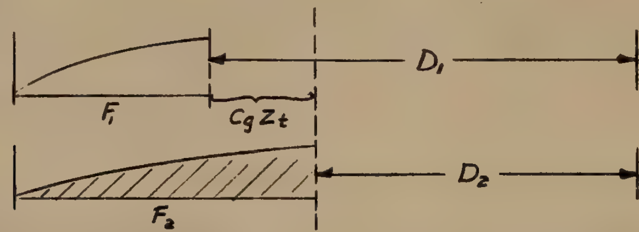
A. The rear of F_2 is even with the rear of F_1 or is ahead of it. . . . . $R_2 \leqq R_1$

1. The front of F_2 is behind the energy front or is abreast of it . . . . $D_2 \geqq D_{m1} - C_{g1}Z_t$

a. The measured fetch F_1 is equal to or less than the minimum fetch F_{m1} . . . . . . $F_1 \leqq F_{m1}$

These criteria encompass eight possible cases.

(1) $R_2 = R_1$, $D_2 = D_{m1} - C_g Z_t$, $F_1 = F_{m1}$



This is a case in which all waves formerly in F_1 have in the time Z_t continually been acted upon by the wind. These waves will advance approximately a distance $C_{g1}Z_t$. Their minimum fetch will be $C_{g1}Z_t$ or (D_1-D_2) longer than F_{m1}, and their minimum duration will be Z_t larger than t_{m1}. That is:

$$F_{m2} = F_2 = F_{m1} + C_{g1}Z_t = F_{m1} + (D_1-D_2)$$

$$\& \; t_{m2} = t_{m1} + Z_t$$

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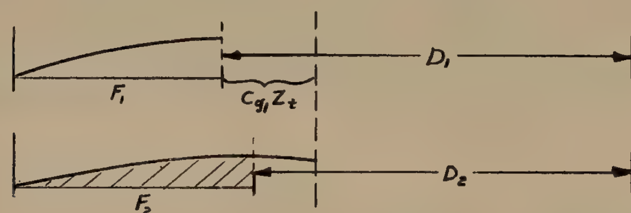
(2) $R_2 = R_1$, $D_2 > D_{m1} - C_{g1}Z_t$, $F_1 = F_{m1}$



This is a similar case, in which only part of the waves formerly in F_1 have in a time Z_t been acted upon by the wind. The maximum wave will be at the head of F_2, for those in front of this point will already be waves of decay. Therefore $F_{m2} = F_2 = F_{m1} + (D_1-D_2)$ which will be less than $F_1+C_{g1}Z_t$. Note $t_{m2} \leqslant t_{m1}+Z_t$, for if this were the minimum duration, the minimum fetch would be larger than F_{m2} above.

---

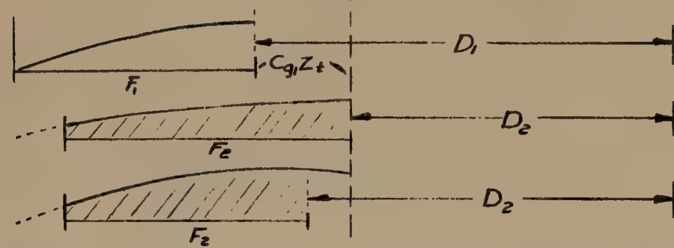
(3) $R_2 < R_1$, $D_2 = D_{m1} - C_{g1}Z_t$, $F_1 = F_{m1}$

&

(4) $R_2 < R_1$, $D_2 > D_{m1} - C_{g1}Z_t$, $F_1 = F_{m1}$

(3)

(4)



These wave distributions may be accounted for by noting that all waves in F_1 have continually been acted on by the wind in the time Z_t. They therefore will grow an amount proportional to the additional length of time that the wind has been blowing (situation 3), if not limited by the fetch available to them (situation 4). We have then:

(3) $F_{m2} = F_{m1} + C_{g1}Z_t = F_{m1} + (D_1 - D_2)$

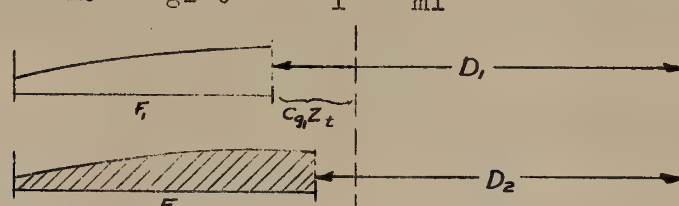
(4) $F_{m2} = F_{m1} + (D_1 - D_2)$

The cases for which $F_1 < F_{m1}$ are analogous.

(1a) $R_2 = R_1$, $D_2 = D_{m1} - C_{g1}Z_t$, $F_1 < F_{m1}$

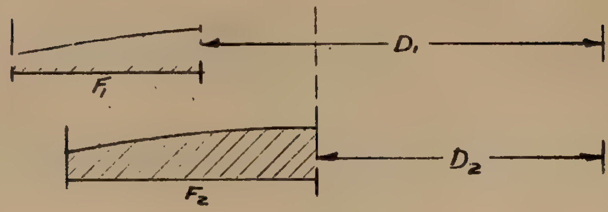
This case is analogous to (1) in that the whole wave pattern of F_1 has continually been acted upon by wind in the time Z_t. And in this case also the limitation imposed on the effective fetch is $F_{m2} = F_{m1} + (D_1 - D_2) = F_{m1} + C_{g1}Z_t$

(2a) $R_2 = R_1$, $D_2 > D_{m1} - C_{g1}Z_t$, $F_1 < F_{m1}$



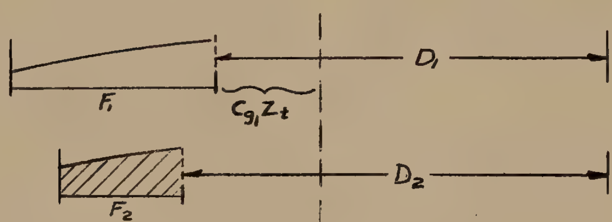
This case is analogous to 2, and $F_{m2} = F_{m1} + (D_1 - D_2)$

(3a) $R_2 < R_1$, $D_2 = D_{m1} - C_{g1}Z_t$, $F_1 < F_{m1}$



This case is similar to 3, and $F_{m2} = F_{m1} + (D_1 - D_2) = F_{m1} + C_{g1}Z_t$

(4a) $R_2 < R_1$, $D_2 > D_{m1} - C_{g1}Z_t$, $F_1 < F_{m1}$



This case is similar to 4, and $F_{m2} = F_{m1} + (D_1 - D_2)$

In all cases above, $D_{m2} = D_2$ since the maximum waves will occur at the head of the fetch F_2.

Summarizing we have:

$$\left.\begin{array}{ll}
(1)\ \&\ (1a) & F_{m2} = F_{m1} + (D_1 - D_2) = F_{m1} + C_{g1}Z_t \\
(2)\ \&\ (2a) & F_{m2} = F_{m1} + (D_1 - D_2) \\
(3)\ \&\ (3a) & F_{m2} = F_{m1} + (D_1 - D_2) = F_{m1} + C_{g1}Z_t \\
(4)\ \&\ (4a) & F_{m2} = F_{m1} + (D_1 - D_2)
\end{array}\right) F_{m2} = F_{m1} + (D_1 - D_2)$$

and

$$D_{m2} = D_2$$

For subsequent forecasts note that the F_{m2} & D_{m2} calculated for the second chart becomes the F_{m1} & D_{m1} for a third chart, unless the wind speed changes between charts 1 and 2 (see page 32).

b. Forecasting Situation (2):

This situation is similar in every way to situation (1) except that the measured fetch F_1 is greater than the minimum fetch F_{m1}. The criteria are:

I. The rear of F_2 is behind the energy front . . . . $R_2 > D_{m1} + C_{g1}Z_t$

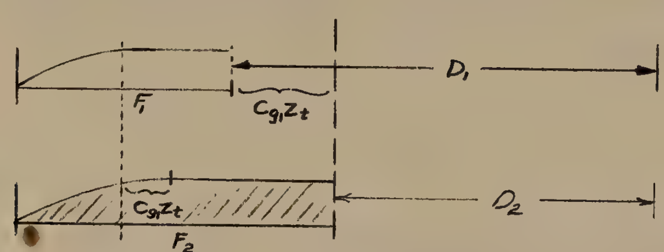
A. The rear of F_2 is even with or ahead of the rear of F_1 . . . . $R_2 \leqq R_1$

1. The front of F_2 is behind or is abreast of the energy front . . . . $D_2 \geqq D_{m1} - C_{g1}Z_t$

2. The measured F_1 is greater than the minimum fetch F_{m1} . . . . $F_1 > F_{m1}$

---

(1) $R_2 = R_1$, $D_2 = D_{m1} - C_{g1}Z_t$, $F_1 > F_{m1}$



This case represents the condition in which all waves in F_1 have been continually acted on by the wind in a time Z_t. The maximum wave in F_2 is a distance $C_{g1}Z_t$ ahead of its former position. The limiting parameter in this case is $F_{m2} = F_{m1} + C_{g1}Z_t$.

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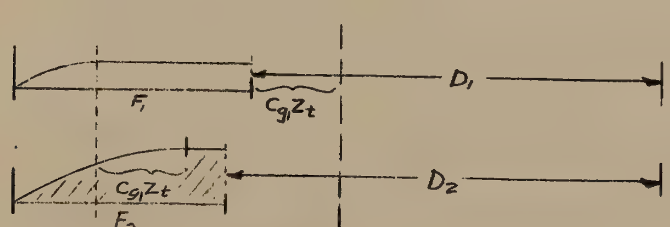
(2) $R_2 = R_1$, $D_2 > D_{m1} - C_{g1}Z_t$ $F_1 > F_{m1}$



This case is similar except that only part of the waves of F_1 have been acted upon by the wind in the time Z_t. However $F_{m2} = F_{m1} + C_{g1}Z_t$ still holds.

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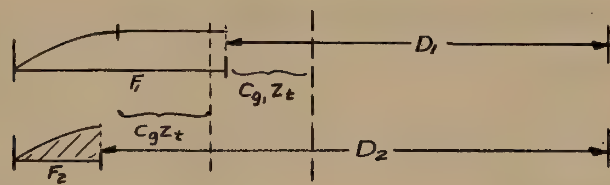
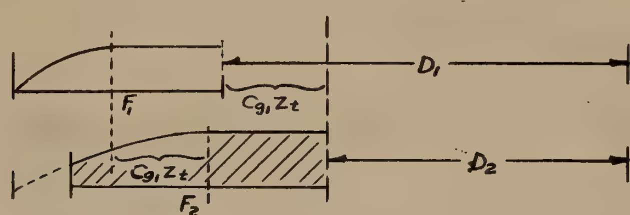
(3) $R_2 = R_1$, $D_2 > D_{m1} - C_{g1}Z_t$, $F_1 > F_{m1}$



In this case the fetch has shrunk so that the front of F_2 determines the position of the maximum waves. F_{m2} is equal to F_2 which we shall write as $F_1 + (D_1 - D_2)$ or $R_1 - D_2$.

The three cases following, in which the rear of F_2 is ahead of the rear of F_1 are analogous except that the area between the backs of F_1 and F_2 is no longer a wave generation area.

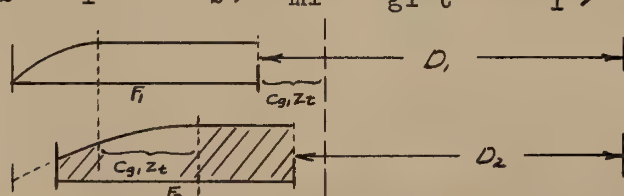
(4) $R_2 < R_1$, $D_2 = D_{m1} - C_{g1}Z_t$, $F_1 > F_{m1}$



$$F_{m2} = F_{m1} + C_{g1}Z_t$$

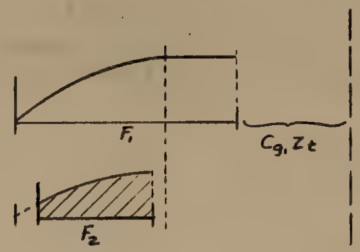
---

(5) $R_2 < R_1$, $D_2 > D_{m1} - C_{g1}Z_t$, $F_1 > F_{m1}$



$$F_{m2} = F_{m1} - C_{g1}Z_t$$

(6) $R_2 < R_1 \quad D_2 > D_{m1} - C_{g1}Z_t \quad F_1 > F_{m1}$



$$F_{m2} = F_1 + (D_1 - D_2) = R_1 - D_2 \neq F_2$$

Summarizing:

(1) & (4) $F_{m2} = F_{m1} + C_{g1}Z_t$

(2) & (5) " " " " "

(3) & (6) $F_{m2} = R_1 - D_2$

and in all the above cases $D_{m2} = D_2$ since the maximum waves are at the head of F_2.

Therefore for forecasting situation (2)

$$\left.\begin{matrix} F_{m2} = F_{m1} + C_{g1}Z_t \quad \text{or} \\ F_{m2} = R_1 - D_2 \end{matrix}\right\} \text{ whichever is smaller}$$

and $D_{m2} = D_2$

For subsequent forecasts note that the F_{m2} & D_{m2} calculated above for the second chart become the F_{m1} & D_{m1} for a third chart, unless the wind speed changes between charts 1 and 2 (see page 32).

c. <u>Forecasting Situation (3):</u>

I. The rear of F_2 is behind the energy front. . . . $R_2 > D_{m1} - C_{g1}Z_t$

A. The rear of F_2 is even with the rear of F_1

or is ahead of it . . . . $R_2 \leqq R_1$

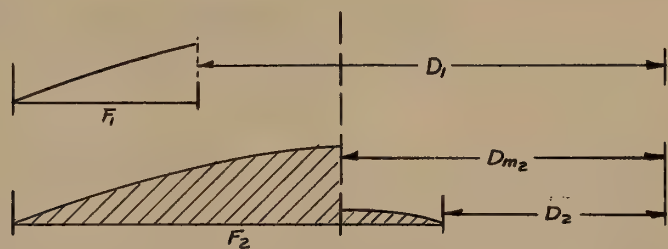
B. The front of F_2 is ahead of the energy front. $D_2 < D_{m1} - C_{g1}Z_t$

In these cases two wave trains will be present, one ahead of and one behind the energy front. That one behind the energy front will in all cases contain the higher waves. This is because the area behind

the energy front will have a longer minimum duration (since waves exist there at the time of chart 1) than the area ahead of the energy front. Only the larger wave train will be analyzed.

Where $F_1 = F_{m1}$

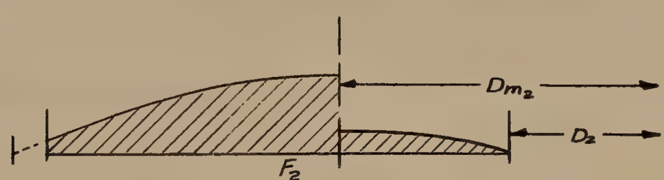
(a) $R_2 = R_1$



Waves in F_1 have been continually subject to wind in the time Z_t. In that time they will grow as shown, and at the time of chart 2 will have a minimum fetch of $F_{m2} = F_{m1} + C_{g1}Z_t$.

Note that $D_{m2} = D_{m1} - C_{g1}Z_t$

(b) $R_2 < R_1$



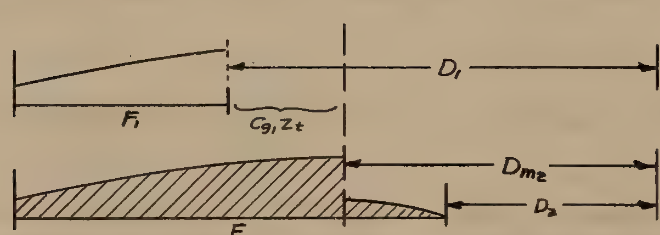
The above analysis holds

$$F_{m2} = F_{m1} + C_{g1}Z_t$$

$$D_{m2} = D_{m1} - C_{g1}Z_t$$

Where $F_1 < F_{m1}$

(c) $R_2 \lesseqgtr R_1$



The above analysis holds, that is:

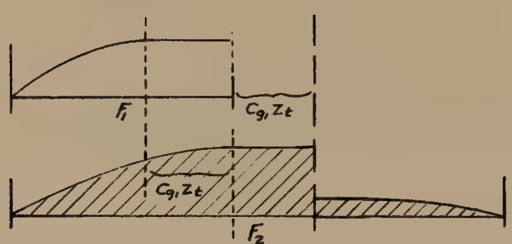
$$F_{m2} = F_{m1} + C_{g1}Z_t$$

$$D_{m2} = D_{m1} - C_{g1}Z_t$$

Note that a forecast made with condition (b) as a starting point on the first weather chart will be analyzed by (c).

Where $F_1 > F_{m1}$

(d) $R_2 \leqq R_1$



The foregoing analysis holds, that is:

$$F_{m2} = F_{m1} + C_{g1}Z_t$$

$$D_{m2} = D_{m1} - C_{g1}Z_t$$

Therefore, in all cases

$$F_{m2} = F_{m1} + C_{g1}Z_t$$

$$D_{m2} = D_{m1} - C_{g1}Z_t$$

Note for subsequent forecasts F_{m2} & D_{m2} above become F_{m1} & D_{m1} for a third chart, unless the wind speed changes between charts 1 and 2 (see page 32).

Also note that if any of the above conditions after Z is the starting point for a subsequent forecast, the maximum waves will not be at the head of F_2. In this case $D_{ml} > D_1$ (see page 31).

d. <u>Forecasting Situations (4), (5), (6), and (7):</u>

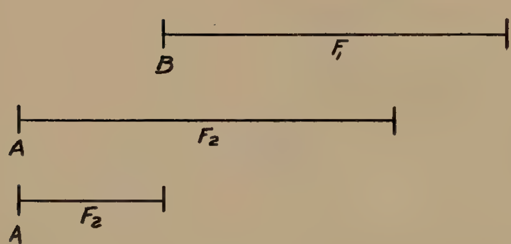
In general:

I. The rear of F_2 is behind the energy

front . . . . $R_2 > D_{ml} - C_{g1} Z_t$

II. The rear of F_2 is behind the rear

of F_1 . . . . $R_2 > R_1$



Wind has been blowing over point B for a time $t \leqq Z_t$ and over point A for zero time. As an approximation, consider wind to have blown over AB a time $\frac{t}{2}$. This will give rise to a wave train in AB with a minimum duration $\frac{t}{2}$, and a minimum fetch corresponding to this minimum duration. This secondary fetch is one in which the fetch available to the secondary winds extends to the front of F_2 (since there is no break of winds at B). However the growth of the waves in this fetch will be limited by the minimum duration $\frac{t}{2}$.

This secondary fetch will ordinarily be of little importance since its minimum duration $\frac{t}{2}$ will be smaller than t_{ml}. It will be important, however, if the front of F_2 is behind the rear of F_1. When this happens, the waves formerly in F_1 will all be waves of decay, and the only new waves being generated are those in F_2.

The three cases considered are:

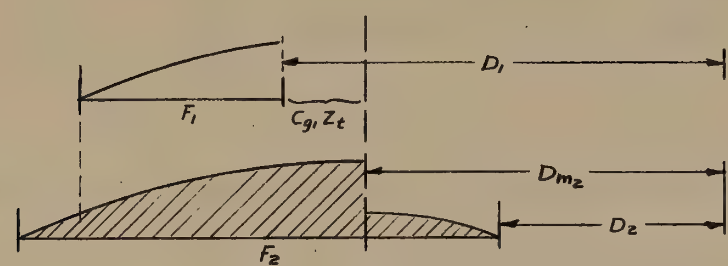
1. The front of F_2 is ahead of the energy front (situation 4).

2. The front of F_2 is ahead of the rear of F_1 but behind the energy front (situations 5 and 6).

3. The front of F_2 is behind the rear of F_1 (situation 7).

---

1. (Situation 4) The front of F_2 is ahead of the energy front . . . . $D_2 < D_{m1} - C_{g1}Z_t$



If the front of F_2 moves ahead of the energy front, the only change over situation 3 will be that the wave train will be lengthened at the rear. The maximum wave will still be a distance $C_{g1}Z_t$ ahead of the front of F_1.

Therefore we have:

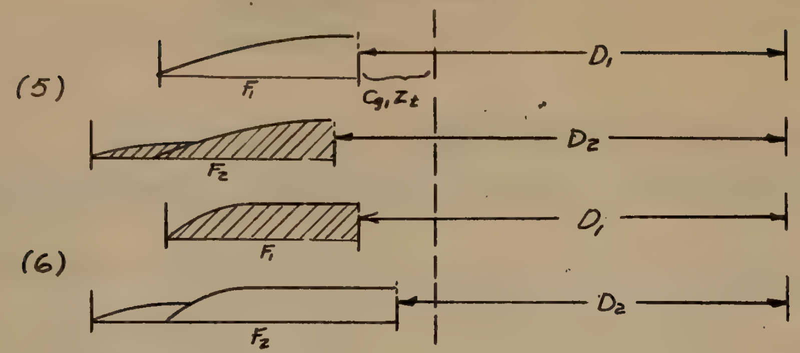
$$F_{m2} = F_{m1} + C_{g1}Z_t$$

$$D_{m2} = D_1 - C_{g1}Z_t$$

---

2. (Situations 5 and 6) The front of F_2 is ahead of the rear of F_1 but behind or abreast of the energy front . . . . $D_2 < R_1$

$$D_2 \geqq D_{m1} - C_{g1}Z_t$$



In these situations the only changes of wave distribution over those of situations 1 and 2 are to lengthen the wave trains at the rear. Therefore, as with situations 1 and 2, these above are classified according to whether the measured fetch F_1 is equal to, greater than, or less than the minimum fetch F_{m1}.

2a. (Situation 5) where $F_1 \leqq F_{m1}$

we have: $F_{m2} = F_{m1} + (D_1 - D_2)$

$D_{m2} = D_2$

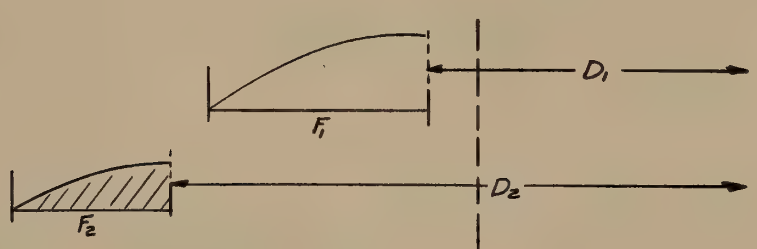
2b. (Situation 6) where $F_1 > F_{m1}$

we have

$$F_{m2} = \begin{cases} F_{m1} + C_{g1} Z_t & \text{or} \\ R_1 - D_2 & \end{cases} \quad \text{whichever is smaller}$$

$D_{m2} = D_2$

3. (Situation 7) The front of F_2 is behind the rear of F_1 . . . . $D_2 > R_1$



The wind has been blowing over point C a time t, where

$$t = \frac{F_2}{R_2 - R_1} Z_t$$

and over point A for zero time. Then as an approximation we will consider AC to be a fetch with a minimum duration of

$$t_{avg} = \frac{F_2 \times Z_t}{(R_2 - R_1) \times 2}$$

Here

$$D_{m2} = D_2$$

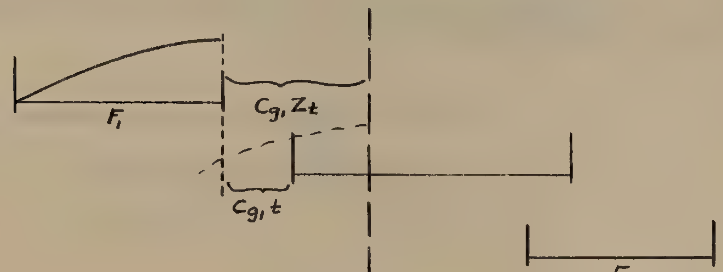
For subsequent forecasts note that the F_{m2}, t_{m2} & D_{m2} calculated above in situations 4, 5, 6, and 7, for the second chart become the F_{m1}, t_m & D_{m1} for a third chart, unless the wind speed changed between charts 1 and 2. (see pages 32 and 36)

e. <u>Forecasting Situation (8):</u>

II. The rear of F_2 moves ahead of the energy front . . . . $R_2 \leqq D_{m1} - C_{g1}Z_t$

In this situation, two wave trains will be present; one behind, and one ahead of the energy front. The wave train behind the energy front will be one of decay entirely, since as soon as the rear of F_2 passes the energy front, these maximum waves, and those behind, will no longer be in a generating area. The wave train ahead of the energy front, is within a generating area, but one of a different type than encountered before. In this advanced generating area, the maximum waves will be at the rear of the fetch.

A. The wave train behind the generating area.



The waves in this area will cease to grow as soon as the rear of F_2 passes the position of the energy front. This will occur some time before the time of the second weather chart, and when it does occur, the waves at the energy front will have grown to their maximum size. Immediately thereafter they will decay.

The minimum duration of these maximum waves will be $t_m + t'$ where t' is the time needed for the back of the fetch to pass the energy front,

i.e. $$C_{g1}t' = \left(\frac{R_1-R_2}{Z_t}\right) t' - F_1 \text{ or } C_{g1}t' = \left(\frac{R_1-R_2}{Z_t}\right) t' - F_{m1} \text{ (if } F_{m1} < F_1)$$

$\left(\frac{R_1-R_2}{Z_t}\right)$ is the speed of the fetch rear.)

solving for t',

$$t' = \frac{F_1 Z_t}{(R_1-R_2)-C_{g1}Z_t} \quad \text{or} = \frac{F_{m1}Z_t}{(R_1-R_2)-C_{g1}Z_t} \quad \text{when } F_{m1} < F_1$$

and since the minimum duration $t_{m2} = t_{m1} + t'$

$$t_{m2} = t_{m1} + \frac{F_1 Z_t}{(R_1-R_2)-C_{g1}Z_t} \quad \text{or } t_{m1} + \frac{F_{m1}Z_t}{(R_1R_2)-C_{g1}Z_t} \quad \text{when } F_{m1} < F_1$$

now, the minimum fetch $F_{m2} \simeq F_{m1} + C_{g1}t'$

therefore

$$F_{m2} = F_{m1} + \frac{F_1\ (C_{g1}Z_t)}{(R_1-R_2)-C_{g1}Z_t} \quad \text{or} \quad F_{m1} + \frac{F_{m1}\ (C_{g1}Z_t)}{(R_1-R_2)-C_{g1}Z_t}$$

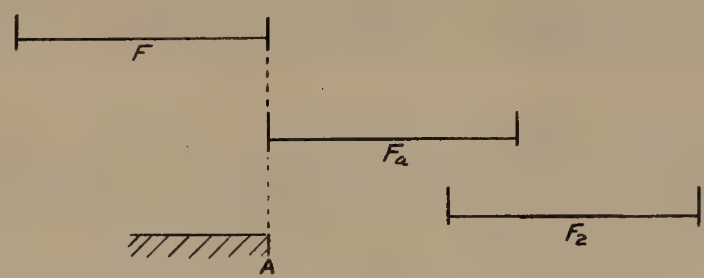
Note that the decay distance for these maximum waves is

$$D_{m2} = D_{m1} - C_{g1}t'$$

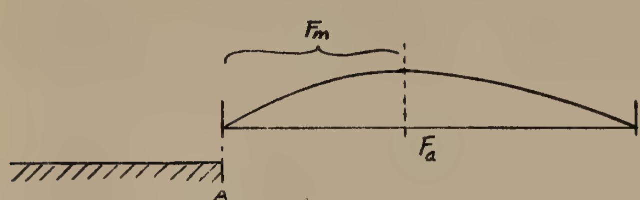
and the time for the start of this decay is the map time of the first chart plus t', i.e. $Z_1 + t'$

B. The wave train ahead of the energy front.

This wave train must be analyzed in a more roundabout fashion than used before.



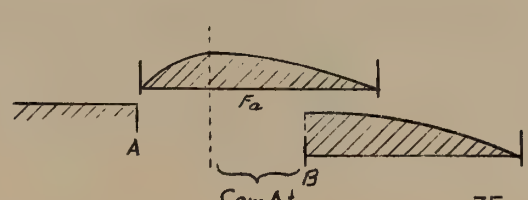
Consider a fetch of constant size, moving to the right at some velocity V_f past a point A representing a point at which waves are always of zero height, (a coastline for example). The position of the fetch F_2 is its final position after a time (say) Z_t, and the position F_a is its position just as the rear passes over point A. When the fetch is at position F_a, the wind will have been blowing over A for a time $t_m = F \times \frac{1}{V_f}$ (where $V_f = \frac{R_1 - R_2}{Z_t}$) represents the velocity of the fetch. With the fetch at position F_a the wave distribution would be as shown below.



Note, F_m is the minimum fetch corresponding to a minimum duration t_m. The maximum waves in the fetch will have C_{gm}, t_m & H_m also corresponding to t_m.

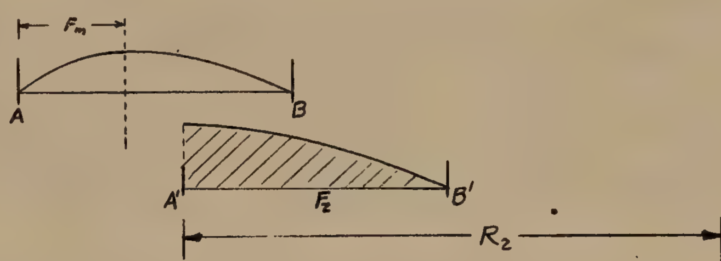
Both the fetch rear and the waves within the fetch are moving, and at some time Δt later the fetch rear will pass the internal "energy front". At that time, the waves at the rear of the fetch are the maximum waves, and will remain so long as the fetch moves with velocity V_f. The time Δt is found by equating

$$V_f \Delta t - F = C_{gm} \Delta t$$



Once the rear of the fetch passes point B we have a condition similar to that of the forecasting situation under discussion, i.e. a fetch is advancing into calm water. In the general case, the fetch size may change but the analysis of the problem is unnecessarily complicated by introducing this changing size. The approximation of constant fetch size may be partially justified on the basis of inaccuracies inherent in the construction of weather maps.

To find the waves at the rear of the fetch (the maximum waves) we reverse the procedure above. That is, at the time of the second chart, the wave distribution will be the final one of the foregoing process. The magnitude of these waves may be determined by finding how far back an imaginery "point A" is.



If t_m, F_m & C_{gm} are the minimum duration, minimum fetch, and group velocity of the waves when the fetch is at F_a, a time Δt before the condition F_2, the "point A" will be a distance $V_f\ \Delta t$ behind A'. The wind will have blown over "point A" a time t_m necessary for the front of the fetch to move from A to B, or a time $\frac{F}{V_f}$. Then in the additional time Δt, the fetch moves to position A'B' (the final position). (Δt is defined by $\Delta t = \frac{F_m}{V_f - C_{gm}}$). At this position, the minimum duration of the waves at the rear of the fetch is

$$t_{m2} = t_m + \Delta t = \frac{F}{V_F} + \frac{F_m}{V_F C_{gm}}$$

$$= \frac{F\,Z_t}{R_1 - R_2} + \frac{F_m\,Z_t}{(R_1 - R_2) - C_{gm} Z_t}$$

and $F_{m2} \simeq F_m + C_{gm}\ \Delta t$

The decay distance is $D_{m2} = D_2 + F_2 = R_2$

Note that for subsequent forecasts, the F_{m2}&D_{m2} calculated above for the second chart become the F_{m1}&D_{m1} for a third chart unless the wind speed changed between charts 1 and 2 (see page 35).

Note too that for subsequent forecasts, the maximum waves are at the rear of the fetch, i.e. $D_{m2} = D_2 + F_2 > D_2$ (see page 31.)

f. Maximum Waves of the First Chart Are Within or at the Rear of F_1:

Of all the foregoing situations only two will give rise to that condition in which the maximum waves are not at the head of F_1. These are situation 3, where the maximum is within F_1, and situation 8, where the maximum wave is at the rear of the fetch. Since, as was noted, these situations become the starting points for subsequent forecasts, they in turn must be analyzed.

The particular forms chosen for the statements of the criteria for all the forecasting situations analyzed have been so generally stated as to include any situation in which the wave is not at the head of the fetch. For example, the criteria for one situation falling within situation 2 are stated and symbolized as follows:

I. The rear of F_2 is behind the energy front: $R_2 > D_{m1} - C_{g1}Z_t$

Note that D_{m1} represents the "decay distance" of the maximum waves in this case within the fetch. (In the previous analysis $D_{m1} = D_1$.)

A. The rear of F_2 is even with the rear of F_1: $R_2 = R_1$

(The position of the maximum wave does not enter.)

1. The front of F_2 is abreast of the energy front:

$$D_2 = D_{m1} - C_{g1}Z_t$$

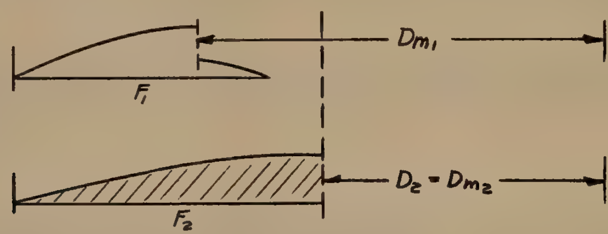
(See I above)

2. The measured F_1 is greater than the minimum fetch F_{m1}:

$$F_1 > F_{m1}$$

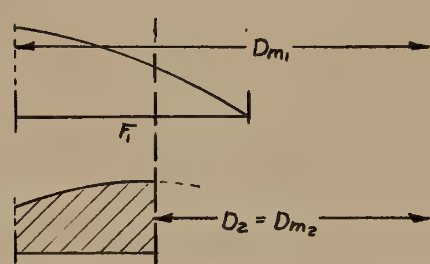
The solutions already presented for situation 2 still hold if the end point of either situations 3 or 8 are the starting points for a forecast whose criteria fit those stated.

Case 1. The wave distribution of the first chart is that of situation 3.



In this case $C_{g1}Z_t$ represents the advance of the <u>maximum</u> waves. All waves in F_1 have been acted on continually <u>by the</u> wind in the time Z_t between charts. The internal maximum, therefore, will grow in this time, and advance a distance $C_{g1}Z_t$. By I.A.1, ($D_2 = D_{m1} - C_{g1}Z_t$) it will appear at the head of F_2, as before.

Case 2. The wave distribution of the first chart is that of situation 8.



It is true that the waves back of the rear of F_1 are waves of decay; but (to the degree of approximation that the advance of the maximum waves' energy front equals $C_{g1}Z_t$) these waves will be as large as the maximum waves, and will advance into the fetch and grow under the influence of the wind within the fetch. The analysis for case 1 will then hold, and the wave pattern will be as shown.

Similar analyses may be performed for the other forecasting situations. The results however will be the same; i.e.,

> the statements of the criteria in the preceding sections include situations in which waves are within or at the rear of the fetch F_1.

g. Winds Change From Weather Chart 1 to Weather Chart 2:

There are two general cases to consider:

1. where U_2 is greater than U_1, or U_2 is less than U_1 but not so much less that U_2 may not supply enough energy to continue generation of waves.

2. U_2 is very much smaller than U_1.

A simple method of determining whether case 1 or case 2 is to be used (when $U_2 \angle U_1$) is given at the end of the discussion of case 1.

Situations 1 through 6

Case 1. U_2 is larger than U_1, or is smaller but not enough so to stop generation of waves.

Bretschneider has shown that for a non-moving fetch F_1 (with a minimum duration t_{m1} and minimum fetch F_{m1}) if, in a time Δt the wind speed changes from U_1 to U_2, for purposes of computation we may consider the change to occur instantaneously after a time $\Delta t/2$.

(The purpose of this device is to permit calculation of the energy imparted to the waves by U_1 with a minimum duration of $t_{m1} + \frac{\Delta t}{2}$. This energy is used to calculate the minimum duration necessary for a wind U_2 to impart this same energy to the waves, and to this new minimum duration is added $\frac{\Delta t}{2}$ to account for the growth of waves after the wind has changed. Note, that in this time if the wind speed had not changed, F_{m1} would have grown to $F_{m1} + C_{g1}\Delta t$ under the influence of U_1 alone.)

Similarly if the wind speed changes from U_1 to U_2 in a time Z_t between weather charts, we may for purposes of computation assume that the change occurs instantaneously after a time $\frac{Z_t}{2}$. If in this time the change in F_{m1} is $F_{m1} + C_{g1}Z_t$ we have the same condition as above and the effect of the changing wind may be readily calculated. This is the case in only part of the forecasting situations analyzed, but the rest of the situations may be treated in a similar manner. That is, if in a time Z_t the minimum fetch changes to

$$F_{m2} = F_{m1} + \Delta F \angle F_{m1} + C_{g1}Z_t$$

we assume that U_1 will continue to blow over the changing fetch for a time $Z_t/2$. In that time the minimum fetch would change to $F_{m1} + \frac{\Delta F}{2} \angle F_{m1} + \frac{C_{g1}Z_t}{2}$. The energy imparted to the waves

will be limited by the minimum fetch $F_{m1} + \frac{\Delta F}{2}$ (which will give waves smaller than those generated in an unlimited fetch -- or at least one with $F_{min} = F_{m1} + \frac{C_{g1} Z_t}{2}$) in the time $\frac{Z_t}{2}$.

We must then find the minimum fetch necessary for U_2 to impart this same energy to the water. Finding this minimum fetch (necessary for U_2 to account for energy imparted to the water by U_1 in the time $t_{m1} + Z_t/2$) is a method of discovering what waves exist at the point $F_{m1} + \frac{\Delta F}{2}$. In the additional time $\frac{Z_t}{2}$ these waves will advance only a distance $\simeq \frac{\Delta F}{2} < \frac{C_{g1} Z_t}{2}$. Therefore to this new minimum fetch we may only add $\frac{\Delta F}{2}$ in accounting for the additional growth of waves under the influence of U_2.

By the above means we may forecast for actual conditions, with the wind speed changing, in situations 1 through 6. The technique is as follows:

1. Calculate $\Delta F = F_{m2} - F_{m1}$
2. Enter plate II with U_1 and move to the intersection of U_1 and $F_{m1} + \frac{\Delta F}{2}$.
3. From this intersection move along a line of constant energy to its intersection with U_2.
4. Moving along U_2 add to this value of fetch, a length $\frac{\Delta F}{2}$
5. Read off at this point t'_{m2} , $F_m'_2$, T_{f2}, H_{f2}

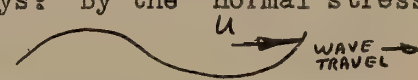
Note that for subsequent forecasts these values for the second chart become the t_{m1}, F_{m1}, T_{f1} & H_{f1} values for a third chart.

If, in step 3, the line of constant energy does not intersect with U_2 we have case 2, where U_2 cannot supply enough energy for further wave generation.

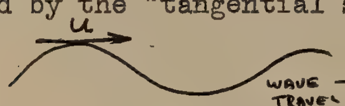
Case 2. $U_2 \not< U_1$

If U_2 is very much smaller than U_1 (or at least so much smaller that U_2 cannot supply the same amount of energy to the water as is supplied by U_1 with a minimum fetch $F_{m1} + \frac{\Delta F}{2}$) the wave generated by U_1 in F_1 decays in the time Z_t, i.e. energy has been lost. If we hold to the assumption that the wind changes from U_1 to U_2 abruptly after a time $\frac{Z_t}{2}$, in this case the waves actually existing after this time will be smaller than those predicted by U_1 with a minimum fetch $F_{m1} + \Delta F/2$. To balance this, the predicted decay of these waves in the additional time $Z_t/2$ should be greater than the actual decay in this time.

Energy is transferred to waves in two ways: By the "normal stress" of the wind blowing against the wave form,



and by the "tangential stress" of the wind pulling at water particles.



If the wind speed equals the wave speed, the normal stress is zero, but energy is still transferred by tangential stress. This comes about because the water particle speed is less than the wave form speed. If we neglect this energy transfer by tangential stress in calculating the wave decay in the case at hand, the decay so predicted will be larger than the actual decay.

Thus we find an "equivalent decay distance" dependent only on the relative wind and wave speeds. This distance is

$$D_{eq} = \left(\frac{D_{m1} - D_{m2}}{2}\right) \left(1 - \frac{U_2}{C'}\right) = D' \left(1 - \frac{U_2}{C'}\right)$$

where C' is the speed of an individual wave in the group whose period T' is determined by U_1 with a minimum fetch of $F' = F_{m1} + \frac{\Delta F}{2}$. In situations 3 and 4, where the maximum significant waves of F_1 are continually under the influence of winds in the time Z_t between charts, this reduces to $D_{eq} = \frac{Z_t}{4}(C' - U_2)$, and in situations 1, 2, 5, and 6, where the maximum significant waves of F_1 are at the head of F_2, this becomes $D_{eq} = \frac{D_{m1} - D_2}{2}\left(1 - \frac{U_2}{C'}\right)$.

For these situations, 1 through 6, (7 and 8 are discussed on pp. 35 & 36) the waves determined from plate II with the use of T', H', F' and D_{eq} are the maximum significant waves within or at the head of the fetch, i.e. with period T_{f2} and height H_{f2}. Their distance from the coast will be, for situations 3 and 4 approximately $D_{m2} = D_{m1} - C_{g1}Z_t$, and for situations 1, 2, 5, and 6 $D_{m2}' = D_2$.

As usual the T_{f2}, H_{F2} and D_{m2}' so determined become the T_{F1}, H_{F1}, and D_{m1} for subsequent forecasting with a third chart.

For additional forecasting with a third chart when the wind either remains the same or again drops off, a similar procedure is followed. To find an additional "equivalent decay distance", it is of sufficient accuracy to average the wind speed of the two charts and calculate either

$$D' = (D_{m1} - D_2)\left(1 - \frac{U_{avg}}{C_1}\right)$$

or

$$D' = C_{g1} Z_t \left(1 - \frac{U_{arg}}{C_1}\right) = \frac{Z_t}{2}(C_1 - U_{avg}) \qquad \text{whichever is smaller}$$

(C_1 is the speed of an individual wave in the group whose period and height are determined by the above method, i.e. $C_1 = 3.04\ T_{F1}$) To this is added the D_{eq} calculated above D_{eq} (for the third chart) $= D'' + D_{eq}$ (for the 2nd chart). The distance so determined, D_{eq}, is the total equivalent decay distance for the waves whose period and height are T' and H' above. The actual distance of these waves from the coast at the time of the third chart is either

$$D_{m2}' = D_{m1} - C_{g1} Z_t = D_{m1} - \frac{Z_t}{2} C_1$$

or

$$D_{m2}' = D_2 \qquad \text{whichever is larger}$$

In other words, in the time between the first and third charts, though the maximum significant waves are at a distance D_{m1} (of the first chart) - D_{m2}' (of the third chart) away from the coast, their significant periods and wave heights have changed as though the waves had moved a distance $D_{eq} + D$ with no wind blowing. (Note: D_{eq} for a 4th chart is the sum of D_{eq} of the third chart and a D" determined as above.)

Situation 8

In situation 8 for the first wave train (that one which becomes a decaying wave train as soon as the rear of F_2 passes the energy front) these same methods may be used if certain substitutions are made for U_2 and Z_t. For U_2 we substitute U_2' equal to the wind speed at the moment the rear of F_2 passes the

energy front, and for Z_t we substitute t' equal to the time needed for the back of the fetch to pass the energy front. These values are:

$$U_2' = \frac{(U_2-U_1)\ F_1}{(R_1-R_2)-C_{g1}Z_t} + U_1 \quad \text{or} \quad U_1 + \frac{(U_2-U_1)\ F_{m1}}{(R_1-R_2)-C_{g1}Z_t}$$

$$t' = \frac{Z_t\ \ F_1}{(R_1-R_2)-C_{g1}Z_t} \quad \text{or} \quad \frac{Z_t\ \ F_{m1}}{(R_1-R_2)-C_{g1}Z_t}$$

For the second wave train (that one ahead of the energy front), to the degree of approximation inherent in the simplified solution presented, we may follow the steps outlined for situations 1 through 6 if we take $F_{m1}=0$. Actually "U" should be changed to some value between U_1 and U_2, but once again, the use of the measured U_1 is correct to the degree of the approximate solution.

Situation 7

For situation 7, a similar degree of approximation holds. The maximum time that the wind has been blowing over any point in F_2 is

$$t = \frac{F_2\ Z_t}{(R_2-R_1)}$$

The wind at this point is at first, $U_2 - \left[(U_2-U_1)\ \left(\frac{F_2}{R_2 - R_1}\right)\right]$

and at the final position of F_2 is U_2

We may use an average wind for forecasting in conjunction with the average t_{avg} given. This average wind $= U_{avg} = U_2 - \frac{(U_2-U_1)\ F_2}{2(R_2-R_1)}$

Note that this is a straightforward forecast, using average values of wind speed and duration.

h. <u>Decay</u>:

The ultimate goal of the foregoing procedures is to forecast (or hindcast) waves for a particular coastal area. So far the methods presented make possible merely the determination of maximum values for significant wave height and period within or at the head of some fetch area at sea. To complete these forecasting procedures, methods must be devised to predict the characteristics of these waves after they have left the generating area and have arrived at the coast.

(1) Fetch is Imparting Energy to the Waves: If a fetch being analyzed is one in which the wind is imparting more energy to the waves than is being lost by these waves, the problem of wave decay need not be discussed for forecasting situations 3 and 4, and the second wave train of situation 8. In these cases, the maximum significant waves determined by use of the methods presented in the preceding sections are wholly within an area of wave generation and will not become decaying waves until they leave this fetch area. In situations 1, 2, 5, and 6 if $D_2 > D_{m1}$, and in situation 7, the maximum significant waves are, at the time of the second weather chart, at the head of F_2. However the fetch front in these cases did not keep pace with the energy front of those waves generated in F_1. That is, these waves leave the fetch area immediately after the time of the first chart and, staying continually ahead of the fetch front, decay. Therefore if further prognostic or synoptic charts indicate that these waves remain outside the influence of the wind (the fetch area) these waves may be decayed to the coast immediately by use of plate II, their decay distance, period, and height as determined from the first synoptic chart. In situations 1, 2, 5, and 6 if $D_2 < D_{m1}$, the above procedure may be followed, and in addition waves decayed from a distance $D_{m1}-D'$, whose periods and heights are T' and H' (found with F' and U_1) may also be determined. The time of the start of decay of these waves is $Z_t/2$ after the time of the first weather chart. Note that the maximum waves at sea (or those waves with maximum energy) are at the head of F_2 and if subsequent charts permit, may in turn be decayed to the coast. The first wave train of situation 8 becomes a train of decay at a time $t' = \dfrac{F_1 Z_t}{(R_1-R_2)-C_{g1}Z_t}$ or

$$t' = \frac{F_{m1}Z_t}{(R_1-R_2)-C_{g1}Z_t} \quad (\text{if } F_{m1} < F_1)$$

after the first chart, with a decay distance of $D_{m2}=D_{m1}-C_{g1}t'$.

(2) Wind Decreases so that Energy is Lost by the Waves: If the fetch being analyzed is one in which more energy is being lost by the waves than is being imparted to them by the wind, the process of decay has already started. The procedures for finding the characteristics of those waves which are still under the influence of wind has already been described, and, indeed, for situations 3 and 4 and the second wave train of situation 8, these are the only waves of interest. However, in situations 1, 2, 5, and 6 if $D_2 < D_{m1}$ and in situation 8 (the first wave train), the maximum waves present at the head of F_2 are of academic interest only, for waves larger than these have already started to decay. The characteristics of one set of decayed waves may be determined for situations 1, 2, 5, and 6 with $D_2 < D_{m1}$ by noting that those waves a distance D_{m1} from the coast at the time of the first chart outrun the fetch immediately. These then, may be decayed to the coast by use of Plate II with T_{F1}, H_{F1}, F_m, and D_{m1}. The characteristics of another greater set of decayed waves, but one arriving

later than those above may be determined as follows. Still assuming that the wind speed between charts changes abruptly in a time $Z_t/2$ after the time of the first chart, for situations 1, 2, 5, and 6 with $D_2 < D_{m1}$ and situation 8, first wave train, the maximum wave generated by U_1 will be at some point approximately D_{m1}-D' from the coast after a time $Z_t/2$, (this is the position of fetch front after that time). The wave so generated will be that whose maximum fetch is $F' = F_{m1} + \frac{\Delta F}{2}$ under the influence of U_1, i.e. it will have period and height of T' and H' respectively. In an additional time $Z_t/2$ this wave will move completely outside the fetch a distance approximately $\frac{C_g Z_t}{2}$ under the influence of no wind, (simple decay). These waves may be decayed to the coast. (Including in this decay the effect of following or opposing winds is usually outside the limits of accuracy of the synoptic charts themselves, and certainly so for a prognostic chart.) The waves still under the influence of the fetch (the maximum waves being at the head of F_2) must have similar procedures applied to them for subsequent charts. Note that these waves at the head of F_2 will always be waves whose periods and heights have changed as though they moved through a decay distance D_{eq} through still air from the point a distance D_{m1}-D' from the coast. In calculating T_D&H_D the total equivalent decay distance should be used, but in computing the travel time t_D, the actual distance from the coast of these waves D_{m2}' must be used.

For situation 7 and situations 1, 2, 5, and 6 where $D_2 > D_{m1}$ those waves originally at F_1 will begin to decay through "still air" immediately after the time of the first weather chart. The characteristics of these waves may be determined by the use of plate II and T_{F1}, H_{F1}, F_{m1}, & D_{m1}.

(3) Wind Speed is Negligible in the Fetch: If the fetch being analyzed is one in which the winds of a second chart are negligible, the characteristics of decayed waves may still be determined by holding to the assumption of wind speed changing abruptly between the first and second charts. If we continue the generation (or decay under the influence of U_1) going on at the time of the first chart, for an additional time $Z_t/2$, and then decay these waves to the coast, the heights and periods so determined may be somewhat larger than they should, but the difference will be small.

For the case in which F_1 is a generating fetch, this decay should take place a time $Z_t/2$ after the first chart time, for a distance of $D_{m1} - C_{g1}Z_t/2$, and with maximum waves of T' & H'.

For the case in which F_1 is a decaying fetch (waves losing energy) the decay of period and wave height should be continued for an additional time $\frac{Z_t}{2}$ using for the new equivalent decay distance

$$D_{eq}' = \frac{C_{g1}Z_t}{2}\left(1 - \frac{U_1}{2xC_1}\right) + D_{eq1} \text{ (of 1st chart)}$$

$$= \frac{Z_t}{8}(2C_1 - U_1) + D_{eq1} \text{ (of 1st chart)}$$

8. Diminution of Swell:

Whenever it becomes necessary to determine when swell will first fall below a certain height, the problem may be solved by considering the relation between wave height at any point in the fetch, the length of the fetch, and the actual duration time. If the minimum fetch is smaller than the measured fetch, waves within the area of constant wave height and period differ from the waves at the head of the fetch only by their distance from the coast. If the wind within the fetch suddenly goes to zero, the variation of these waves in height, period and time of arrival at the end of decay may be determined by merely changing the decay distance through which these waves move.

In like manner, the variation with decay of those waves between the rear of the fetch and a point F_{min} ahead of the fetch rear may be determined by varying the distance through which the waves in any particular part of this area must move in decaying. These waves however will also vary in height and period within the fetch area, that is, they will vary in height and period with their position in relation to the fetch rear. To determine the waves within the fetch a distance (say) d behind the head of the minimum fetch, it is only necessary to decrease the magnitude of the minimum fetch by an amount d, and use this fetch length and the wind within the fetch to determine wave characteristics. If now the wind within the fetch suddenly goes to zero, the decay distance of these waves is the sum of d and the distance to the head of the minimum fetch.

9. Forecasting Methods and Procedures:

The procedures outlined on the following pages are for the most part purely mechanical graph reading and tabulating steps. For each forecasting situation, tabulation is made of the height, period, and position of the maximum significant wave within or at the head of the measured fetch F_2. In order to make forecasts for specific coastal area, the changes these waves undergo in moving from the fetch to the coastal areas must also be calculated.

To do this, the time that waves leave a fetch area, (if at all) and the decay distance to this point of leaving must both be known. A hindcaster always has this information at hand, since he deals with historical weather situations. A forecaster on the other hand must often guess (a knowing guess, to be sure) at what the weather pattern will be subsequent to the last synoptic chart at hand.

In either case, when this information is obtained, characteristics of waves at a particular area for which forecasts are to be made may be found by use of Plate II, the use of which is illustrated by the typical example on the Plate, and the procedures of Section IIIB-7h.

Table III B-3

Definitions of Symbols

U_1 =	Wind speed	first chart
F_1 =	Measured fetch	first chart
D_1 =	" distance from front of fetch to coast	" "
T_{F1} =	Maximum significant wave period in fetch F_1	" "
H_{F1} =	" " " height " " F_1	" "
C_{g1} =	$1.52 \times T_{F1}$ = group velocity of these waves	" "
C_1 =	$3.04 \times T_{F1}$ = individual wave vel. of these waves	" "
	And the above with subscript 2 for the second chart	
Z_t =	Time between charts 1 & 2	
t_{m1} =	Minimum duration	first chart
F_{m1} =	" fetch	" "
D_{m1} =	"Decay distance" of the max. significant waves	" "
D_{m2} =	" " " " " " "	second chart
t_{m2} =	Minimum duration second chart if $U_1 = U_2$	
F_{m2} =	" fetch " " " "	
t_{m2}' =	" duration " " " $U_1 \neq U_2$	
F_{m2} =	" fetch " " " " " "	
t_D =	Time of decay	
T_D =	Period at end of decay	
H_D =	Wave ht. " " "	
t_{avg} =	a minimum duration $= \dfrac{F_2 Z_t}{2(R_2-R_1)}$	see procedures situation 7
U_{avg} =	a wind velocity $= U_2 - \dfrac{(U_2-U_1)F_2}{2(R_2-R_1)}$	see procedures situation 7

t' = a certain time to use instead of $Z_t = \dfrac{F_1 Z_t}{(R_1-R_2)-C_{g1}Z_t}$) see procedures situation 8 first wave train

U_2' = " " wind speed " " " $U_2 = U_1 + \dfrac{(U_2-U_1)F_1}{(R_1-R_2)-C_{g1}Z_t}$

$F = \dfrac{F_1+F_2}{2}$) see procedures situation 8 second wave train

t_m = a certain minimum duration $= \dfrac{F\ Z_t}{(R_1-R_2)}$

F_m = the minimum fetch corresponding to U_1 & t_m

C_{gm} = " group velocity of waves " " " "

$\triangle t$ = a certain time used in situation 8 $= \dfrac{F_m Z_t}{(R_1-R_2)-C_{gm}Z_t}$

T' = intermediate value of wave period) see procedures situations 1-6.

H' = " " " " height)

a. General:

(i) Calculate or bring from chart 1 and tabulate

V_1, F_1, D_1, D_{m1}, F_{m1}, t_{m1}, T_{F1}, H_{F1}, C_{g1}

(Note: these values are the preceding chart's

U_2, F_2, D_2, D_{m2}, F_{m2}')

(ii) From chart 2, calculate and tabulate U_2, F_2, D_2, C_g, Z_t

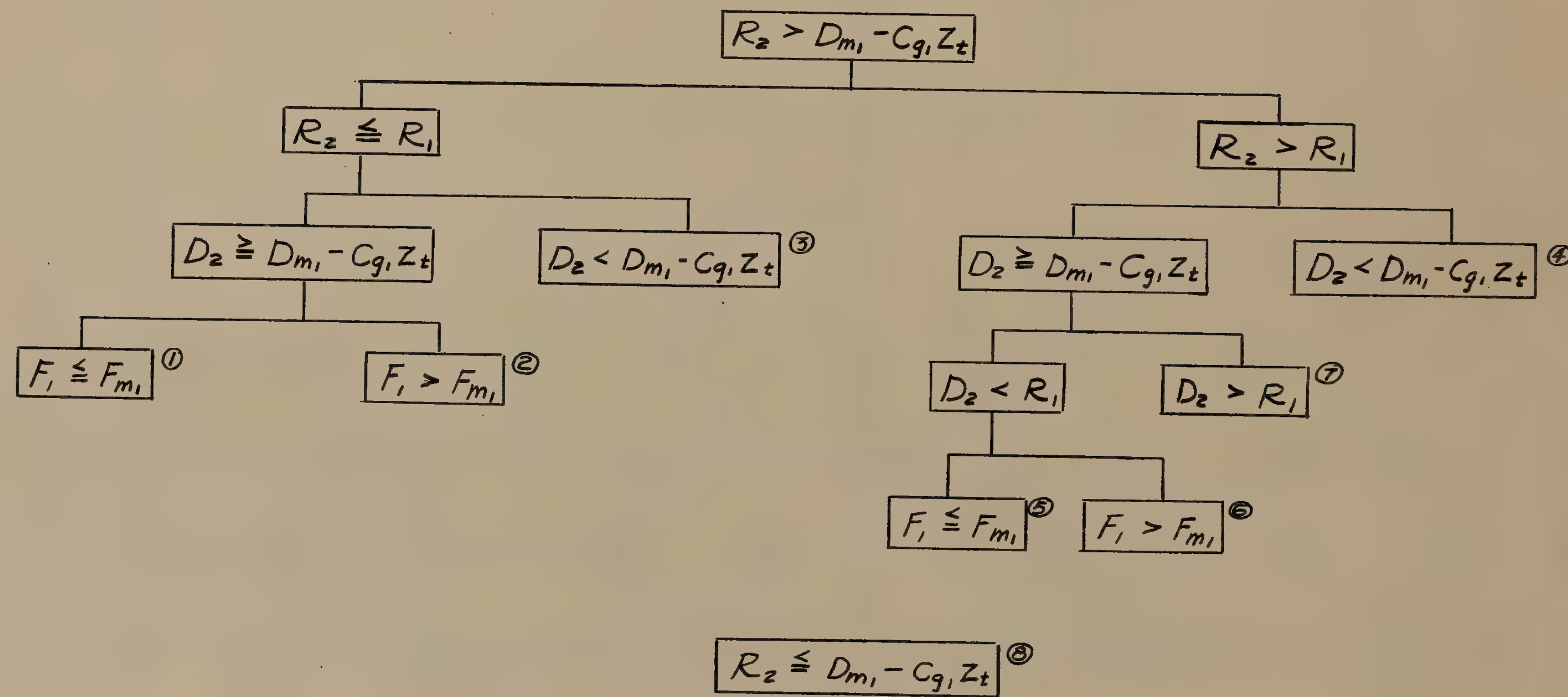
(iii) Always compute $R_1 = D_1 + F_1$, $R_2 = D_2 + F_2$, $D_{m1} - C_{g1}Z_t$

(iv) Classify the situation by the "genealogical chart" on page 42

(v) Note, calculate, and record the forecasting parameters as given on pages 43 and 44

(vi) Follow the procedures as outlined on pages 45 through 49

A. "GENEALOGICAL CHART" FOR WAVE FORECASTING



Circled numbers refer to Forecasting Situations (see following chart and Proceedures)

Table III B-4

Situation	Situation #	Forecasting Parameters
$R_2 > D_{m1}-C_{g1}Z_t$ $R_2 \leqq R_1$ $D_2 \geqq D_{m1}-C_{g1}Z_t$ $F_1 \leqq F_{m1}$	(1)	$F_{m2} = F_{m1} + (D_1-D_2)$ $D_{m2} = D_2$
$R_2 > D_{m1}-C_{g1}Z_t$ $R_2 \leqq R_1$ $D_2 \geqq D_{m1}-C_{g1}Z_t$ $F_1 > F_{m1}$	(2)	$F_{m2} = (F_{m1} + C_{g1}Z_t$ or $R_1 - D_2)$ whichever is smaller $D_{m2} = D_2$
$R_2 > D_{m1}-C_{g1}Z_t$ $R_2 \leqq R_1$ $D_2 < D_{m1}-C_{g1}Z_t$	(3)	$F_{m2} = F_{m1} + C_{g1}Z_t$ $D_{m2} = D_{m1}-C_{g1}Z_t$
$R_2 > D_{m1}-C_{g1}Z_t$ $R_2 > R_1$ $D_2 < D_{m1}-C_{g1}Z_t$	(4)	$F_{m2} = F_{m1} + C_{g1}Z_t$ $D_{m2} = D_{m1}-C_{g1}Z_t$
$R_2 > D_{m1}-C_{g1}Z_t$ $R_2 > R_1$ $D_2 \geqq D_{m1}-C_{g1}Z_t$ $D_2 < D_1 + F_1$ $F_1 \leqq F_{m1}$	(5)	$F_{m2} = F_{m1} + (D_1-D_2)$ $D_{m2} = D_2$

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Security Information

Situation	Situation #	Forecasting Parameters
$R_2 > D_{m1} - C_{g1}Z_t$ $R_2 > R_1$ $D_2 \geqq D_{m1} - C_{g1}Z_t$ $D_2 < R_1$ $F_1 > F_{m1}$	(6)	$F_{m2} = (F_{m1} + C_{g1}Z_t$ or) whichever ( ) is $(R_1 - D_2$) smaller $D_{m2} = D_2$
$R_2 > D_{m1} - C_{g1}Z_t$ $R_2 > R_1$ $D_2 \geqq D_{m1} - C_{g1}Z_t$ $D_2 > R_1$	(7)	$U_{avg} = U_2 - \frac{(U_2 - U_1) F_2}{2(R_2 - R_1)}$ $t_{avg} = \frac{F_2 Z_t}{2(R_2 - R_1)}$ $D_{m2} = D_2$
$R_2 \leqq D_{m1} - C_{g1}Z_t$	(8)	There are two wave trains. The first becomes one entirely of decay at time $Z_1 + \frac{F_1 Z_t}{(R_1 - R_2) - C_{g1}Z_t}$ Use $F_{m2} = F_{m1} + \frac{F_1 (C_{g1}Z_t)}{(R_2 - R_2) - C_{g1}Z_t}$ & for U_2 use $U_2' = U_1 + \frac{(U_2 - U_1) F_1}{(R_1 - R_2) - C_{g1}Z_t}$ for the second use $F_{m1} = 0$ $F_{m2} = F_m + C_{gm}\Delta t$ (See definition of terms for definition of F_m, C_{gm} & Δt.)

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b. <u>Situations 1 through 6:</u>

Parameters are presented as F_{m2} & D_{m2}. Use one of the following methods of forecasting.

A. Generation.

If $U_2 > U_1$ or $U_2 < U_1$ (see below B. and C.)

1. Find $\Delta F = F_{m2} - F_{m1}$
2. Enter Plate I with U_1 and move along U_1 to $F' = F_{m1} + \Delta F/2$
3. From this point follow a constant energy line to its intersection with U_2. (If it doesn't intersect, see below.)
4. Moving along U_2 add to the value of fetch at this point a length $\Delta F/2$.
5. Read off at this point and tabulate t_{m2}', F_{m2}', T_{F2}, H_{F2}
6. For situations 1, 2, 5 and 6, tabulate T', H', F' & $D' = \dfrac{D_{m1} - D_2}{2}$ and see page 46, Decay, Case I.

B. Generation

If in step 3 above, the constant energy line does not intersect with U_2, follow the procedure below (If $U_2 = 0$ see page 47, Decay, Case IIIa.)

3' Using T' from step 2 above and Plate I, tabulate T', H', F', C' & $D' = \dfrac{D_{m1} - D_{m2}}{2}$

4' Calculate and tabulate $D_{eq} = D'(1 - U_2/C')$

5' With D_{eq}, T', H', & F', from Plate II find T_D/T_F & H_D/H_F

6' Multiply these values by T' & H' respectively and tabulate the period and height so determined as T_{F2} & H_{F2} and calculate C_{g2}

7' For situations 1, 2, 5, and 6 see page 46, Decay, Case I

For additional forecasting with subsequent charts, if the wind remains the same or again drops off in an additional time Z_t, but not to zero (if $U_2=0$ or is negligible, see page 48, Decay, Case IIb.):

1. Retabulate R', H', F', C' & D' of step B.3'; tabulate T_{F2}, H_{F2}, C_{g2} & D_{m2} of step B.6' as T_{F1}, H_{F1}, C_{g1}, & D_{m1} and D_{eq} of step B.4' as D_{eq1}

2. Calculate $U_{avg} = \frac{U_1+U_2}{2}$ & either $D_{m1}-D_2$ or $C_{g1}Z_t$ whichever is smaller

3. Calculate and tabulate as D", $(D_{m1}-D_2)\left(1-\frac{U_{avg}}{2C_{g1}}\right)$ or $C_{g1}Z_t\left(1-\frac{U_{avg}}{2C_{g1}}\right)$ whichever is smaller.

4. Add and tabulate as D_{eq}, $D'' + D_{eq1}$

5. With this value and T', H', & F' from Plate II find T_D/T_F & H_D/H_F

6. Multiply these values by T' & H' respectively and tabulate the period and height so determined as T_{F2} & H_{F2}, and calculate C_{g2}.

7. Calculate and tabulate as D_{m2}, $D_{m1} - C_{g1}Z_t$ or D_2 whichever is larger.

8. If, in step 7, $D_2 \geqq D_{m1}-C_{g1}Z_t$, see page 47, Decay, Case II. Repeat this procedure if wind again remains the same or drops off in an additional time Z_t on a subsequent chart.

C. Decay.

Case I. U_2 adds energy to waves, or waves lose energy, but $U_2 \neq 0$. - situations 1, 2, 5 and 6.

a - 1. Using T_{F1}, H_{F1}, F_{M1}, & D_{m1} enter Plate II and read off T_D/T_F & H_D/H_F

2. Multiply these values by T_{F1} & H_{F1} respectively and tabulate the period and height so determined as T_D & H_D.

3. Figure III B-13 using T_D & D_{m1} find t_d

4. By adding t_d to Z_1 calculate and tabulate the time of arrival.

In addition, if $D_2 \angle D_{m1}$ find another decayed train as follows:

b - 1. Using T', H', F', & $D_{1,2} = D_{m1} - D'$ enter Plate II and read off T_D/T_F, H_D/H_F

2. (as above)

3. (as above)

4. Calculate the time of arrival by adding t_d (step 3) to $Z_1 + Z_t/2$

Case II. For additional forecasting; waves already decaying $D_2 \geqq D_{m1} - C_{g1} Z_t$

a - 1. and 2. With T', H', F', & $(D_{1,2} - D_{eq1})$ as decay distance find T_D & H_D as in Case I, steps a - 1. and 2.

3. From Fig. III B-13 calculate the travel time t_d by use of D_{m1} as decay distance, and T_D above.

4. Calculate the time of arrival by adding the t_d so found to Z_1

In addition, if $D_2 \angle D_{m1}$ find another decayed train as follows:

b - 1. and 2. With T', H', F' & $\left[D_{12} - \left(\frac{D_{eq1} + D_{eq}}{2}\right)\right]$ as decay distance find T_D, H_D, & t_d as above, steps 1 and 2.

3. From Fig. III B-13 calculate the travel time by use of $\frac{D_{m1} + D_{m2}}{2}$ as decay distance and T_D above.

4. Calculate the time of arrival by adding the t_d so found to $Z_1 + Z_t/2$.

Case III. U_2 is negligible.

a. F_1 is a generating fetch.

1. and 2. Using T', H', F' & $D_{1,2} = \left(D_{m1} - \frac{C_{g1}Z_t}{2}\right)$ as decay distance find T_D & H_D as in steps a-1 and 2, Case I

3. With the decay distance and T_D above, calculate t_d

4. Find the time of arrival by adding t_d to $Z_1 + Z_t/2$

b. F_1 is a decaying fetch.

1. Find $D_{eq}' = \frac{Z_t}{8}(2C_1 - U_1) + D_{eq1}$

2 and 3. With T', H', F' & $D_{1,2} - D_{eq}'$ as decay distance, find T_D & H_D as in steps a-1. and 2., Case I.

4. From Fig. III B-13 use T_D above and $D_{m1} - \frac{C_{g1}Z_t}{2}$ as decay distance to find t_d

5. Find the time of arrival by adding t_d above to $Z_1 + Z_t/2$

c. Situation 7

A. Generation.

1. Calculate $U_{arg} = U_2 - \frac{(U_2 - U_1)\,F_2}{2(R_2 - R_1)}$ and $t_{arg} = t_{m2}' - \frac{Z_t F_2}{2(R_2 - R_1)}$

2. With these values of wind speed and minimum duration read from Plate I T_{F2}, H_{F2}, F_{m2}'

B. Decay.

1. Using T_{F1}, H_{F1}, F_{m1}, & D_{m1} enter Plate II and read off T_D/T_F & H_D/H_F

2 - 4. (same as Case I 2-4.)

d. Situation 8

A. First Wave Train - Generation

1. Calculate $U_2' = U_1 + \frac{(U_2 - U_1)F_{m1} \text{ or } F_1}{(R_1 - R_2) - C_{g1}Z_t}$ and $t' = \frac{Z_t F_{m1} \text{ or } F_1}{(R_1 - R_2) + C_{g1}Z_t}$

(use the smaller value of F_1 & F_{m1})

2. Using t' instead of Z_t and the wind speed from step 1 U_2' instead of U_2, follow the procedures outlined for situation 3.

B. Decay.

1. Using T_{F2}, H_{F2}, F_{m2}, & D_{m2} determined above, find T_D, H_D & t_d from Plate II and Fig. III B-13.

2. Find the time of arrival by adding the t_d so determined to $Z_1 + t'$

C. Second Wave Train - Generation.

1. Calculate $F = \frac{F_1 + F_2}{2}$ and $t_m = \frac{F\,Z_t}{R_1 - R_2}$

2. From Plate I, at the intersection of t_m with U_1 read off F_m & T_m.

3. Calculate $C_{gm} = 1.52\,T_m$ and $\Delta t = \frac{Z_t F_m}{(R_1 - R_2) - C_{g1} Z_t}$

4. From this find $F_{m2} = F_m + C_{gm}\,\Delta t$

5. Using $F_{m1} = 0$ and F_{m2} from step 4, follow the procedures outlined for situation 3.

D. Decay.

As with situation 3, decay is no problem.

10. Application - Step-by-step Forecast of Sea and Swell:

The following example is given to illustrate the procedure used in forecasting sea and swell. The forecast (or hindcast) is based on a short series of weather maps analyzed by the U.S. Weather Bureau, Washington, D.C., for facsimile transmission. Figures III B-14 to III B-19 show reproductions of portions of North Pacific Ocean weather maps from which the particular storm was selected. The forecast is made for Pt. Arguello, California, where a wave recorder (Mark III) had been in operation, the wave records of which are used to compare with the forecast (hindcast). These weather maps are 12-hourly surface maps (except one 6-hourly map) period of 1230 GMT 25 Oct. 1950 to 1230 GMT 27 Oct. 1950.

The forecast is made by selecting the storm shown in Figure III B-16 - 1230Z, 26 Oct. 1950. This storm has suddenly intensified from that shown on the previous map, Figure III B-15, and swung about so that there is an effective fetch with high winds for Pt. Arguello. The storm gradually decreases in intensity until, as is shown in Figure III B-18, winds in the fetch area are negligible.

The resulting wave forecasts made from this series of maps are shown in Figure III B-20 together with the recorded wave heights and periods. No refraction corrections have been applied to the forecasted wave heights.

Step I

Weather chart of 26 Oct. 1230Z (Fig. III B-16)

Fetch #	2	
Δn	65° of latitude	
R°	7° cyclonic	
Lat	38°	
BF	8.10	
ΔT_{sa}	+1° F	
V_g	70 knots	
U/V_g	0.59	
U_2	41 knots	The Fetch became effective sometime between 0030 & 1230 26 Oct., say at 0630, hence the 6 hr. duration
D_2	660 NM	
F_2	360 NM	
	6 hr. dur.	

D_{m2}	660	The Fetch is assumed to be non-moving; therefore, plate I immediately gives these values. Note that the sub-script -2 is used for the current chart.
F_{m2}'	58	
t_{m2}'	6	
T_{F2}	8.9	
H_{F2}	17	
C_{g2}	13.5	

Step II

Weather chart of 27 Oct. 0030Z

Fetch #	2	
Δn	.70° Lat.	
R°	10° cyc.	
Lat	37°	
BF	9	
ΔT_{sa}	+1°F	
V_g	67 kn.	
U/V_g	0.60	
U_2	40 kn.	
D_2	430	
F_2	470	
U_1	41	These values are retabulated from step I, in which their subscript was 2.
D_1	660	
F_1	360	
D_{m1}	660	
F_{m1}	58	
t_{m1}	6	
T_{F1}	8.9	
H_{F1}	17	
C_{g1}	13.5	
Z_t	12 hr.	the time between charts
$C_{g1}Z_t$	162 NM	

Step II (continued)

R_2	900	The relative values of these parameters determine the forecasting situation, see chart p. 42.
$D_{m1}-C_{g1}Zt$	498	
R_1	1020	
situation #	(3)	
F_{m2}	220	Determined in accordance with table III B-4
$\Delta F/2$	81	See procedures situation 1 through 6, paragraph A.
F'	139	
D_{m2}'	498	
t_{m2}'	230	
T_{F2}	17	
H_{F2}	27	
C_{g2}	19.0	

Step III

Weather chart of 27 Oct. 0630Z

Fetch #	2
Δn	1.0
R^o	straight
Lat	36.5°
ΔT_{sa}	0°
V_g	47
U/V_g	0.62

U_2	29
D_2	420
F_2	460

U_1	40	See Step II
D_1	430	
F_1	470	
D_{m1}	498	
F_{m1}	230	
t_{m1}	17	
T_{F1}	12.5	
H_{F1}	27	
C_{g1}	19.0	

Z_t	6
$C_{g1}Z_t$	114
R_2	880
$D_{m1}-C_{g1}Z_t$	384
R_1	900
situation #	(2)

Decay with T_{F1}, H_F, etc.

T_D/T_F	1.29
H_D/H_F	0.41
T_D	16.2
H_D	11'
t_d	20 hrs.
date	27 Oct.
time	1230 PST

Note the 8 hr difference between GMT & PST

F_{m2}	344
$\Delta F/2$	57
F'	287

T'	13
H'	29
C'	39.5
D'	39
Deq	10
T/TF	~1.0
H/HF	~1.0

Because the wind speed between charts dropped from 40 kn. to 29 kn. the procedures of sit. 1 through 6, paragraph B must be followed. Step #7' of this paragraph calls for the calculation of two sets of values for decay; one using subscript $_1$ parameters, and one with primed parameters

D_{m2}	420
T_{F2}	13
H_{F2}	29
C_{g2}	19.8

Decay with T' H' etc.

D	459
T_D/T_F	1.25
H_D/H_F	0.47
T_D	16.2
H_D	13.6
t_d	18.5 hrs.
date	27 Oct.
Time	1400 PST

Note that $D=D_{m1}-D'$

Step IV

Weather chart of 1230Z

The winds in the fetch area under consideration have become negligible. No map reductions need be performed. The procedure to be followed is that of Case III b.

D_{m1}	420	The preceding step's D_{m2} & C_{g2}
C_{g1}	19.8	
C_1	39.6	$= 2\ C_{g1}$
Deq'	48	see case III b 1
T'	13	
H'	29	
F'	287	
$D_{(for\ T\&H)}$	372	$= D_{m1} - Deq'$
T_D/T_F	1.21	
H_D/H_F	0.48	
T_D	15.8	
H_D	13.9	
D(for t_d)	360	$= D_{m1} - \frac{C_{g1} Z_t}{2}$
t_d	15	
Date	27 Oct.	
time	1630	

Note D for change in T & H is not the same as D for travel time t_d

Step V

Diminution of Swell
(Section B-8)

The procedure is a step-by-step decrease of the minimum fetch available to the last generating wind (in this case 40 knots) and an increase by the same amount of both the

effective decay distance for periods & heights (D for T & H) and the actual decay distance.

	(1)	(2)	(3)
U	40	40	40
"F'"	187	87	50
T'	11.8	9.8	8.3
H'	25	20	15
DT,H	472	572	609
T_D/T_F	1.31	1.41	1.49
H_D/H_F	0.38	0.29	0.21
T_D	15.5	13.8	12.4
H_D	9.5	5.8	3.2
D_{td}	460	560	597
t_d	19.5	27	32
Date	27 Oct.	28 Oct.	28 Oct.
time	2100PST	0430PST	0930

Step VI

A secondary fetch

After the main storm has been dealt with the weather charts must be reviewed for secondary fetches. One is noted as F_2' on Fig. III B-18. It has its beginnings in the upper part of the preceding charts F_2. This generation area is analyzed in a manner similar to that applied to the major storm. Map reductions will not be tabulated. For lack of information, the decay in column (4) is computed with D = 750.

(1)		(2)		(3)		(4)		Diminution of swell (5)	
Date	27 Oct.	Date	27 Oct.	Date	27 Oct.		(27 Oct)		
time	0030Z	time	0630Z	time	1230Z		(1830Z)		
U_2	35	U_2	38.5	U_2	32.5		neglig.		
D_2	700	D_2	600	D_2	750				
F_2	200	F_2	300	F_2	420				
	3 hr. duration	U_1	35	U_1	38.5				
D_{m2}'	700	D_1	700	D_1	600				
F_{m2}'	20	F_1	200	F_1	300				
t_{m2}'	3	D_{m1}	700	D_{m1}	644	D_{m1}	750	D	850
T_{F2}	6.2	F_{m1}	20	F_{m1}	70	F_{m1}	220	F	120
H_{F2}	9	t_{m1}	3	t_{m1}	7	t_{m2}'	18		
C_{g2}	9.4	T_{F1}	6.2	T_{F1}	9	T_{F2}	10.8	T	9.4
		H_{F1}	9	H_{F1}	17	H_{F2}	19	H	17
		C_{g1}	9.4	C_{g1}	13.7	C_{g2}	16.4		
		Z_t	6	Z_t	6	Z_t	6		
		$C_{g1}Z_t$	56	$C_{g1}Z_t$	82	$C_{g1}Z_t$	82		
		R_2	900	R_2	1170				
		$D_{m1}-C_{g1}Z_t$	644	$D_{m1}-C_{g1}Z_t$	562				
		R_1	900	R_1	900				
		situation #	(3)	situation #	(6)				
		F_{m2}	76	F_{m2}	150				
		$\Delta F/2$	28	$\Delta F/2$	40				
		F'	48	F'	110				

(1)	(2)		(3)		(4)		(5)	
	D_{m2}'	644	D_{m2}'	750				
	F_{m2}'	70	F_{m2}'	220				
	t_{m2}'	7	t_{m2}'	18				
	T_{F2}	9	T_{F2}	10.8				
	H_{F2}	17	H_{F2}	19				
	C_{g2}	13.7	C_{g2}	16.4				
			D	644	D	750	D	850
			$T_D/T_F (T_{F1})$	1.33	T_D/T_F	1.30	T_D/T	1.40
			$H_D/H_F (H_{F1})$	0.30	H_D/H_F	0.32	H_D/H	0.25
			T_D	12.0	T_D	14.1	T_D	13.2
			H_D	5.1	H_D	6.1	H_D	4.2
			t_d	35	t_d	35	t_d	42.5
			date	28 Oct.	date	29 Oct.	date	29 Oct.
			time	0930Z	time	0130	time	0900

Step VII

Secondary Fetch

Another fetch, that one marked "secondary fetch" on Fig. III B 16 & 17 may also be analyzed.

Date	26 Oct.	Date	27 Oct.	Date	27 Oct.
Time	1230Z	time	0030Z	time	0630
U_2	36	U_2	30	U_2	0
D_2	400	D_2	150		
F_2	425	F_2	350		
	3 hr. dur.	U_1	36		
D_{m2}'	400	D_1	400		

F_{m2}'	20	F_1	425		
t_{m2}'	3	D_{m1}	400	D_{m1}	283
T_{F2}	6.4	F_{m1}	20	F_{m1}	180
H_{F2}	9	t_{m1}	3	t_{m1}	16
C_{g2}	9.7	T_{F1}	6.4	T_{F1}	10
		H_{F1}	9	H_{F1}	10
		C_{g1}	9.7	C_{g1}	15.2
		Z_t	12	Z_t	6
		$C_{g1}Z_t$	117	$C_{g1}Z_t$	91
		R_2	500		
		$D_{m1}-C_{g1}Z_t$	283		
		R_1	825		
		sit.#	(3)		
		F_{m2}	137		
		$\Delta F/2$	58	$\Delta F/2$	45
		F'	78	F'	225
				T'	10.5
		D_{m2}'	283	H'	17.5
		F_{m2}'	180		
		t_{m2}'	16		
		T_{F2}	10		
		H_{F2}	17		
		C_{g2}	15.2		
				D	238
				T_D/T_F	1.21
				H_D/H_F	0.51
				T_D	12.7
				H_D	9
				t_d	12.5
				date	27 Oct.
				time	0800

SUMMARY OF FORECAST

Date	27 Oct.	27 Oct.	27 Oct.	27 Oct.	27 Oct.	28 Oct.	28 Oct.
Time PST	0800	1230	1400	1630	2100	0430	0930
T_D	12.7	16.2	16.2	15.8	15.5	13.8	12.4
H_D	9	11	13.6	13.9	9.5	5.8	3.2
Direction	WWNW	WWNW	WWNW	WWNW	WWNW	WWNW	WWNW

Date	28 Oct.	29 Oct.	29 Oct.
time PST	0930	0130	0900
T_D	12.0	14.1	13.2
H_D	5.1	6.1	4.2
Direct	NW	NW	NW

11. References:

III B-1. Arthur, R.S. - "Procedure for Computing Surface Wind Velocities" - Scripps Institution of Oceanography, Wave Report 35, La Jolla, California, February 1945.

III B-2. Basquin, E.A. - "The Diminution of Ocean Swell" - Unpublished manuscript, 1951.

III B-3. Bretschneider, C.L. - "The Generation and Decay of Wind Waves in Deep Water" - Institute of Engineering Research, University of California, Series 29, Issue 46, Berkeley, California, August 1951, (unpublished). RESTRICTED

III B-3a. Bretschneider, C.L. - "Revised Wave Forecasting Curves and Procedures" - Institute of Engineering Research, University of California, Series 29, Issue 47, Berkeley, California, September 1951, (unpublished). RESTRICTED

III B-4. Kaplan, Kenneth - "A Practical Manual for Wave Forecasting" - Institute of Engineering Research, University of California, Series 29, Issue 40, Berkeley, California, September 1951, (unpublished). RESTRICTED.

III B-5. Petterssen, S. - "Weather Analysis and Forecasting" - McGraw Hill, New York, New York, 1940.

III B-6. Putz, R.R. - "Wave Height Variability: Prediction of Distributed Function" - Institute of Engineering Research, University of California, Series 3, Issue 318, Berkeley, California, December 1950.

III B-7. Sverdrup, H. U. and Munk, W.H. - "Wind Sea and Swell: Theory of Relations for Forecasting" - U.S. Navy, Hydrographic Office Pub. No. 601, March 1947.

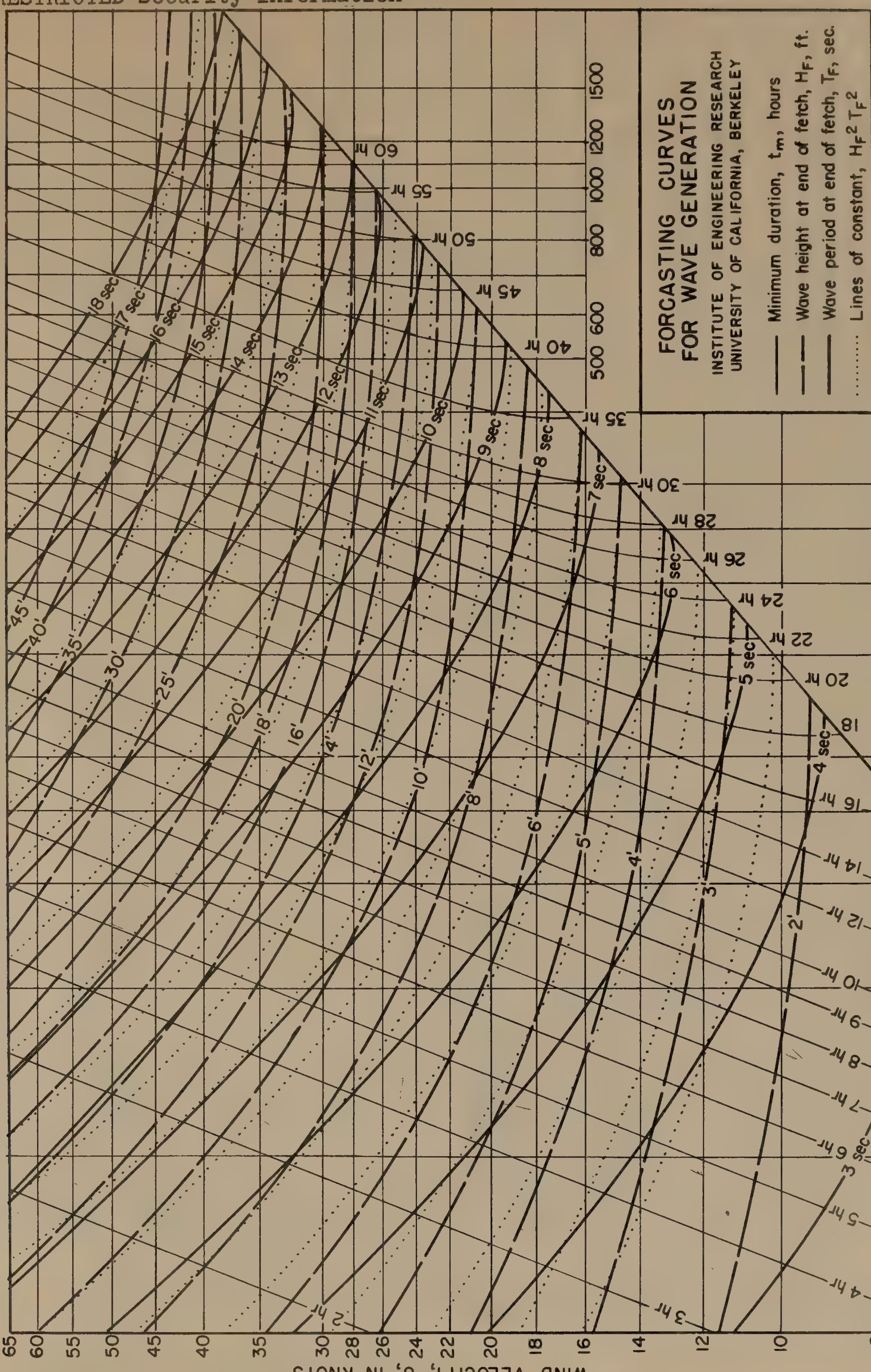
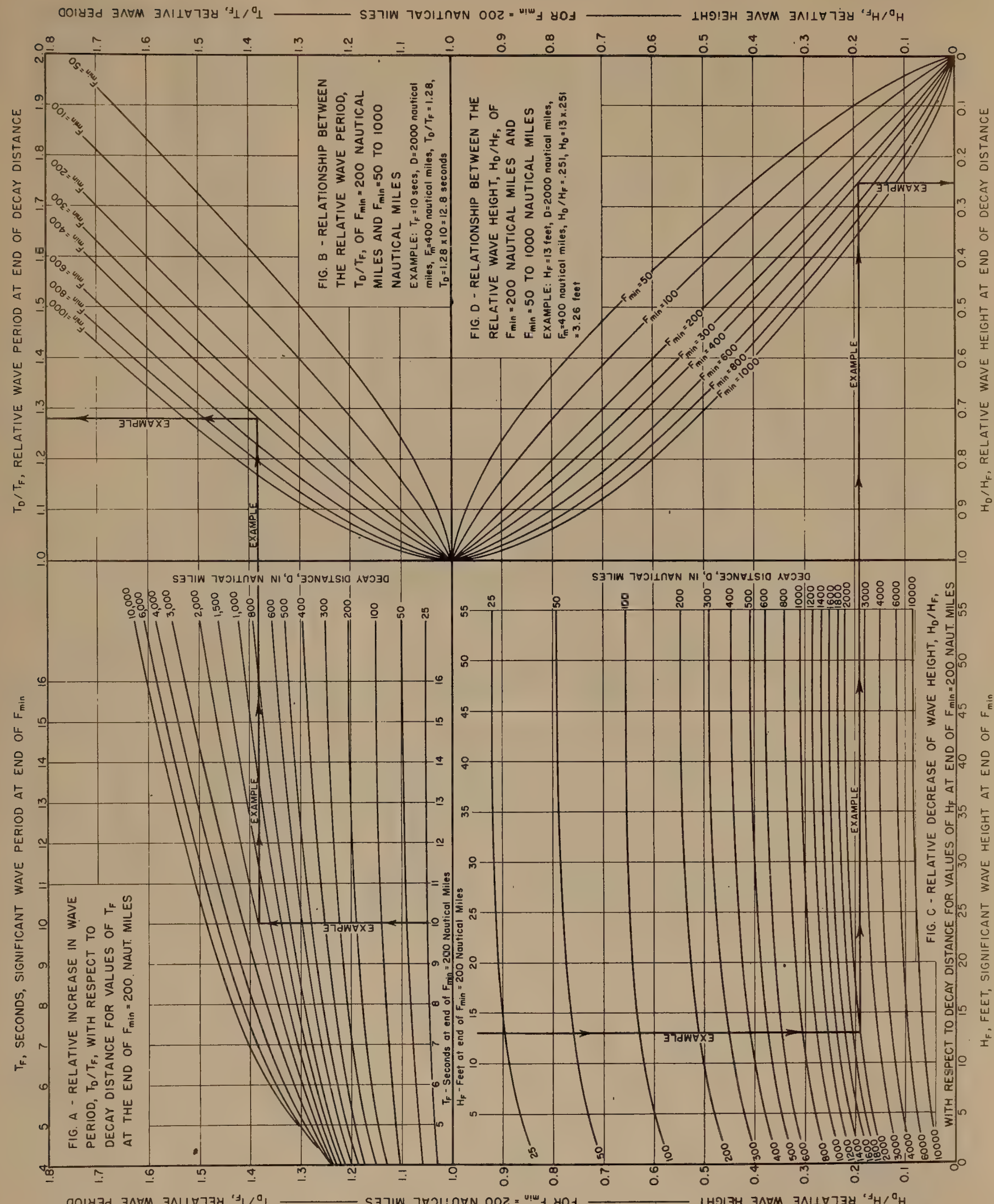
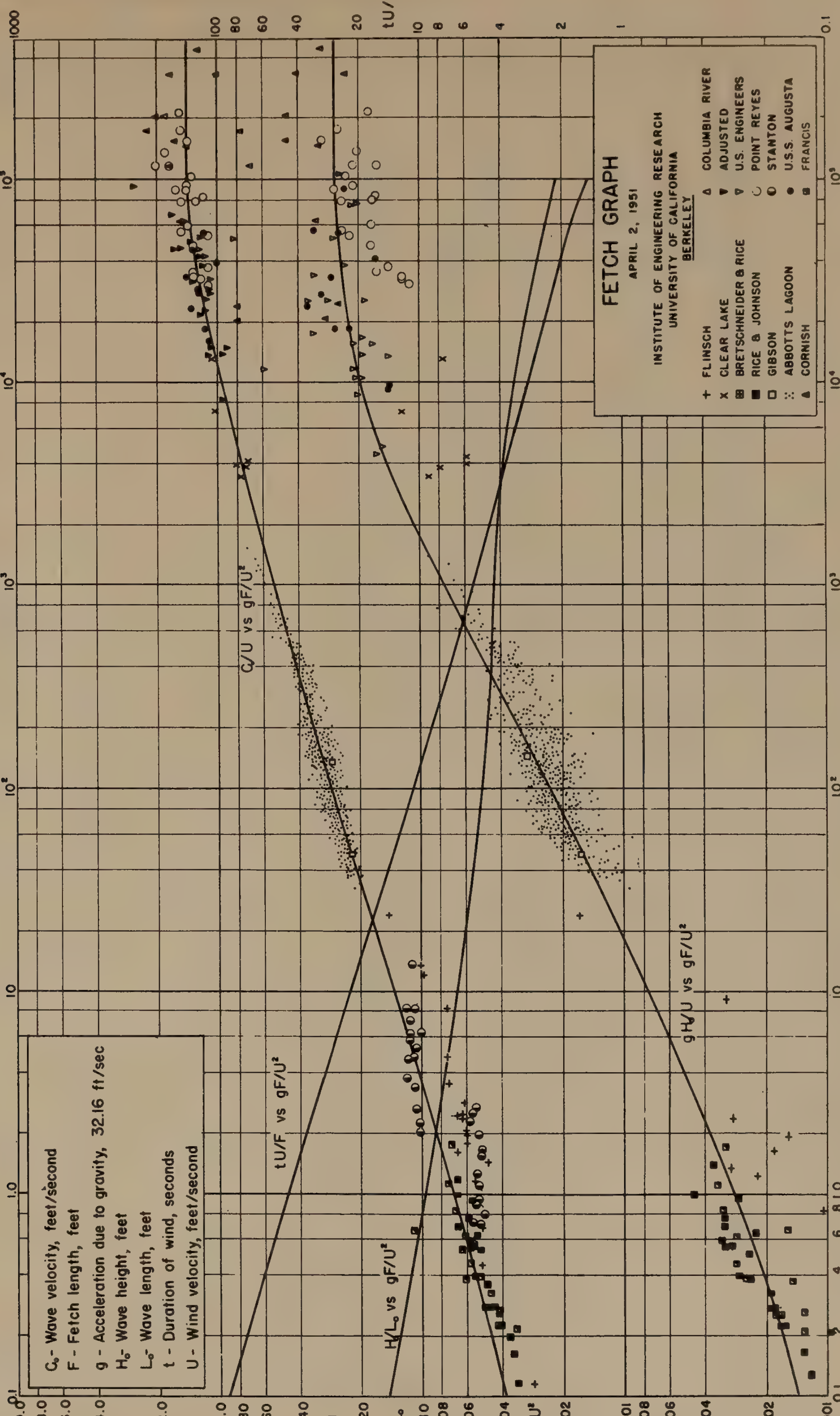

FORCASTING CURVES
FOR WAVE GENERATION
INSTITUTE OF ENGINEERING RESEARCH
UNIVERSITY OF CALIFORNIA, BERKELEY
Minimum duration, t_m, hours
Wave height at end of fetch, H_F, ft.
Wave period at end of fetch, T_F, sec.
Lines of constant, $H_F^2 T_F^2$
FETCH DISTANCE, F, IN NAUTICAL MILES
WIND VELOCITY, U, IN KNOTS

PLATE I



UNIVERSITY OF CALIFORNIA HYD - 5430 - 155

PLATE II



HYD-5155-155

FIGURE III B-1

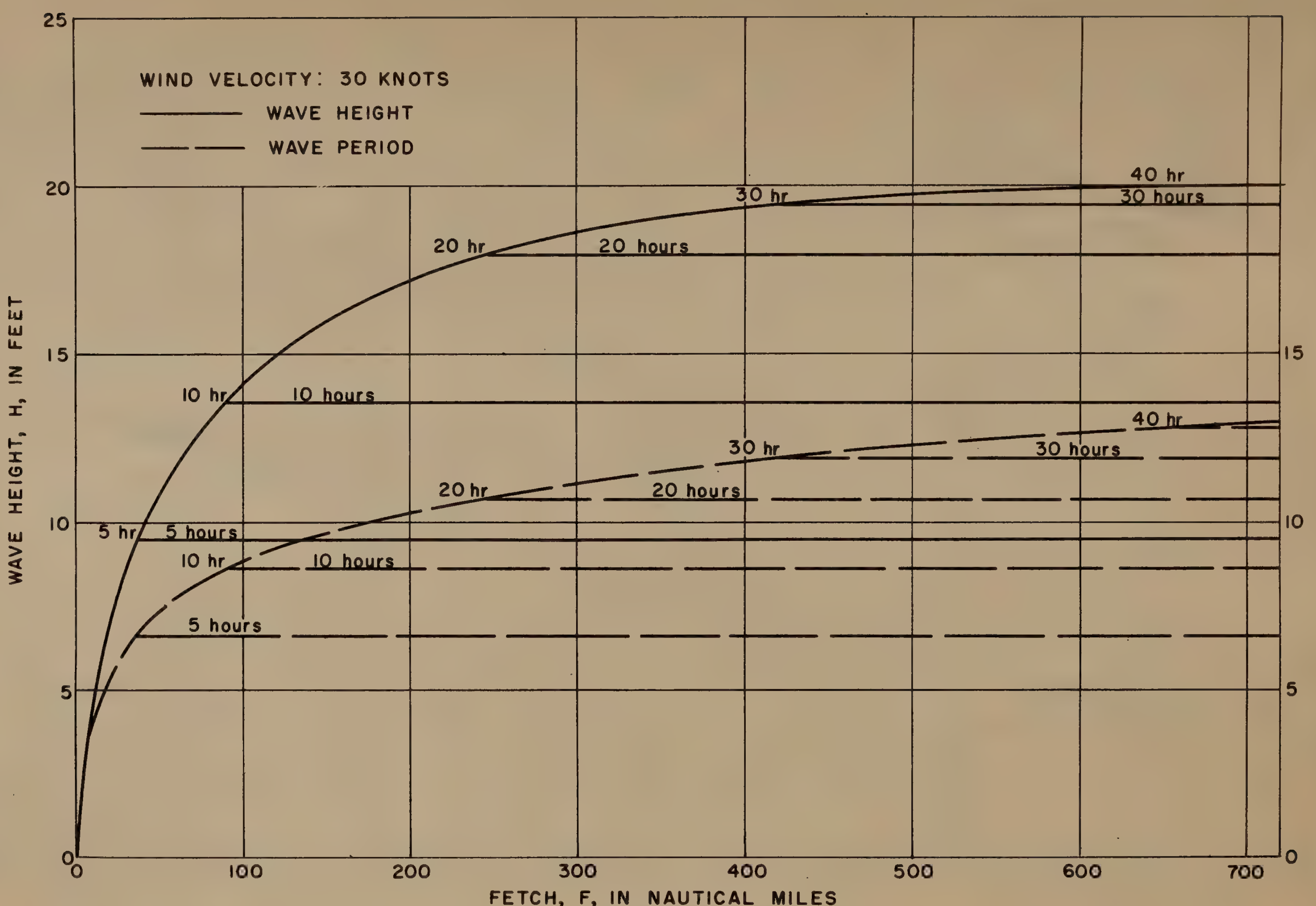


FIG. III B-2 - Wave Height and Wave Period as functions of distance from coast line at intervals from 5 hours to 40 hours after a wind of 30 knots started to blow over an undisturbed water surface.

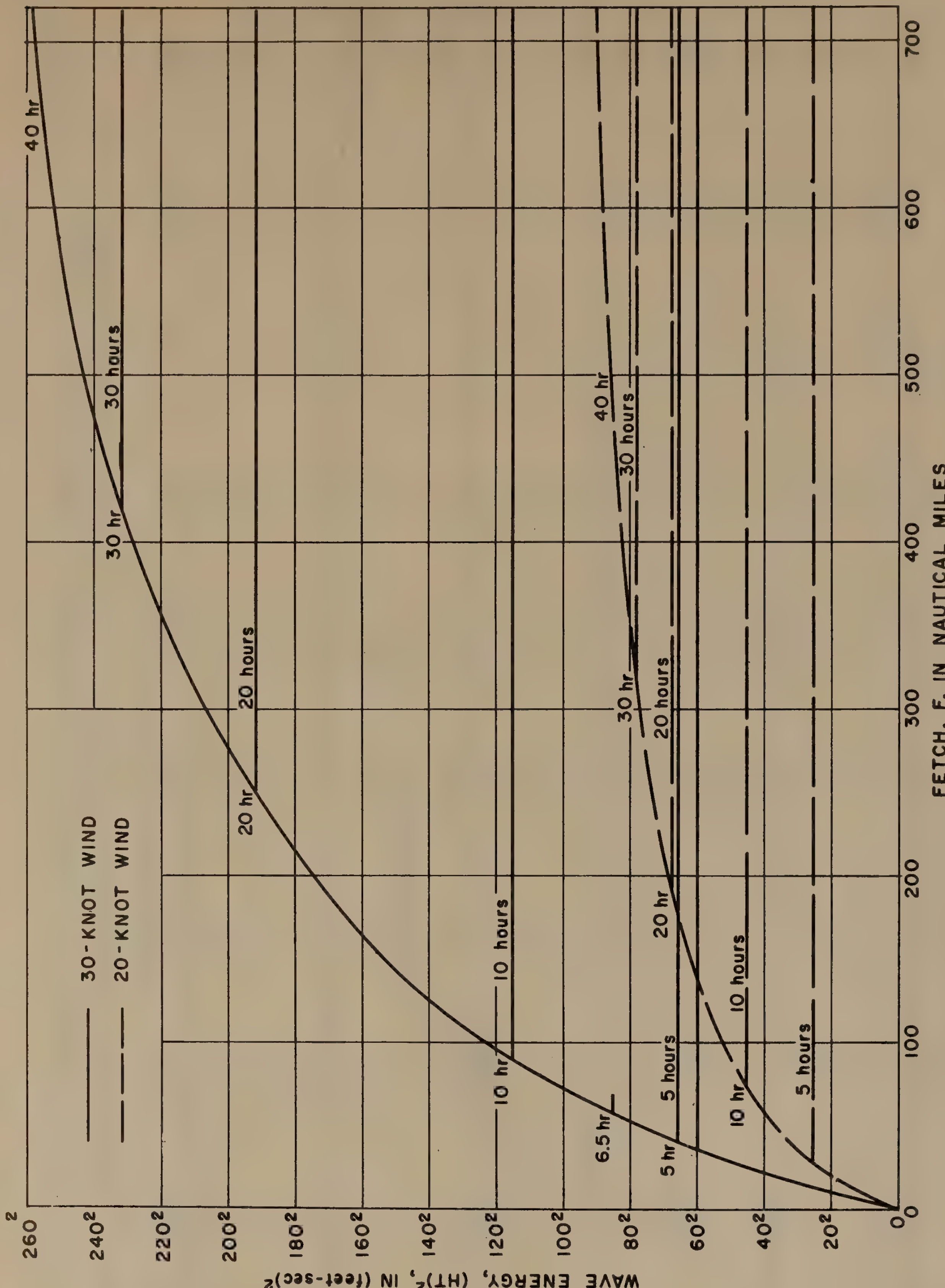


FIG. IIIB-3 — Significant Wave Energy $(HT)^2$ as a function of distance from coast line at intervals from 5 hours to 40 hours after a wind of 30 knots and 20 knots started to blow over an undisturbed water surface.

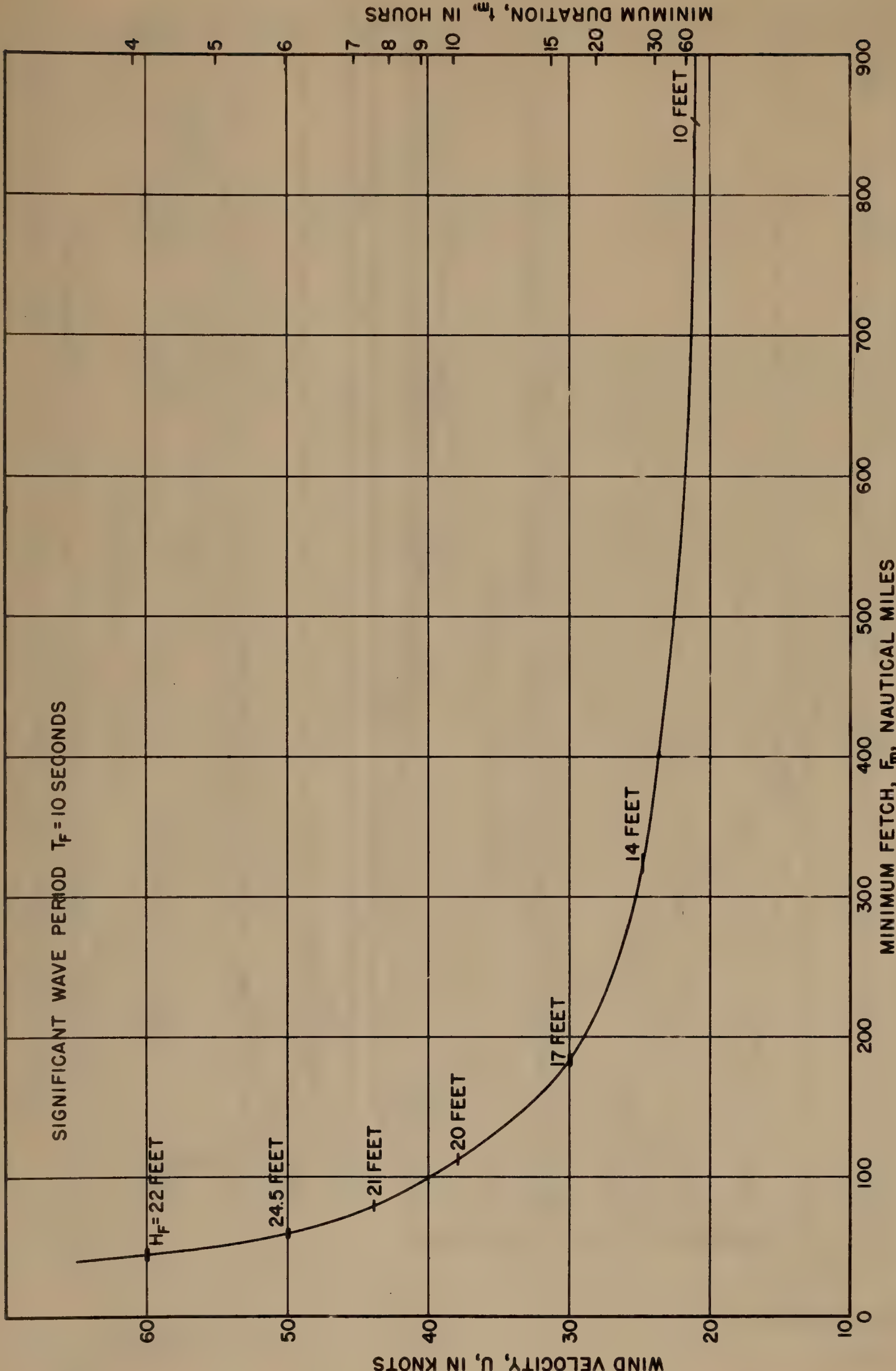


FIG. IIIB-4 -- WIND VELOCITY VERSUS MINIMUM FETCH FOR A 10-SECOND SIGNIFICANT WAVE PERIOD FOR INTERVALS OF SIGNIFICANT WAVE HEIGHT BETWEEN 10 FEET & 22 FEET.

HYD. 5439-155-00

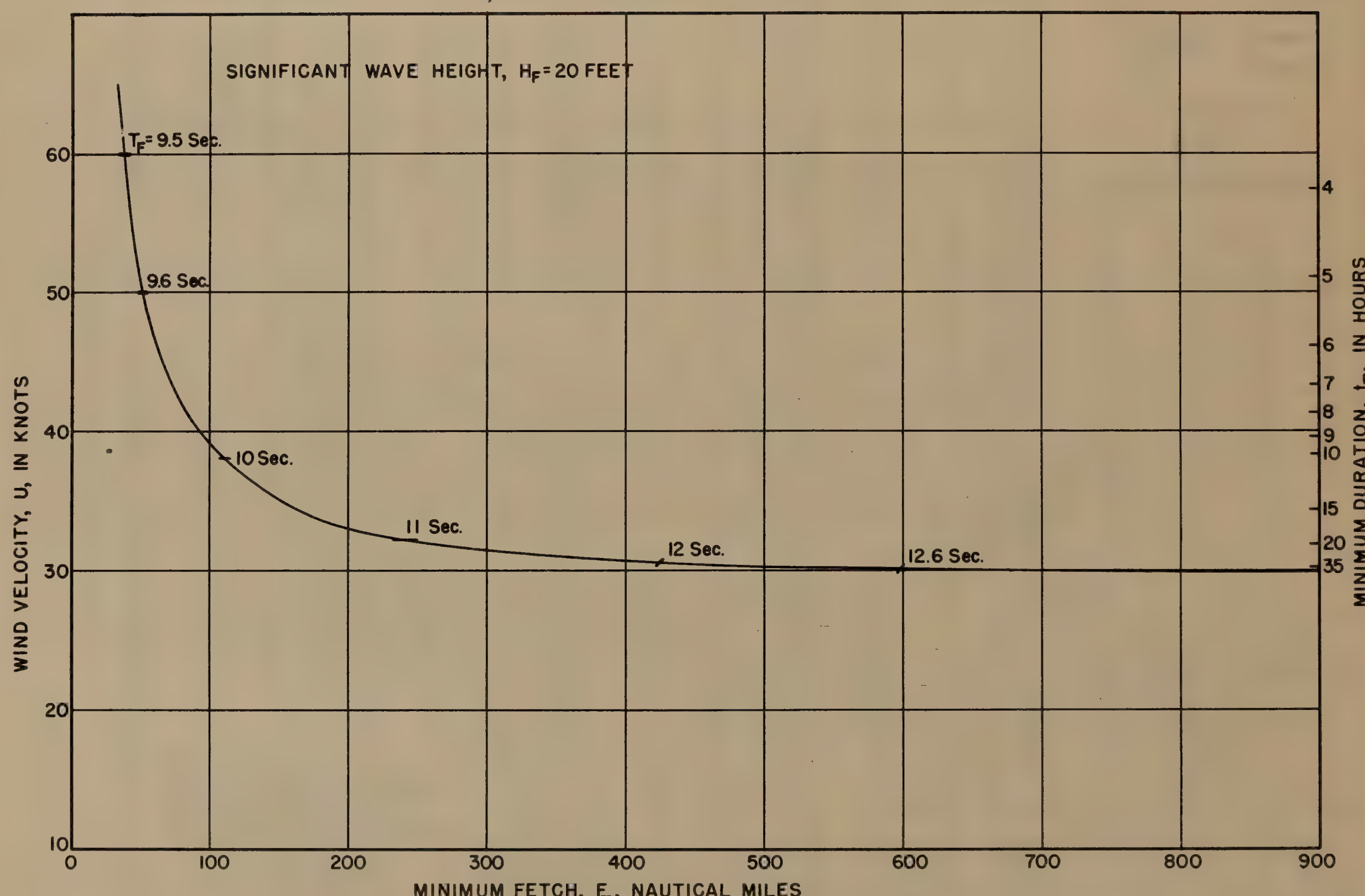
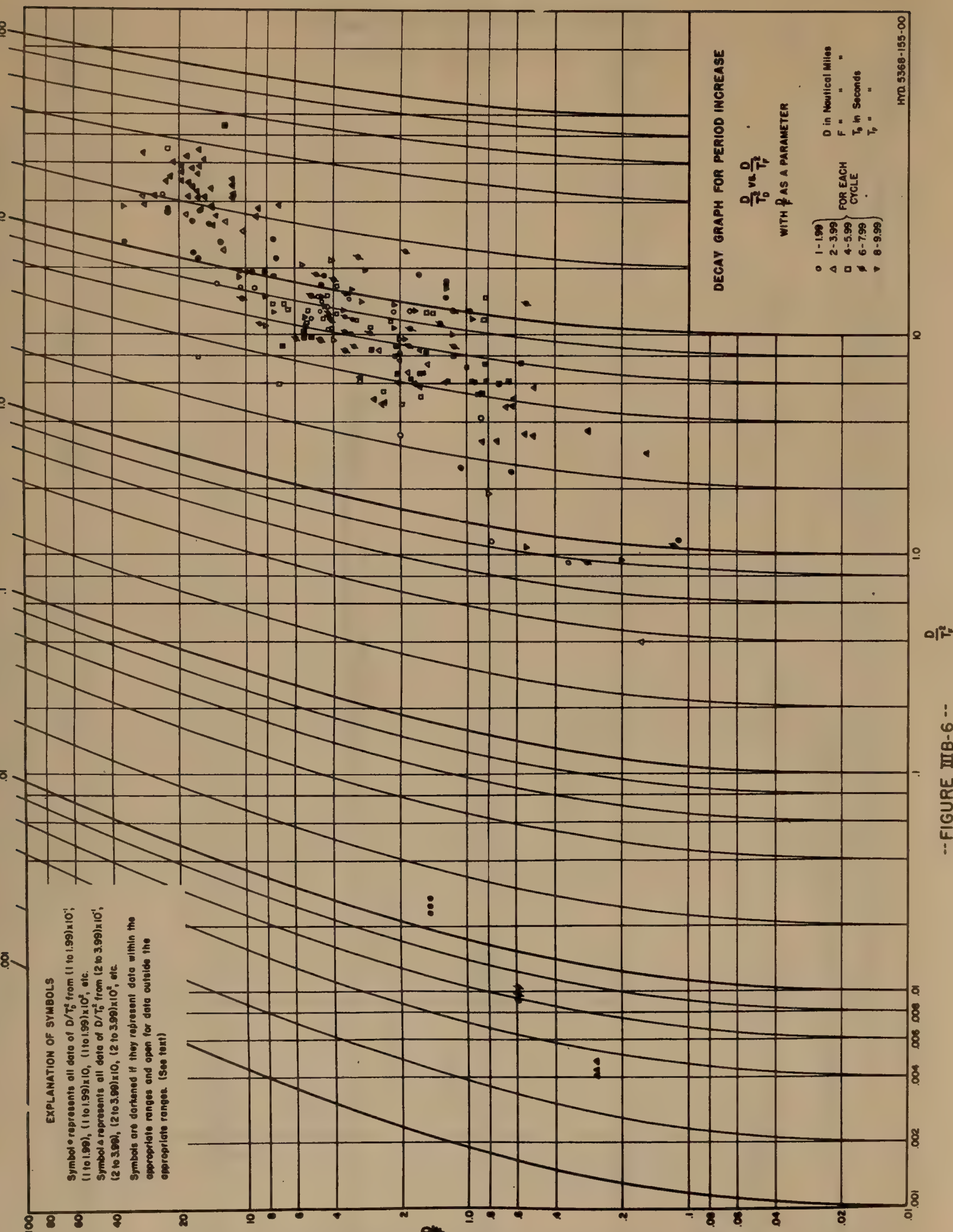


FIG. IIIB-5--WIND VELOCITY VERSUS MINIMUM FETCH FOR A 20-FOOT SIGNIFICANT WAVE HEIGHT FOR INTERVALS OF SIGNIFICANT WAVE PERIOD FROM 9.5 TO 12.6 SECONDS.

HYD. 5440-155-00



--FIGURE IIIB-6 --

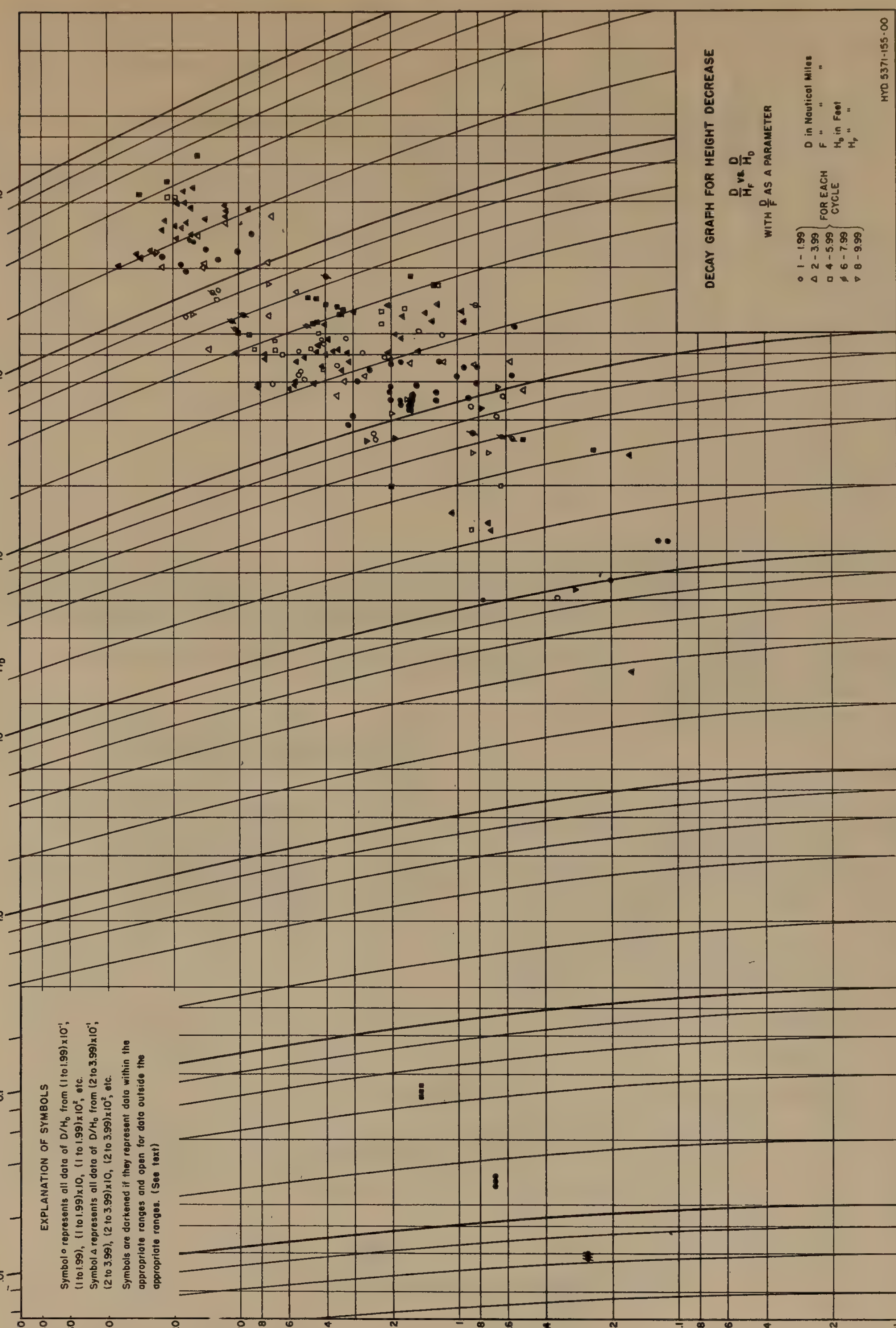
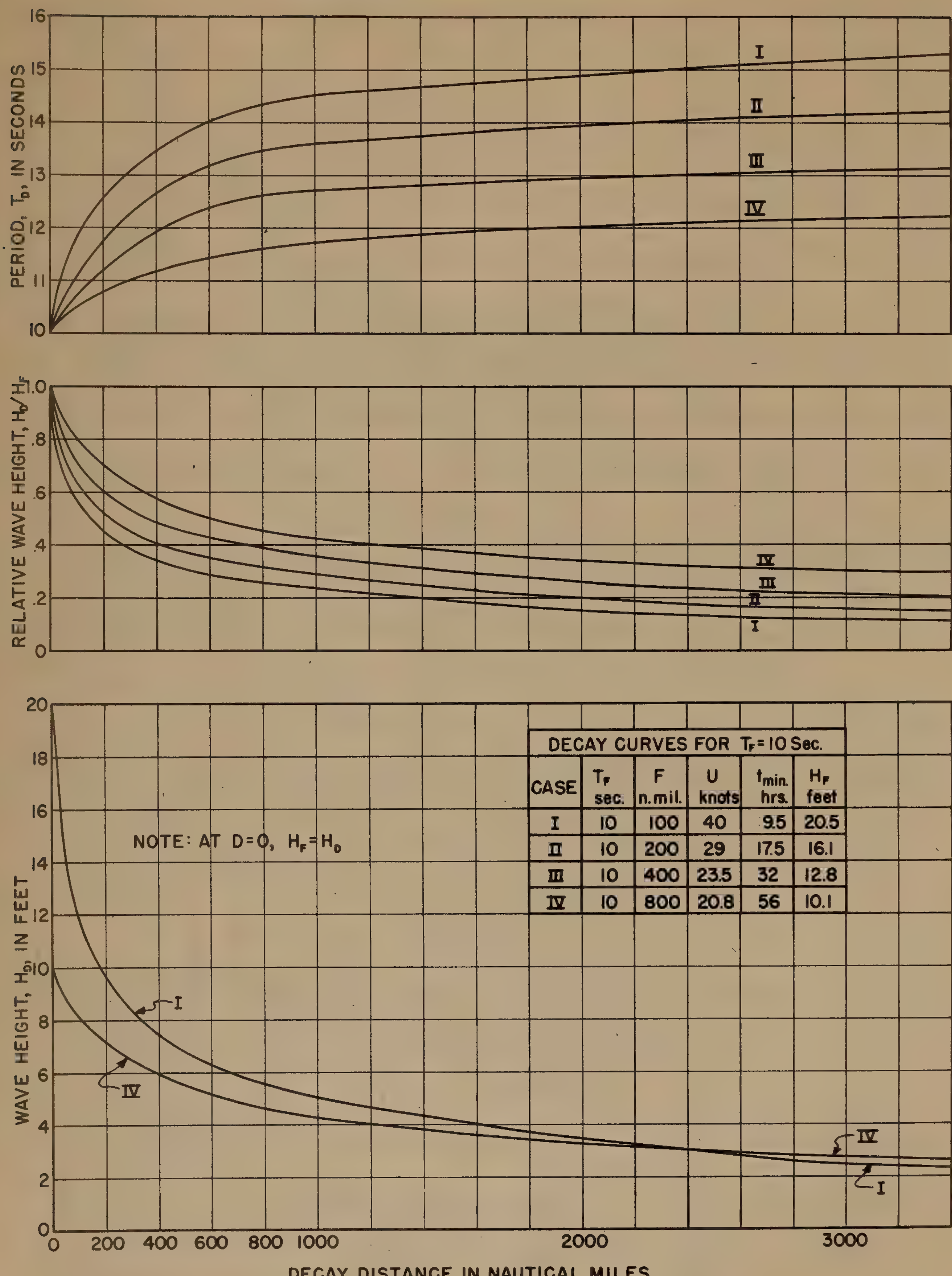
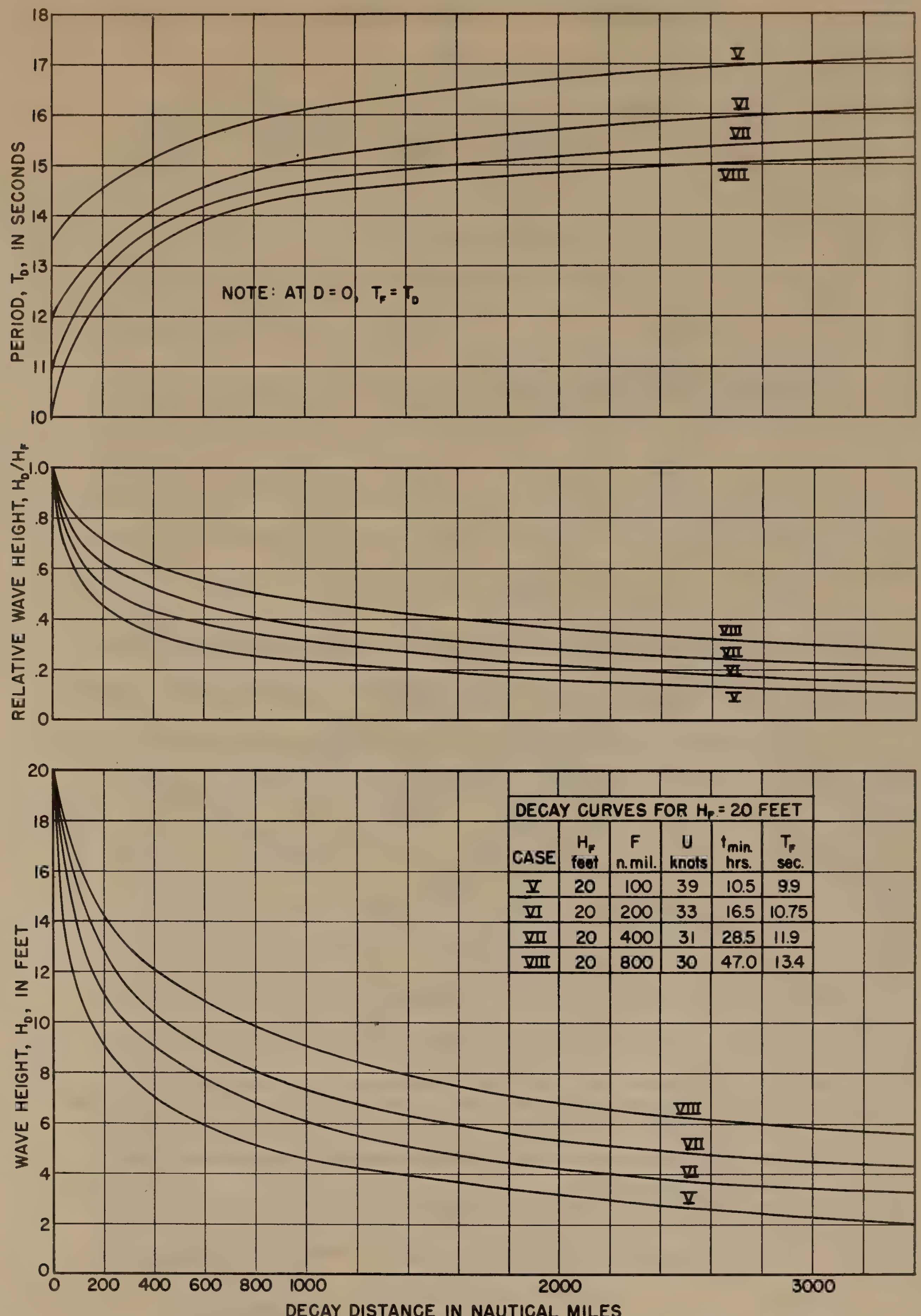


FIGURE III B-7



DECAY CURVES FOR T_F = 10 Sec.					
CASE	T_F sec.	F n.mil.	U knots	$t_{min.}$ hrs.	H_F feet
I	10	100	40	9.5	20.5
II	10	200	29	17.5	16.1
III	10	400	23.5	32	12.8
IV	10	800	20.8	56	10.1

FIG. IIIB-8



DECAY CURVES FOR H_F = 20 FEET

CASE	H_F feet	F n. mil.	U knots	$t_{min.}$ hrs.	T_F sec.
V	20	100	39	10.5	9.9
VI	20	200	33	16.5	10.75
VII	20	400	31	28.5	11.9
VIII	20	800	30	47.0	13.4

 FIGURE III B-9

GEOSTROPHIC WIND SCALE

$$V_g = \frac{1}{2\Omega\rho\sin\phi}\,\frac{\Delta p}{\Delta n}$$

FOR Δp = 5mb & 3mb

Δn = Degrees Latitude

p = 1013.3 mb

T = 10° C

ρ = 1.26 gm.cm.$^{-3}$

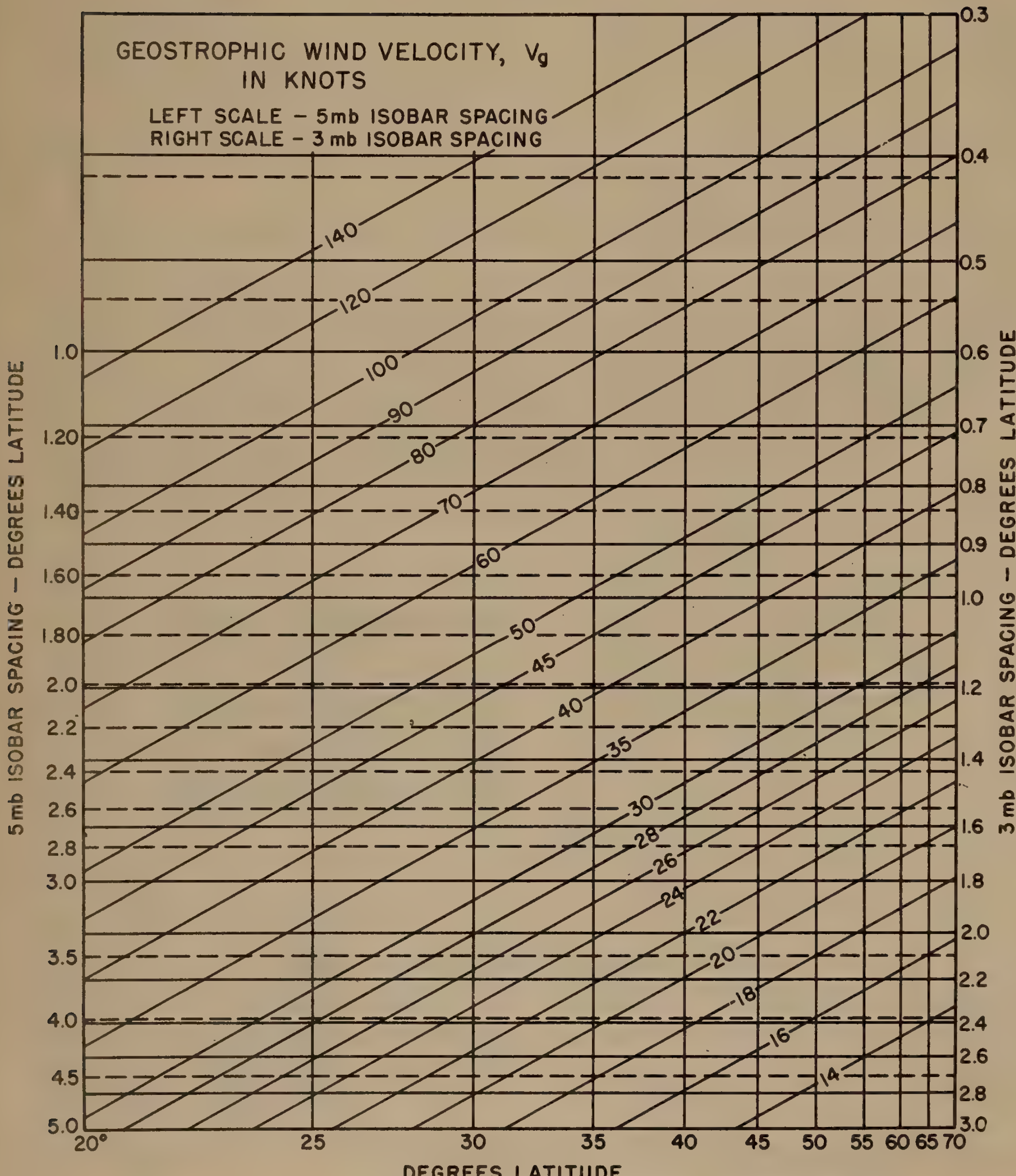
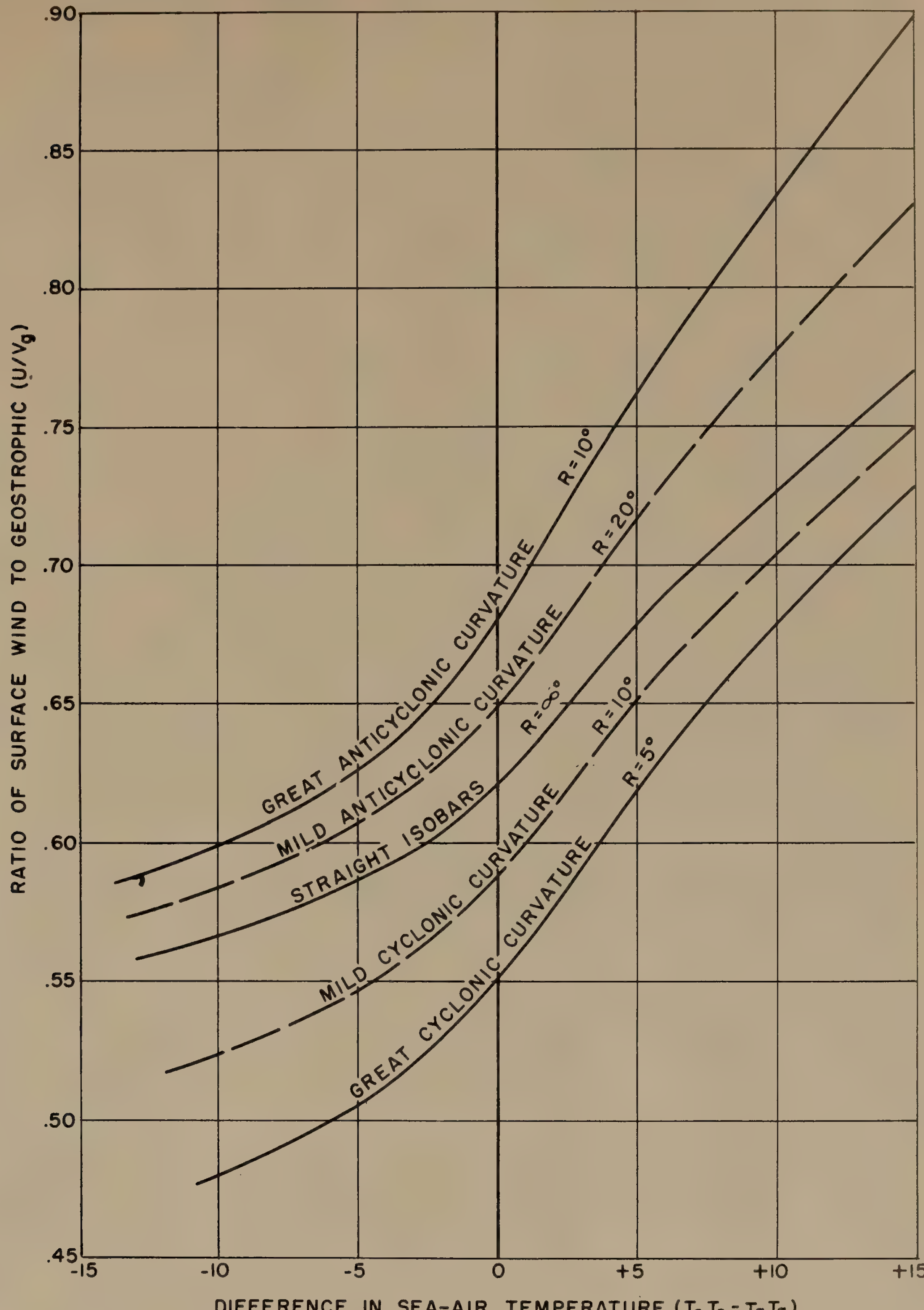


FIGURE III B-10

HYD. 5264-155-00



SURFACE WIND SCALE

FIGURE IIIB-11

HYD-5444-155

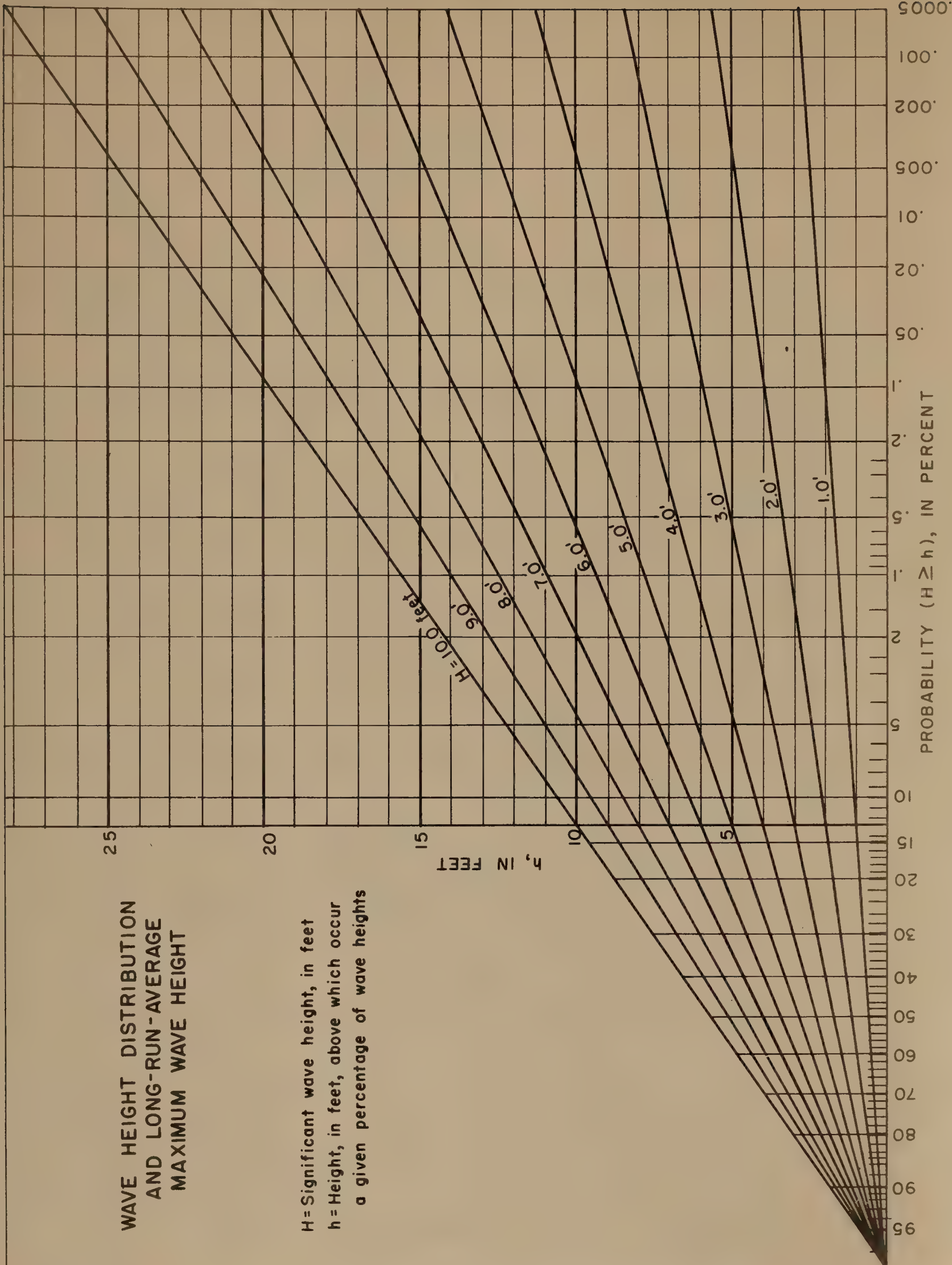


FIGURE III B-12

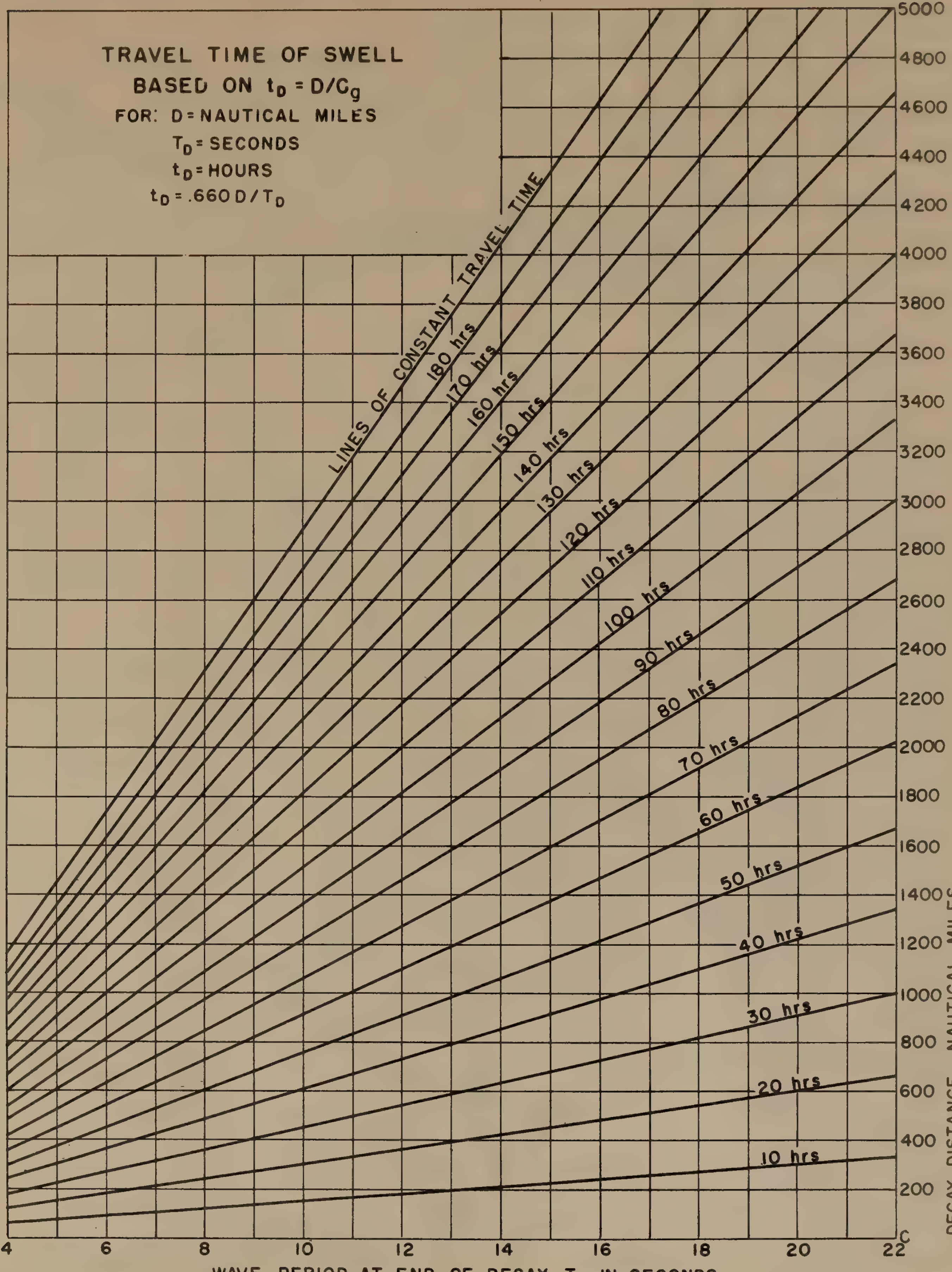


FIGURE IIIB-13
HYD-5433-155

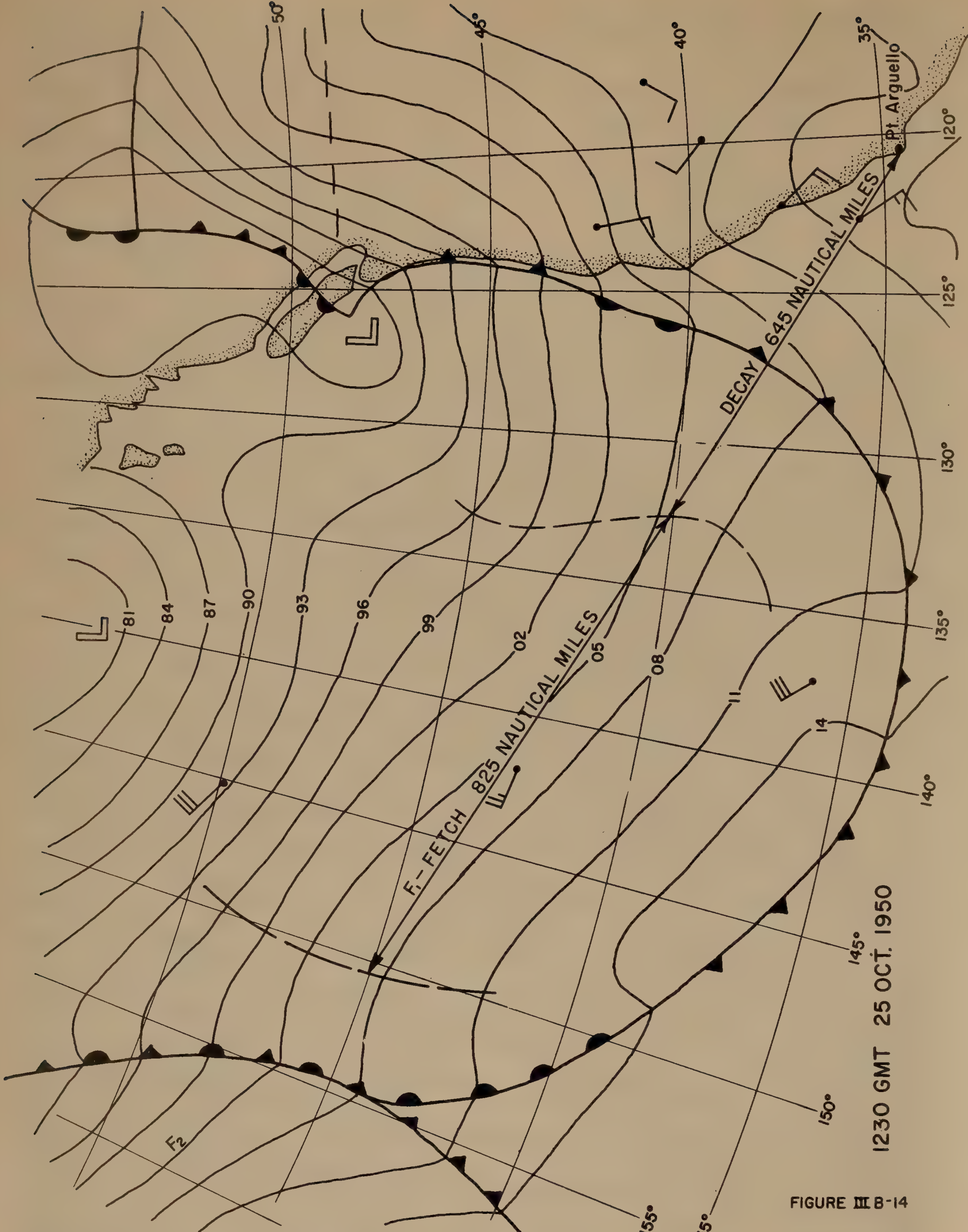


FIGURE III B-14

HYD-5445-155

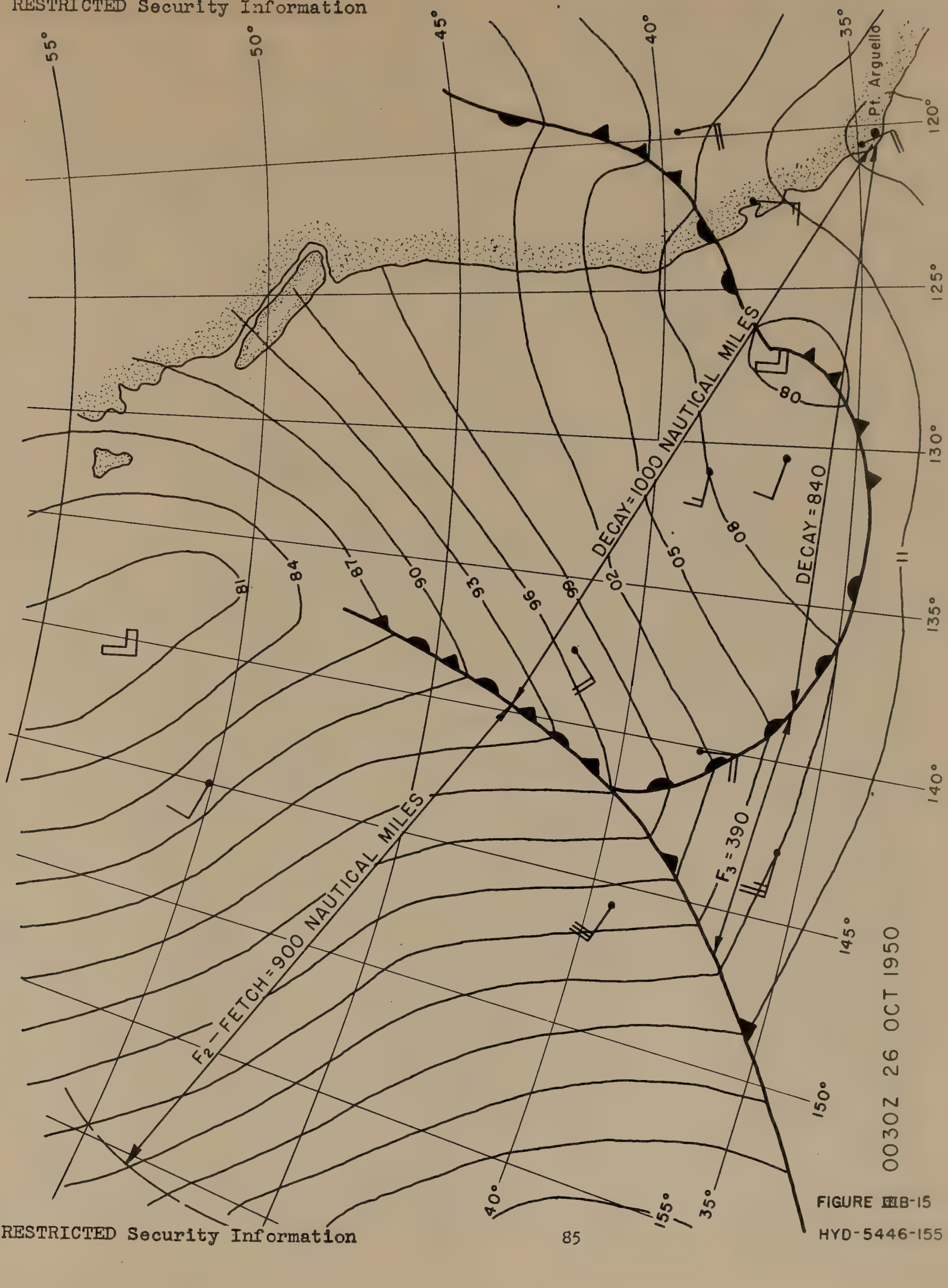


FIGURE IIIB-15

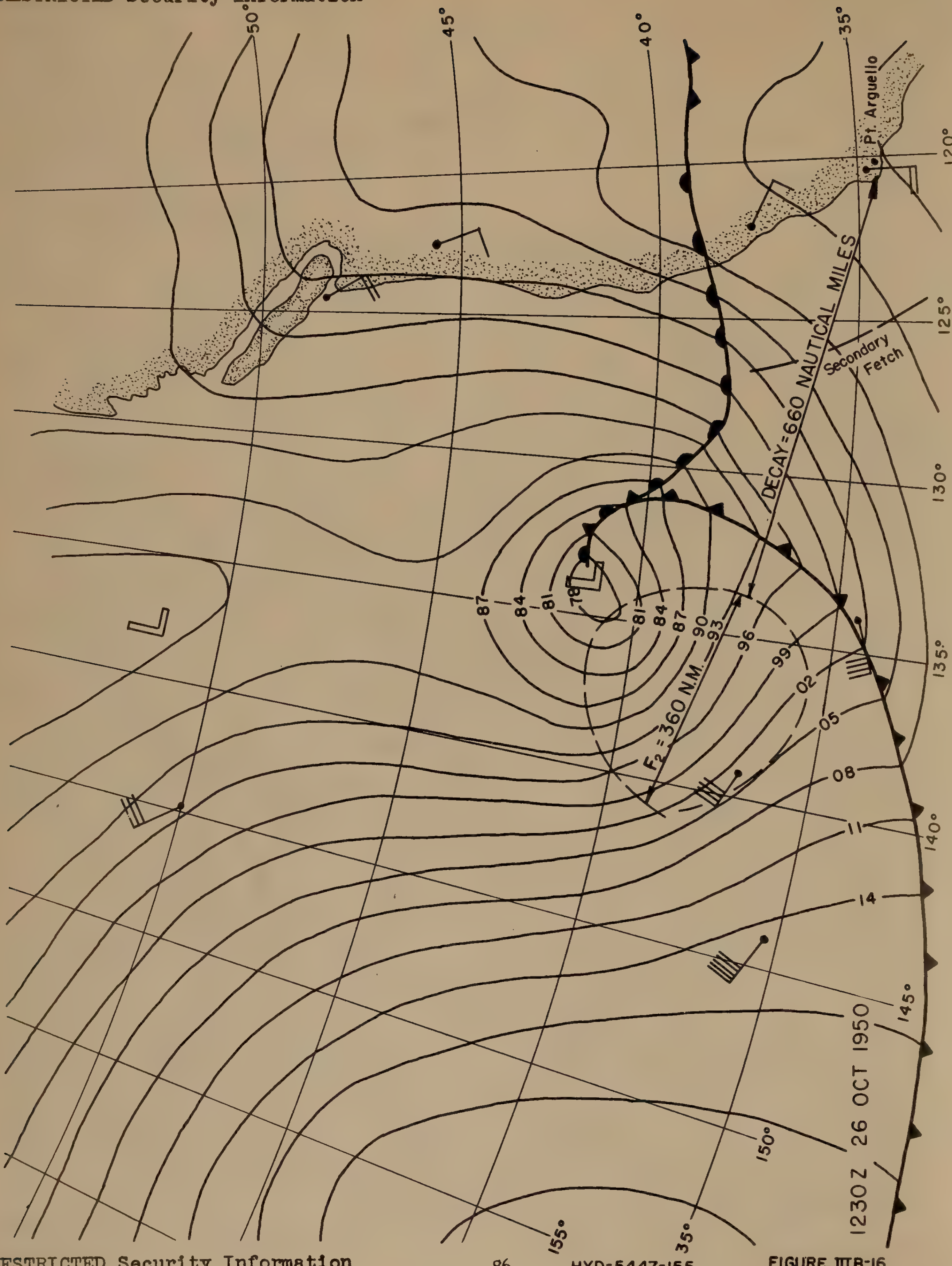


FIGURE IIIB-16

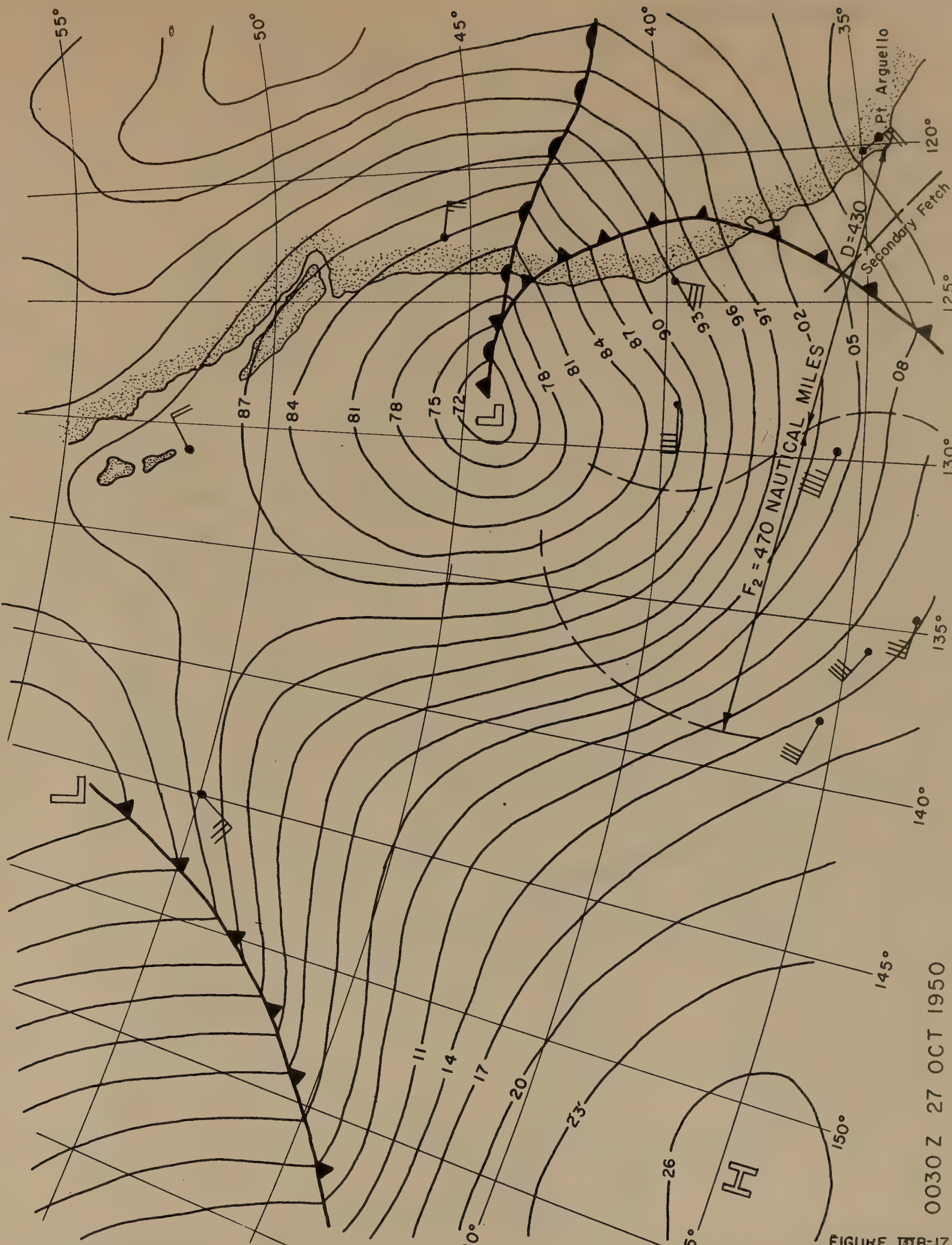
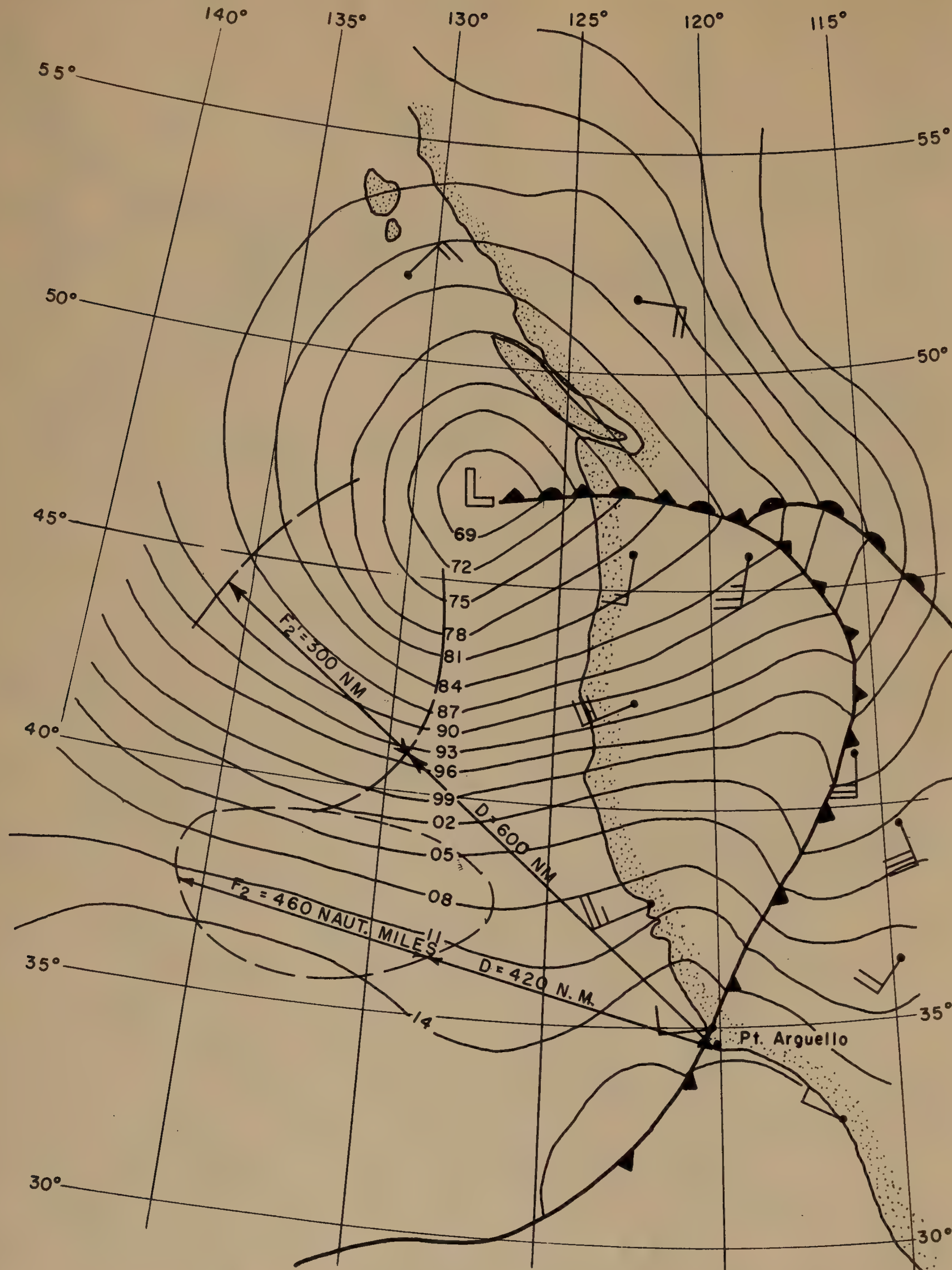
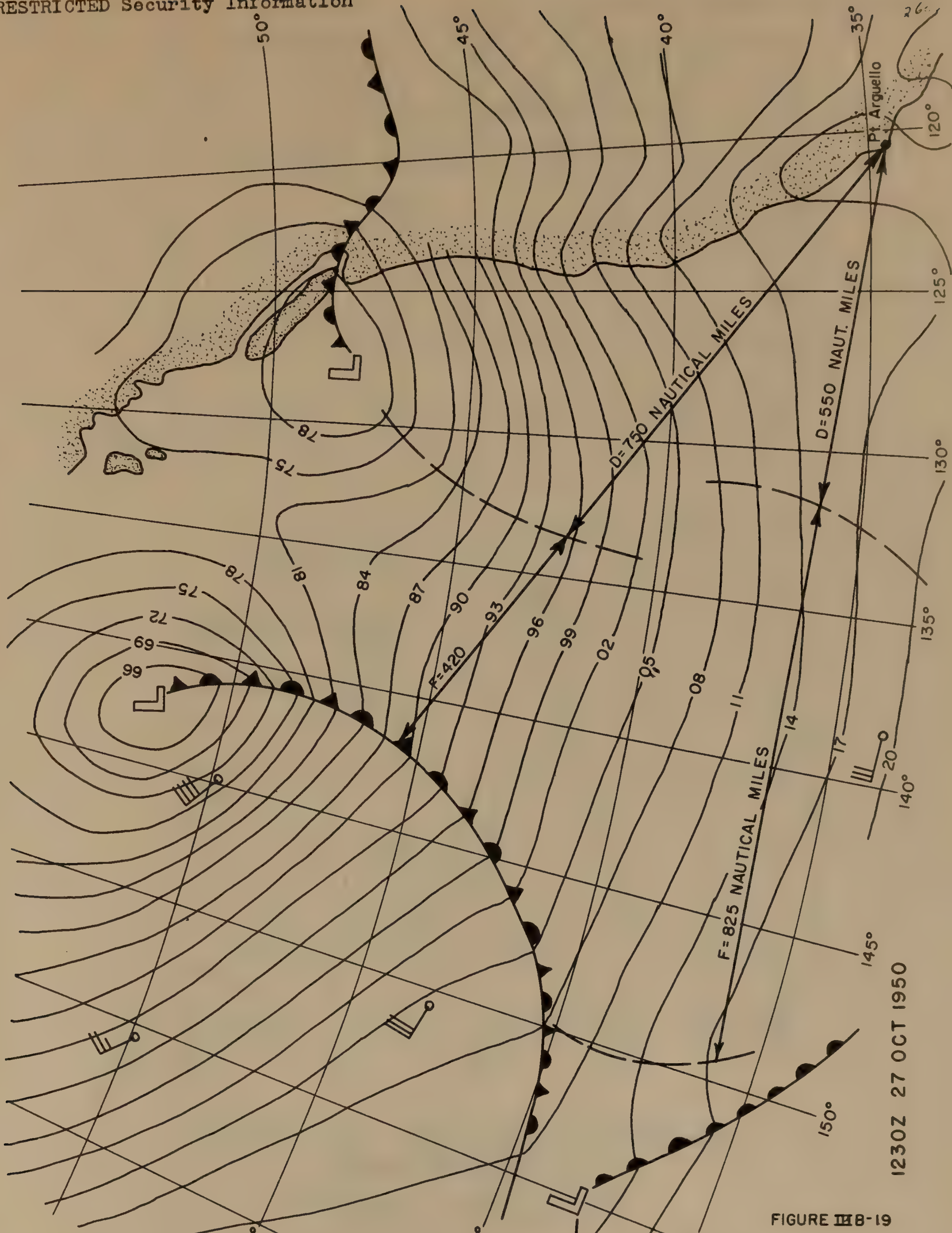


FIGURE IIIB-17
HYD-5448-155



0630Z 27 OCT 1950

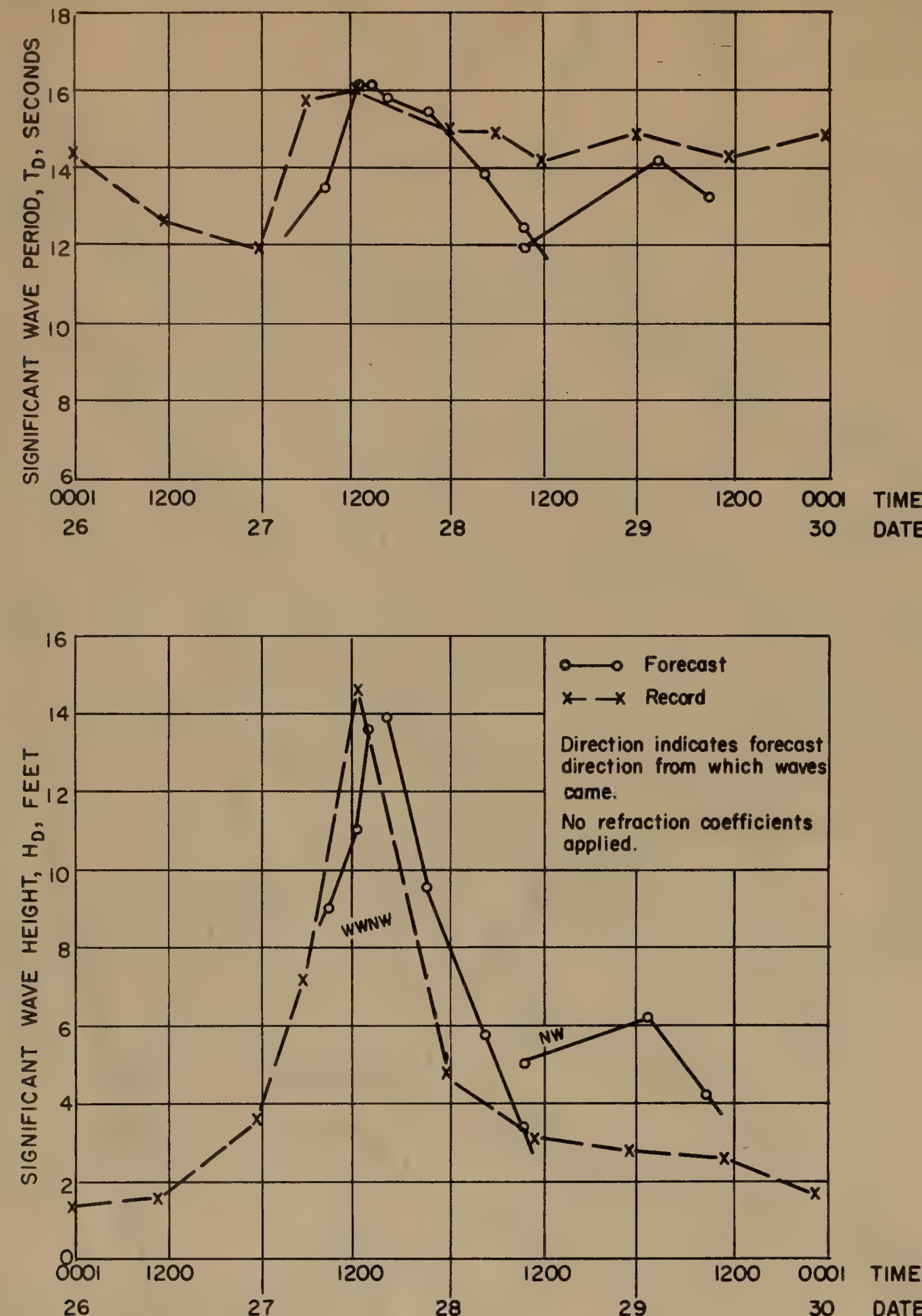
FIGURE IIIB-18



1230Z 27 OCT 1950

FIGURE IIIB-19

HYD-5459-155



COMPARISON OF DEEP WATER WAVE FORECAST
AND RECORDED WAVES
FOR PT. ARGUELLO, CALIFORNIA
OCTOBER 27-29, 1950

FIGURE IIIB-20

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION III: WAVE FORECASTING

C. SURF FORECASTING

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C. SURF FORECASTING

1. Refraction and Diffraction Effects

As waves approach shore from deep water the crests are bent and tend to conform to the bottom contours. This action results from the reduction of wave velocity as the depth decreases. The phenomenon is known as wave refraction because of its similarity to the refraction of light and sound. This subject is fully discussed in Section I-D of this Manual. Because of refraction the energy of waves converges or diverges, with a corresponding increase or decrease in wave height. The amount of change is described by the refraction coefficient. In wave forecasting refraction diagrams can be constructed to determine refraction coefficients which correct deep-water heights to shallow-water heights.

Wave diffraction is that phenomenon in which water waves are propagated into a sheltered region by a barrier, such as a breakwater, which interrupts a portion of an otherwise regular wave train. A knowledge of diffraction behavior has important applications in the design and location of breakwaters for harbors and in determining the distribution of wave energy along beaches located in the lee of headlands and offshore islands. This subject is fully discussed in Section I-E of this Manual.

2. Breaker Characteristics and Effects

The classic description of wave behavior in shallow water states that as a depth equal to half the length of the wave (L/2) is reached the wave begins to "feel" bottom and is retarded (Reference III D-2). To an observer probably the first noticeable effect is a bending or refraction of the wave, closely followed by peaking of the wave crest. As the wave reaches quite shoal water the wave peaks up rapidly, and the crest appears to be a mound, while the trough appears long and flat. Finally, it becomes unstable and breaks.

The wave changes form radically when it breaks, and apparently regardless of its prior characteristics, assumes the appearance of a small tidal bore as it rushes foaming up the beach. The recession of the wave from its upper limit of travel on the beach resembles simple sheet flow without wave characteristics.

The theory of wave behavior in shallow water is applicable only for wave travel from deep water to a point several wave lengths seaward of the breakers. In order to predict breaker heights it is necessary to rely upon relationships established by laboratory experiment.

A recent study (Reference III C-1) has resulted in a simple procedure for determining breaker heights from known deep-water waves. This is a modification of the basic forecasting method (Reference III C-3) introducing the effect of beach slope, and is

fully discussed in Section I-F of this Manual. The data used are based upon Reference III C-3 but modified to include recent findings. Figures III C-1 and 2 are used for this purpose. An example of the procedure will be carried through in detail to illustrate the various steps involved.

Information that is known to the forecaster includes the forecast deep-water wave height, the forecast deep-water wave direction, the forecast wave period, the beach contours and tide stage, and the refraction diagrams of the area.

a. Example:

Assume:

H_o = 9.0 feet, T = 12 seconds, Beach Slope = 1/20 = 0.05. The procedure to obtain the breaker height is made by successive approximations. Note that H_o/T^2 is used in this presentation rather than H_o/L. The period, T, is obtained from the forecast. The conversion to L_o is eliminated to simplify the computations. The details are as follows:

(i) Evaluate $H_o/T^2 = 9.0/(12)^2 = 0.0625$

(ii) This value, uncorrected for refraction, is used as a first approximation to determine the depth of breaking. At $H_o/T^2 = 0.0625$, enter Figure III C-1 to obtain:

$$d_b/H_o = 1.55$$

(iii) Since H_o = 9.0 feet, then: $d_b = H_o \times 1.46 = (9.0)(1.46) = 13.1$ feet.

(iv) From the refraction diagrams for the given conditions of wave period and direction, the value of K for this water depth is found. Assume: K = 0.80 for this example.

(v) The effective deep-water wave height is: $H_o' = KH_o = (0.80)(9.0) = 7.2$ feet.

(vi) The revised value of H_o/T^2 is then H_o'/T^2. $H_o'/T^2 = 7.2/(12)^2 = .050$

(vii) With this revised H_o'/T^2, enter Figure III C-1 to obtain: $d_b'/H_o' = 1.60$.

(viii) Since H_o' = 7.2 feet, then: $d_b' = H_o \times 1.60 = (7.2)(1.60) = 11.5$ feet.

(ix) From the refraction diagram, determine a new value of K at the second approximation of the depth of step viii. Usually the revised K will not differ appreciably from that originally used. If the two K values differ by more than 5 percent, a third approximation should be made by repeating steps iv through viii.

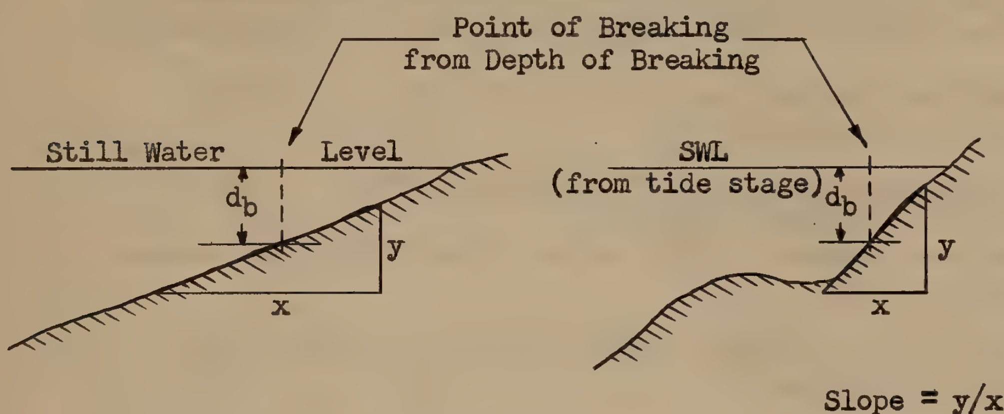
(x) For this example, the assumption is made that the K values differ by less than 5 percent and hence K will be taken as 0.80.

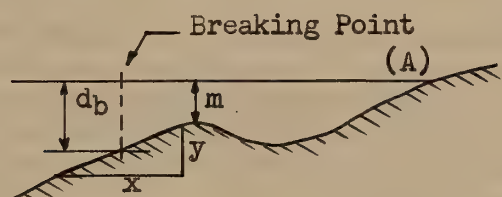
(xi) From the beach contours and the tide stage, locate the point of breaking at d_b' (In this example, at d_b' = 11.5 feet). Determine an average slope of the beach in this region. This determination of the average beach slope is a matter of judgment to establish an effective slope to correct the breaker height for a slope effect. In this example, assume the average slope in the region of breaking is 1/20 (0.05).

(xii) The effective deep-water wave height is then 7.2 feet (from step (v)). The effective wave characteristic is: $H_o'/T^2 = 7.2/(12)^2 = .050$, as in step (vi). Enter Figure III C-2 with $H_o'/T^2 = .050$ to obtain: $H_b'/H_o' = 1.62$ from the 1:20 beach slope wave. Hence: $H_b = (H_o')(1.62) = (7.2)(1.62) = 11.7$ feet, where H_b is the breaker height, corrected for refraction and shoaling effects.

b. Discussion of Limitations:

(1) Effective Beach Slope: Figures III C-1 and 2 were obtained from experiments in the laboratory on plane beaches. The effect of large scale irregularities in the beach profile is as yet undetermined. In step (xi) of the example, the effective beach slope was determined. An example of three beach profiles is shown to illustrate the determination of the effective slope.





NOTE: Waves may break at d_b as shown, reform after passing the bar, and break again nearer the shore. If d_b is less than m, the waves should run into the shore and break in the region A.

(2) Application to Beach Slopes Greater than 1:10 or Less Than 1:50: The investigations on which Figures III C-1 and 2 were based have not been extended to slopes steeper than 1:10 or flatter than 1:50. Until information is obtained in these ranges, it is suggested that, for beaches with a slope greater than 1:10, the breaker curve for this 1:10 slope be used, and, for beaches with a slope less than 1:50, the 1:50 breaker curve be used.

3. References:

III C-1. Iversen, H. W., Crooke, R.C., Larocco, M. J., and Wiegel, R. L., 1950, Beach slope effect on breakers and surf forecasting, Tech Report No. 155-38, Inst. of Engineering Research, University of California, Berkeley, 15 pp.

III C-2. Mason, M. A., 1950, The transformation of waves in shallow water, Institute of Coastal Engineering, Long Beach, California, 18 pp.

III C-3. U.S. Navy Hydrographic Office, 1944, Breakers and surf--principles in forecasting, H. O. Pub. No. 234, Washington, 55 pp.

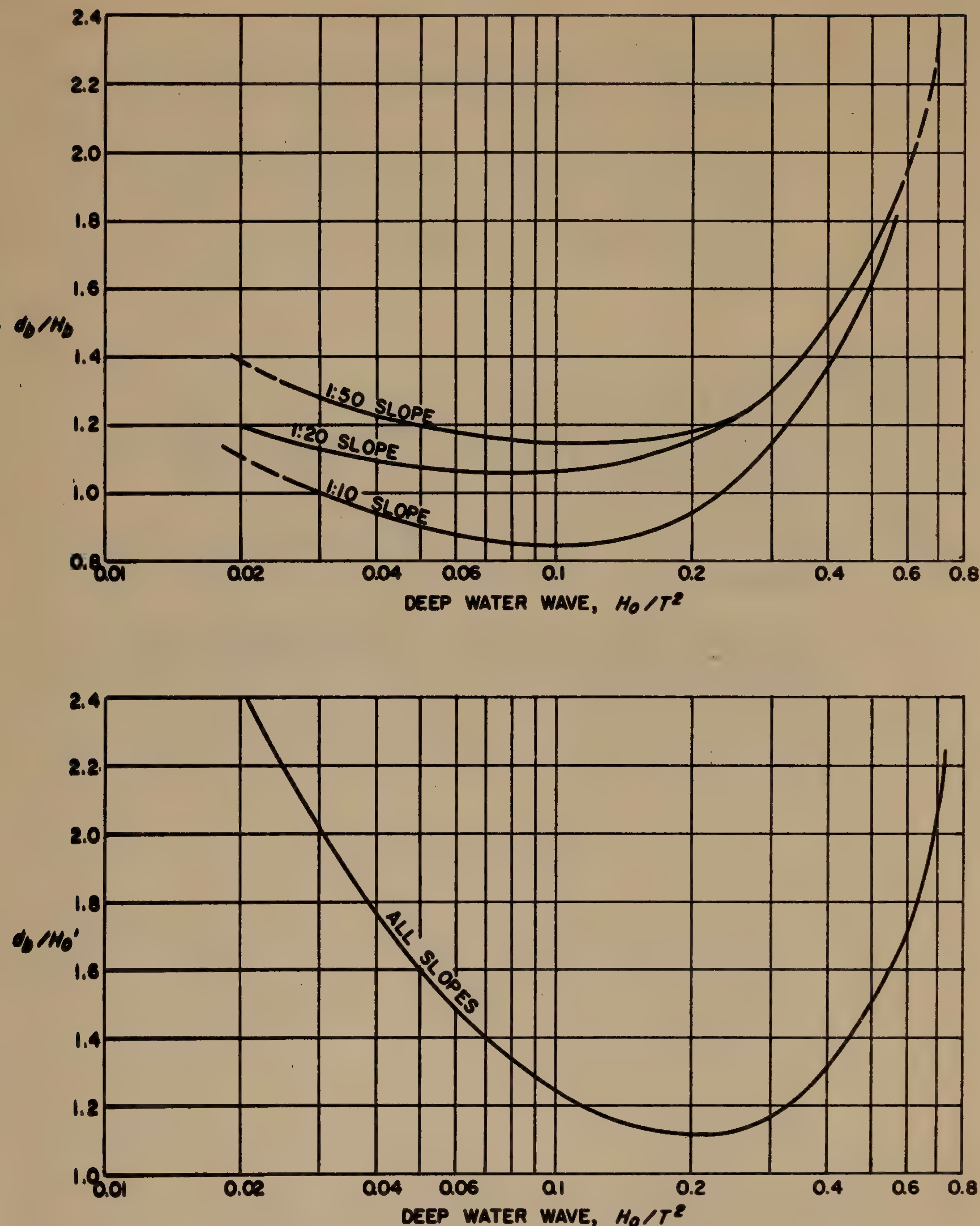


FIG. IIIC-1 - BREAKER DEPTH INDEX

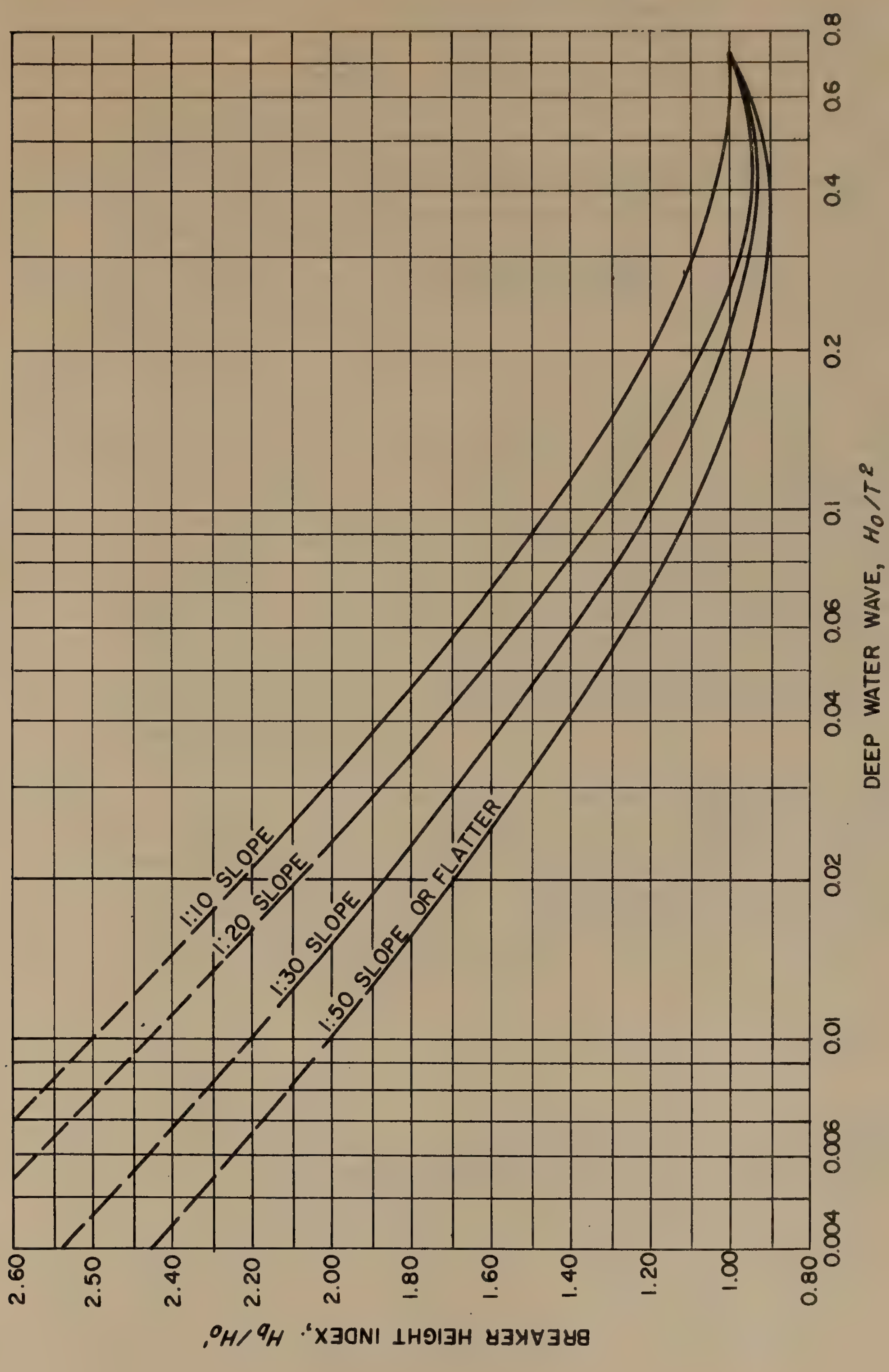


FIG. IIIC-2 - BREAKER HEIGHT INDEX AS A FUNCTION OF DEEP WATER WAVE AND BEACH SLOPE

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MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION III: WAVE FORECASTING

D. EFFECT OF WAVE VARIABILITY

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D. EFFECT OF WAVE VARIABILITY

1. Discussion:

Because of the irregularity of ocean waves in height and period, it is necessary to have a statistical expression for wave forecasting purposes. (This subject has been fully discussed in Section I-H of this Manual.) Mention has already been made of the significant wave height and the significant wave period, these being defined as the average height and period of the highest one-third of the waves. All wave forecasts are ordinarily made for significant wave heights and periods.

A summary of the relations between significant wave heights and other statistical wave measures has been made (Ref. III D-1 and Section I-H), which indicate the following. The ratio of the significant wave height to the average wave height is 1.57. The ratio of the average of the highest 10 percent of the wave heights to the significant wave height is 1.29. And the ratio of the maximum wave height occurring in a twenty minute period to the significant wave height is 1.87. All of these figures have been obtained from many months of wave records on both the Atlantic and Pacific Coasts. These ratios have been found to hold reasonably constant at different points of investigation, so that having a given significant wave height, considerable other information is therefore available about the wave distribution and variability from knowing these ratios.

The significant wave period does not describe the period-distribution as does the significant wave height. Thus, although wave heights have been found to follow a simple mathematical distribution even though arriving from two or more storm areas, wave periods do not follow a simple distribution if more than one generating area exists. Additional information is needed to adequately describe wave periods.

2. References:

III D-1. Snodgrass, F.E. and Stiling, D.E., 1951, Analysis of wave records, Report No. HE-116-307, Institute of Engineering Research, University of California, Berkeley, 13 pp.

MANUAL OF AMPHIBIOUS OCEANOGRAPHY

SECTION III: WAVE FORECASTING

E. LIMITATIONS AND CONCLUSIONS

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E. LIMITATIONS AND CONCLUSIONS

It has been the purpose of this section of the Manual to describe the basic principles and procedures involved in wave forecasting. It can be seen that the subject, although largely developed in only the last 10 years, has been reduced to a large extent to purely mechanical procedures. Therefore, it is now possible to prepare wave forecasts for any oceanographic area or location provided the basic data required, including synoptic weather maps and bottom contour configurations, are available.

To one not familiar with wave forecasting procedures, the rather complex organization of the wave cases presented in Sections IIIB-7 and IIIB-9 may seem rather confusing. However, it will be seen upon becoming familiar with the terminology and symbols involved, that each situation once classified lends itself to straightforward forecasting procedures. In learning wave forecasting, some time will be involved in becoming acquainted with the various situations and learning to recognize them. After this initial period, however, the method is rapid and easy to apply.

It is well to emphasize here that a certain amount of judgment is required in wave forecasting. To one unfamiliar with delineating fetch and decay area, discrepancies between wave forecasts and observations may occasionally be rather large. However, with experience and practice, the experienced forecaster will usually be able to hold such discrepancies to a minimum. The methods outlined herein are not complete and certainly are not foolproof. Further studies indicate revisions continually, so that as time goes on these forecasting plates and procedures will need to be modified. Nevertheless, considering the development of the subject in the past few years, enormous strides have been made and most of the future work will make only minor improvements or refinementsin the basic procedures now in use.

As mentioned previously, Figures III C-1 and 2, concerned with forecasting waves at the beach, have been obtained from experiments in the laboratory on plane sloping beaches. One of the limitations in the use of these figures is that of large scale irregularities in the beach profile. Beaches that have large ridges or steps in their profiles modify both the breaker height and the point of breaking. Ordinarily, if the ridges are below the breaker depth, d_b, the breakers will not be changed, but if the ridges rise to such an extent that the depth is less than d_b, then there may be partial breaking at more than one point. Further studies of these special cases are needed. It can be seen in Figure III E-2 that beach slopes from 1/10 to 1/50 only have been included. For slopes greater or less than these values, no definite information on breaker heights is known. It is suggested, however, that for beaches slopes greater than 1/10, the correction factor for the 1/10 slope be used, and for slopes less than 1/50, the 1/50 correction factor be used. Certain errors may also be incurred

in breaker heights using Figures III C-1 and 2 because bottom friction and percolation in the porous bottom of natural beaches have the tendency to reduce the wave amplitude as it travels shoreward. Hence, natural breaker heights would be expected to be lower than those on an impervious, smooth bottom as used in the laboratory investigations.

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